

*Foundations of Modern Macroeconomics*

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Chapter 13: New Keynesian Economics

## Aims of this lecture

- Monopolistic competition as a micro-foundation for the multiplier (is it Keynesian?)
- Monopolistic competition and welfare-theoretic aspects
  - the marginal costs of public funds and the multiplier
- Monetary non-neutrality and price adjustment costs
- Nominal and real rigidity: definitions and interaction

## The “Keynesian multiplier”

- Literature is related to the quantity rationing approach of the 1970s and 1980s
- Key question: Who sets the prices?
  - auctioneer? Fictional *deus ex machina*
  - price setting firms? Monopolistic competition
- Develop simple static macro model with monopolistic competition
- Study two cases:
  - flexible prices
  - sticky prices

## A static model with monopolistic competition in the goods market

- households, many small firms, government
- horizontal product differentiation
- single production factor: labour

## Representative Household

- Household utility:

$$U \equiv C^\alpha (1 - L)^{1-\alpha}, \quad 0 < \alpha < 1$$

- $C$  is *composite* consumption
- $L$  is labour supply ( $1 - L$  is leisure)
- $U$  is utility

- Composite consumption: S-D-S preferences:

$$C \equiv N^\eta \left[ N^{-1} \sum_{j=1}^N C_j^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$

- $N$  is number of product varieties
- $C_j$  is variety  $j$
- $\infty \gg \theta > 1$ : close but imperfect substitutes
- $\eta \geq 1$ : preference for diversity (“taste for variety”). Spread given production over as many varieties as possible if  $\eta > 1$ .

- Household budget constraint:

$$\sum_{j=1}^N P_j C_j = W^N L + \Pi - T$$

- $P_j$  is price of variety  $j$
  - $W^N$  is nominal wage rate
  - $\Pi$  is profit income of the household (from MC sector)
  - $T$  is lump-sum taxes
- Household chooses  $L, C_j$  (for  $j = 1, 2, \dots, N$ ) to maximize  $U$  subject to the budget constraint and taking as given  $W^N$  and  $P_j$  (for  $j = 1, 2, \dots, N$ ).

● Solutions:

$$\begin{aligned}
 PC &= \alpha [W^N + \Pi - T] \\
 W^N [1 - L] &= (1 - \alpha) [W^N + \Pi - T] \\
 \left(\frac{C_j}{C}\right) &= N^{-(\theta+\eta)+\eta\theta} \left(\frac{P_j}{P}\right)^{-\theta} \quad (j = 1, \dots, N)
 \end{aligned}$$

–  $P$  is a true price index depending on the  $P_j$ 's and on  $N$ :

$$P \equiv N^{-\eta} \left[ N^{-\theta} \sum_{j=1}^N P_j^{1-\theta} \right]^{1/(1-\theta)}$$

- $W^N + \Pi - T$  is *full income*
- CD preferences imply constant spending shares
- demand for variety  $j$  is price elastic ( $\theta$  is the elasticity)

## Representative firm

- technology:

$$Y_j = \begin{cases} 0 & \text{if } L_j \leq F \\ (1/k) [L_j - F] & \text{if } L_j \geq F \end{cases}$$

- $Y_j$  is output of firm  $j$  (producing variety  $j$ )
- $L_j$  is labour used by firm  $j$
- $1/k$  is the (constant) marginal product of labour
- $F > 0$  is fixed costs (“overhead labour”): increasing returns to scale at firm level

- Profit definition:

$$\Pi_j \equiv P_j Y_j - W^N [kY_j + F]$$

- $\Pi_j$  is profit of firm  $j$
  - $P_j Y_j$  is revenue of firm  $j$
  - $W^N L_j = W^N [kY_j + F]$  is costs of firm  $j$
- We anticipate that  $P_j$  depends on output by firm  $j$  (and on competitors' output),  $P_j = P_j(Y_j)$ , and adopt the Cournot assumption (firm  $j$  takes other firms' output as given)
  - The choice problem is:

$$\text{Max}_{\{Y_j\}} \Pi_j = P_j(Y_j)Y_j - W^N [kY_j + F]$$

- The optimal decision rule is:

$$\frac{d\Pi_j}{dY_j} = P_j + Y_j \left( \frac{\partial P_j}{\partial Y_j} \right) - W^N k = 0 \Rightarrow$$

$$P_j = \mu_j W^N k \quad (a)$$

- (a): price is set equal to a gross markup,  $\mu_j$ , times marginal (labour) cost,  $W^N k$
- The gross markup is:

$$\mu_j \equiv \frac{\epsilon_j}{\epsilon_j - 1}, \quad \epsilon_j \equiv -\frac{\partial Y_j}{\partial P_j} \frac{P_j}{Y_j}$$

The higher is  $\epsilon_j$ , the lower is  $\mu_j$  (lower market power)

## Government

- levies lump-sum tax,  $T$ , on household
- employs (useless) civil servants,  $L_G$
- consumes a composite good,  $G$ , defined analogously to  $C$ :

$$G \equiv N^\eta \left[ N^{-1} \sum_{j=1}^N G_j^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$

- $\eta$  same as in  $C$ : price index same
- $\theta$  same as in  $C$ : price elasticity same

- Assume government is a cost minimizer: chooses  $G_j$  (for  $j = 1, 2, \dots, N$ ) in order to “produce” a given level of  $G$  at least cost.
- Derived government demand for variety  $j$  is:

$$\frac{G_j}{G} = N^{-(\theta+\eta)+\eta\theta} \left( \frac{P_j}{P} \right)^{-\theta} \quad (j = 1, \dots, N)$$

## Some loose ends

- Demand facing firm  $j$  is:

$$\begin{aligned}
 Y_j &= C_j + G_j \\
 &= \underbrace{(C + G) N^{-(\theta+\eta)+\eta\theta}}_{(a)} \underbrace{\left(\frac{P_j}{P}\right)^{-\theta}}_{(b)}
 \end{aligned}$$

- (a): shift factors affecting firm  $j$ 's demand
- (b): relative price factor affecting firm  $j$ 's demand

- Demand elasticity is constant (and equal to  $\theta$ ):

$$\mu_j = \frac{\theta}{\theta - 1} = \mu \quad (\text{for all } j = 1, \dots, N)$$

- Symmetric model: for all  $j = 1, \dots, N$  we have:

$$P_j = \mu W^N k = \bar{P}, \quad Y_j = \bar{Y}, \quad L_j = \bar{L}$$

- Aggregate quantity index,  $Y$ , is:

$$Y \equiv \frac{\sum_{j=1}^N P_j Y_j}{P}$$

- Labour market equilibrium (LME):

$$L = L_G + \sum_{j=1}^N L_j$$

- Summary of the model is provided in **Table 13.1**. [briefly run through table; LME implied by model via Walras Law]
- We can use  $W^N$  as the numeraire (everything measured in wage units)

**Table 13.1. A simple macro model with monopolistic competition**

$$Y = C + G \tag{T1.1}$$

$$PC = \alpha I_F, \quad I_F \equiv [W^N + \Pi - T] \tag{T1.2}$$

$$\Pi \equiv \sum_{j=1}^N \Pi_j = \theta^{-1} P Y - W^N N F \tag{T1.3}$$

$$T = PG + W^N L_G \tag{T1.4}$$

$$P = N^{1-\eta} \bar{P} = N^{1-\eta} \mu W^N k \tag{T1.5}$$

$$W^N (1 - L) = (1 - \alpha) I_F \tag{T1.6}$$

$$P_V = \left( \frac{P}{\alpha} \right)^\alpha \left( \frac{W^N}{1 - \alpha} \right)^{1-\alpha}, \quad V = \frac{I_F}{P_V} \tag{T1.7}$$

## Balanced-budget multipliers

- Short-run multiplier ( $N$  fixed—no entry of new firms)
  - financed with lump-sum taxes,  $T \uparrow$
  - financed by firing civil servants,  $L_G \downarrow$  (proxy for “bond financing” in a static model)
- Long-run multiplier ( $N$  variable—free entry of firms)

## Short-run multiplier, I

- $N = N_0$  (fixed); GBC:  $dG = d(T/P)$
- aggregate *consumption function* is:

$$C = \underbrace{\alpha [1 - N_0 F - L_G]}_{c_0} W + (\alpha/\theta)Y - \alpha G$$

- $c_0$  is fixed in the short run
- $W \equiv W^N/P$  is fixed in the short run (in the SE  $P$  depends on  $N$  only)
- $C$  depends on  $Y$  via the profit channel (as  $Y \uparrow \Rightarrow$  aggregate profit income,  $\Pi \uparrow \Rightarrow C \uparrow$  (and  $(1 - L) \uparrow$ )
- $G$  effect is due to taxation ( $G \uparrow \Rightarrow T \uparrow, C \downarrow$  (and  $(1 - L) \downarrow$ )
- The effect of an increase in  $G$  is illustrated in **Figure 13.1**.

