Foundations of Modern Macroeconomics

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Chapter 2: Dynamics in Aggregate Demand and Supply

Aims of this lecture

The principal aim of this lecture is to study the "intrinsic dynamics" in IS-LM type models.

Particularly, we look at the following examples:

- The Adaptive Expectations Hypothesis [AEH] and stability in the AD-AS model
- Hysteresis and path dependency [self study and possibly in work groups]
- Investment theory and the interaction between the *stock* of capital (K) and the *flow* of investment (I). This is yet another important building block for the course.
- The government budget restriction, stability, stock-flow interaction, and multipliers under different financing methods

What do we mean by stability?

System returns to equilibrium following an exogenous shock.

Why are we so interested in stable models?

- unstable models are rather useless
- the Samuelsonian “correspondence principle” is very handy
- “backward looking” stability arises naturally in IS-LM type models and is easy to handle
- “forward looking” stability is a more recently developed form of stability but it can also be handled relatively easily.

The AEH and stability

Assume that we have a simple continuous-time model in the tradition of the Neo-Keynesian Synthesis:

$$\begin{aligned} Y &= AD(\underset{+}{G}, \underset{+}{M/P}), \qquad AD_G > 0, \quad AD_{M/P} > 0 \\ Y &= Y^* + \phi\left[P - P^e\right], \quad \phi > 0, \\ \dot{P}^e &= \lambda\left[P - P^e\right], \quad \lambda > 0. \end{aligned}$$

where $\dot{P}^e \equiv dP^e/dt$ and Y^* is full employment output (that level of output which is consistent with full employment in the labour market)

- the AD curve depends positively on both government consumption (G) and on the level of real money balances (M/P)

- the AS curve is upward sloping in the short run because of expectational errors
- the expected price levels adapts gradually to expectational errors

The effect of an increase in government consumption

- See **Figure 2.1** for the graphical derivation. key effects:
  - $G \uparrow$ so that IS and AD both shift up
  - P^e is given so that short-run equilibrium is at point A
  - in point A, $P^e \neq P$ (expectational disequilibrium)
  - since $P > P^e$, $\dot{P}^e > 0$ and AS_{SR} starts to shift up
  - economy moves gradually along the AD curve from A to E_1
- We can conclude from the graph that the model is stable!

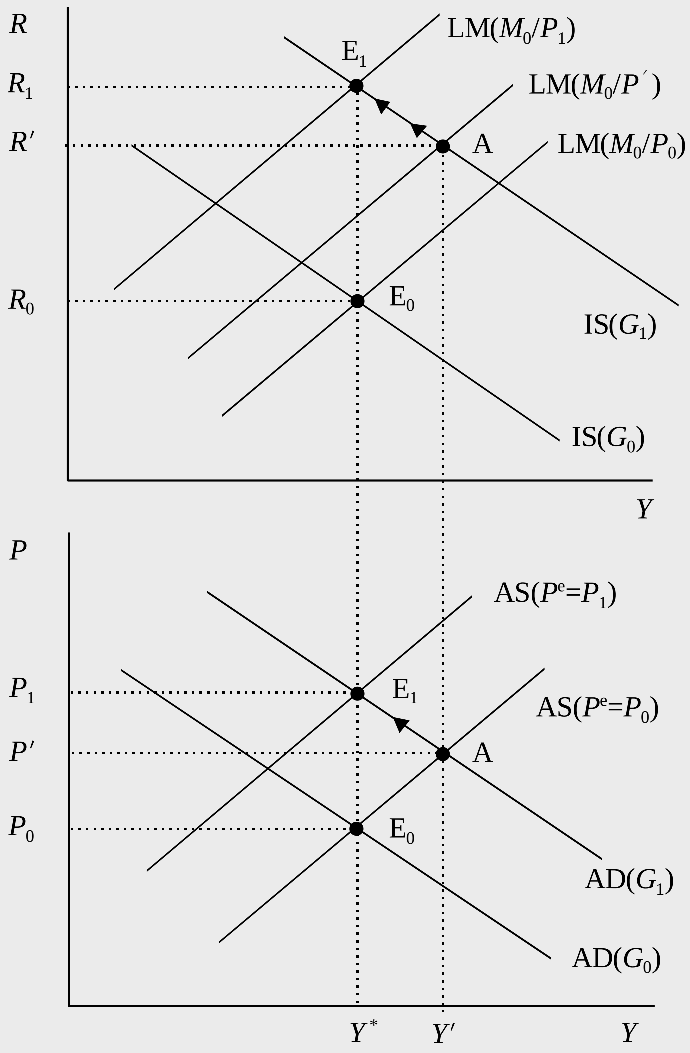

Figure 2.1: Fiscal Policy Under Adaptive Expectations

Can we do this analytically?

