Foundations of Modern Macroeconomics

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Chapter 4: Anticipation Effects and Economic Policy

Aims of this lecture

- Complete our discussion of the forward looking theory of investment [commenced in Chapter 2]
- Study the effects of investment stimulation by the government
- Study a dynamic IS-LM theory with an endogenous term structure of interest rates

Dynamic investment theory

- Redo basic model in continuous time

- Real profit:

$$\pi(t) \equiv F(N(t), K(t)) - w(t)N(t) - p^I(t)\left[1 - s_I(t)\right]\Phi(I(t)),$$

  - F is CRTS

  - Φ is the adjustment cost function

- Quadratic adjustment cost function:

$$\Phi(I(t)) = I(t) + b\,[I(t)]^2$$

  - $b > 0$ so that:
  - $\Phi(0) = 0$, $\Phi_I = 1 + 2bI > 0$, and $\Phi_{II} = 2b > 0$

- Capital accumulation:

$$\dot{K}(t) = I(t) - \delta K(t)$$

  - $\delta > 0$ is the depreciation rate

- Value of the firm:

$$\begin{aligned} V(0) &\equiv \int_0^\infty \pi(t) e^{-rt} dt \\ &= \int_0^\infty \Big[F(N(t), K(t)) - w(t)N(t) - [1 - s_I(t)]\,\Phi(I(t))\Big] e^{-rt} dt \end{aligned}$$

- Firm must choose paths for labour demand, investment, and the capital stock such that the value of the firm is maximized, given the constraints imposed by (a) the capital accumulation identity and (b) the initial capital stock ($K(0)$)

How to solve this problem

- Set up so-called *Hamiltonian* expression [similar to Lagrangian]

$$\mathcal{H}(t) \equiv e^{-rt}\Big[F(N(t), K(t)) - w(t)N(t) - [1 - s_I(t)]\,\Phi(I(t)) \\ + q(t)\,[I(t) - \delta K(t)]\Big]$$

  - $q(t)$ is a co-state variable [similar to a Lagrange multiplier]
  - $N(t)$ and $I(t)$ are control variables
  - $K(t)$ is the state variable

- First-order (necessary) condition for employment:

$$\frac{\partial \mathcal{H}(t)}{\partial N(t)} = e^{-rt}\Big[F_N(N(t), K(t)) - w(t)\Big] = 0$$

  - since $e^{-rt} > 0$ we get the usual result: $w = F_N$

- First-order (necessary) condition for investment:

$$\frac{\partial \mathcal{H}(t)}{\partial I(t)} = e^{-rt}\Big[q(t) - (1 - s_I(t))\Phi_I(I(t))\Big] = 0 \quad \Rightarrow$$

$$\underbrace{q(t)}_{(a)} = \underbrace{(1 - s_I(t))\Phi_I(I(t))}_{(b)} \qquad (*)$$

(a) shadow price of installed capital (marginal benefit of investment)

(b) net marginal cost of investing

- For the quadratic adjustment cost function, (*) becomes very simple:

$$\Phi_I(I(t)) = 1 + 2bI(t) = \frac{q(t)}{1 - s_I(t)} \quad \Rightarrow$$

$$I(t) = \frac{1}{2b}\left[\frac{q(t)}{1 - s_I(t)} - 1\right]$$

- The first-order (necessary) condition for the capital stock:

$$\frac{d\left[q(t)e^{-rt}\right]}{dt} = -\frac{\partial \mathcal{H}(t)}{\partial K(t)} \quad \Longrightarrow$$

$$e^{-rt}\left[\dot{q}(t) - rq(t)\right] = -e^{-rt}\Big[F_K(N(t), K(t)) - \delta q(t)\Big] \quad \Longrightarrow$$

$$\dot{q}(t) = (r + \delta)q(t) - F_K(N(t), K(t)) \qquad (*)$$

- An intuitive way to write (*) is in the form of an arbitrage equation:

$$\underbrace{\frac{\dot{q}(t) + F_K(N(t), K(t))}{q(t)}}_{(a)} = \underbrace{r + \delta}_{(b)}$$

(a) the return to installed capital consists of a capital gain ($\dot{q}$) plus the marginal product of capital (F_K). By dividing the return by q we obtain a *rate* of return

(b) the opportunity cost of invested funds consists of the rate of interest on other assets (r) plus the rate of depreciation (δ) [capital evaporates]

The effects of investment stimulation measures

Policy question: What happens if the government subsidizes investment spending by firms?

- Summary of the model developed so far [drop time index where no confusion is possible]

$$\dot{K} = I(\underset{+}{q}, \underset{+}{s_I}) - \delta K,$$

$$\dot{q} = (r + \delta)q - F_K(\underset{+}{N}, \underset{-}{K}),$$

$$w = F_N(\underset{-}{N}, \underset{+}{K}).$$

- There are three ways to interpret the model
  - (a) At firm level: w is constant
  - (b) At the economy-wide level:
    - (i) exogenous labour supply (labour scarcity)
    - (ii) endogenous labour supply (labour supply effects)

Case (a): Capital-investment dynamics at the level of a firm

- If w is constant then so is the marginal product of labour [since $w = F_N$]
- Since F features CRTS [homogeneous of degree **one**] it follows that $F_N(N, K)$ is homogeneous of degree **zero**, i.e. we can write $F_N(N, K) = F_N(1, K/N)$.
- Hence, the labour demand equation can be written as $w = F_N(1, K/N)$
- Since w is constant so is the optimal capital-labour ratio for the firm (K/N)
- But then the marginal product of capital, F_K, is also constant [since $F_K(N, K) = F_K(1, K/N)$]
- So our model at firm level simplifies to:

$$\dot{K} = I(\underset{+}{q}, \underset{+}{s_I}) - \delta K,$$

$$\dot{q} = (r + \delta)q - F_K,$$

where F_K is a constant.

- We can derive the phase diagram of this model in **Figure 4.1**.

