Foundations of Modern Macroeconomics

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Chapter 9: Search in the Labour Market

Aims of this lecture

- How can we explain unemployment *duration*?
- What policies can be used to reduce equilibrium unemployment?
- Can the search model explain the persistence in the unemployment rate?

A simple model of search behaviour

- Matching function:

$$XN = G(\underset{+}{UN}, \underset{+}{VN}),$$

  - X is the matching rate
  - N is the number of workers
  - U is the unemployment rate
  - V is the vacancy rate
  - $G(.,.)$ features CRTS (i.e. $G(UN, VN) = NG(U, V) = NVG(U/V, 1)$.
    Example: Cobb-Douglas matching function: $XN = (UN)^{\alpha}(VN)^{1-\alpha}$
  - Further properties: $G_U, G_V > 0$; $G_{UU}, G_{VV} < 0$; $G_{UU}G_{VV} - G_{UV}^2 > 0$

- Instantaneous probability of a vacancy being filled:

$$q \equiv \frac{\text{number of matches}}{\text{number of vacancies}} = \frac{G(UN, VN)}{VN} = \frac{VNG(UN/VN, 1)}{VN} = G(U/V, 1) \equiv q(\underset{-}{\theta}),$$

where θ is the indicator for labour market pressure:

$$\theta \equiv \frac{V}{U}$$

  – if θ is high then there are relatively many vacancies so firms with a vacancy find it hard to get a match with an unemployed job seeker (q is low)

  – if θ is low then there are relatively few vacancies so firms with a vacancy find it easy to get a match with an unemployed job seeker (q is high)

– For later use: the elasticity of the $q(\theta)$ function:

$$\eta(\theta) \equiv -\frac{\theta}{q}\frac{dq}{d\theta} = \frac{G_U}{\theta q} \Rightarrow \quad 0 < \eta(\theta) < 1,$$

- Instantaneous probability of an unemployed job seeker finding a job:

$$\begin{aligned} f &\equiv \frac{\text{number of matches}}{\text{number of unemployed}} \\ &= \frac{G(UN, VN)}{UN} \\ &= \frac{VNG(UN/VN, 1)}{UN} = \theta q(\theta) \equiv f(\underset{+}{\theta}), \end{aligned}$$

– if θ is high then there are relatively few unemployed workers so unemployed job seekers find it easy to locate a firm with a vacancy (f is high)

– if θ is low then there are relatively many unemployed workers so unemployed job seekers find it hard to locate a firm with a vacancy (f is low)

– For later use: the elasticity of the $f(\theta)$ function:

$$\frac{\theta}{f}\frac{df}{d\theta} = \left[q(\theta) + \theta\frac{dq}{d\theta}\right]\frac{\theta}{\theta q(\theta)} = 1 + \frac{\theta}{q}\frac{dq}{d\theta} = 1 - \eta(\theta) > 0$$

- Note the intimate link between the probabilities facing the two searching parties, i.e. firms with a vacancy and unemployed job seekers. [Two sides of the same coin]

- We now already have some duration definitions:

– Expected duration of a job vacancy:

$$\frac{1}{q(\theta)}$$

– Expected duration of unemployment spell:

$$\frac{1}{f(\theta)}$$

- Inflow/outflow equilibrium

$$\underbrace{s(1-U)Ndt}_{(a)} = \underbrace{\theta q(\theta)UNdt}_{(b)}, \qquad (*)$$

where s is the (exogenous) job destruction rate (due to idiosyncratic match-productivity shocks

(a) (expected) flow into unemployment

(b) (expected) flow out of unemployment

NB 1 Note: large numbers, so frequencies and probabilities coincide

NB 2 Equation (*) implies equilibrium unemployment rate:

$$U = \frac{s}{s+\theta q(\theta)} = \frac{s}{s+f(\theta)}$$

Remainder of the model solved as follows

(A) Firm behaviour

(B) Worker behaviour

(C) Wage setting

(D) Market equilibrium

(A) Firm behaviour

- Analyze single-job firms (risk-neutral owner)
- Focus on intuitive "derivation"
- Firms with a vacancy have the following arbitrage equation:

$$\underbrace{rJ_V}_{(a)} = \underbrace{-\gamma_0 + q(\theta)\left[J_O - J_V\right]}_{(b)}$$

  - J_V is the value of a (firm with a) vacancy; r is the interest rate
  - γ_0 is the search cost of the firm with a vacancy
  - J_O is the value of (a firm with) an occupied job

(a) capital cost of the asset

(b) return on the asset: "dividend" [search costs] plus expected capital gain [finding a worker, upgrading from vacancy to a filled job]

- Assumption: free entry of firms with a vacancy:

$$\begin{aligned} J_V &= 0 \;\Rightarrow\; 0 = -\gamma_0 + q(\theta) J_O \;\Rightarrow \\ J_O &= \frac{\gamma_0}{q(\theta)} \end{aligned}$$