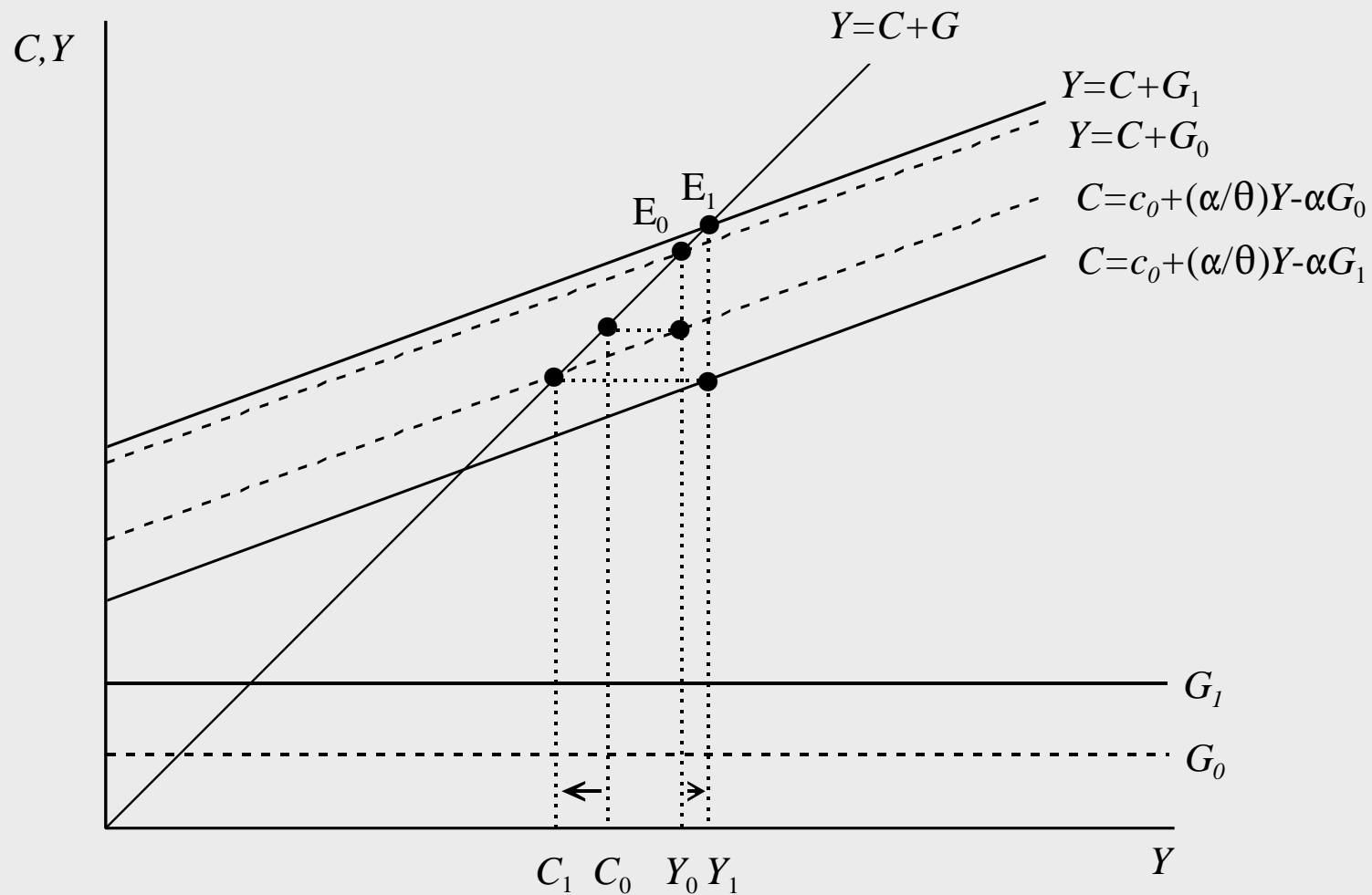


Figure 13.1: Government Spending Multipliers

- Effect on output:

$$\begin{aligned} \left( \frac{dY}{dG} \right)_T^{SR} &= \left( \frac{\theta d\Pi}{PdG} \right)_T^{SR} \\ &= (1 - \alpha) \left[ 1 + \sum_{i=1}^{\infty} (\alpha/\theta)^i \right] = \frac{1 - \alpha}{1 - \alpha/\theta} > 1 - \alpha \end{aligned}$$

- degree of monopoly,  $\frac{1}{\theta}$ , does magnify the expansionary effect!

- Effect on consumption:

$$-\alpha < \left( \frac{dC}{dG} \right)_T^{SR} = - \left( \frac{\theta - 1}{\theta - \alpha} \right) \alpha < 0$$

- inconsistent with Haavelmo b-b multiplier (where  $C$  is unaffected)!

- Effect on employment:

$$0 < W \left( \frac{dL}{dG} \right)_T^{SR} = \left( \frac{\theta - 1}{\theta - \alpha} \right) (1 - \alpha) < 1 - \alpha$$

- labour supply effect explains output expansion (rather classical mechanism)

## Short-run multiplier, II

- $N = N_0$  (fixed); GBC:  $dG = -W dL_G$
- aggregate *consumption function* is:

$$C = \alpha [1 - N_0 F] W + (\alpha/\theta)Y - \alpha(T/P)$$

- $T/P$  is constant (by assumption)

- Effects of increase in government consumption:

$$\left(\frac{dY}{dG}\right)_{L_G}^{SR} = \left(\frac{\theta d\Pi}{P dG}\right)_{L_G}^{SR} = \left[1 + \sum_{i=1}^{\infty} (\alpha/\theta)^i\right] = \frac{1}{1 - \alpha/\theta} > 1$$

$$\left(\frac{dC}{dG}\right)_{L_G}^{SR} = \frac{\alpha}{\theta - \alpha} > 0$$

$$W \left(\frac{dL}{dG}\right)_{L_G}^{SR} = - \left(\frac{1 - \alpha}{\theta - \alpha}\right) < 0$$

- $\frac{dY}{dG}$  exceeds unity as consumption rises ( $\frac{dC}{dG} > 0$ )!
- labour supply falls (wealth effect) but output expansion made possible by release of labour from the unproductive to the productive sector.