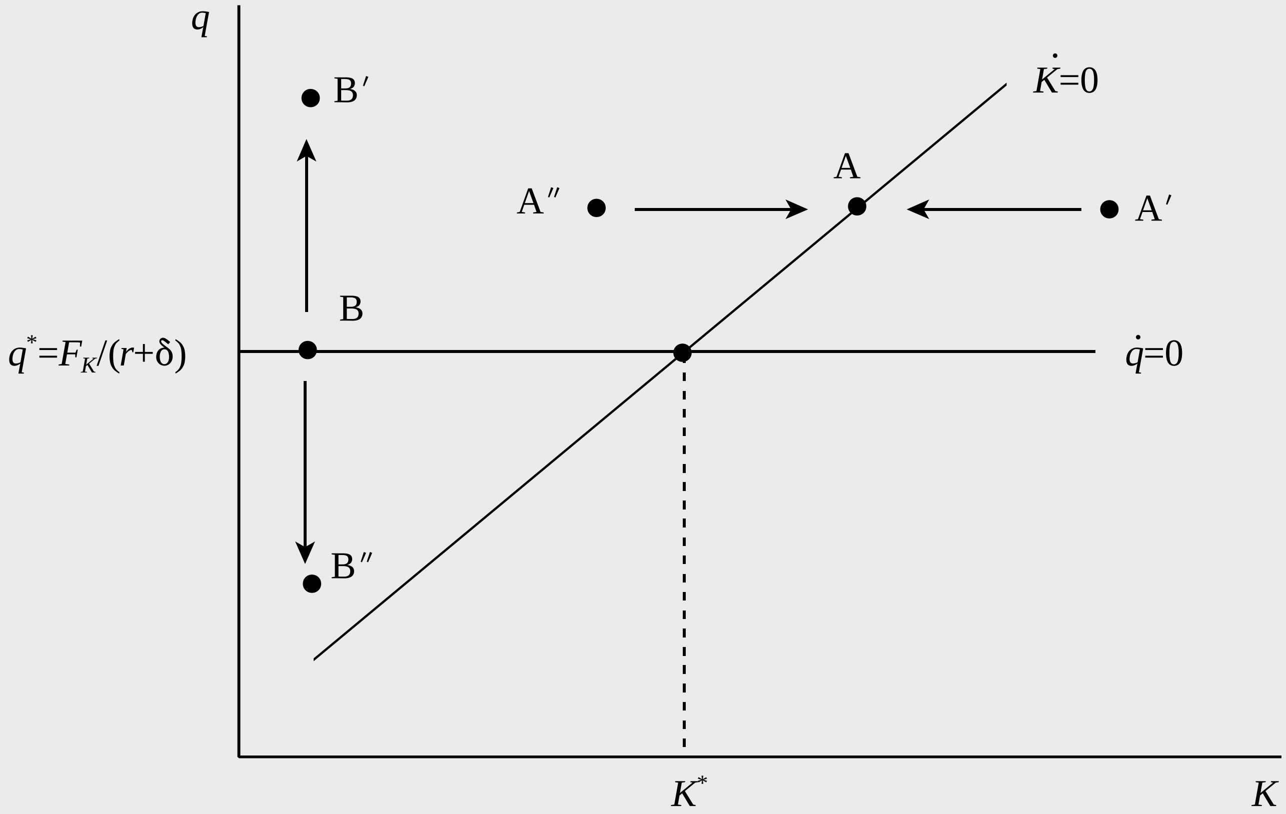

Figure 4.1: Investment With Constant Real Wages

- Start with the $\dot{K} = 0$ line: combinations of q and K for which net investment is zero ($I(q, s_I) = \delta K$).
  - slope of this line is obtained in the usual fashion:

$$\left(\frac{\partial q}{\partial K}\right)_{\dot{K}=0} = \frac{\delta}{I_q} > 0$$

$\Rightarrow$ the line is upward sloping

  - for points off the $\dot{K} = 0$ line we have:

$$\frac{\partial \dot{K}}{\partial K} = -\delta < 0$$

$\Rightarrow$ for points to the right (left) of the $\dot{K} = 0$ line gross investment is less than (more than) replacement investment and net investment is negative (positive). This is indicated with horizontal arrows in Figure 4.1

- Now look at the $\dot{q} = 0$ line: combinations of q and K for which there are no capital gains or losses ($q = F_K/(r+\delta)$).
  - slope of this line:

$$\left(\frac{\partial q}{\partial K}\right)_{\dot{q}=0} = 0$$

$\Rightarrow$ the line is horizontal

  - for points off the $\dot{q} = 0$ line we have:

$$\frac{\partial \dot{q}}{\partial q} = r + \delta > 0$$

$\Rightarrow$ for points above (below) the $\dot{q} = 0$ line the shadow price of capital is higher (lower) than its long-run equilibrium value (of $F_K/(r+\delta)$) so that part of the rate of return on installed capital is explained by capital gains (losses). Hence, $\dot{q} > 0$ (< 0) for point above (below) the $\dot{q} = 0$ line. See the vertical arrows in Figure 4.1

- By combining all the information derive so far we obtain **Figure 4.2**. Let us derive (heuristically) the properties of the model
  - there is a unique steady state where the $\dot{q} = 0$ line intersects the $\dot{K} = 0$ line (in point E$_0$)
  - by combining the "arrow" information we get the dynamic forces operating in the four regions [see the hands of the clock]
  - we can try out some arbitrary trajectories in the various regions. None of them seem to go to the equilibrium at E$_0$!
  - but that is not quite right! The $\dot{q} = 0$ line itself is a stable trajectory (leading back to E$_0$).
- We call the unique stable trajectory the **saddle path**. In this particular model the saddle path is equal to the $\dot{q} = 0$ line [in the other models this will no longer hold]

