

*Foundations of Modern Macroeconomics*

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Chapter 10: Macroeconomic Policy,  
Credibility, and Politics

## Aims of this lecture

- What do we mean by *dynamic inconsistency*?
- How can reputation effects help in solving the problem?
- Why do we appoint conservative central bankers?
- Why does taxing capital lead to dynamic inconsistency?

## Dynamic inconsistency

- Monetary policy: policy maker exploits the Lucas supply curve
- The Lucas supply curve is:

$$y = \bar{y} + \alpha [\pi - \pi^e] + \epsilon, \quad \alpha > 0,$$

- $y$  ( $\bar{y}$ ) is the logarithm of (full employment) output
  - $\pi$  is actual inflation
  - $\pi^e$  is expected inflation
  - $\epsilon$  is a stochastic supply shock [observable to policy maker but not to public]
- LSC can be inverted:

$$\pi = \pi^e + (1/\alpha) [y - \bar{y} - \epsilon]$$

In terms of **Figure 10.1** the LSC curves are upward sloping lines with a vertical intercept at the level of  $\pi^e$ .

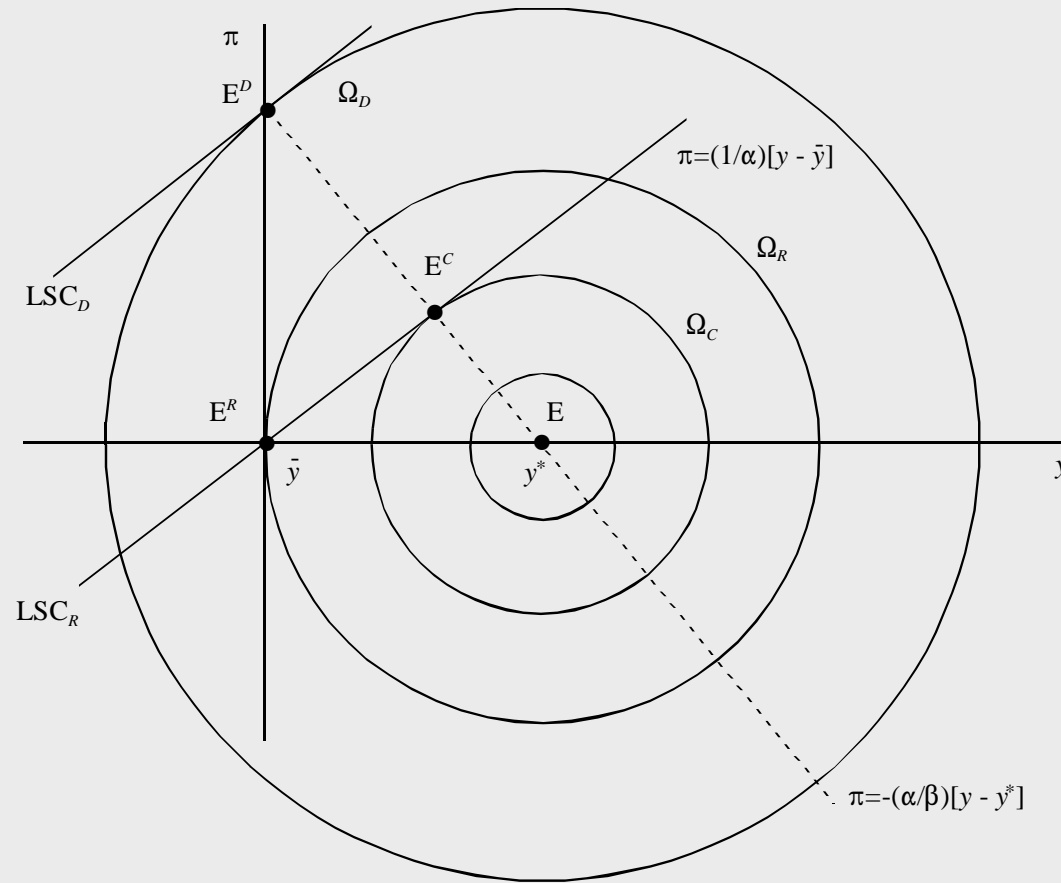


Figure 10.1: Consistent and Optimal Monetary Policy

- The objective function of the policy maker [Social Welfare Function]:

$$\Omega \equiv \frac{1}{2} [y - y^*]^2 + \frac{\beta}{2} \pi^2, \quad \beta > 0,$$