- This is useful if the model is too complicated to analyze graphically.
- stability holds in our model provided $\dot{P}^e$ dies out (goes to zero). In a *phase diagram* the stable and unstable cases look like in **Figure A**.
- from the diagram we conclude that we must show that for a stable model the phase diagram slopes downward:

$$\frac{\partial \dot{P}^e}{\partial P^e} < 0 \qquad \text{(stability condition)}$$

- note that a model may be non-linear. All we do is prove *local stability,* i.e. stability close to an equilibrium

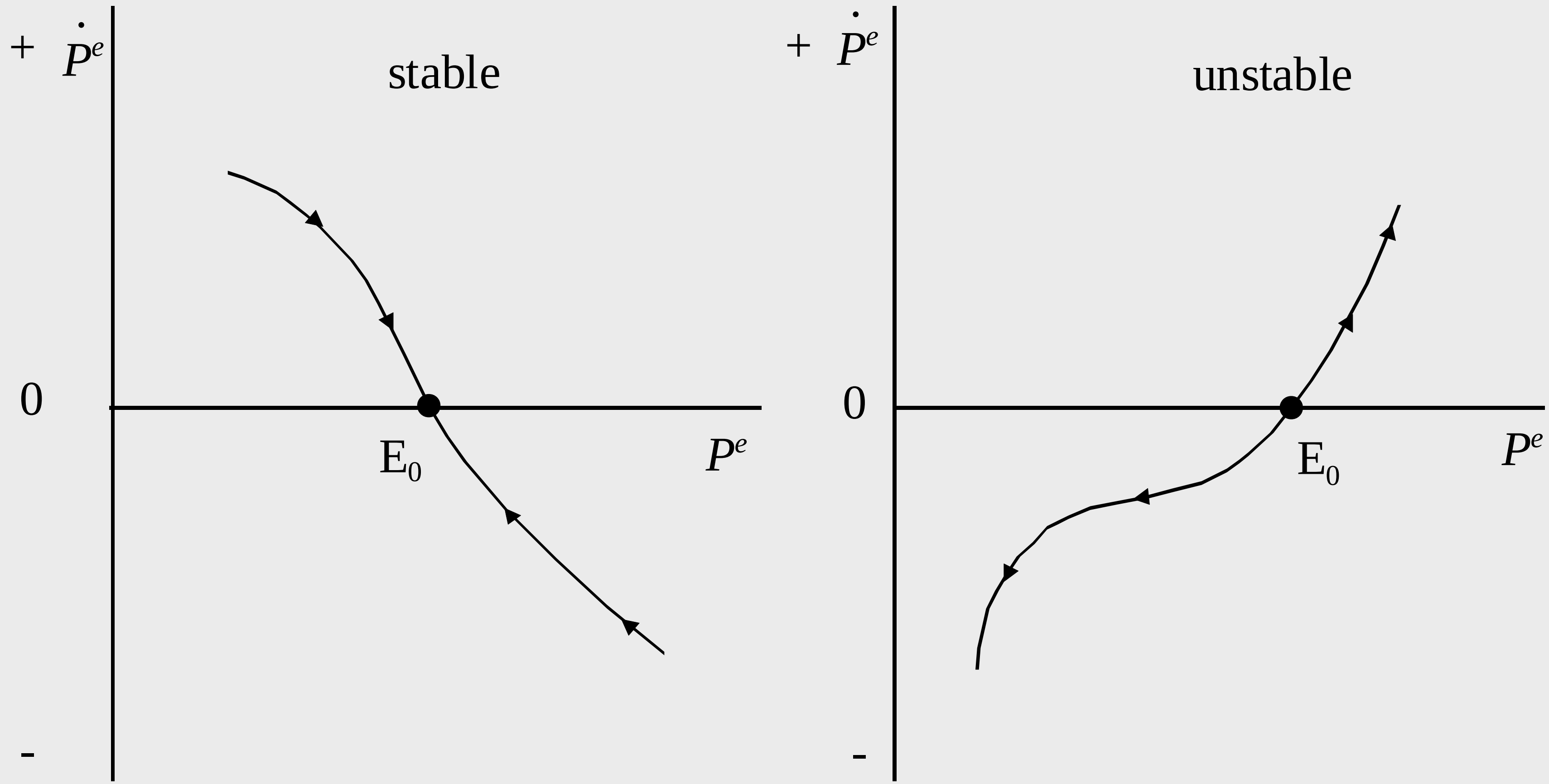

Figure A: Phase Diagram

- In our model we must take into account that P depends on P^e. We use AD and AS to find $\partial P/\partial P^e$ with our implicit function trick:

$$dY = AD_G dG + AD_{M/P}(M/P)\left(\frac{dM}{M} - \frac{dP}{P}\right),$$
$$dY = \phi\left[dP - dP^e\right] + dY^*.$$

and solve for dP:

$$dP = \frac{\phi dP^e + AD_G dG + AD_{M/P}(1/P)dM - dY^*}{\phi + AD_{M/P}(M/P^2)}$$

- We conclude that $\partial P/\partial P^e = \phi/\left[\phi + (M/P^2)AD_{M/P}\right]$ which is between 0 and 1.

- The AEH implies:

$$
\begin{aligned}
d\dot{P}^e &= \lambda\left[dP - dP^e\right] \\
&= \lambda\left[\frac{AD_G dG + AD_{M/P}(1/P)\left[dM - (M/P)dP^e\right] - dY^*}{\phi + AD_{M/P}(M/P^2)}\right] \\
\Leftrightarrow &\quad \dot{P}^e = \Omega\left(\underset{-}{P^e}, \underset{+}{G}, \underset{+}{M}, Y^*\right).
\end{aligned}
$$

- We conclude that $\partial\dot{P}^e/\partial P^e < 0$ so that the model is stable

- We can integrate the stability analysis with the fiscal policy shock in **Figure 2.2**