  Hence, the value of a filled job equals the expected cost of creating it [i.e. the cost of filling a vacancy]

- Firms with an occupied job have the following arbitrage equation:

$$\underbrace{rJ_O}_{(a)} = \underbrace{[F(K,1) - (r+\delta)K - w] - sJ_O}_{(b)} \qquad (*)$$

  – $F(K,1)$ is the output of the single-job firm (note $L=1$ substituted)

  – firm rents capital at rental rate $r+\delta$

  – firm hires labour at wage rate w [to be determined below]

(a) capital cost of the asset

(b) return on the asset, consisting of the “dividend” [profit, i.e. output left over after capital and labour have been paid] plus the expected capital gain [experiencing a shock by which the match is destroyed: downgrading from filled job to vacancy]

$\longrightarrow$ The firm hires capital such that J_O is maximized:

$$\max_{\{K\}} (r+s)J_O \equiv F(K,1) - (r+\delta)K - w \Rightarrow \quad \text{(**)}$$
$$F_K(K,1) = r+\delta$$

- Since $J_O = \gamma_0 / q(\theta)$ and $F(K,1) = F_K K + F_L$ we can combine (*) and (**):

$$\frac{(r+s)\gamma_0}{q(\theta)} = F(K,1) - F_K(K,1)K - w \quad \Rightarrow$$

$$\underbrace{\frac{F_L(K,1) - w}{r+s}}_{(a)} = \underbrace{\frac{\gamma_0}{q(\theta)}}_{(b)} \qquad \text{(ZP condition)}$$

(a) the value of an occupied job, equalling the present value of rents (accruing to the firm during the job's existence) using the risk-of-job-destruction-adjusted discount rate, $r+s$, to discount future rents

(b) expected search costs

NB since firm search costs are positive ($\gamma_0 > 0$) it follows that $w < F_L$ (workers do not get their marginal product!!)

(B) Worker behaviour

- Risk-neutral / infinitely-lived worker
- Cares only for the present value of present and future income stream
- Receives wage w when employed and "unemployment benefit" z when unemployed
- Unemployed worker's arbitrage equation is:

$$\underbrace{rY_U}_{(a)} = \underbrace{z + \theta q(\theta) \left[Y_E - Y_U \right]}_{(b)} \qquad (*)$$

  - Y_U is the human wealth of the unemployed worker (who is looking for a job)
  - Y_E is the human wealth of the employed worker

(a) capital cost of the asset

(b) return on the asset: "dividend" [unemployment benefits] plus expected capital gain [finding a job and upgrading from unemployment to being employed]

- Employed worker's arbitrage equation is:

$$\underbrace{rY_E}_{(a)} = \underbrace{w - s\left[Y_E - Y_U\right]}_{(b)} \tag{**}$$

  – capital cost of the asset

  – return on the asset, consisting of the "dividend" [the wage] plus the expected capital gain [losing one's job due to a shock and downgrading from being employed to being unemployed]

- Combining (*) and (**) yields:

$$rY_U = \frac{(r+s)z + \theta q(\theta)w}{r+s+\theta q(\theta)},$$

$$rY_E = \frac{sz + \left[r + \theta q(\theta)\right]w}{r+s+\theta q(\theta)} = \frac{r(w-z)}{r+s+\theta q(\theta)} + rY_U,$$

(C) Wage setting

- Generalized wage bargaining over the wage between the firm and the worker
- Expected gain from striking a deal
  - to the firm:

$$rJ_O^i = F(K_i, 1) - (r+\delta)K_i - w_i - sJ_O^i \quad \Rightarrow$$

$$J_O^i = \frac{F_L(K_i, 1) - w_i}{r+s}$$

  - to the worker:

$$r\left(Y_E^i - Y_U\right) = w_i - s\left[Y_E^i - Y_U\right] - rY_U$$

- Bargaining is over a wage, w_i, which maximizes Ω:

$$\max_{\{w_i\}} \Omega \equiv \beta \log \left[Y_E^i - Y_U\right] + (1-\beta) \log \left[J_O^i - \underbrace{J_V}_{=0} \right],$$

where $0 < \beta < 1$ represents the (relative) bargaining power of the worker and Y_U and $J_V = 0$ are the threat points of, respectively the worker and the firm

- Maximization yields the *rent sharing rule*:

$$Y_E^i - Y_U = \left(\frac{\beta}{1-\beta} \right) \left[J_O^i - J_V \right] \qquad (*)$$

There are two ways to turn the rent sharing rule into a wage equation [details in book]:

1) after some substitutions we get:

$$w_i = (1-\beta) r Y_U + \beta F_L(K_i, 1)$$

- worker gets a weighted average of the reservation wage (rY_U) and the marginal product of labour (F_L)

2) in symmetric situation we have $K_i = K$ and $w_i = w$ for all firm/worker pairs:

$$w = (1-\beta) z + \beta \left[F_L(K, 1) + \theta \gamma_0 \right] \qquad \text{(WS curve)}$$