- $y^*$  is desired output target of the policy maker
  - $y^* > \bar{y}$ ; policy maker deems  $\bar{y}$  to be too low [overly ambitious?  $\bar{y}$  distorted?]
  - $\beta$  measures the relative inflation-aversion of the policy maker [ high  $\beta$  is a right-winger]
- Policy maker chooses  $\pi$  (by monetary policy) and thus  $y$  to minimize  $\Omega$  subject to the Lucas supply curve.
  - The Lagrangian is:

$$\min_{\{\pi, y\}} \mathcal{L} \equiv \frac{1}{2} [y - y^*]^2 + \frac{\beta}{2} \pi^2 + \lambda [y - \bar{y} - \alpha(\pi - \pi^e) - \epsilon]$$

- first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial y} = (y - y^*) + \lambda = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \pi} = \beta\pi - \alpha\lambda = 0,$$

where  $\lambda$  is the Lagrange multiplier.

- Combing the two FONCs yields the “social expansion path” [combinations of  $\pi$  and  $y$  for which  $\Omega$  is minimized]:

$$y - y^* = -(\beta/\alpha)\pi \Leftrightarrow (*)$$

$$\pi = -(\alpha/\beta)[y - y^*]$$

In terms of Figure 10.1, the FONC is a downward sloping [dashed] line through  $y^*$ .

- The optimal solution under **discretionary policy** is computed by combining (\*) with the constraint and solving for the inflation rate,  $\pi_D$ :

$$\pi_D = \frac{\alpha^2 \pi^e + \alpha [y^* - \bar{y} - \epsilon]}{\alpha^2 + \beta} \quad (\#)$$

In terms of Figure 10.1, all points on the line between  $E^D$  and E are solutions for  $\pi_D$  for a particular expected price level ( $\pi^e$ ).

- By invoking the rational expectations hypothesis [REH] we find a unique solution for the inflation rate under discretionary policy. Derivation:
  - REH says

$$\pi^e = E\pi_D \quad (\text{REH})$$

– From (#) we get:

$$E\pi_D = \frac{\alpha^2 \pi^e + \alpha \left[ y^* - \bar{y} - \overbrace{E\epsilon}^{=0} \right]}{\alpha^2 + \beta} = \pi^e,$$

so that we can solve for  $\pi^e$ :

$$\pi^e = \left( \frac{\alpha}{\beta} \right) [y^* - \bar{y}] \quad (**)$$

– Substituting (\*\*) into (#) and the LSC we find the actual inflation rate:

$$\pi_D = (\alpha/\beta) [y^* - \bar{y}] - \left( \frac{\alpha}{\alpha^2 + \beta} \right) \epsilon,$$

$$y_D = \bar{y} + \left( \frac{\beta}{\alpha^2 + \beta} \right) \epsilon$$

In Figure 10.1 this is represented by point  $E^D$ .

- But the discretionary solution  $(\pi_D, y_D)$  is sub-optimal!! If the policy maker commits to a zero-inflation rule  $(\pi_R = 0)$  and households would expect it to stick to the rule [so that  $\pi^e = 0$  also] then output would be:

$$y_R = \bar{y} + \epsilon$$

In terms of Figure 10.1 the rule-based solution  $(\pi_R, y_R)$  is found in point  $E^R$ . [Later on we shall use “R” to denote reputation]. Social welfare is higher in  $E^R$  than in  $E^D$ .

- But unfortunately the rule-based solution is  $(\pi_R, y_R)$  inconsistent!! If the policy maker is able to convince the public that it will follow the rule [so that  $\pi^e = 0$ ] then the policy maker is tempted to produce “surprise inflation” to steer the economy towards  $y^*$ . In terms of Figure 10.1 the “cheating solution” [subscript C] lies at point  $E^C$ . We find:

$$\pi_C = \frac{\alpha [y^* - \bar{y} - \epsilon]}{\alpha^2 + \beta},$$

$$y_C = \left( \frac{\beta}{\alpha^2 + \beta} \right) \bar{y} + \left( \frac{\alpha^2}{\alpha^2 + \beta} \right) y^* + \left( \frac{\beta}{\alpha^2 + \beta} \right) \epsilon$$

- It follows from the diagram that:

$$\Omega_D > \Omega_R > \Omega_C > 0$$

- *Discretion*: satisfies REH but is sub-optimal [worst of all cases]
- *Rule*: optimal and satisfies REH. But is open to temptation and thus not credible
- *Cheating*: closest to bliss but inconsistent with REH.