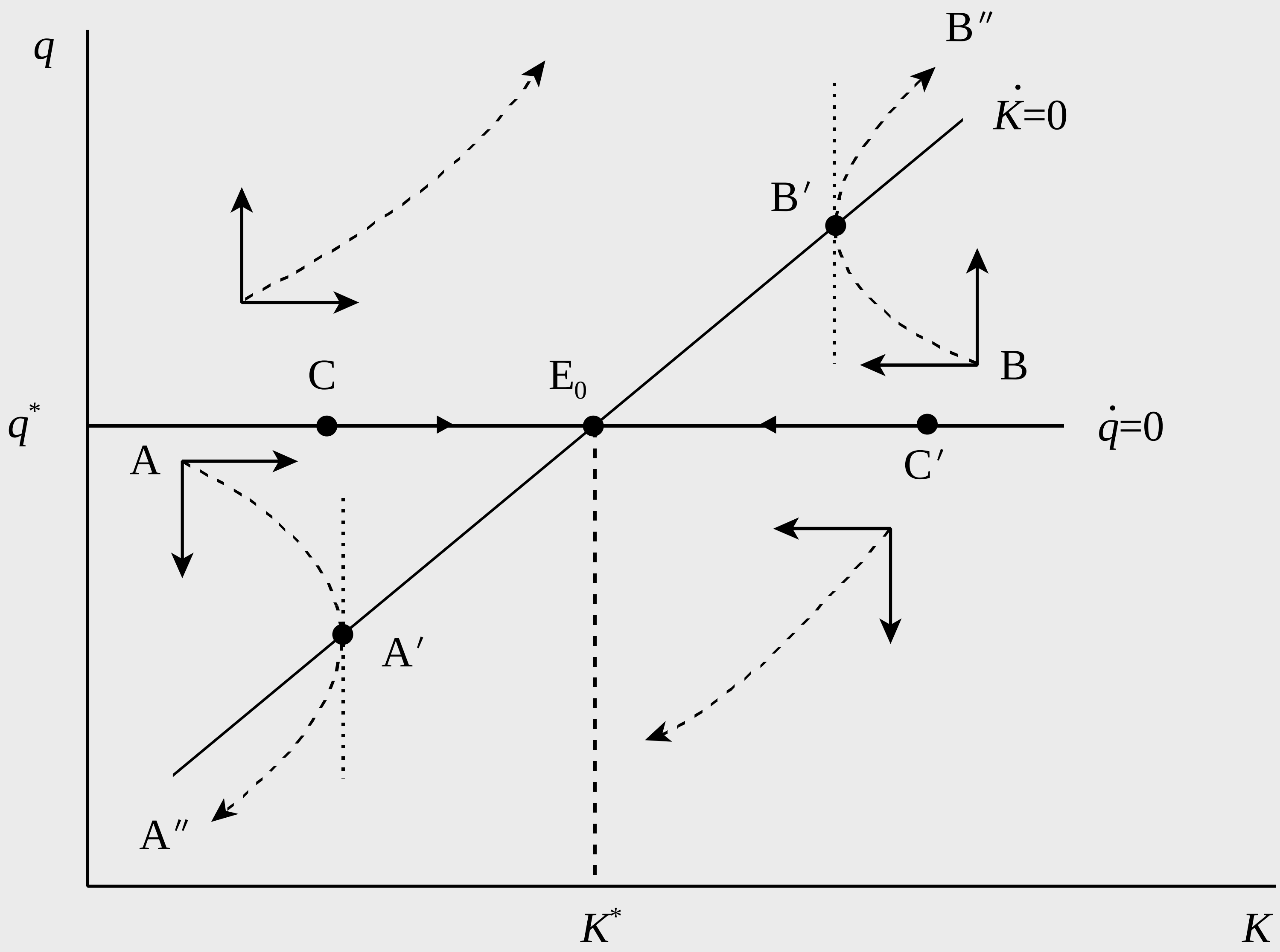

Figure 4.2: Derivation of the Saddle Path

Policy experiment 1

- We can now use the model to conduct some policy experiments. The first case we study is that of an *unanticipated* and *permanent* increase in the investment subsidy
  - "unanticipated" because announcement date (t_A) and implementation date (t_I) are the same [agents cannot prepare for the policy measure and are taken by surprise]
  - "permanent" because policy maker announces that the policy measure is permanent and the agents believe it
  - the increase in the investment subsidy lowers the cost of investing to firms and shifts the $\dot{K} = 0$ line to the right in **Figure 4.3**. In formal terms:

$$\left(\frac{\partial q}{\partial s_I}\right)_{\dot{K}=0} = -\frac{I_s}{I_q} < 0$$

  - the new long-run equilibrium is at E_1

– the adjustment occurs *gradually* along the saddle path from E_0 to E_1 [see the arrows]