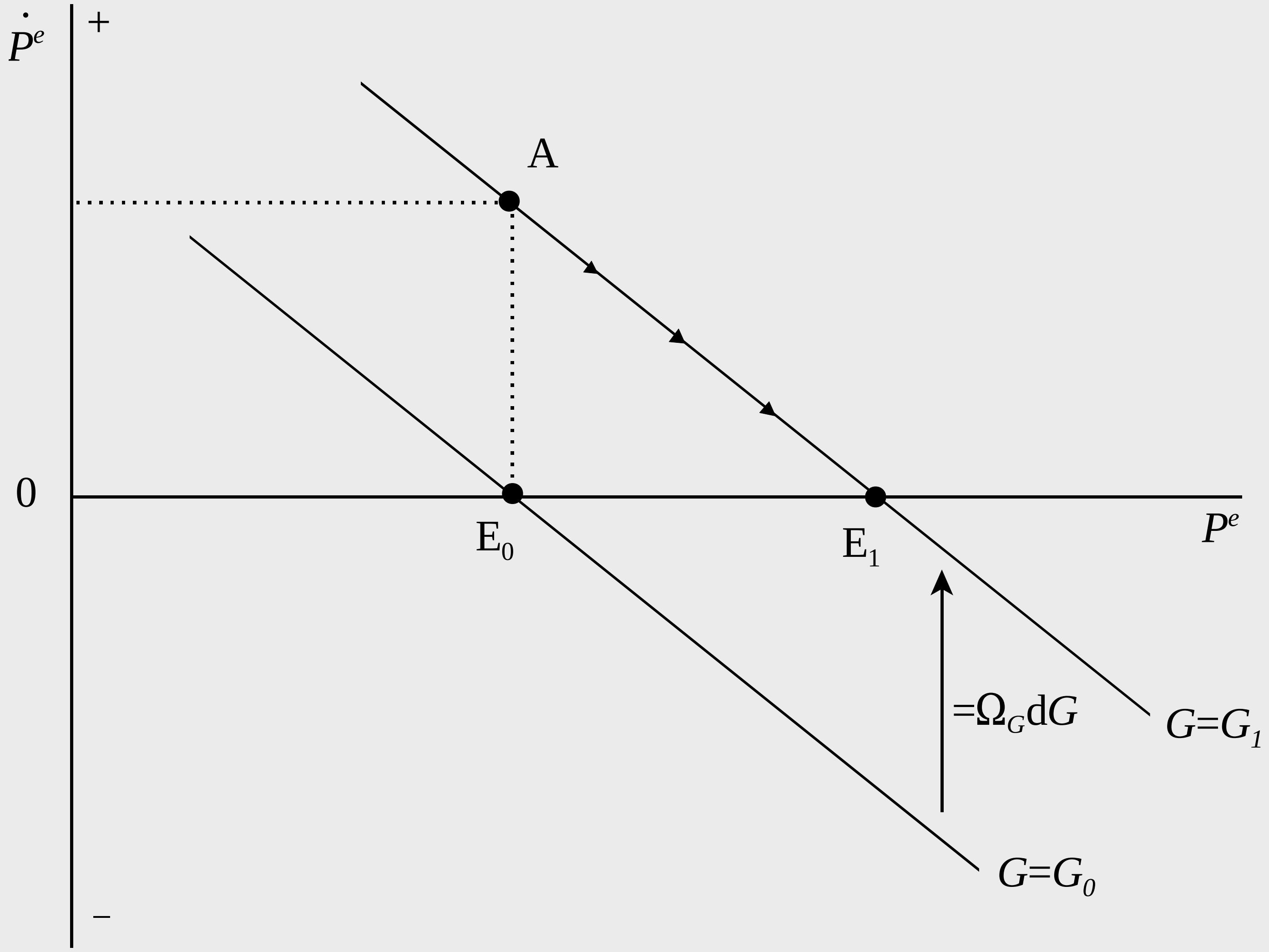

Figure 2.2: Stability and Adaptive Expectations

**** Self test ****

Phase diagrams are very important in modern macroeconomics. Make absolutely sure you feel confident working with them! If you don't understand these simple (one-dimensional) phase diagrams you will have trouble later on!

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Building block: A first look at investment theory

[Recall our earlier building blocks: demand for labour by firms, supply of labour by households, demand for money by households] We are now going to start the development of a theory of investment, i.e. the accumulation of *capital goods* (such as machines, PCs, buildings, etcetera) by firms. Basic ingredients:

- adjustment cost model
- firms now choose both employment (as in Chapter 1) *and* investment
- simplifying assumptions: static expectations, perfect competition
- production function still given by:

$$Y_t = F(N_t, K_t)$$

  - need a time subscript because the investment decision is a dynamic decision
  - choices made now affect outcomes in the future

- example: just like the decision to educate oneself

- timing: K_t is the capital *stock* at the beginning of period t

- properties as before: positive but diminishing marginal products ($F_N > 0$, $F_K > 0$, $F_{NN} < 0$, and $F_{KK} < 0$), cooperative factors ($F_{NK} > 0$), and CRTS

- accumulation identity:

$$\underbrace{K_{t+1} - K_t}_{1} = \underbrace{I_t}_{2} - \underbrace{\delta K_t}_{3},$$

Net investment (term 1) equals gross investment (term 2) minus depreciation of existing capital (term 3).

- the representative firm's manager maximizes the present value of net payments to owners of the firm ("share holders") using the market rate of interest to discount future payments [Modigliani-Miller Theorem]

- profit in period t is:

$$\Pi_t = \underbrace{PF(N_t, K_t)}_{1} - \underbrace{WN_t}_{2} - \underbrace{P^I I_t}_{3} - \underbrace{bP^I I_t^2}_{4}, \tag{1}$$

Profit (or cash flow) equals revenue (term 1) minus the wage bill (term 2) minus the purchase cost of new capital (term 3) minus the quadratic adjustment costs (term 4). See **Figure 2.3**.