## Reputation as an enforcement mechanism

- Idea presented by Barro & Gordon (1983). *Key idea:*
  - monetary policy is like a prisoners' dilemma [PD] game. If we only consider solution consistent with the REH then  $(\pi_R, y_R)$  is preferable over  $(\pi_D, y_D)$  but society nevertheless ends up with the worst case.
  - repeated interactions may help mitigate the PD problem. Barro and Gordon suggest that the reputation of the policy maker may act as an enforcement mechanism which makes the rule-based solution credible
- Model is inherently dynamic [reputation is an asset that can be accumulated or decumulated!]

- The social welfare function is now:

$$V \equiv \Omega_0 + \frac{\Omega_1}{1+r} + \frac{\Omega_2}{(1+r)^2} + \dots = \sum_{t=0}^{\infty} \frac{\Omega_t}{(1+r)^t},$$

where  $r$  is the discount factor [interest rate] and  $\Omega_t$  is:

$$\Omega_t \equiv \frac{1}{2} [y_t - y^*]^2 + \frac{\beta}{2} \pi_t^2,$$

- The Lucas supply is deterministic:

$$y_t = \bar{y} + \alpha [\pi_t - \pi_t^e], \quad \alpha > 0.$$

- Again we look at three types of solution, discretion [D], rule-based [R], and cheating [C].

## Policy under discretion

- From our previous discussion we see that under discretion we would have:

$$\pi_{D,t} = (\alpha/\beta) [y^* - \bar{y}]$$

- So that:

$$V^D \equiv \left( \frac{1+r}{r} \right) \Omega_D$$

$$\Omega_D \equiv \frac{1}{2} \left( \frac{\alpha^2 + \beta}{\beta} \right) [\bar{y} - y^*]^2$$

## Policy under a constant-inflation rule

- The policy maker follows the rule  $\pi_t = \pi_R$  [a constant]. The REH implies

$$E\pi_t = \pi_R$$

- From our earlier discussion we find that:

$$\Omega_R = \frac{1}{2} [\bar{y} - y^*]^2$$

can be generalized [for a non-zero  $\pi_R$ ] to:

$$\Omega_R(\pi_R) = \Omega_R + \frac{\beta}{2} \pi_R^2$$

- The social welfare function under the rule-based solution is:

$$V^R(\pi_R) \equiv \left( \frac{1+r}{r} \right) \left[ \Omega_R + \frac{\beta}{2} \pi_R^2 \right]$$

## Cheating solution

- If the policy maker manages to make the agent expect that the rule will be followed [ $\pi^e = \pi_R$ ] then he has the incentive to cheat by exploiting the Lucas supply curve associated with  $\pi^e = \pi_R$ . The result is:

$$\pi_C = \frac{\alpha^2 \pi_R + \alpha [y^* - \bar{y}]}{\alpha^2 + \beta}$$

$$y_C = \left( \frac{\beta}{\alpha^2 + \beta} \right) \bar{y} + \left( \frac{\alpha^2}{\alpha^2 + \beta} \right) y^* - \left( \frac{\alpha \beta}{\alpha^2 + \beta} \right) \pi_R$$

so that the objective function under cheating is:

$$\Omega_C(\pi_R) = \frac{1}{2} \left[ \left( \frac{\beta}{\alpha^2 + \beta} \right) [\bar{y} - y^*] - \left( \frac{\alpha \beta}{\alpha^2 + \beta} \right) \pi_R \right]^2 + \frac{\beta}{2} \left[ \left( \frac{\alpha^2}{\alpha^2 + \beta} \right) \pi_R + \left( \frac{\alpha}{\alpha^2 + \beta} \right) [y^* - \bar{y}] \right]^2,$$

## Reputation

- We now introduce the following **reputation mechanism** [“tit-for-tat”]

$$\pi_t^e = \begin{cases} \pi_R & \text{if } \pi_{t-1} = \pi_{t-1}^e \\ \pi_{D,t} & \text{if } \pi_{t-1} \neq \pi_{t-1}^e \end{cases}$$