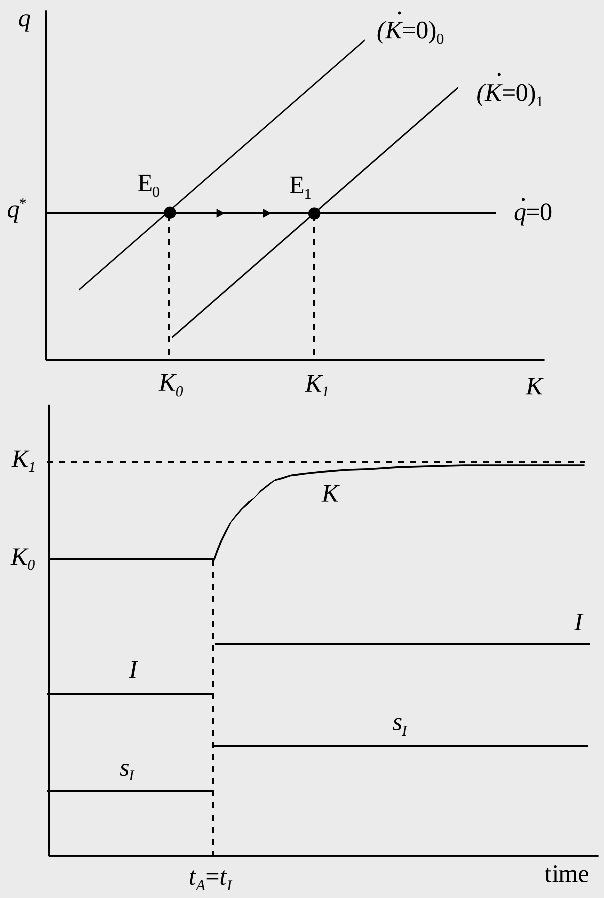

Figure 4.3: Unanticipated and Permanent Increase in the Investment Subsidy

Policy experiment 2

- The second exercise with the model concerns an *unanticipated* and *permanent* increase in the interest rate.
  - the increase in the interest rate reduces the long-run equilibrium level for q because the future marginal products of capital are discounted more heavily. Hence, the $\dot{q} = 0$ line shifts down in **Figure 4.4**
  - the new long-run equilibrium is at E_1
  - adjustment path is an immediate jump in q from E_0 to A and impact [because K is predetermined and can only move gradually]. This is a "financial correction" in the light of new information [concerning the interest rate]
  - economy must jump to the new saddle path because that is [by definition] the only trajectory leading to the new equilibrium

– during transition the economy moves gradually along the saddle path from point A to point E_1

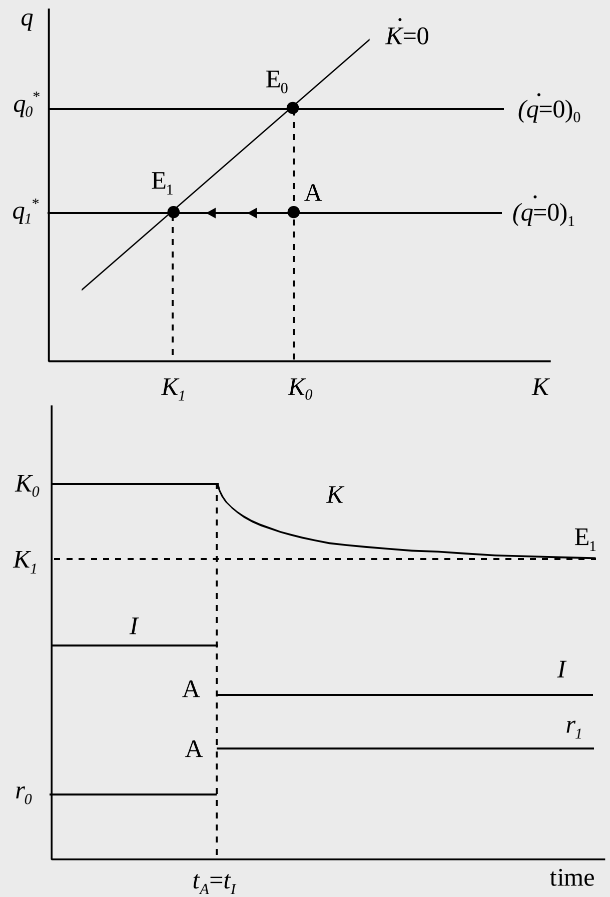

Figure 4.4: Unanticipated and Permanent Increase in the Rate of Interest

Policy experiment 3

- The third exercise with the model concerns an *anticipated* and *permanent* increase in the interest rate. Agents hear [at announcement time t_A] that the rate of interest will increase permanently at some later date [implementation date t_I]
  - "anticipated" because announcement date (t_A) and implementation date (t_I) are not the same [agents can prepare partially for the shock; the news arrives at time t_A]
  - intuitive solution principle:
    - ∗ discrete jump in q only allowed when news arrives [which is at time t_A]
    - ∗ K is predetermined at impact [accumulated in the past]
    - ∗ when shock occurs [at time t_I] the economy must be on the stable trajectory to the new equilibrium [the saddle path]

  * between t_A and t_I the economy must be on a trajectory which reaches the saddle path at exactly the right time [at t_I]. Since the shock has not occurred yet, dynamics of the old equilibrium [E_0] determine the laws of motion.

- In **Figure 4.5** we deduce the equilibrium adjustment path from E_0 to A to B to E_1
- Intuition for why q falls over time. Integrating the arbitrage equation $\dot{q} + F_K = (r + \delta)q$ from t to ∞ yields the expression for $q(t)$ at some time t

$$q(t) \equiv \int_t^\infty F_K(\tau) \exp\left[-\int_t^\tau [r(s) + \delta]\, ds\right] d\tau$$

  Hence, $q(t)$ represents the discounted present value of marginal capital productivities.

  * If something happens to the interest rate in the future $q(t)$ reacts immediately
  * As time gets closer to implementation of the shock, few years of low discounting remain so that $q(t)$ falls over time