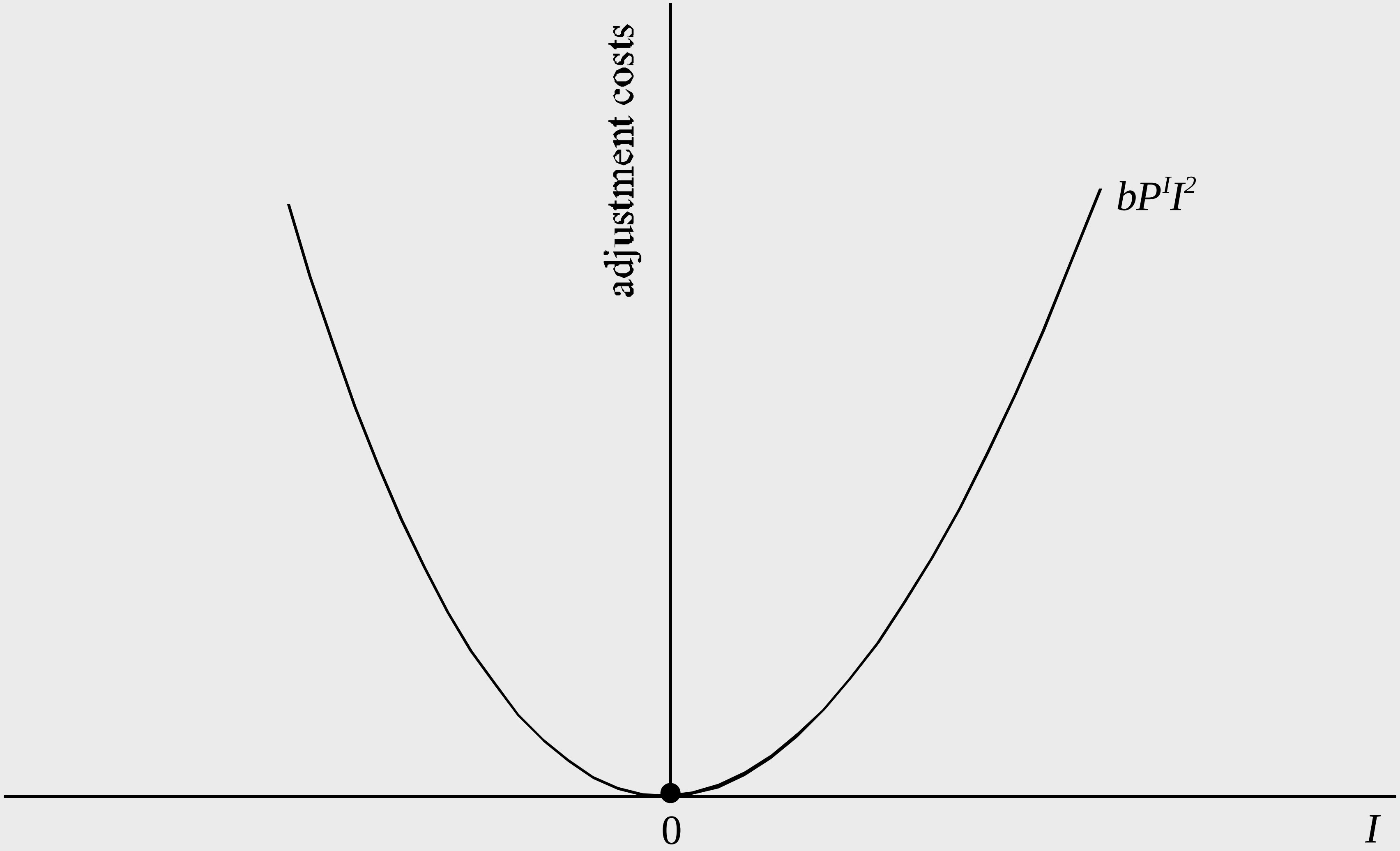

Figure 2.3: Adjustment Costs of Investment

- Let us call the planning period "today" and normalize it to $t = 0$.
- The value of the firm in the stock market is:

$$\begin{aligned}\bar{V}_0 &\equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+R}\right)^t \Pi_t \\ &= \sum_{t=0}^{\infty} \left(\frac{1}{1+R}\right)^t \left[PF(N_t, K_t) - WN_t - P^I I_t - bP^I I_t^2\right].\end{aligned}$$

- The firm must choose paths for N_t and K_t (and thus for Y_t) such that $\bar{V}_0$ is maximized subject to the accumulation identity (and the initial capital stock, K_0)

- To solve the problem we use the method of Lagrange multipliers. The Lagrangian is:

$$\mathcal{L}_0 \equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+R} \right)^t \left[PF(N_t, K_t) - WN_t - P^I I_t - bP^I I_t^2 \right]$$
$$- \sum_{t=0}^{\infty} \frac{\lambda_t}{(1+R)^t} \left[K_{t+1} - (1-\delta) K_t - I_t \right],$$

  – we need a whole *path* of Lagrange multipliers–λ_t is the one relevant for the constraint in period t. Note that we scale the Lagrange multipliers in order to facilitate interpretation later on

- first-order necessary conditions [FONCs] (for $t = 0, 1, 2, ...$):

$$\frac{\partial \mathcal{L}_0}{\partial N_t} = \left(\frac{1}{1+R}\right)^t [PF_N(N_t, K_t) - W] = 0,$$

$$\frac{\partial \mathcal{L}_0}{\partial K_{t+1}} = \left(\frac{1}{1+R}\right)^t \left[\frac{PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1-\delta)}{1+R} - \lambda_t\right] = 0,$$

$$\frac{\partial \mathcal{L}_0}{\partial I_t} = \left(\frac{1}{1+R}\right)^t [-P^I - 2bP^I I_t + \lambda_t] = 0,$$

where we must note the timing in the expression for $\partial \mathcal{L}_0 / \partial K_{t+1}$!