- worker gets a weighted average of the unemployment benefit (z) and the match surplus ($F_L + \gamma_0 \theta$)
- The match surplus consists of the marginal product of labour plus the expected search costs that are saved if the deal is struck [$\theta \equiv V/U$ so that $\gamma_0 \theta \equiv \gamma_0 V/U$ represents the average hiring costs per unemployed worker]

(D) Market equilibrium

- Summary of the model

$$F_K(K,1) = r + \delta \quad \text{(a)}$$

$$\frac{\gamma_0}{q(\theta)} = \frac{F_L\left[K(r+\delta),1\right] - w}{r+s} \quad \text{(b)}$$

$$w = (1-\beta)z + \beta\left[F_L\left[K(r+\delta),1\right] + \theta\gamma_0\right] \quad \text{(c)}$$

$$U = \frac{s}{s+\theta q(\theta)} \quad \text{(d)}$$

- Endogenous: K, w, θ, and U; Exogenous: r, s, γ_0, and δ
- Model is recursive and can thus be solved sequentially:
  - (a) yields K^* as a function of $r+\delta$ [$K^* = F_K^{-1}(r+\delta)$]
  - (b)-(c) with $K = K^*$ inserted only depend on (and determine) w^* and θ^*
  - (d) once θ^* is known equation (d) determines U^*

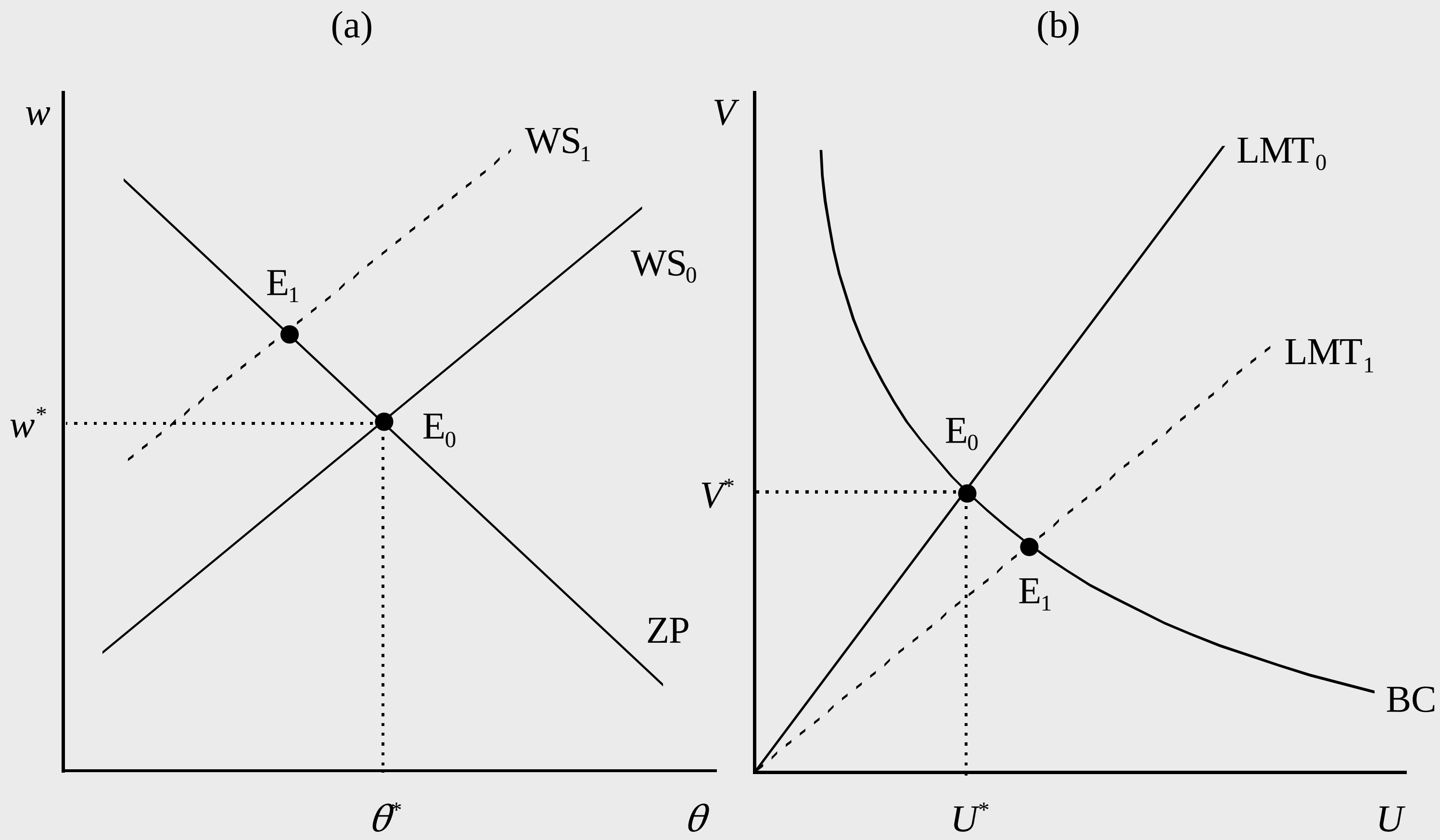

Figure 9.1: Search Equilibrium in the Labour Market

Graphical analysis

- The model can be represented graphically in **Figure 9.1**
- ZP curve: [equation (b)] supply of vacancies under free entry/exit of firms
  - slopes downwards in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{ZP} = \frac{(r+s)\gamma_0}{q(\theta)^2} q'(\theta) < 0.$$