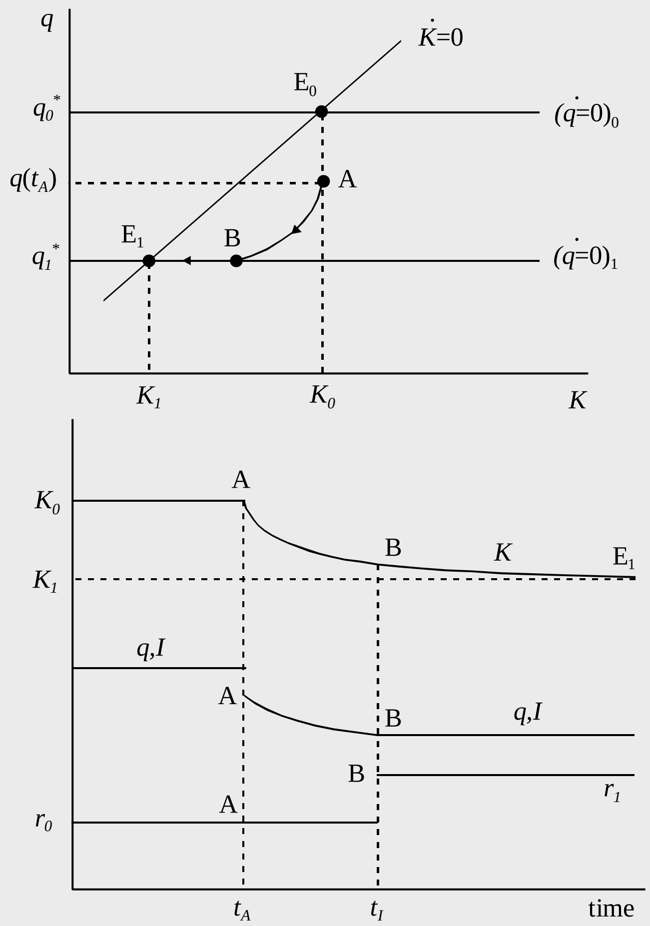

Figure 4.5: Anticipated and Permanent Increase in the Rate of Interest

Case (b)-(i): Capital-investment dynamics in the aggregate economy

- As a second case we interpret our investment model at the level of the aggregate economy. Instead of assuming a constant real wage [which is hard to justify in this case] we assume that the supply of labour is exogenous ($N = 1$)
- The model that we wish to analyze is:

$$\dot{K} = I(\underset{+}{q}, \underset{+}{s_I}) - \delta K,$$

$$\dot{q} = (r + \delta)q - F_K(1, \underset{-}{K}),$$

where we have substituted $N = 1$ in the expression for the marginal product of capital

- The key complication is that F_K is no longer constant but diminishing in K [the more capital is added the scarcer is labour]
  - As a result the $\dot{q} = 0$ line is downward sloping:

$$\left(\frac{\partial q}{\partial K}\right)_{\dot{q}=0} = \frac{F_{KK}}{r+\delta} < 0$$

  - For points above (below) the $\dot{q} = 0$ line there are capital gains (losses):

$$\frac{\partial \dot{q}}{\partial q} = r + \delta > 0$$

- Using the same tricks as before we can deduce that the saddle path is now downward sloping–see **Figure 4.6**

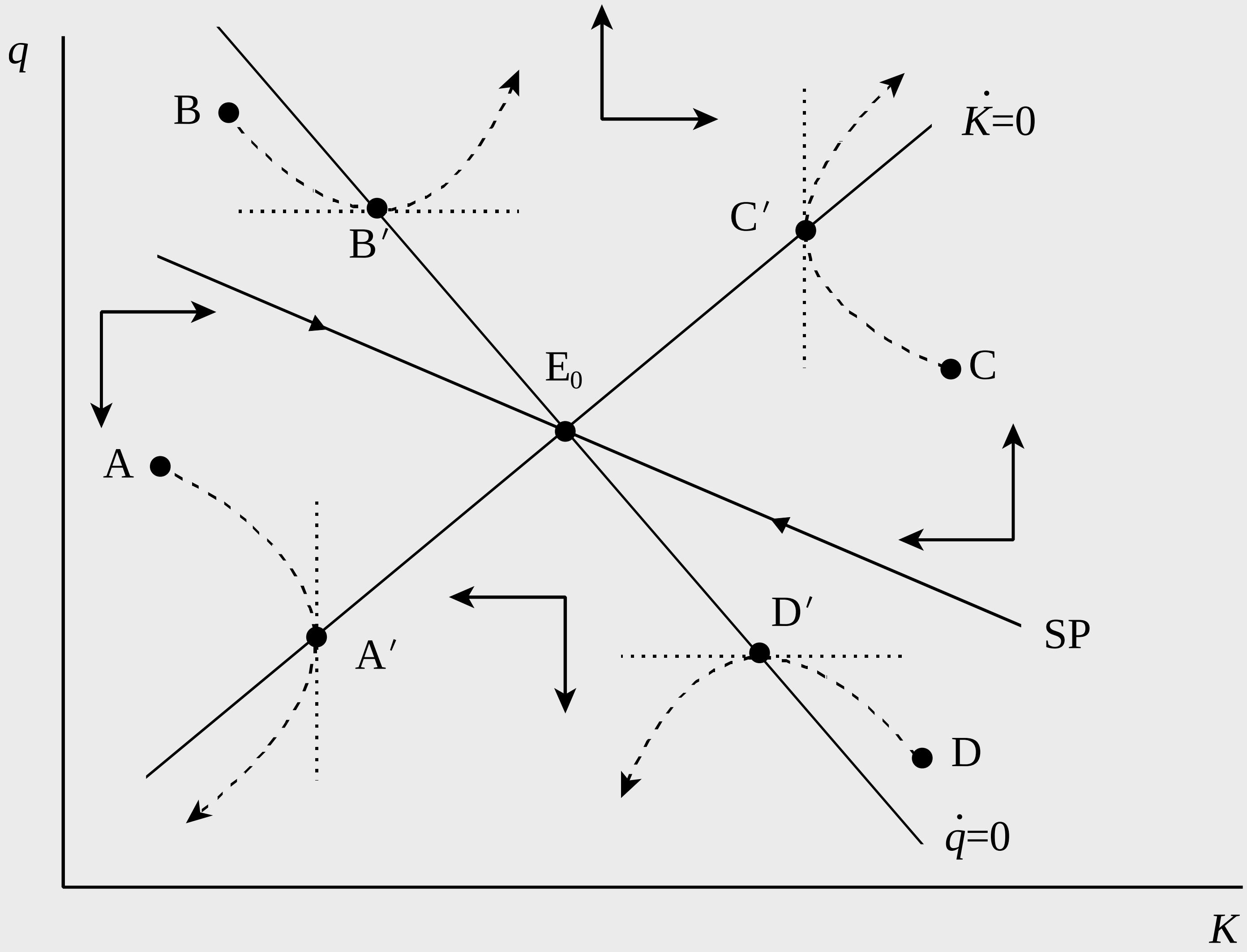

Figure 4.6: Investment With Full Employment in the Labour Market

Policy experiment 1

- First policy shock to be studied concerns an *anticipated* and *permanent* abolition of the investment subsidy [as occurred in the Netherlands in the 1980s]
  - The $\dot{K} = 0$ line shifts to the left and the long-run equilibrium shifts from E_0 to E_1 in **Figure 4.7**
  - Following our "intuitive solution method" we deduce that the adjustment path is from E_0 to A to B to E_1
  - We reach the intuitively appealing conclusion that *investment rises at impact* [enjoy the subsidy while it exists]