- if the policy maker did in the last period what the public expected him to do ( $\pi_{t-1} = \pi_{t-1}^e$ ) then this policy maker has credibility and the public expects that the rule inflation rate ( $\pi_R$ ) will be produced in the present period
- if the policy maker did not do in the last period what the public expected him to do ( $\pi_{t-1} \neq \pi_{t-1}^e$ ) then this policy maker has no credibility and the public expects that the discretionary inflation rate ( $\pi_{D,t}$ ) will be produced in the present period.
- the public adopt the “tit-for-tat” strategy in the repeated prisoner’s dilemma game that it plays with the policy maker. If the policy maker “misbehaves” it gets punished by the public for one period.

- Consider a policy maker in period 0 which kept its promise and produced the rule inflation in the period before [i.e. in period -1 it set  $\pi_{-1} = \pi_R$ ]. This policy maker has credibility in period 0 and the public expects  $\pi_0^e = \pi_R$ . The policy maker can do two things in period 0:
  - keep its promise and maintain its reputation [produce  $\pi_0 = \pi_R$ ]. No punishment takes place!
  - cheat in period 0 by producing  $\pi_C$  in that period [*temptation* is present because  $\Omega_R(\pi_R) > \Omega_C(\pi_R)$ ]. But because he broke his promise, the public punishes the policy maker and expect the discretionary solution next period [ $\pi_1^e = \pi_D$ ]. This involves *punishment* because  $\Omega_D > \Omega_R(\pi_R)$  in period 1. In period 1 the public expects  $\pi_1^e = \pi_D$  and, given this expectation, it is optimal for the policy maker to produce  $\pi_D$ . So policy maker has reputation again in period 2 [as it kept its promise in period 1] and the public expects  $\pi_2^e = \pi_R$ .

- The benefits of cheating [*temptation*] are:

$$\begin{aligned}
 T(\pi_R) &\equiv \Omega_R(\pi_R) - \Omega_C(\pi_R) \\
 &= \frac{1}{2} [\bar{y} - y^*]^2 + \frac{\beta}{2} \pi_R^2 - \frac{1}{2} \left[ \left( \frac{\beta}{\alpha^2 + \beta} \right) [\bar{y} - y^*] - \left( \frac{\alpha\beta}{\alpha^2 + \beta} \right) \pi_R \right]^2 \\
 &\quad - \frac{\beta}{2} \left[ \left( \frac{\alpha^2}{\alpha^2 + \beta} \right) \pi_R + \left( \frac{\alpha}{\alpha^2 + \beta} \right) [y^* - \bar{y}] \right]^2
 \end{aligned}$$

- The costs of cheating [*punishment*] are:

$$\begin{aligned}
 P(\pi_R) &\equiv \frac{\Omega_D - \Omega_R(\pi_R)}{1 + r} \\
 &= \left[ \frac{1}{2} \left( \frac{\alpha^2 + \beta}{\beta} \right) [\bar{y} - y^*]^2 - \frac{1}{2} [\bar{y} - y^*]^2 - \frac{\beta}{2} \pi_R^2 \right] \left( \frac{1}{1 + r} \right) \\
 &= \left[ \frac{1}{2} \frac{\alpha^2}{\beta} [\bar{y} - y^*]^2 - \frac{\beta}{2} \pi_R^2 \right] \left( \frac{1}{1 + r} \right),
 \end{aligned}$$

- In **Figure 10.2** we plot these two curves as a function of the rule inflation rate  $\pi_R$ .
  - rule inflation rates between 0 and  $\pi_R^*$  and the ones exceeding  $\pi_D$  are such that the policy maker will always deviate from the rule. The temptation is too big.
  - rule inflation rates between  $\pi_R^*$  and  $\pi_D$  are enforceable. The punishment exceeds the temptation and it is not worthwhile to deviate from the rule.
  - Since social welfare depends negatively on inflation, the optimal enforceable inflation rate is the lowest enforceable one, i.e.  $\pi_R^*$ .
  - If the interest rate rises,  $P(\pi_R)$  rotates counter-clockwise and the optimal enforceable inflation rate rises. Punishment more heavily discounted.