    Intuition: $w \downarrow$ increases the value of an occupied job [raises the right-hand side of (b)]. To restore the zero-profit equilibrium the expected search cost for firms (the left-hand side of (b) must also increase, i.e. $q(\theta) \downarrow$ and $\theta \uparrow$.

  - shifts up as $\gamma_0 \downarrow$ or as $s \downarrow$

- WS curve: [equation (c)] wage setting curve
  - upward sloping in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{WS} = \beta\gamma_0 > 0$$

    Intuition: the worker receives part of the search costs that are foregone when he strikes a deal with a firm with a vacancy.

  - shifts up as $z \uparrow$ or $\gamma_0 \uparrow$

- In panel (a) the intersection of ZP and WS yields the equilibrium (w^*, θ^*) combination. This is the ray from the origin in panel (b).

- The Beveridge curve (BC) is given by equation (d). It can be linearized in (V, U) space as follows:

$$\tilde{V} = \left(\frac{1}{1-\eta}\right)\tilde{s} - \left(\frac{s+f\eta}{f(1-\eta)}\right)\tilde{U},$$

  where $\tilde{U} \equiv dU/U$, $\tilde{V} \equiv dV/V$, and $\tilde{s} \equiv ds/s$.

  - BC slopes down: for a given unemployment rate, $V \downarrow$ leads to a fall in the instantaneous probability of finding a job ($f \downarrow$), i.e. for points below the BC curve the unemployment rate is less than the rate required for flow equilibrium in the labour market ($U < s/(s+f)$). To restore flow equilibrium the $U \uparrow$
  - shifts to the right as $s \uparrow$

Shock 1: Increase in the unemployment benefit

- Suppose that $z \uparrow$
- In Figure 9.1 this shock is illustrated.
  - WS curve to the left
  - equilibrium from E_0 to E_1
  - $w^* \uparrow$ and $\theta^* \downarrow$
  - in panel (b) the LMT ratio rotates clockwise
  - $V \downarrow$ and $U \uparrow$