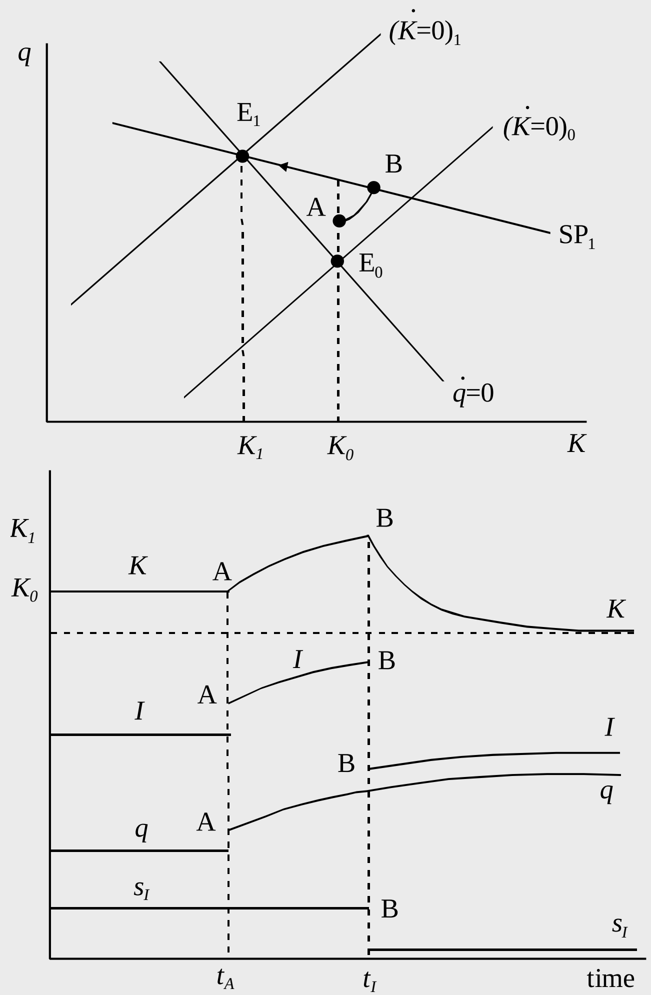

Figure 4.7: Anticipated Abolition of the Investment Subsidy

Policy experiment 2

- Second policy shock to be studied concerns an *unanticipated* and *temporary* increase of the investment subsidy [as is sometimes used to boost the economy]
  - By "temporary" we mean that the policy maker announces at time $t_A = t_I$ that the policy shock will be undone at some future date t_E
  - The $\dot{K} = 0$ line shifts to the left [if the shock were permanent, the long-run equilibrium would shift from E_0 to E_1 in **Figure 4.8**]
  - While the higher subsidy is in place [between t_A and t_E] the equilibrium E_1 dictates the laws of motion
  - Following out "intuitive solution method" we deduce that the adjustment path is from E_0 to A to B to E_1
  - Intuitive conclusion: *investment rises at impact* ["make hay while the sun shines"] (temporary shock has higher impact effect on investment than permanent shock)

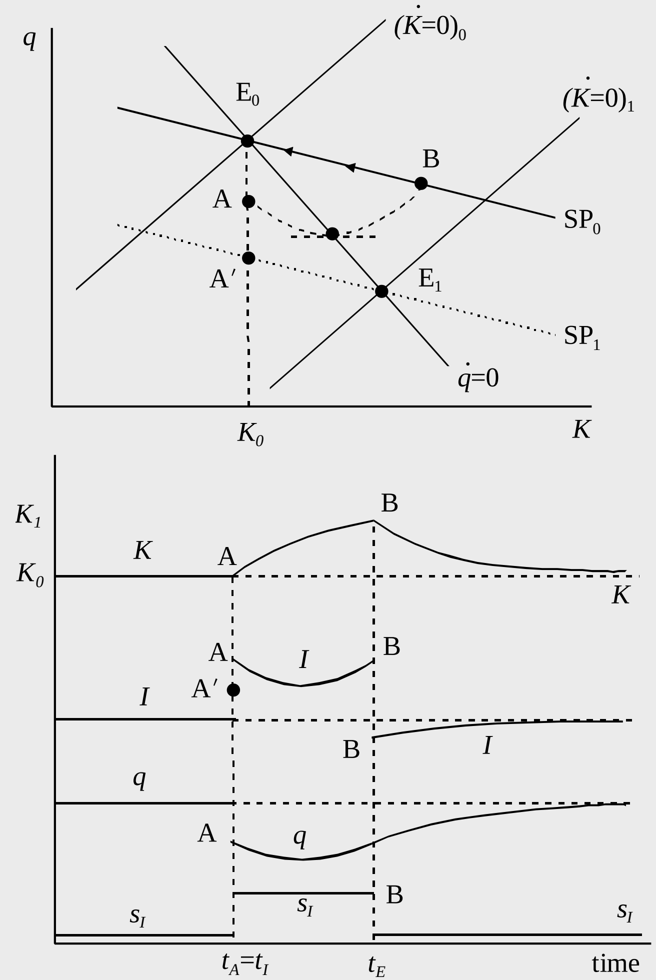

Figure 4.8: A Temporary Increase in the Investment Subsidy

Case (b)-(ii): Capital-investment dynamics in the aggregate economy

- As a third case we interpret our investment model at the level of the aggregate economy. Instead of assuming a constant real wage we assume that the supply of labour is endogenous and depends on the after-tax wage rate

- The model that we wish to analyze is:

$$\dot{K} = I(\underset{+}{q}, \underset{+}{s_I}) - \delta K,$$

$$\dot{q} = (r + \delta)q - F_K(\underset{+}{N}, \underset{-}{K}),$$

$$w = F_N(\underset{-}{N}, \underset{+}{K}),$$

$$g(\underset{+}{N}) = w(1 - t_L),$$

- In **Figures 4.9** and **4.10** we study the effects on investment and the capital stock of a decrease in the labour income tax rate, t_L
  - For a given capital stock, the decrease in the tax rate stimulates labour supply [because the substitution effect dominates the income effect by assumption] so that employment increases [see **Figure 4.10**]
  - Since capital and labour are cooperative factors of production the marginal product of capital rises and the $\dot{q} = 0$ line shifts to the right in **Figure 4.9**
  - The adjustment path is from E_0 to A to E_1 in both figures
  - Hence, measures which impact directly on the labour market also have an induced effect on investment and capital accumulation!