Interpretation

- there are no adjustment costs on labour. Hence the firm can vary employment freely in each period such that:

$$PF_N(N_t, K_t) = W$$

- the FONC for investment yields (for adjacent periods t and $t+1$):

$$\begin{aligned} \lambda_t &= P^I\left[1 + 2bI_t\right] \\ \lambda_{t+1} &= P^I\left[1 + 2bI_{t+1}\right] \end{aligned}$$

- ....so that the FONC for capital becomes:

$$PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1-\delta) - \lambda_t(1+R) = 0 \;\Rightarrow$$

$$PF_K(N_{t+1}, K_{t+1}) + (1-\delta)P^I\left[1 + 2bI_{t+1}\right] - (1+R)P^I\left[1 + 2bI_t\right] = 0 \;\Rightarrow$$

$$I_{t+1} - \left(\frac{1+R}{1-\delta}\right) I_t + \frac{PF_K(N_{t+1}, K_{t+1}) - P^I(R+\delta)}{2bP^I(1-\delta)} = 0. \qquad (\sharp)$$

– the final expression is an *unstable* difference equation because the coefficient for I_t is greater than 1 (as $R > 0$ and $0 < \delta < 1$)

– in general $I_t \to +\infty$ or $I_t \to -\infty$. But these are economically non-sensical solutions because adjustment costs for the firm will explode and thus firm profits and the value of the firm will go to $-\infty$

– but ($\sharp$) pins down only one economically sensible investment policy, namely the constant policy, for which $I_{t+1} = I_t$. Solving ($\sharp$) for this policy yields:

$$I = \frac{1}{2b}\left[\frac{PF_K(N,K)}{P^I(R+\delta)} - 1\right], \qquad (\spadesuit)$$

where we have dropped the time subscripts to indicate that ($\spadesuit$) is a steady-state investment policy [we analyze the non-steady-state case in Lecture 4]

- Let us assume that $P^I = P$ (single good economy; no investment subsidy). Then (♠) simplifies to:

$$I = \frac{1}{2b}\left[\frac{F_K(N,K)}{R+\delta} - 1\right],$$

  - if there are no adjustment costs ($b \longrightarrow 0$) then the firm expresses a demand for *capital*. The demand for investment is not well-defined in that case, because there is no punishment for the firm in adjusting its stock of capital freely (i.e. $I_t \to +\infty$ or $I_t \to -\infty$ are no longer disastrous in that case).
  - Formally, if $b \longrightarrow 0$ then so must the term in square brackets:

$$\frac{F_K(N,K)}{R+\delta} - 1 = 0 \qquad \Longleftrightarrow \qquad F_K = R + \delta$$

  - notice the parallel with the expression for labour demand in this case [the firm rents the use of the capital goods]

- With adjustment costs, however, we have a well-defined investment equation which we write generally as:

$$I = I(\underset{-}{R}, \underset{-}{K}, \underset{+}{Y}), \quad I_R < 0, \; I_K < 0, \; I_Y > 0,$$

[For example, if $Y = N^{\alpha} K^{1-\alpha}$ (with $0 < \alpha < 1$) then $F_K = (1-\alpha)Y/K$]

- We can study the stock-flow interaction on the demand side of the economy, in the IS-LM model. The model is:

$$\begin{aligned} Y &= C(Y - T(Y)) + I(R, K, Y) + G, \\ M/P &= l(Y, R), \qquad (P \text{ is fixed}) \\ \dot{K} &= I(R, K, Y) - \delta K. \end{aligned}$$

- IS-LM equilibrium yields:

$$\begin{aligned} Y &= AD(\underset{-}{P}, \underset{-}{K}, \underset{+}{G}, \underset{+}{M}), \\ R &= H(\underset{+}{P}, \underset{-}{K}, \underset{+}{G}, \underset{-}{M}). \end{aligned}$$

****** Self test ******

Draw IS-LM diagrams to rationalize these effects. Use **Figure 2.4** to do so.