## Long-run multiplier

- Following a fiscal shock there are excess profits to be gained ( $\Pi > 0$ )
- In absence of barriers to entry one would expect entry of new firms
- Ad hoc entry/exit rule:

$$\dot{N} = \gamma_N (\Pi/P) = \gamma_N [\theta^{-1}Y - WNF], \quad \gamma_N > 0$$

- What is the long-run multiplier? Assume there are no civil servants ( $L_G = 0$ )
- Goods market equilibrium (GME) line:

$$\begin{aligned} Y &= \alpha [1 - NF] W + (\alpha/\theta)Y + (1 - \alpha)G \\ &= \left[ \frac{\alpha(1 - NF)}{\mu k(1 - \alpha/\theta)} \right] N^{\eta-1} + \left[ \frac{1 - \alpha}{1 - \alpha/\theta} \right] G \end{aligned} \quad \text{(GME)}$$

- we have used the pricing rule:

$$W = \frac{N^{\eta-1}}{\mu k}$$

- The zero-profit (ZP) condition is:

$$Y = \frac{\theta F N^{\eta}}{\mu k} \quad (\text{ZP})$$

- In **Figure 13.2** we illustrate the impact, transitional, and long-run effect of a tax-financed increase in government consumption.
- ZP slopes up
  - there is entry (exit) of firms to the left (right) of the ZP line (see the horizontal arrows)

- slope of GME is ambiguous due to interplay of offsetting effects
  - diversity effect if  $\eta > 1$ : renders slope positive
  - fixed-cost effect (for  $F > 0$ ): renders the slope negative
  
- two special cases for GME:
  - standard S-D-S preferences (set  $\eta = \mu$ ): GME slopes up (see Figure 13.2).  
 Long-run multiplier is larger than the short-run multiplier:

$$\begin{aligned}
 \left(\frac{dY}{dG}\right)_T^{LR, \eta=\mu} &= \frac{1 - \alpha}{1 - \left(\frac{\mu-1}{\mu}\right) [\alpha + (1 - \alpha)\omega_C]} \\
 &= \frac{1 - \alpha}{1 - (1/\theta) [\alpha + (1 - \alpha)\omega_C]} > \frac{1 - \alpha}{1 - \alpha/\theta} \equiv \left(\frac{dY}{dG}\right)_T^{SR}
 \end{aligned}$$

- no PFD at all (set  $\eta = 1$ ): GME slopes down. Long-run multiplier “vanishes”:

$$0 < \left( \frac{dY}{dG} \right)_T^{LR, \eta=1} = (1 - \alpha) < \frac{1 - \alpha}{1 - \alpha/\theta} \equiv \left( \frac{dY}{dG} \right)_T^{SR}$$

- Diversity effect shows up in the “aggregate production function” for this economy, relating  $Y$  to  $L$ :

$$Y = \left( \frac{(\theta F)^{1-\eta}}{\mu k} \right) L^\eta$$

- $\eta > 1$  implies IRTS at the aggregate level
- some hard-core Keynesians argue that IRTS are (or should be) the central element of Keynesian economics (PFD is one simple mechanism)

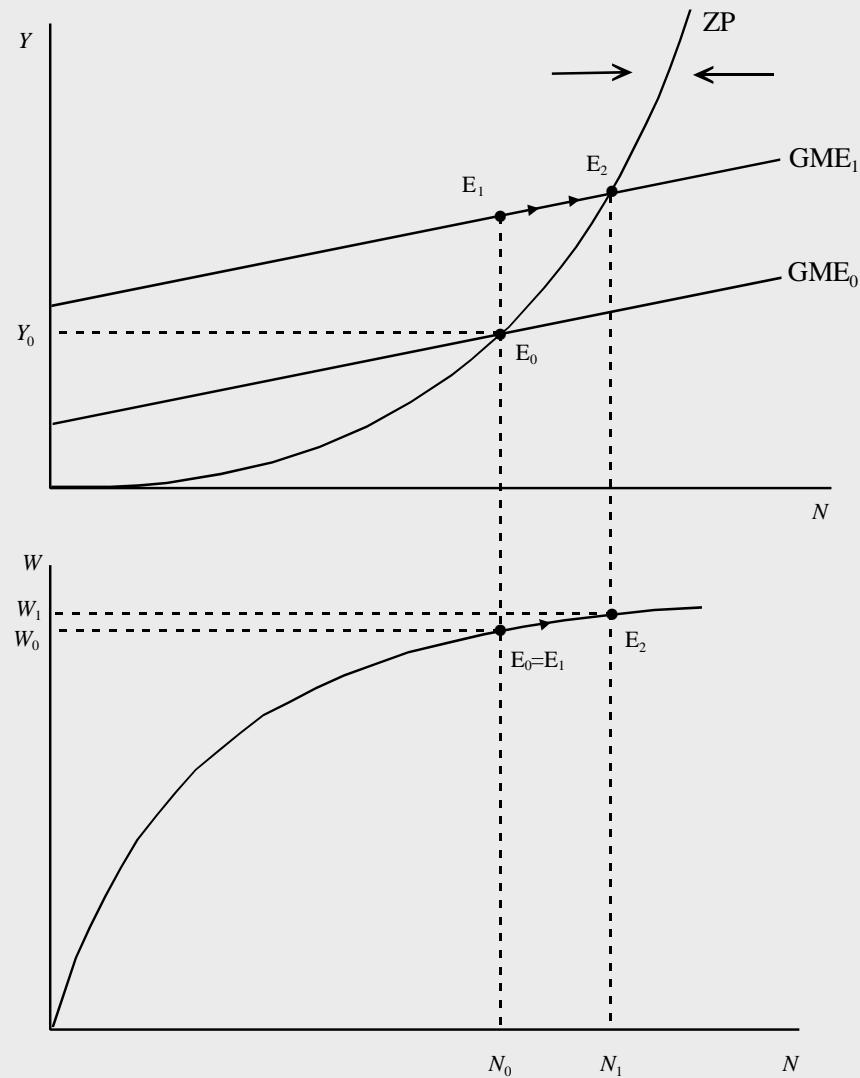


Figure 13.2: Multipliers and Firm Entry

## Welfare effects

- establish link between the multiplier and welfare
- look at short-run multiplier only
- handy tool: the indirect utility function (IUF):

$$V \equiv \frac{I_F}{P_V} \equiv \frac{W + \Pi/P - T/P}{P_V/P}$$

$$\frac{P_V}{P} \equiv \frac{W^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$$

## Tax-financed fiscal policy

- substitute GBC and profit definition into IUF:

$$V \equiv \frac{[1 - NF - L_G]W + (1/\theta)Y - G}{P_V/P}$$

- Recall:  $N$ ,  $W$ ,  $P$ , and  $P_V$  all fixed in the short run
  - $V$  rises with  $Y$ : output too low from a social point of view
  - $V$  falls with  $G$ : taxation hurts
- differentiate  $V$  w.r.t.  $G$ :

$$\left(\frac{dV}{dG}\right)_T^{SR} = \left(\frac{P}{P_V}\right) \left[ \frac{1}{\theta} \left(\frac{dY}{dG}\right)_T^{SR} - 1 \right] = - \left(\frac{P}{P_V}\right) \left(\frac{\theta - 1}{\theta - \alpha}\right) < 0$$

- fiscal policy is not welfare increasing (contra Keynes claim about empty bottles).

Reasons for this un-Keynesian result:

- flexible wages and clearing labour market
  - every unit of labour is productive
- let us reconsider the bond-financed case again

## Bond-financed fiscal policy

- extra spending financed by firing unproductive civil servants ( $dG = -W dL_G$ )
- for this case the IUF is:

$$V \equiv \frac{[1 - NF] W + (1/\theta)Y - T/P}{P_V/P}$$

- $T/P$  is fixed
- only output effect remains
- differentiate  $V$  w.r.t.  $G$ :

$$\left(\frac{dV}{dG}\right)_{L_G}^{SR} = \left(\frac{P}{P_V}\right) \frac{1}{\theta} \left(\frac{dY}{dG}\right)_{L_G}^{SR} = \left(\frac{P}{P_V}\right) \left(\frac{1}{\theta - \alpha}\right) > 0$$

- Fiscal policy increases welfare!
  - labour shifted from unproductive to productive activities
  - but: a tax cut would improve welfare even more:

$$\begin{aligned}\left(\frac{dV}{d(T/P)}\right)_{LG}^{SR} &= \left(\frac{P}{P_V}\right) \left[ \frac{1}{\theta} \left(\frac{dY}{d(T/P)}\right)_{LG}^{SR} - 1 \right] \\ &= \left(\frac{P}{P_V}\right) \left(\frac{\theta}{\theta - \alpha}\right) > 0\end{aligned}$$

## Monopolistic competition and money

- turn from real to monetary model
- usual short-cut trick: put money in the utility function.
  - Money saves on shoe-leather costs
  - Shopping costs depend on leisure and money
  - Money makes shopping easier (saves valuable leisure)
- Household utility function:

$$U \equiv [C^\alpha (1 - L)^{1-\alpha}]^\beta \left( \frac{M}{P} \right)^{1-\beta}, \quad 0 < \alpha, \beta < 1,$$

where  $M$  is *nominal* money balances.