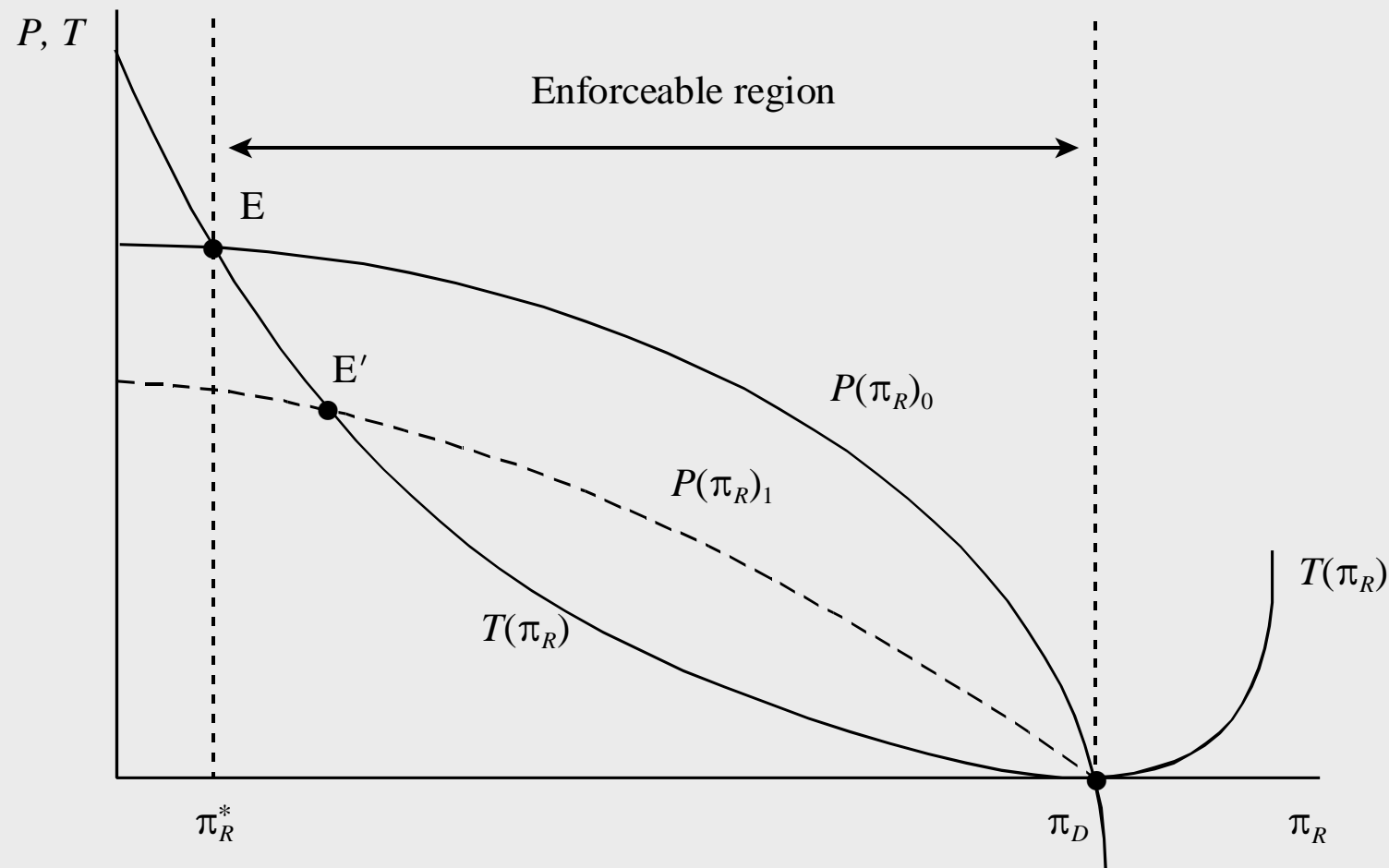


Figure 10.2: Temptation and Enforcement

## Voting and optimal inflation

- Rogoff (1985) and Alesina & Grilli (1992) ask themselves why central bankers tend to be conservative economists.
- The median voter model of A & G can be used to cast some light on this issue.  
Which agent is elected to head the central bank?