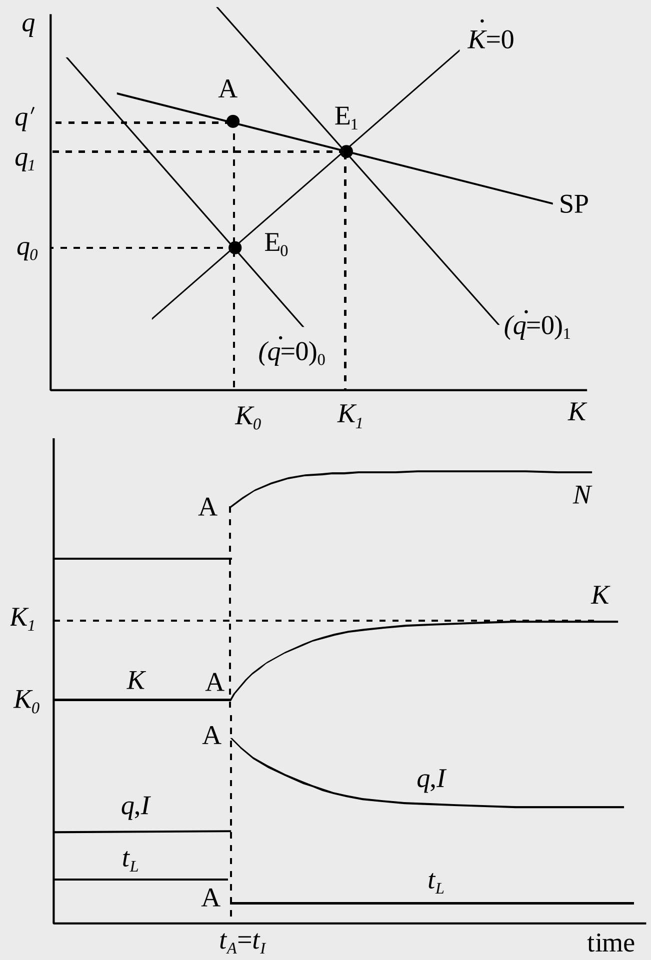

Figure 4.9: Labour Taxation, Employment and Investment

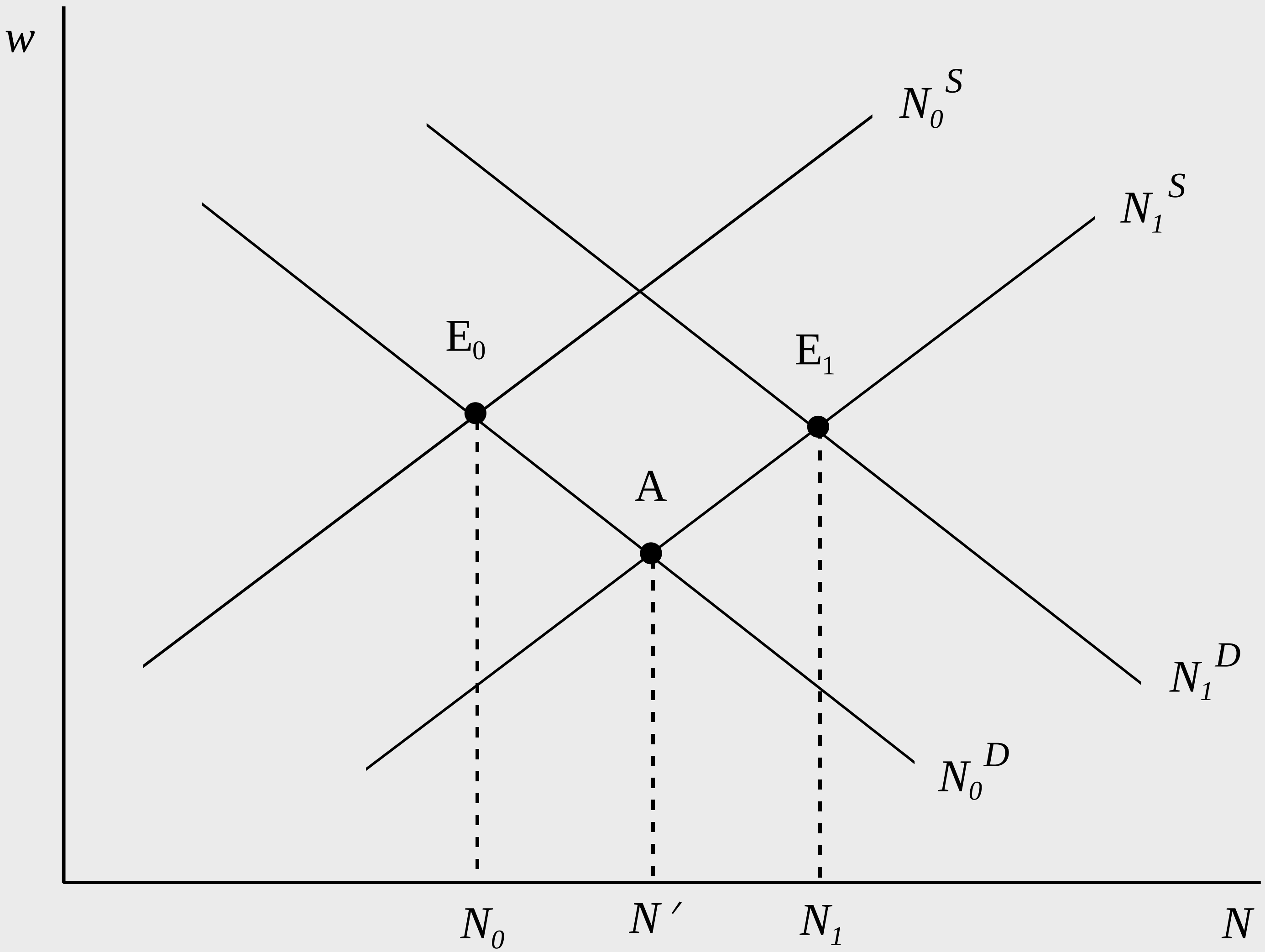

Figure 4.10: Employment Effect

A dynamic IS-LM model

- One of the things some economists do not like about the IS-LM model is the lack of (forward looking) dynamics [Blinder-Solow is an example of backward looking dynamics].
- It is not difficult, however, to add interesting dynamic effects to the IS-LM model. We study the model of the New Keynesian economist Olivier Blanchard
- The model is described by the following equations:
  - Aggregate demand for goods depends on Tobin's q ($a > 0$), on aggregate production in the economy (Y, $0 < \beta < 1$), and on government consumption

$$Y^D = aq + \beta Y + G$$

  - Production is changed only gradually ($\sigma > 0$):

$$\dot{Y} = \sigma \left[Y^D - Y \right]$$

- Money market equilibrium (R_S is the interest rate on short term securities):

$$M/P = kY - lR_S, \quad k > 0, \; l > 0,$$

- Term structure of interest rates (R_L is the yield on perpetuities):

$$R_S = R_L - (1/R_L)\dot{R}_L \tag{*}$$

- Arbitrage equation between shares and short bonds:

$$\frac{\dot{q} + \pi}{q} = R_S \tag{**}$$

- Profits depend positively on aggregate output:

$$\pi = \alpha_0 + \alpha_1 Y,$$

- Especially (*) and (**) need some further comment.
  - There are three financial assets: shares, short bonds, perpetuities
  - All assets are perfect substitutes in the portfolios of investors
  - Yields on three assets must be same
    - yield on short bonds is R_S
    - yield on shares is $(\dot{q}+\pi)/q$