- Household budget constraint:

$$PC + W^N(1 - L) + M = M_0 + W^N + \Pi - T,$$

where  $M_0$  is *initial* money balances (accumulated in the previous period).

- Household chooses  $C$ ,  $L$ , and  $M$  to maximize  $U$  subject to the budget constraint.

Solutions:

$$PC = \alpha\beta I_F$$

$$I_F \equiv M_0 + W^N + \Pi - T$$

$$W^N(1 - L) = \beta(1 - \alpha)I_F$$

$$M = (1 - \beta)I_F$$

- Assume that the policy maker maintains a constant money supply. Money market equilibrium (MME) is then:

$$M = M_0$$

- The monetary monopolistic competition model is summarized in **Table 13.2**. Some remarks:
  - $M_0$  features in the indirect utility function (IUF, eqn (T2.8))
  - helicopter drop of money,  $dM_0 > 0$ , has no welfare effects
  - money is neutral / classical dichotomy
  - $dM_0 > 0$  inflates nominal variables but leaves real variables unchanged
  - in and of itself, monopolistic competition does not cause monetary non-neutrality

**Table 13.2. A simple monetary monopolistic competition model**

$$Y = C + G \tag{T2.1}$$

$$C = \alpha\beta(I_F/P), \quad I_F/P \equiv M_0/P + W^N/P + \Pi/P - T/P \tag{T2.2}$$

$$\Pi/P \equiv \theta^{-1}Y - (W^N/P)NF \tag{T2.3}$$

$$T/P = G + (W^N/P)L_G \tag{T2.4}$$

$$P/W^N = \mu k N^{1-\eta} \tag{T2.5}$$

$$(W^N/P)(1 - L) = \beta(1 - \alpha)(I_F/P) \tag{T2.6}$$

$$M_0/P = (1 - \beta)(I_F/P) \tag{T2.7}$$

$$V = \frac{I_F}{P_V}, \quad P_V = \left(\frac{P}{\alpha\beta}\right)^{\alpha\beta} \left(\frac{W^N}{\beta(1-\alpha)}\right)^{\beta(1-\alpha)} \left(\frac{P}{1-\beta}\right)^{1-\beta} \tag{T2.8}$$

## Properties of the monetary monopolistic competition model

- Model can be reduced to two schedules
- Focus on the short run:  $N$  and  $W$  are fixed.
- Goods market equilibrium (GME) locus:

$$Y = \frac{\alpha [1 - NF - L_G] W + (1 - \alpha)G}{1 - \alpha/\theta}$$

- Money market equilibrium (MME) locus:

$$\frac{M_0}{P} = \left( \frac{1 - \beta}{\beta} \right) \left[ [1 - NF - L_G] W + (1/\theta)Y - G \right]$$

- Classical dichotomy:
  - GME fixes  $Y$  independently from  $M_0$
  - MME then fixes  $P$

- Effects on lump-sum tax financed fiscal policy:

$$\begin{aligned}
 0 &< \left( \frac{dY}{dG} \right)_T^{SR} = \frac{1 - \alpha}{1 - \alpha/\theta} < 1 \\
 \left( \frac{dW^N}{W^N} \right)_T^{SR} &= \left( \frac{dP}{P} \right)_T^{SR} = \left( \frac{d\bar{P}}{\bar{P}} \right)_T^{SR} \\
 \left( \frac{dM_0/P}{dG} \right)_T^{SR} &= - \left( \frac{M_0}{P^2} \right) \left( \frac{dP}{dG} \right)_T^{SR} = - \frac{(1 - \beta)(\theta - 1)}{\beta(\theta - \alpha)} < 0
 \end{aligned}$$

- Monetary part of the model is more Classical than Keynesian!

## Sticky prices and monetary non-neutrality

- under which conditions would a price-setting agent change his price or keep it unchanged?
- key ingredient of the New Keynesian approach: non-trivial price adjustment costs (remember Modigliani (1944)?)
- Two types of price adjustment costs:
  - menu costs (non-convex): fixed cost per price change (e.g. informing dealers, reprinting price lists or “menu’s”, etcetera)
  - convex costs: costs depending on the size of the price change (e.g. adverse reactions by customers to large price changes)

## Menu costs

- Develop simplified version of the Blanchard-Kiyotaki model (competitive labour market)
- Focus on the short run: fixed number of firms ( $N$ )
- Household utility is additively separable in  $(C, M/P)$  and  $L$ :

$$U(C, M/P, L) \equiv U^1(C, M/P) - U^2(L)$$

$$= C^\alpha (M/P)^{1-\alpha} - \gamma_L \left[ \frac{L^{1+1/\sigma}}{1+1/\sigma} \right], \quad 0 < \alpha < 1$$

- $\sigma > 0$  regulates labour supply elasticity
- $C$  is composite differentiated good ( $\eta = 1$ : no diversity preference)

- Household budget restriction:

$$PC + M = W^N L + M_0 + \Pi - T (\equiv I)$$

- Use two-stage budgeting:

- *stage 1*: maximizing  $U^1(C, M/P)$  subject to  $PC + M = I$  yields:

$$PC = \alpha I$$

$$M = (1 - \alpha)I$$

$$V^1(I/P) = \alpha^\alpha (1 - \alpha)^{1-\alpha} (I/P)$$

- *stage 2*: maximizing  $V^1(I/P)$  (the IUF associated with stage 1 problem) subject to  $I \equiv W^N L + M_0 + \Pi - T$  yields:

$$L = \left( \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{\gamma_L} \right)^\sigma \left( \frac{W^N}{P} \right)^\sigma \tag{a}$$

$$\frac{I}{P} = \left( \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{\gamma_L} \right)^\sigma \left( \frac{W^N}{P} \right)^{1+\sigma} + \frac{M_0 + \Pi - T}{P}$$

- Key feature of the labour supply equation (a): no income effect. Substitution effect parameterized by  $\sigma$ . For future reference:
  - $\sigma$  large: near horizontal labour supply equation. Small change in  $W$  causes large change in  $L$ . High degree of *real rigidity*. [empirically problematic]
  - $\sigma$  small: near vertical labour supply equation. Small change in  $L$  causes large change in  $W$ . Low degree of real rigidity. [empirically realistic]

- Firms face demand from private sector and from the government (same elasticity; no diversity effect)

$$Y_j(P_j, P, Y) = \left(\frac{P_j}{P}\right)^{-\theta} \left(\frac{Y}{N}\right)$$

- Aggregate demand is:

$$Y = C + G = \left(\frac{\alpha}{1 - \alpha}\right) \left(\frac{M}{P}\right) + G$$

- $G$  raises aggregate demand
- if  $P$  is somehow fixed (e.g. due to menu costs), then  $M$  will also raise aggregate demand

- Technology of the differentiated product firm is slightly more general than before:

$$Y_j = \begin{cases} 0 & \text{if } L_j \leq F \\ \left[ \frac{L_j - F}{k} \right]^\gamma & \text{if } L_j \geq F \end{cases}$$

- we had  $\gamma = 1$  but now also allow for  $0 < \gamma < 1$ .
- $\gamma$  regulates curvature of the marginal cost curve ( $\gamma < 1$ , MC falls with output, and AC is *U*-shaped)
- Firm chooses its price,  $P_j$ , in order to maximize its profit:

$$\Pi_j(P_j, P, Y) \equiv \underbrace{P_j Y_j(P_j, P, Y)}_{\text{revenue}} - \underbrace{W^N \left[ k (Y_j(P_j, P, Y))^{1/\gamma} + F \right]}_{\text{total cost}}$$