- Person  $i$  has the following cost function:

$$\Omega_i \equiv \frac{1}{2} [y - y^*]^2 + \frac{\beta_i}{2} \pi^2, \quad (*)$$

- Note that  $\beta_i$  appears in (\*). The higher is  $\beta_i$  the more “right wing” we call this person.
- The Lucas supply curve is still given by:

$$y = \bar{y} + \alpha [\pi - \pi^e] + \epsilon, \quad \alpha > 0,$$

- If person  $i$  would be the central banker then he/she would set inflation according to:

$$\pi_D^i = \left( \frac{\alpha}{\beta_i} \right) [y^* - \bar{y}] - \left( \frac{\alpha}{\alpha^2 + \beta_i} \right) \epsilon,$$

$$y_D^i = \bar{y} + \left( \frac{\beta_i}{\alpha^2 + \beta_i} \right) \epsilon.$$

- Assume that the distribution of  $\beta_i$  across the population is as in **Figure 10.3**. The person with preference parameter  $\beta_M$  is the *median voter* and effectively decides the election. [There is a single issue and preferences are single-peaked, so the median voter theorem holds]

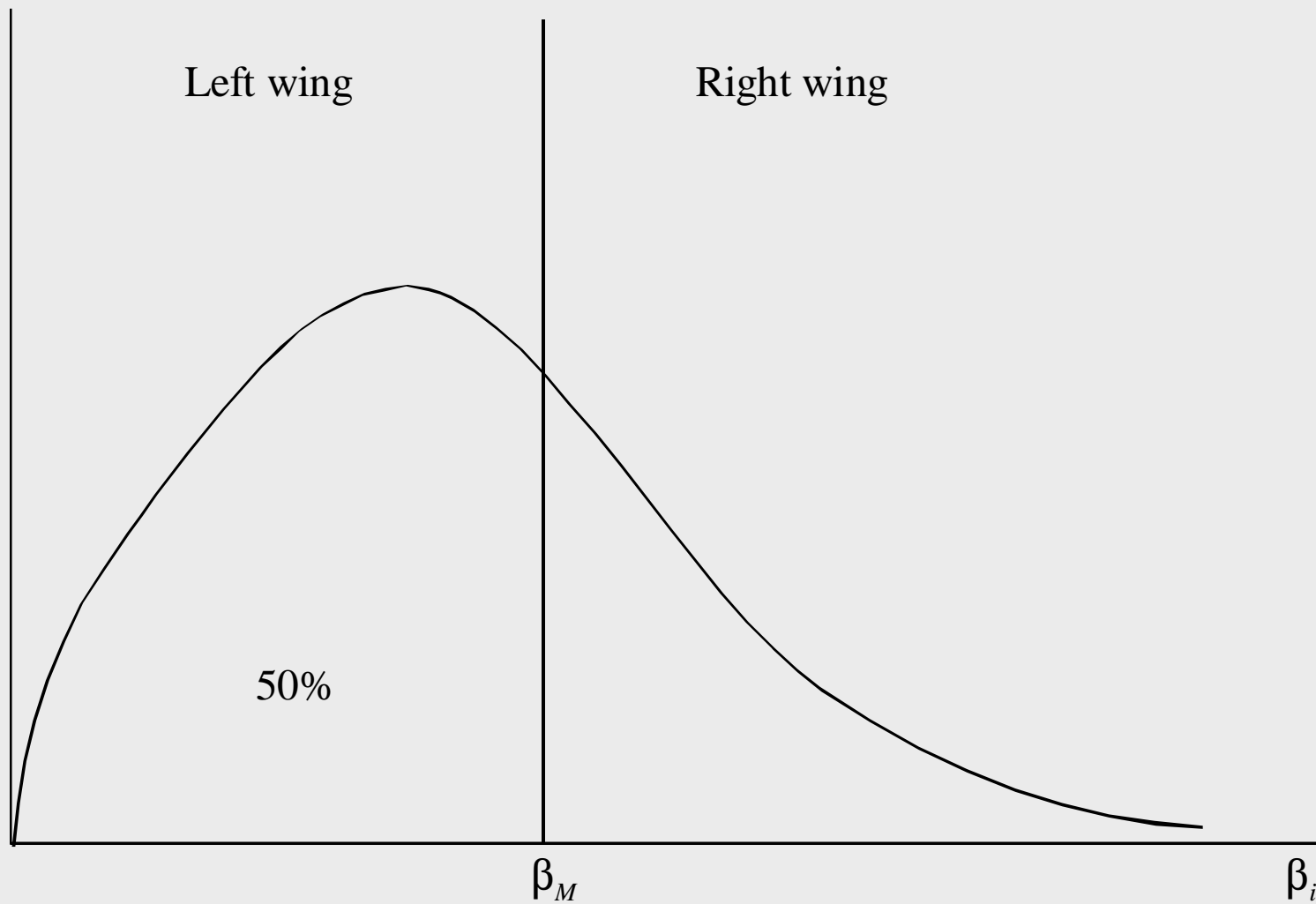


Figure 10.3: Leftwingers and Rightwingers

- The median voter's cost function is:

$$\begin{aligned} \Omega_M &\equiv \frac{1}{2} E \left[ (y_D^i - y^*)^2 + \beta_M (\pi_D^i)^2 \right] \\ &= \frac{1}{2} E \left[ \underbrace{\left( \bar{y} - y^* + \left( \frac{\beta}{\alpha^2 + \beta} \right) \epsilon \right)^2}_{(a)} + \underbrace{\beta_M}_{(b)} \underbrace{\left( \frac{\alpha}{\beta} (y^* - \bar{y}) - \left( \frac{\alpha}{\alpha^2 + \beta} \right) \epsilon \right)^2}_{(c)} \right] \\ &= \frac{1}{2} \left[ 1 + \beta_M \left( \frac{\alpha}{\beta} \right)^2 \right] (\bar{y} - y^*)^2 + \frac{1}{2} \left[ \frac{\beta^2 + \beta_M \alpha^2}{(\alpha^2 + \beta)^2} \right] \sigma^2, \end{aligned}$$

→ median voter cannot observe  $\epsilon$  but he knows how banker of type  $\beta$  reacts to  $\epsilon$