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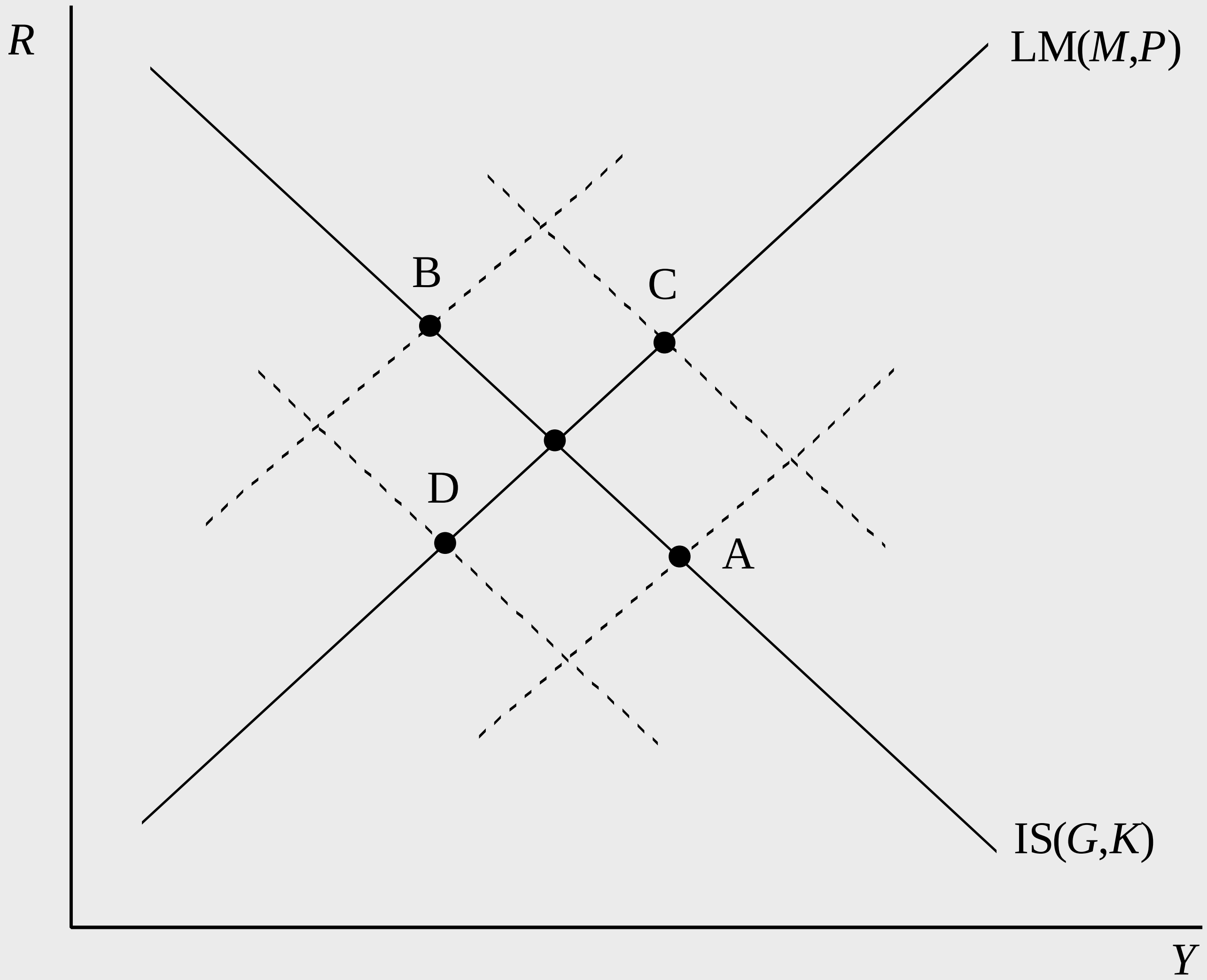

Figure 2.4: Comparative static effects in the IS-LM model

- Hence, the capital dynamics is governed by:

$$\dot{K} = I\big[\underbrace{H(P,K,G,M)}_{R},K,\underbrace{AD(P,K,G,M)}_{Y}\big] - \delta K$$

- Note that the capital stock, K, appears in no less than *four* places on the right-hand side. Hence, checking stability (by computing $\partial\dot{K}/\partial K$ and proving it is negative) is much more difficult. A graphical approach will not help!!!

- Formally we find:

$$\begin{aligned} d\dot{K} &= I_R dR + I_K dK + I_Y dY - \delta dK \\ &= I_R \left[H_K dK + H_G dG \right] + I_K dK + \\ &\quad + I_Y \left[AD_K dK + AD_G dG \right] - \delta dK \\ &= \Big[\underset{-}{I_R} \underset{-}{H_K} + \underset{-}{I_K} + \underset{+}{I_Y} \underset{-}{AD_K} - \underset{+}{\delta} \Big] dK \\ &\quad + \left[I_R H_G + I_Y AD_G \right] dG, \end{aligned} \qquad (\sharp)$$

- Not at all guaranteed that the term in square brackets on the right-hand side is negative (as is required for stability); the term $I_R H_K > 0$ which is a "destabilizing" influence.
- Appeal to the Samuelsonian Correspondence Principle [believe and use only stable models] and simply assume that $\partial \dot{K} / \partial K < 0$ This gets you information that is useful to determine the long-run effect of fiscal policy

- From (♯) we find that, assuming stability, $d\dot{K}=0$ in the long run so that the long-run effect on capital is:

$$\left(\frac{dK}{dG}\right)^{LR} = \frac{\underset{-}{I_R}\underset{+}{H_G} + \underset{+}{I_Y}\underset{+}{AD_G}}{-\left[I_R H_K + I_K + I_Y AD_K - \delta\right]}$$

where the denominator is positive for the stable case. The long-run effect on capital of an increase in government consumption is ambiguous.

- Heated debate in the 1970s between monetarists (like Friedman) and Keynesians (like Tobin) [a.k.a. the "battle of the slopes"]:
  - Friedman: a strong interest rate effect on investment ($|I_R|$ large), and a large effect on the interest rate but a small effect on output of a rise in government spending (H_G large and AD_G small). Consequently, a monetarist might suggest that $\partial\dot{K}/\partial G$ is negative.
  - Tobin: $|I_R|$ small, H_G small, and AD_G large, so that $\partial\dot{K}/\partial G > 0$