- Bertrand assumption: firm takes prices of close competitors as given ( $P$  is an aggregate of these prices)

- The optimal price for firm  $j$  satisfies the FONC:

$$\begin{aligned}
 \frac{d\Pi_j(P_j, P, Y)}{dP_j} &= [P_j - MC_j] \left( \frac{\partial Y_j(P_j, P, Y)}{\partial P_j} \right) + Y_j(P_j, P, Y) \\
 &= Y_j(P_j, P, Y) \left[ 1 + \left( \frac{P_j - MC_j}{P_j} \right) \left( \frac{P_j}{Y_j(\cdot)} \frac{\partial Y_j(\cdot)}{\partial P_j} \right) \right] \\
 &= Y_j(P_j, P, Y) \left[ 1 - \theta \left( \frac{P_j - MC_j}{P_j} \right) \right] = 0 \tag{a}
 \end{aligned}$$

- We derive from (a) that the optimal price is a markup times marginal cost ( $MC_j$ ), i.e.  $P_j = \mu MC_j$  or:

$$P_j = \left( \frac{\mu k}{\gamma} \right) W^N Y_j^{(1-\gamma)/\gamma}, \quad \mu = \frac{\theta}{\theta - 1} > 1$$

- without menu costs optimal pricing rule under Cournot and Bertrand same (not so with menu costs!!)
- relative price of firm  $j$  depends on  $Y_j$  and on the real wage,  $W \equiv W^N / P$ :

$$\frac{P_j}{P} = \left( \frac{\mu k}{\gamma} \right) W Y_j^{(1-\gamma)/\gamma}$$

This is where the aggregate labour market comes into play.

- Model is summarized in **Table 13.3**.

**Table 13.3. A simplified Blanchard-Kiyotaki model (no menu costs)**

$$Y = C + G \tag{T3.1}$$

$$C = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{M_0}{P} \right) = \begin{cases} \alpha \left[ \omega^{-\sigma} \left( \frac{W^N}{P} \right)^{1+\sigma} + \frac{M_0}{P} + \frac{\Pi}{P} - G \right] & (\text{if } \sigma < \infty) \\ \alpha \left[ \left( \frac{W^N}{P} \right) L + \frac{M_0}{P} + \frac{\Pi}{P} - G \right] & (\text{if } \sigma \rightarrow \infty) \end{cases} \tag{T3.2}$$

$$\Pi/P \equiv \left( \frac{\mu - \gamma}{\mu} \right) Y - (W^N/P)NF \tag{T3.3}$$

$$P/W^N = (\mu k/\gamma) \left( \frac{Y}{N} \right)^{(1-\gamma)/\gamma} \tag{T3.4}$$

$$\frac{W^N}{P} = \begin{cases} \omega L^{1/\sigma} & (\text{if } \sigma < \infty) \\ \omega & (\text{if } \sigma \rightarrow \infty) \end{cases} \tag{T3.5}$$

**Notes:**  $\omega \equiv \gamma_L [\alpha^\alpha (1 - \alpha)^{1-\alpha}]^{-1} > 0$  and  $\mu \equiv \theta/(\theta - 1)$ .

## The flex-price version of the B-K model

- Money is neutral: doubling  $M_0$  doubles all nominal variables ( $W^N, P, \Pi$ ) but leaves the real variables ( $Y, C, L, M_0/P, \Pi/P, W^N/P$ ) unaffected.
- Fiscal policy completely ineffective. There is no income effect in labour supply, so concomitant tax increase does not affect employment:  $\frac{dY}{dG} = \frac{dL}{dG} = \frac{dW}{dG} = 0$  and  $\frac{dC}{dG} = -1$  (one-for-one crowding out of private by public consumption)!
- Flex-price B-K model is hyper-classical indeed.

## The menu-cost insight

- small costs of changing one's actions can have large allocational and welfare effects
- or: “small deviations from rationality make significant differences to equilibria”  
[Akerlof & Yellen].
- In macro-context: following a shock to aggregate demand, is it possible that:
  - (a) price stickiness is privately efficient?
  - (b) price stickiness exists in general equilibrium?
  - (c) price stickiness has first-order effect on economic welfare?

- In our version of the B-K model we verify the various parts of the “menu-cost agenda”:
  - part (a) easy: application of the envelope theorem
  - part (b) tricky: intricate general equilibrium effects (interaction nominal and real rigidity)
  - part (c) follows once (a)-(b) are covered

## Can it be rational not to change one's price?

- individual firm cares about its profits only
- optimal price chosen by firm  $j$  satisfies:

$$\frac{P_j^*}{P} = \left[ \left( \frac{\mu k}{\gamma} \right) \left( \frac{W^N}{P} \right) \left( \frac{Y}{N} \right)^{(1-\gamma)/\gamma} \right]^{\gamma/[\gamma+\theta(1-\gamma)]}$$

- we have combined the optimal pricing rule with the firm's demand function
- $P$ ,  $Y$ , and  $W^N$  are all taken as exogenous by the firm (as is  $N$ )
- we can write  $P_j^* = P_j^*(P, Y, W^N)$
- The optimized profit function of firm  $j$  can be written as:

$$\Pi_j^*(P, Y, W^N) \equiv P_j^*(\cdot) Y_j(P_j^*(\cdot), P, Y) - W^N \left[ k [Y_j(P_j^*(\cdot), P, Y)]^{1/\gamma} + F \right]$$

- By differentiating  $\Pi_j^*(\cdot)$  w.r.t. aggregate demand,  $Y$ , we find the envelope result:

$$\begin{aligned}
 \frac{d\Pi_j^*(\cdot)}{dY} &= \left[ [P_j^*(\cdot) - MC_j^*(\cdot)] \left( \frac{\partial Y_j(P_j, P, Y)}{\partial P_j} \right)_{P_j=P_j^*} + Y_j(P_j^*(\cdot), P, Y) \right] \times \\
 &\quad \left( \frac{dP_j^*(\cdot)}{dY} \right) + [P_j^*(\cdot) - MC_j^*(\cdot)] \left( \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \right) \\
 &= \left[ \frac{\partial \Pi_j(\cdot)}{\partial P_j} \right]_{P_j=P_j^*} \left( \frac{dP_j^*(\cdot)}{dY} \right) + [P_j^*(\cdot) - MC_j^*(\cdot)] \left( \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \right) \\
 &= [P_j^*(\cdot) - MC_j^*(\cdot)] \left( \frac{\partial Y_j(P_j^*(\cdot), P, Y)}{\partial Y} \right) \equiv \frac{\partial \Pi_j(\cdot)}{\partial Y}
 \end{aligned}$$

- To a first-order of magnitude, the effect on the profit of firm  $j$  of a change in aggregate demand is the same whether or not firm  $j$  changes its price optimally following the aggregate demand shock. *Hence, small menu costs will prevent price adjustment by firm  $j$ .*

## Graphical representation

- The menu-cost result is illustrated in **Figure 13.3**.
- initially aggregate demand is  $Y_0$  and optimum is at point A.
- assume  $Y$  rises (to  $Y_1$ ): ceteris paribus  $(W^N, P)$ :
  - profit is higher for all  $P_j$  and (provided  $\gamma < 1$ ) and new optimum is at point B (north-east from A—output expansion increases marginal cost).
  - keeping the old price costs firm  $j$  DC in foregone profits. This is small because “objective functions are flat at the top”
  - we have completed part (a)! Next we work on the GE repercussions.