(a) output gap a central banker of type  $\beta$  would create

(b) evaluated from the point of view of the median voter

(c) inflation a central banker of type  $\beta$  would create

- The median voter elects central banker such that  $\Omega_M$  is minimized by choice of  $\beta$ .
- The first-order condition is:

$$\frac{d\Omega_M}{d\beta} = \frac{1}{2} \left[ 2\beta_M \left( \frac{\alpha}{\beta} \right) \left( \frac{-\alpha^2}{\beta} \right) \right] (\bar{y} - y^*)^2 + \frac{1}{2} \left[ \frac{2(\alpha^2 + \beta)^2\beta - 2(\beta^2 + \beta_M\alpha^2)(\alpha^2 + \beta)}{(\alpha^2 + \beta)^4} \right] \sigma^2 = 0 \Rightarrow$$

$$\frac{d\Omega_M}{d\beta} = - \left( \frac{\beta_M}{\beta} \right) \left( \frac{\alpha}{\beta} \right)^2 (\bar{y} - y^*)^2 + \left[ \frac{(\beta - \beta_M)\alpha^2}{(\alpha^2 + \beta)^3} \right] \sigma^2 = 0.$$

- It follows that the optimal  $\beta$  exceeds  $\beta_M$ . The median voter delegates the conduct of monetary policy to someone more conservative than he is himself. This way the median voter commits to a lower inflation rate.

## Dynamic consistency and capital taxation

- Dynamic inconsistency can also play a role in fiscal policy. We give the example of capital taxation.
- Two-period model ( $t = 1, 2$ )
- Household utility:

$$U \equiv \frac{C_1^{1-1/\epsilon_1}}{1-1/\epsilon_1} + \left( \frac{1}{1+\rho} \right) \left[ C_2 + \alpha \left( \frac{(1-N_2)^{1-1/\epsilon_2}}{1-1/\epsilon_2} \right) + \beta \left( \frac{G_2^{1-1/\epsilon_3}}{1-1/\epsilon_3} \right) \right]$$

- Technology:

$$F(N_t, K_t) = aN_t + bK_t$$

- production factors perfect substitutes
- inessential production factors
- constant marginal products

- Resource constraints:

$$C_1 + [K_2 - K_1] = bK_1$$

$$C_2 + G_2 = F(N_2, K_2) + K_2 = aN_2 + (1 + b)K_2$$

Note that these are expressions like “ $Y = C + I + G$ ”