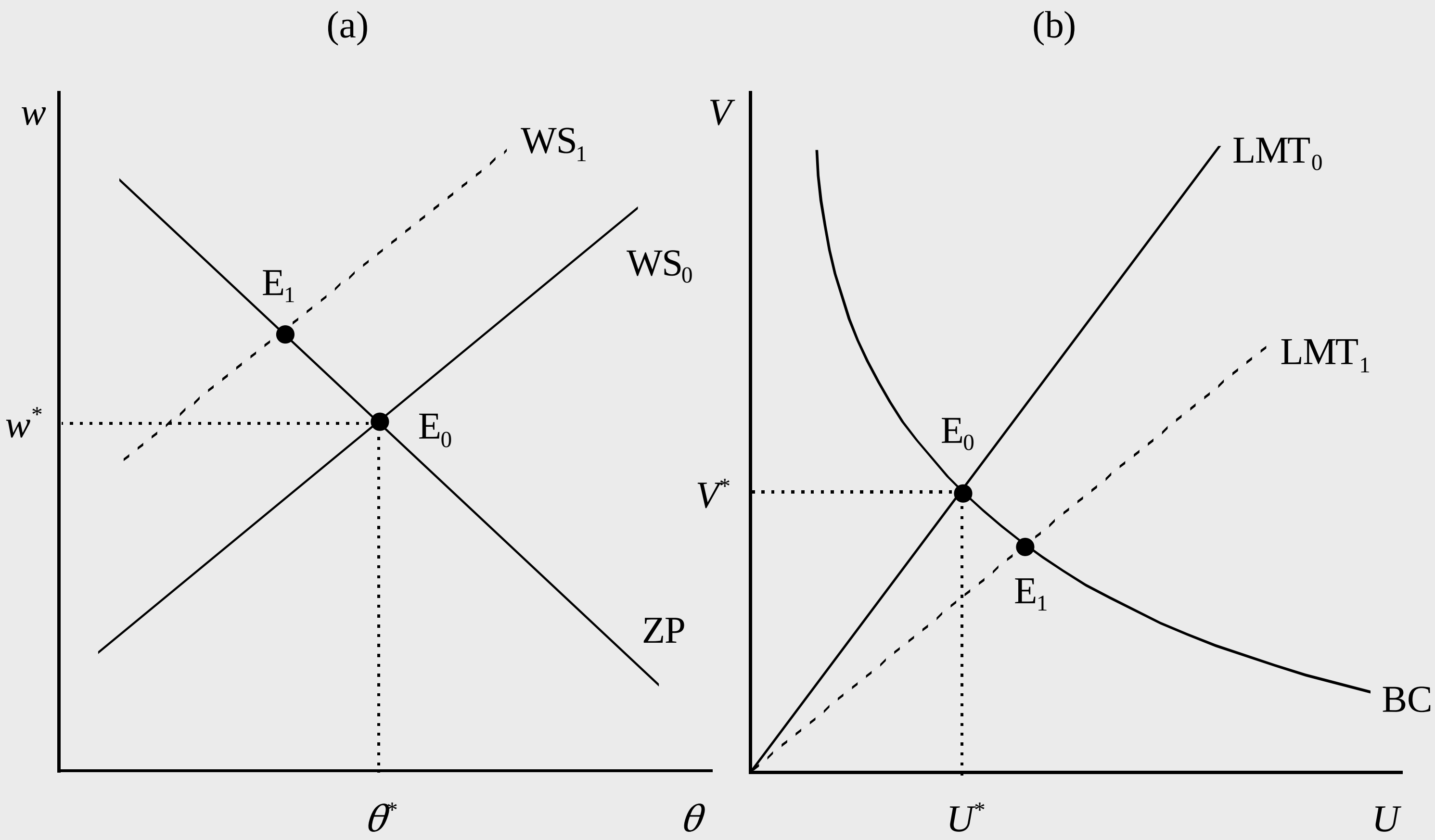

Figure 9.1: Search Equilibrium in the Labour Market

Shock 2: Increase in the job destruction rate

- Suppose that $s \uparrow$
- ZP curve down in panel (a) of **Figure 9.2**
- equilibrium from E_0 to E_1
- $w^* \downarrow$ and $\theta^* \downarrow$
- in panel (b) the LMT ratio rotates clockwise **and** BC shifts outwards [dominant effect]
- $V \uparrow$ and $U \uparrow$