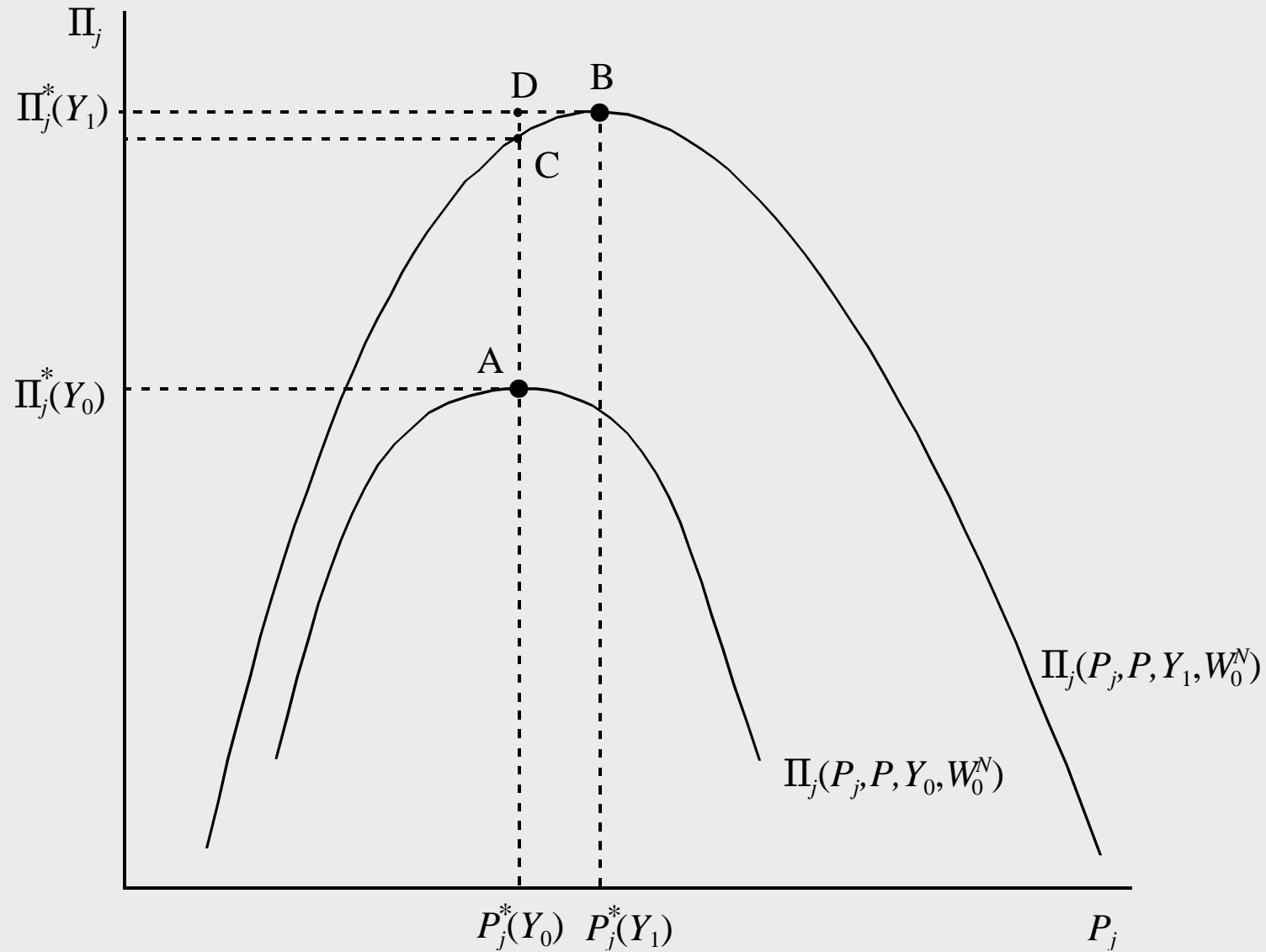


Figure 13.3: Menu Costs

## General equilibrium effects

- all firms are in the same position as firm  $j$  is in, so they all want to expand output following an increase in aggregate demand.
  - Where does the required labour come from?
  - Will there be cost increases because labour is scarce?
- Two cases:
  - $\sigma$  large (highly elastic labour supply): menu cost equilibrium exists
  - $\sigma$  finite/low (moderate labour supply elasticity): general equilibrium effects destroy menu cost equilibrium (simulations in **Tables 13.4-13.5**)

**Table 13.4: Menu Costs and the Markup**

	$\mu = 1.10$			$\mu = 1.25$		
$\Delta M = 0.05$	menu costs	welfare gain	ratio	menu costs	welfare gain	ratio
$\sigma_Y = 0.1$						
$\sigma = 0.2$	20.44	28.6	1.40	18.10	29.1	1.61
$\sigma = 0.5$	7.85	28.9	3.68	6.96	29.4	4.22
$\sigma = 1$	3.95	29.0	7.35	3.51	29.5	8.40
$\sigma = 2.5$	1.69	29.1	17.18	1.51	29.5	19.49
$\sigma = 5$	0.94	29.1	30.80	0.86	29.6	34.37
$\sigma = 10^6$	0.20	29.1	146.12	0.20	29.6	145.73

**Table 13.4: Menu Costs and the Markup (continued)**

	$\mu = 1.50$			$\mu = 2$		
$\sigma = 0.2$	15.23	29.8	1.96	11.53	30.6	2.65
$\sigma = 0.5$	5.87	30.0	5.11	4.55	30.8	6.76
$\sigma = 1$	2.99	30.1	10.06	2.35	30.8	13.12
$\sigma = 2.5$	1.32	30.1	22.80	1.06	30.8	29.12
$\sigma = 5$	0.76	30.1	39.56	0.63	30.9	48.68
$\sigma = 10^6$	0.21	30.1	144.67	0.21	30.9	144.95

**Table 13.5: Menu Costs and the Elasticity of Marginal Cost**

	$\sigma_Y = 0$			$\sigma_Y = 0.05$		
$\Delta M = 0.05$	menu	welfare	ratio	menu	welfare	ratio
$\mu = 1.25$	costs	gain		costs	gain	
$\sigma = 0.2$	17.44	29.2	1.67	17.72	29.2	1.65
$\sigma = 0.5$	6.61	29.4	4.45	6.76	29.4	4.35
$\sigma = 1$	3.17	29.5	9.31	3.34	29.5	8.84
$\sigma = 2.5$	1.19	29.5	24.73	1.36	29.5	21.69
$\sigma = 5$	0.52	29.6	56.72	0.70	29.6	42.23
$\sigma = 10^6$	$\rightarrow 0$	29.6	$\rightarrow \infty$	0.04	29.6	672.74

**Table 13.5: Menu Costs and the Elasticity of Marginal Cost (continued)**

	$\sigma_Y = 0.1$			$\sigma_Y = 0.2$		
$\sigma = 0.2$	18.10	29.1	1.61	18.54	29.1	1.57
$\sigma = 0.5$	6.96	29.4	4.22	7.34	29.4	4.00
$\sigma = 1$	3.51	29.5	8.40	3.84	29.5	7.67
$\sigma = 2.5$	1.51	29.5	19.49	1.83	29.5	16.16
$\sigma = 5$	0.86	29.6	34.37	1.15	29.5	25.60
$\sigma = 10^6$	0.20	29.6	145.73	0.49	29.6	60.60

## The menu-cost equilibrium (MCE)

- assume  $\sigma \rightarrow \infty$  so that the real wage is rigid: if  $P$  does not change (because all firms keep their old prices) then neither does the nominal wage  $W^N$
- our partial equilibrium story is equivalent to the general equilibrium effects—see Figure 13.3. Part (b) is confirmed.
- Properties of the menu cost equilibrium:
  - fiscal policy is highly effective
  - monetary policy is highly effective
  - both policies have first-order welfare effects. Part (c) is confirmed.