## First-best command optimum

- A benevolent social planner would choose  $C_1$ ,  $C_2$ ,  $N_2$ , and  $G_2$  such that household utility is maximized subject to the consolidated resource constraint:

$$C_1 + \frac{C_2 + G_2 - aN_2}{1 + b} = (1 + b)K_1$$

- The solutions are:

$$C_1 = \left( \frac{1 + b}{1 + \rho} \right)^{-\epsilon_1} .$$

$$1 - N_2 = (a/\alpha)^{-\epsilon_2} ,$$

$$G_2 = \beta^{-\epsilon_3} .$$

- The FBCO can be decentralized [i.e. reproduced in a free market setting] provided the policy maker has access to lump-sum taxes.

## Second-best problem

- What happens if lump-sum tax is not available and only distorting taxes can be used to obtain revenue [needed to pay for the public good]?

- The GBC becomes:

$$G_2 = t_K b K_2 + t_L a N_2$$

- The market solution becomes:

$$C_1 = \left( \frac{1 + b(1 - t_K)}{1 + \rho} \right)^{-\epsilon_1},$$

$$C_2 = a(1 - t_L) + (1 + b) [1 + b(1 - t_K)] K_1 \\ - (1 + \rho)^{\epsilon_1} [1 + b(1 - t_K)]^{1-\epsilon_1} - \alpha^{\epsilon_2} [a(1 - t_L)]^{1-\epsilon_2},$$

$$1 - N_2 = \left( \frac{a(1 - t_L)}{\alpha} \right)^{-\epsilon_2}.$$

Non-zero  $t_K$  and/or  $t_L$  drive the market solution away from the FBCO. We cannot set  $t_L = t_K = 0$  because that would imply zero  $G$  [which is not optimal]. What do we do?

- ... We trade off the distortions in the tax system as well as we can by choosing  $G$ ,  $t_L$ , and  $t_K$  such that welfare of the household is maximized *given the absence of lump-sum taxes!*
- The optimality conditions are the GBC plus:

$$\beta G_2^{-1/\epsilon_3} = \eta, \tag{a}$$

$$\eta = \frac{1}{1 - \left(\frac{t_L}{1-t_L}\right) \epsilon_L}, \tag{b}$$

$$\eta = \frac{1}{1 - \left(\frac{t_K}{1-t_K}\right) \epsilon_K}, \tag{c}$$

- $\eta$  is the marginal cost of public funds [MCPF]
  - $\epsilon_L$  is the uncompensated wage elasticity of labour supply
  - $\epsilon_K$  is the uncompensated interest elasticity of gross saving
  - Equation (a) is the “modified Samuelson rule”
- Equations (b) and (c) can be solved for the optimal tax rates:

$$\frac{t_L}{1 - t_L} = \left(1 - \frac{1}{\eta}\right) \frac{1}{\epsilon_L}, \quad (*)$$

$$\frac{t_K}{1 - t_K} = \left(1 - \frac{1}{\eta}\right) \frac{1}{\epsilon_K}, \quad (\#)$$

The intuition is as follows: the objective is to tax in the least distorting fashion by taxing most heavily the most inelastic tax base (e.g. if  $\epsilon_L = 0$  then  $1/\epsilon_L \rightarrow \infty$ ,  $\eta = 1$ , and  $t_K = 0$ . Labour income source of inelastic tax base in this special case)

- BUT!!! In the general case, with both taxes non-zero, taxing labour in period 2 is not efficient. Once period 2 comes along,  $K_2$  is inelastic and  $t_L = 0$  and  $t_K > 0$  is optimal. Hence, solutions in (\*) and (#) are dynamically inconsistent.
- To find the consistent solution we would have to work backwards. we know that  $t_L = 0$  and  $t_K > 0$  in period 2. Then we can figure out what  $t_L$  and  $t_K$  should be in the first period.

## Punchlines

- Dynamic inconsistency is all around us
- In the context of monetary policy a reputational mechanism can make a rule-based inflation rate enforceable.
- The median voter can commit to a lower inflation rate by electing a central banker who is more conservative than himself.
- The optimal taxes on labour and capital suffer from the dynamic inconsistency problem.