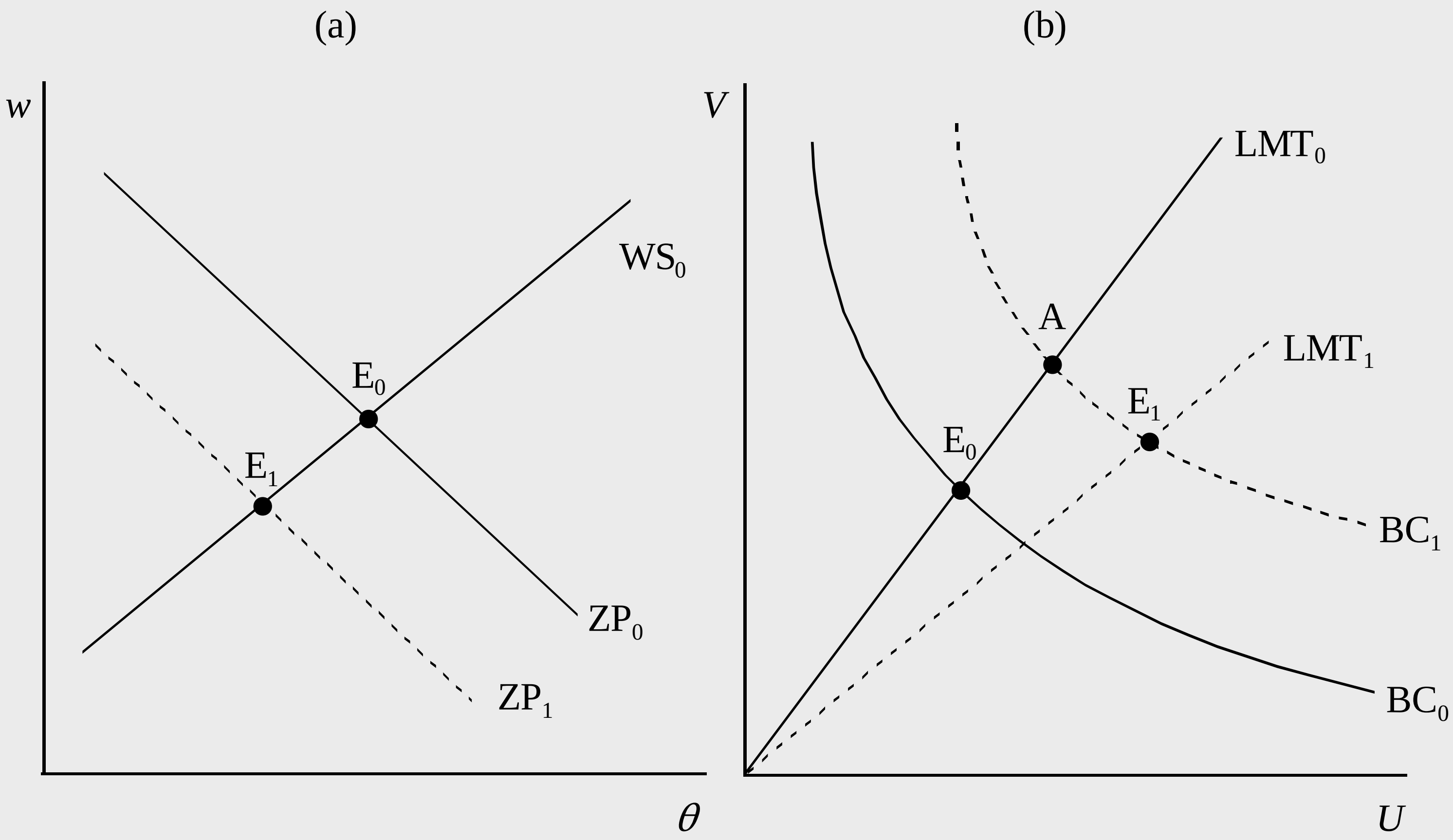

Figure 9.2: Higher Job Destruction Rate

Further policy shocks in the search model: Labour taxes

- The effects of labour taxes; t_E levied on firms t_L levied on households
- The model becomes:

$$
\begin{aligned}
\frac{\gamma_0}{q(\theta)} &= \frac{F_L\left[K(r+\delta),1\right] - w(1+t_E)}{r+s} \\
w &= (1-\beta)\left(\frac{z}{1-t_L}\right) + \beta\left(\frac{F_L\left[K(r+\delta),1\right] + \theta\gamma_0}{1+t_E}\right) \\
U &= \frac{s}{s+\theta q(\theta)}
\end{aligned}
$$

- In **Figure 9.3** the effects of the payroll tax increase are analyzed ($t_E \uparrow$)
  - WS curve to the right
  - ZP curve to the left
  - equilibrium from E_0 to E_1 and $w^* \downarrow$ and $\theta^* \downarrow$

– in panel (b) the LMT ratio rotates clockwise

– $V \downarrow$ and $U \uparrow$

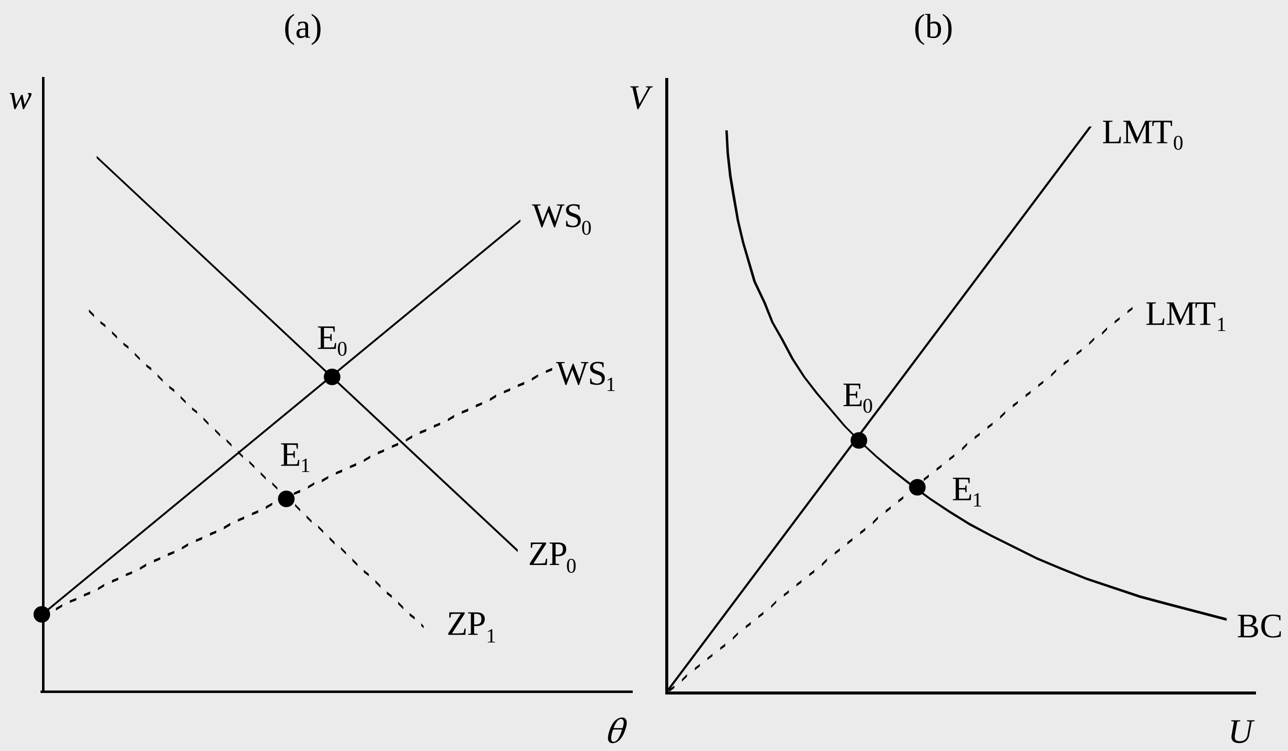

Figure 9.3: Higher Payroll Tax

- In **Figure 9.4** the effects of the labour income tax increase are analyzed ($t_L \uparrow$)
  - WS curve to the left [z untaxed!]
  - equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \downarrow$
  - in panel (b) the LMT ratio rotates clockwise
  - $V \downarrow$ and $U \uparrow$