## Fiscal policy in the MCE

- in the MCE, the model can be condensed to:

$$Y = C + G$$

$$C = \left( \frac{\alpha}{1 - \alpha} \right) (M_0/P) = \alpha [Y + M_0/P - G]$$

where  $P$  is fixed (because all firms keep their price unchanged)

- Fiscal policy: a lump-sum tax financed increase in  $G$  is quite effective:

$$\begin{aligned} \left(\frac{dY}{dG}\right)_T^{MCE} &= 1 \\ \left(\frac{dC}{dG}\right)_T^{MCE} &= \left(\frac{d(M_0/P)}{dG}\right)_T^{MCE} = 0 \\ (W^N/P) \left(\frac{dL}{dG}\right)_T^{MCE} &= \frac{1}{\mu} \left(\frac{dY}{dG}\right)_T^{MCE} = \frac{\theta - 1}{\theta} > 0 \end{aligned}$$

- $G \uparrow$  causes shift in aggregate demand
  - $Y_j \uparrow$  (for all firms) but  $P_j$  (and thus  $P$ ) unaffected
  - labour supply horizontal, so  $W^N$  unchanged but  $L \uparrow$
  - household income rise (both wage income and profit income)–multiplier effect
- Looks exactly like the Haavelmo multiplier (mentioned in Chapter 1)

## Monetary policy in the MCE

- Helicopter drop of money balances:  $dM_0 > 0$  also increases output, consumption, and employment:

$$P \left( \frac{dY}{dM_0} \right)^{MCE} = P \left( \frac{dC}{dM_0} \right)^{MCE} = \mu W^N \left( \frac{dL}{dM_0} \right)^{MCE} = \frac{\alpha}{1 - \alpha} > 0$$

- $M_0 \uparrow$  causes increase in  $C$  (wealthier households)
- $Y_j \uparrow$  (for all firms) but  $P_j$  (and thus  $P$ ) unaffected
- labour supply horizontal, so  $W^N$  unchanged, but  $L \uparrow$
- household income rise (both wage income and profit income)–multiplier effect

## Welfare effects of policy in the MCE

- In the menu-cost equilibrium, the hyper-classical model becomes hyper-Keynesian (strong effects of policy)
- But: what are the welfare effects of fiscal and monetary policy?

- Indirect utility function (IUF):

$$\begin{aligned}
 V &= \alpha^\alpha (1 - \alpha)^{1-\alpha} \left[ Y + \frac{M_0}{P} - G \right] - \gamma_L L \\
 &= \alpha^\alpha (1 - \alpha)^{1-\alpha} \left[ \frac{M_0 + \Pi}{P} - G \right] + \left[ \alpha^\alpha (1 - \alpha)^{1-\alpha} \left( \frac{W^N}{P} \right) - \gamma_L \right] L \\
 &= \alpha^\alpha (1 - \alpha)^{1-\alpha} \left[ \frac{M_0 + \Pi}{P} - G \right] \tag{a}
 \end{aligned}$$

- used  $\Pi \equiv PY - W^N L$  in the first step
- used labour supply,  $\gamma_L = \alpha^\alpha (1 - \alpha)^{1-\alpha} \left( \frac{W^N}{P} \right)$ , in second step: labour supply set optimally so variation in  $L$  causes no first-order welfare effect.
- From (a) we conclude that both policies cause first-order welfare effect.

- Fiscal policy:

$$\begin{aligned}
 \left(\frac{dV}{dG}\right)_T^{MCE} &= \alpha^\alpha(1-\alpha)^{1-\alpha} \left[ \left(\frac{dY}{dG}\right)_T^{MCE} - 1 \right] - \gamma_L \left(\frac{dL}{dG}\right)_T^{MCE} \\
 &= -\frac{\gamma_L}{\mu} \left(\frac{P}{W^N}\right) \\
 &= -\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\mu} < 0
 \end{aligned}$$

- $\frac{dY}{dG} = 1$  does not come for free as the household must supply more hours of labour
- net effect on welfare is negative

- Monetary policy:

$$\begin{aligned}
 \left( \frac{dV}{dM_0} \right)^{MCE} &= \alpha^\alpha (1 - \alpha)^{1-\alpha} \left[ \frac{1}{P} + \left( \frac{d(\Pi/P)}{dM_0} \right)^{MCE} \right] \\
 &= \frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{P} \left[ 1 + P \left( \frac{dY}{dM_0} \right)^{MCE} - W^N \left( \frac{dL}{dM_0} \right)^{MCE} \right] \\
 &= \underbrace{\frac{\alpha^\alpha (1 - \alpha)^{1-\alpha}}{P}}_{(a)} \underbrace{\left[ 1 + \left( \frac{1}{\theta} \right) \left( \frac{\alpha}{1 - \alpha} \right) \right]}_{(b)} > 0
 \end{aligned}$$

- (a): marginal utility of nominal income (positive)
  - (b), first term: *liquidity effect* (also exists in competitive model)
  - (b), second term: *profit effect* (specific to B-K model)
  - total effect on welfare is positive, more so the larger is the degree of monopoly ( $\frac{1}{\theta}$ )
- NOTE: the liquidity effect holds because the competitive monetary equilibrium is sub-optimal: one should not economize on fiat money!! (Friedman's satiation result)

## The general MCE equilibrium

- now we assume that labour supply features a finite elasticity:  $0 < \sigma \ll \infty$
- analytical results no longer possible: GE effects too complicated
- calibrate the model and simulate robustness of menu-cost result to variations in:
  - the value of  $\sigma$
  - the value of  $\mu$  (the gross monopoly markup)
  - the value of  $\sigma_Y \equiv \frac{1-\gamma}{\gamma}$  (the elasticity of the marginal cost function)
- the calibration exercise allows the evaluation of big shocks (inframarginal)
- assume that the menu costs take the form of labour input  $Z$  (e.g. shop assistants changing price tags)

- following a monetary shock there are two scenarios:
  - (FA) full adjustment: all firms change their price and incur the menu costs

$$\Pi^{FA} = \left( \frac{\mu - \gamma}{\mu} \right) PY - W^N N (F + Z)$$

- (NA) non-adjustment: all firms keep their price unchanged

$$\Pi^{NA} = P_0 Y - W^N [kY^{1/\gamma} N^{1-1/\gamma} + NF]$$

- In the simulations we find minimum value of  $Z$  (labeled  $Z_{MIN}$ ) for which non-adjustment is an equilibrium (for which  $\Pi^{NA} > \Pi^{FA}$ ). See **Tables 13.4-13.5**.
  - the entry “menu costs” is defined as follows:

$$\text{menu costs} = 100 \times \left( \frac{N_0 (W^N)^{NA} Z_{MIN}}{P_0 Y^{NA}} \right)$$

- the entry “welfare gain” is defined as follows:

$$\text{welfare gain} = 100 \times \left( \frac{V^{NA} - V_0}{U_C Y^{NA}} \right)$$

- the entry “ratio” is defined as  $\frac{\text{welfare cost}}{\text{menu cost}}$
- example from Table 13.4:  $\mu = 1.1$ ,  $\sigma_Y = 0.1$ , and  $\sigma = 10^6$ . Menu costs amounting to no more than 0.20% of revenue (tiny) will make non-adjustment of prices an equilibrium in the sense that  $\Pi^{NA} > \Pi^{FA}$ ! The welfare effect is 29.1% of output (huge). Small menu costs have large welfare effects.

- Other key features of the simulation results:
  - welfare measure relatively constant
  - the markup does not affect menu costs and ratio very much
  - the labour supply elasticity exerts a very strong effect on menu costs and the ratio. Intuition: if  $\sigma$  is low, then output expansion drives up wages (production costs) which makes non-adjustment less likely to be optimal.
- Table 13.5 has basically very similar results: the key role is played by the labour supply elasticity.

## Evaluation of the menu-cost idea

- runs into same trouble as the RBC literature does: we simply do not observe a high  $\sigma$
- Ball & Romer: both *nominal rigidity* (menu cost) and some kind of *real rigidity* (e.g. high  $\sigma$ , customer market, or efficiency wage labour market) are needed to get the menu-cost equilibrium
- Rotemberg mentions some further problems:
  - MC equilibrium may not be unique
  - may equally well apply to quantities instead of prices (makes price adjustment more likely)
  - MC insight does not generalize easily to dynamic setting (our next theories do not have that problem)

## Quadratic price adjustment costs

- Convex adjustment costs: quadratic in price change
- Derive approximate pricing rule in two steps:
  - determine path of equilibrium prices  $\{P_{j,\tau}^*\}_{\tau=0}^{\infty}$  which the firm would set in the absence of price-adjustment costs (PACs). This is the desired “target” the firm will aim for.
  - next determine the quadratic approximation of the profit function around this target price path and incorporate PACs.