    - yield on perpetuities is $(1+\dot{P}_B)/P_B$, where $P_B = 1/R_L$ is the price of a perpetuity paying 1 guilder each period.

- Model summary:

$$\dot{Y} = \sigma\left[aq - bY + G\right], \quad b \equiv 1-\beta,\ 0<b<1,$$

$$R_S = (k/l)Y - (1/l)(M/P),$$

$$R_S = \frac{\dot{q}+\alpha_0+\alpha_1 Y}{q}.$$

- Model can be illustrated graphically with the aid of **Figure 4.11**
  - $\dot{Y} = 0$ line is upward sloping. For points above (below) the line investment is too high (low) and output gradually rises (falls). See horizontal arrows in Figure 4.11
  - slope of the $\dot{q} = 0$ line is ambiguous:

$$\left(\frac{\partial q}{\partial Y}\right)_{\dot{q}=0} = \frac{\alpha_1 - qk/l}{R_S}$$

    In the steady state $q = (\alpha_0 + \alpha_1 Y)/R_S$ and a rise in Y raises both the numerator and the denominator. If the LM curve is relatively steep (so that k/l is high) then the interest rate effect dominates and the $\dot{q} = 0$ line slopes down. This is called the “bad news case” by Blanchard
  - there is a unique saddle-point stable equilibrium at E_0
  - the saddle path is downward sloping

- Illustration: an *anticipated* and *permanent* increase in government consumption (Figure 4.11). Initially a perverse effect on output!

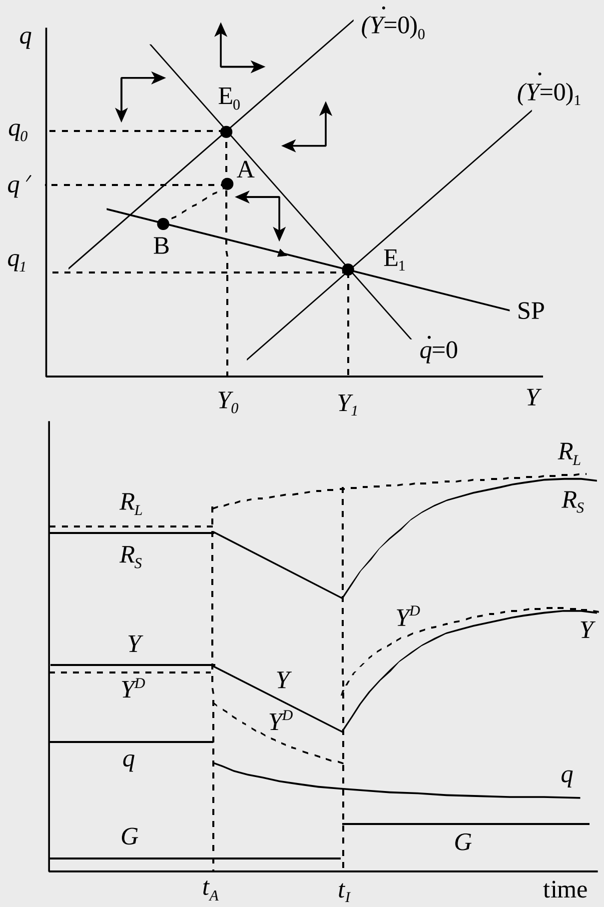

Figure 4.11: Anticipated Fiscal Policy

Punchlines

- key concept saddle-point stability
- timing crucially important
  - when is the news received by the agents?
  - when does the shock actually happen?
  - is the shock (believed to be) permanent?
- intuitive solution principle can often yield the solution
- policies can often have perverse effects (initially) due to forward looking behaviour of agents