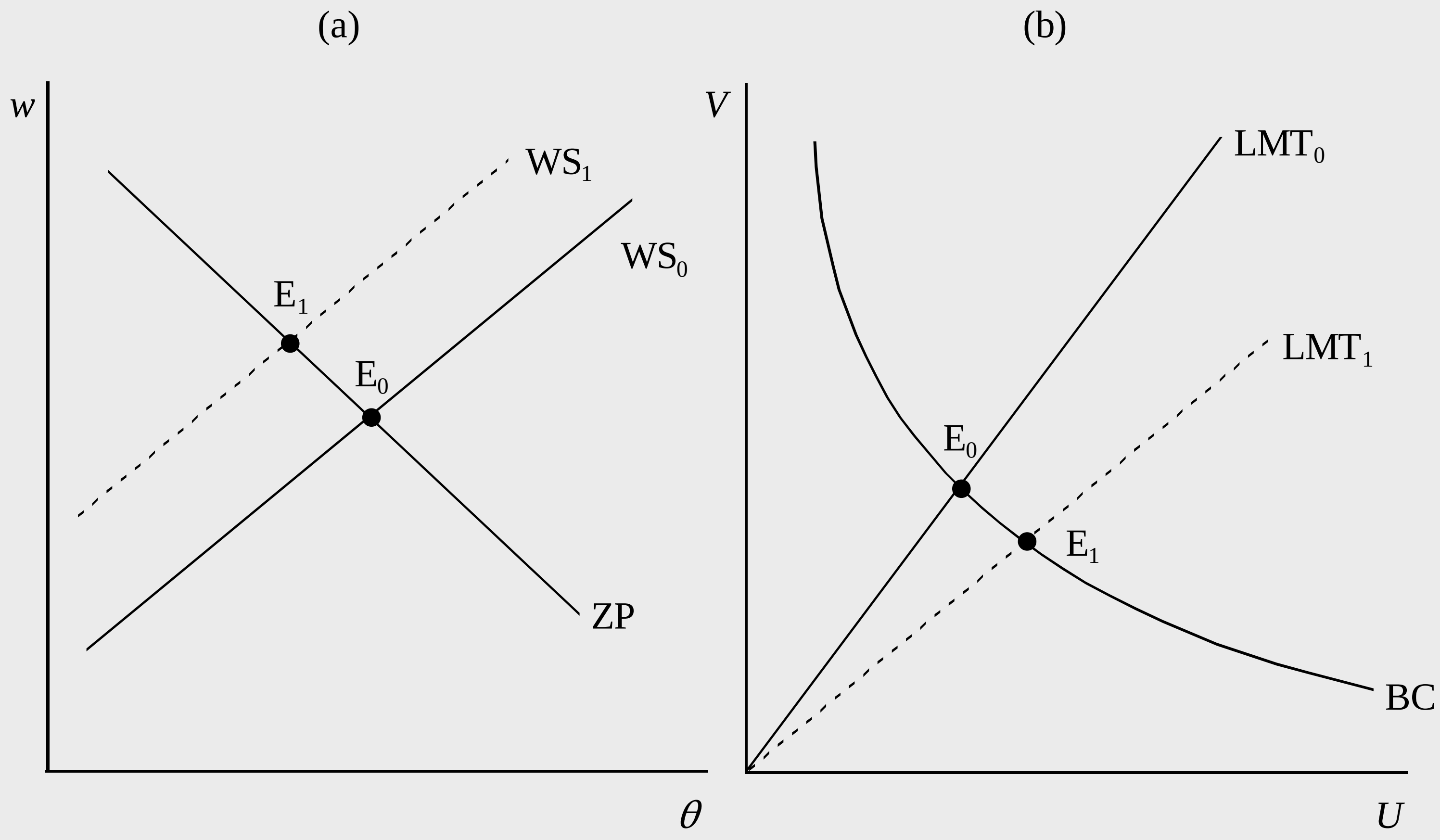

Figure 9.4: Higher Labour Income Tax

Further policy shocks in the search model: Deposits on labour

- workers as empty pop bottles
- deposit scheme: firm pays a deposit b to the government when it fires a worker, to be refunded b when it (re-) hires that (or another) worker
- Model becomes:

$$\frac{F_L(K,1) - w + rb}{r+s} = \frac{\gamma_0}{q(\theta)}$$

$$w = (1-\beta)z + \beta\left[F_L(K,1) + rb + \theta\gamma_0\right]$$

$$U = \frac{s}{s+\theta q(\theta)}$$

Hence, the capital value of the deposit (rb) acts as a subsidy on the use of labour!