- The objective function of the firm is then:

$$\Omega_0 = \sum_{\tau=0}^{\infty} \left( \frac{1}{1+\rho} \right)^{\tau} \left[ \underbrace{(p_{j,\tau} - p_{j,\tau}^*)^2}_{(a)} + c \underbrace{(p_{j,\tau} - p_{j,\tau-1})^2}_{(b)} \right]$$

- stay as close as possible to target path:  $\Omega_0$  should be minimized
- $p_{j,\tau} \equiv \log P_{j,\tau}$  (actual price);  $p_{j,\tau}^* \equiv \log P_{j,\tau}^*$  (target price)
- $\rho$  is the firm's discount factor
- (a): intratemporal cost of setting the “wrong” price
- (b): intertemporal costs associated with changing the price (annoyed customers, etcetera)

- The firm minimizes  $\Omega_0$  by choosing the appropriate sequence of prices,  $\{p_{j,\tau}\}_{\tau=0}^{\infty}$ .

The FONC is:

$$\begin{aligned} \frac{\partial \Omega_0}{\partial p_{j,\tau}} &= \left( \frac{1}{1+\rho} \right)^\tau [2(p_{j,\tau} - p_{j,\tau}^*) + 2c(p_{j,\tau} - p_{j,\tau-1})] \\ &- \left( \frac{1}{1+\rho} \right)^{\tau+1} [2c(p_{j,\tau+1} - p_{j,\tau})] = 0 \end{aligned}$$

or:

$$p_{j,\tau+1} - \left[ 1 + (1+\rho) \left( \frac{1+c}{c} \right) \right] p_{j,\tau} + (1+\rho)p_{j,\tau-1} = - \left( \frac{1+\rho}{c} \right) p_{j,\tau}^* \quad (\text{a})$$

- equation (a) is a second-order difference equation in  $p_{j,\tau}$ . We need two boundary conditions:
  - initial condition:  $p_{j,-1}$  is pre-determined (set in the past)
  - terminal condition

- The pricing rule in the planning period ( $p_{j,0}$ ) is then:

$$p_{j,0} = \lambda_1 p_{j,-1} + (1 - \lambda_1) \left[ \left( \frac{\lambda_2 - 1}{\lambda_2} \right) \sum_{\tau=0}^{\infty} \left( \frac{1}{\lambda_2} \right)^{\tau} p_{j,\tau}^* \right] \quad (b)$$

- $0 < \lambda_1 < 1$  is the stable characteristic root of (a)
- $\lambda_2 > 1$  is the unstable characteristic root of (a)
- actual price weighted average of the past price and a long-run target price
- Note that (b) contains both backward-looking and forward-looking elements.  
Anticipated changes in  $p_{j,\tau}^*$  will immediately have an effect on the current price.

## Staggered price setting

- Guillermo Calvo (and co-workers) have devised an alternative approach to price stickiness (red light-green light model)
- Price contracts are staggered (old idea of Phelps and Taylor)
- No separate price-adjustment costs
- Duration of price contract is stochastic via a Poisson process: each period “nature” draws a signal to each firm:
  - “green light” with probability  $\pi$ : go ahead and adjust your contract price (optimally)
  - “red light” with probability  $1 - \pi$ : continue to charge your present contract price

- Objective function of a firm which has just received a green light:

$$\begin{aligned} \Omega_0 = & (p_{j,0} - p_{j,0}^*)^2 + \left( \frac{1}{1 + \rho} \right) \left[ \pi (p_{j,1} - p_{j,1}^*)^2 + (1 - \pi) (p_{j,0} - p_{j,1}^*)^2 \right] \\ & + \left( \frac{1}{1 + \rho} \right)^2 \left[ \pi^2 (p_{j,2} - p_{j,2}^*)^2 + \pi(1 - \pi) (p_{j,1} - p_{j,2}^*)^2 \right. \\ & \left. + (1 - \pi)^2 (p_{j,0} - p_{j,2}^*)^2 \right] + \text{higher-order terms.} \end{aligned}$$

- period  $\tau = 0$ : you can set your price at  $p_{j,0}$  (take intratemporal costs into account)
- period  $\tau = 1$ : you may get a green or a red light. In latter case, you keep old price,  $p_{j,0}$ . In the former case, you can re-optimize and determine  $p_{j,1}$
- period  $\tau = 2$ : three possibilities ....

- Collecting terms involving  $p_{j,0}$  we get:

$$\begin{aligned} \Omega_0 &= (p_{j,0} - p_{j,0}^*)^2 + \left(\frac{1 - \pi}{1 + \rho}\right) (p_{j,0} - p_{j,1}^*)^2 + \left(\frac{1 - \pi}{1 + \rho}\right)^2 (p_{j,0} - p_{j,2}^*)^2 + \dots \\ &= \sum_{\tau=0}^{\infty} \left(\frac{1 - \pi}{1 + \rho}\right)^{\tau} (p_{j,0} - p_{j,\tau}^*)^2 + \text{uninteresting terms} \end{aligned} \tag{a}$$

– pricing friction shows up as heavier discounting: if  $\pi \approx 1$  you have almost perfect price flexibility. If  $\pi \approx 0$  you attach higher weight to future deviation costs.

- The firm chooses  $p_{j,0}$  in order to minimize  $\Omega_0$ . The FONC is:

$$p_{j,0} \sum_{\tau=0}^{\infty} \left(\frac{1 - \pi}{1 + \rho}\right)^{\tau} = \sum_{\tau=0}^{\infty} \left(\frac{1 - \pi}{1 + \rho}\right)^{\tau} p_{j,\tau}^*$$

- we get:

$$p_0^n = \left( \frac{\pi + \rho}{1 + \rho} \right) \sum_{\tau=0}^{\infty} \left( \frac{1 - \pi}{1 + \rho} \right)^{\tau} p_{\tau}^* \quad \text{(new price)}$$

- Firms facing a red light maintain their old prices:

$$p_{-s}^n = \left( \frac{\pi + \rho}{1 + \rho} \right) \sum_{\tau=0}^{\infty} \left( \frac{1 - \pi}{1 + \rho} \right)^{\tau} p_{\tau-s}^* \quad \text{(price set } s \text{ period ago)}$$

- Given the Poisson process and the assumption of a large number of firms we know that  $\pi(1 - \pi)^s$  is the fraction of firms which last set its price  $s$  periods ago. We can aggregate all prices to derive an expression for the aggregate price level:

$$\begin{aligned}
 p_0 &= \pi p_0^n + \pi(1 - \pi)p_{-1}^n + \pi(1 - \pi)^2 p_{-2}^n + \pi(1 - \pi)^3 p_{-3}^n + \dots \\
 &= \pi \sum_{s=0}^{\infty} (1 - \pi)^s p_{-s}^n \\
 &= \pi p_0^n + (1 - \pi)p_{-1}
 \end{aligned}$$

or:

$$p_0 = (1 - \pi)p_{-1} + \pi \left[ \left( \frac{\pi + \rho}{1 + \rho} \right) \sum_{\tau=0}^{\infty} \left( \frac{1 - \pi}{1 + \rho} \right)^{\tau} p_{\tau}^* \right]$$

- actual price is weighed average of new price and past price
- Rotemberg and Calvo approaches “observationally equivalent” (yield same macro pricing equation). Rotemberg estimates that in the US 8% of all prices are adjusted each quarter (mean time between price adjustments in three years).

## Punchlines

- general equilibrium monopolistic competition (MC) model provides micro-foundations for multiplier
- intimate link between multiplier and welfare effects [pre-existing distortion]
- the existence of MC does **not** render money neutral! We need price-adjustment costs
- the menu cost insight: small deviations from rationality can have large macroeconomic and welfare effects [need both nominal and real rigidity]
- practical models: convex adjustment costs [macroeconomic price level becomes backward-looking state variable]