- In **Figure 9.5** we show the effects of $b \uparrow$
  - ZP curve to the right
  - WS curve up
  - equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \uparrow$
  - in panel (b) the LMT ratio rotates counterclockwise
  - $V \uparrow$ and $U \downarrow$
- The system works to combat unemployment!

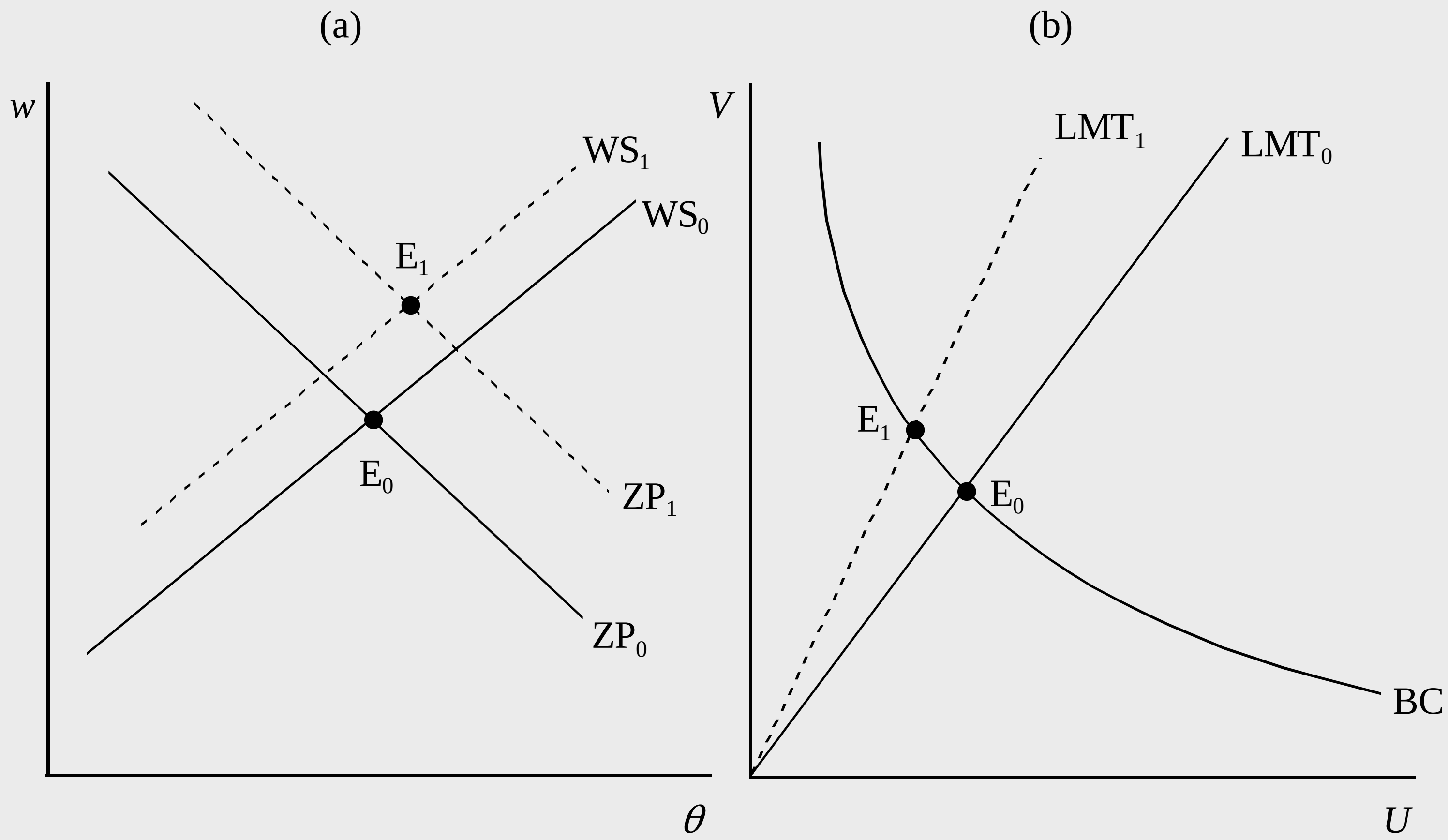

Figure 9.5: Higher Deposit on Labour

Encore: Unemployment persistence in the search model

- One of the stylized facts of the labour market: high persistence in the unemployment rate
- Pissarides argues that loss of skills during unemployment can explain this phenomenon
  - unemployed lose human capital [“skills”]
  - are thus less attractive to firms, vacancy supply falls
  - more long-term unemployment

Punchlines

- Central elements of the search model:
  - search frictions
  - matching function
  - wage negotiations
  - Beveridge curve
- Attractive model which abandons notion of the aggregate labour market
- Holds up well empirically