- In **Figures 2.5 and 2.6** illustrate the two cases

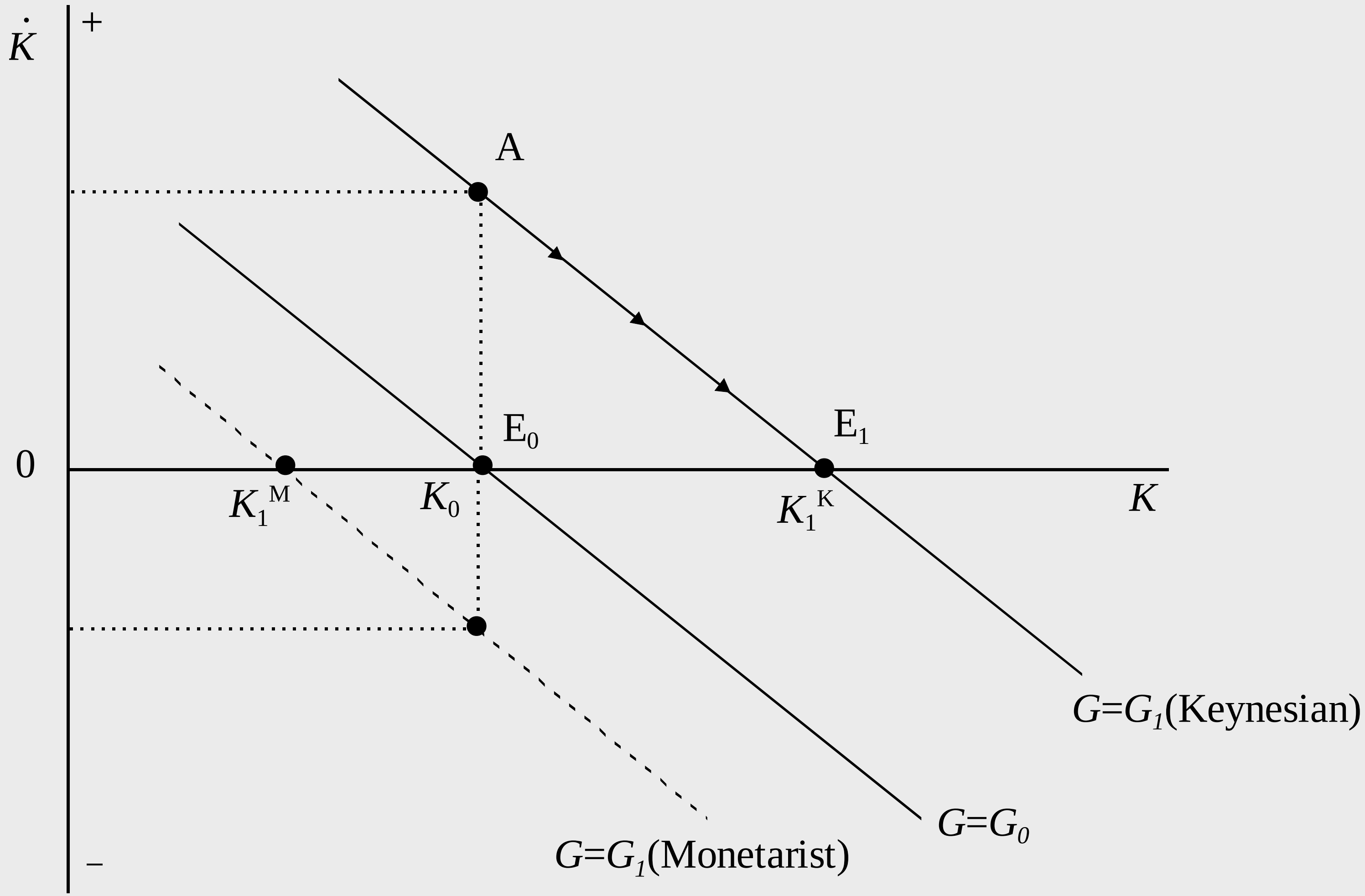

Figure 2.5: The Capital Stock and Public Consumption

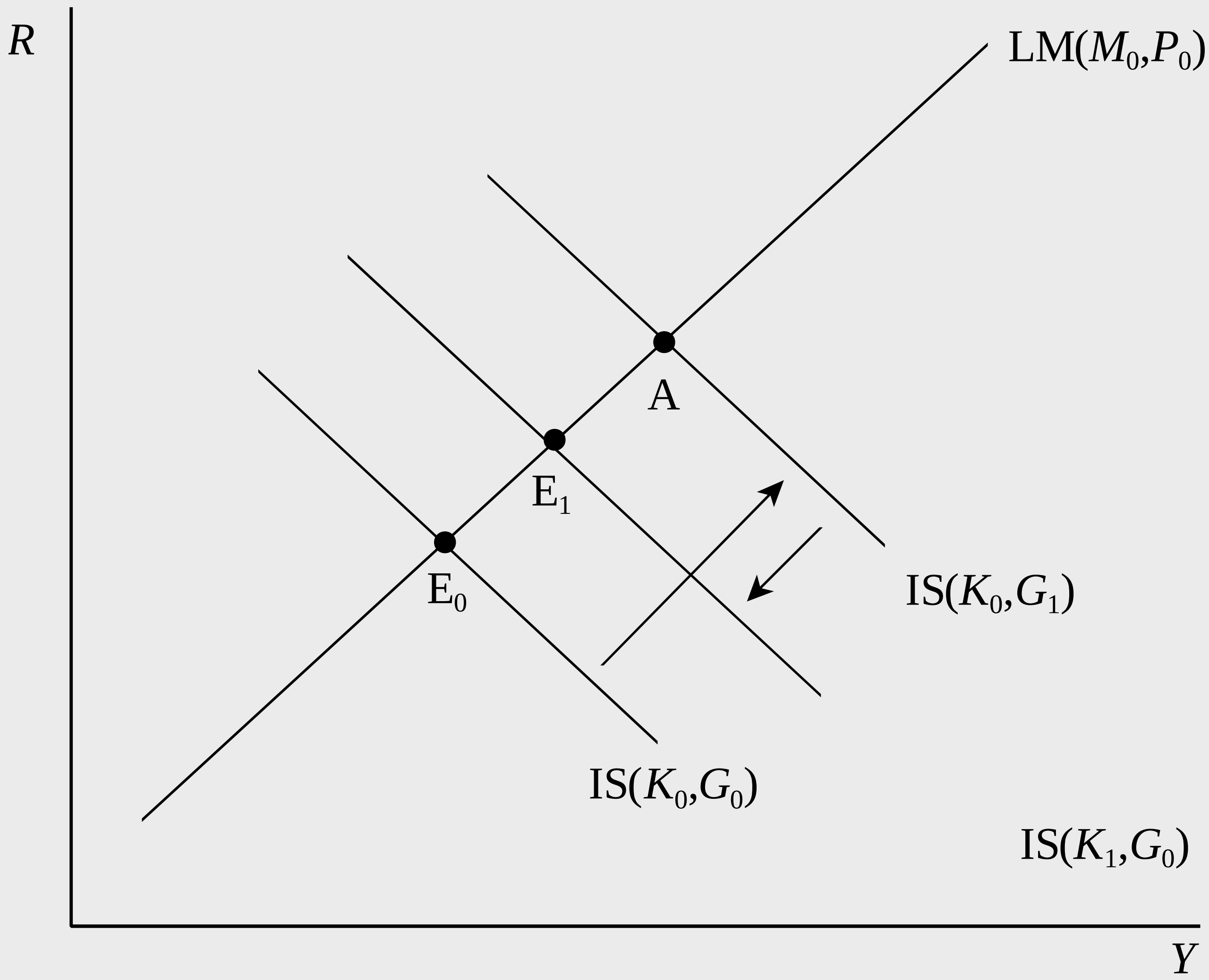

Figure 2.6: Capital and Keynesian Effects of Fiscal Policy

Intrinsic dynamics and the government budget constraint

- IS-LM is a little strange because
  - it combines flow concepts (IS) and stock concepts (LM) in one diagram
  - it cannot be used to study effect of government financing method
- Blinder and Solow (1973) show how the IS-LM model can be extended with a government budget restriction. With their model we can study:
  - money creation
  - tax financing
  - bond financing

Key ingredients of the Blinder-Solow model

- fixed price level, $P = 1$ (horizontal AS curve)
- special type of bond, the consol, pays 1 euro from now until perpetuity
- if the interest rate is R the price of the bond would be:

$$P_B = \int_0^\infty 1 e^{-R\tau} d\tau = -(1/R)\left[e^{-R\tau}\right]_0^\infty = 1/R$$

- if there are B consols in existence than the “coupon payments” at each instant is $B \times 1$ euros
- if the government emits new consols, $\dot{B} > 0$, then it receives $\dot{B} \times P_B$ in revenue from the bond sale
- if the government issues new money, then $\dot{M} > 0$

- the government budget constraint is:

$$G + B = T + \dot{M} + (1/R)\dot{B}$$

government consumption plus coupon payments equals tax revenue plus money issuance plus revenue from new bond sales

- other changes to the IS-LM model:

$$\begin{aligned} T &= T(Y+B), \;\; 0 < T_{Y+B} < 1 \\ A &\equiv \bar{K} + M/P + B/R \\ C &= C(Y+B-T, A), \quad 0 < C_{Y+B-T} < 1, \; C_A > 0 \\ M/P &= l(Y, R, A), \quad l_Y > 0, \; l_R < 0, \; 0 < l_A < 1 \end{aligned}$$

- new IS curve

$$Y = C\big[\underbrace{Y + B - T(Y+B)}_{1}, \underbrace{\bar{K} + M/P + B/R}_{2}\big] + I(R) + G,$$

where term 1 is household disposable income, and term 2 is total wealth

- "quasi-reduced form" expressions for Y and R can be derived in the usual way

$$\begin{aligned} Y &= \textit{AD}(\underset{+}{G}, \bar{K}, \underset{?}{B}, \underset{+}{M}), \\ R &= H(\underset{+}{G}, \bar{K}, \underset{+}{B}, M) \end{aligned}$$

- two pure cases, pure money financing ($\dot{M} \neq 0$ and $\dot{B} = 0$) and pure bond financing ($\dot{M} = 0$ and $\dot{B} \neq 0$). Key issues:
  - is the model stable?
  - relation between financing method and the government spending multiplier

Pure money financing $(\dot{M} \neq 0,\ \dot{B} = 0)$

- money financing is stable:

$$\frac{\partial \dot{M}}{\partial M} \equiv -T_{Y+B}\mathit{AD}_M < 0$$

- boost in government consumption causes an initial government deficit

$$\frac{\partial \dot{M}}{\partial G} \equiv (1 - T_{Y+B}\mathit{AD}_G) > 0$$

- long-run multiplier exceeds short-run multiplier

$$\left(\frac{dY}{dG}\right)_{MF}^{LR} \equiv \frac{1}{T_{Y+B}} > \mathit{AD}_G \equiv \left(\frac{dY}{dG}\right)_{MF}^{SR}$$

- economic intuition: both IS and LM shift out, $Y\uparrow$, $T(Y)\uparrow$, deficit closes and $\dot{M} = 0$

- see **Figure 2.7** for the graphical illustration

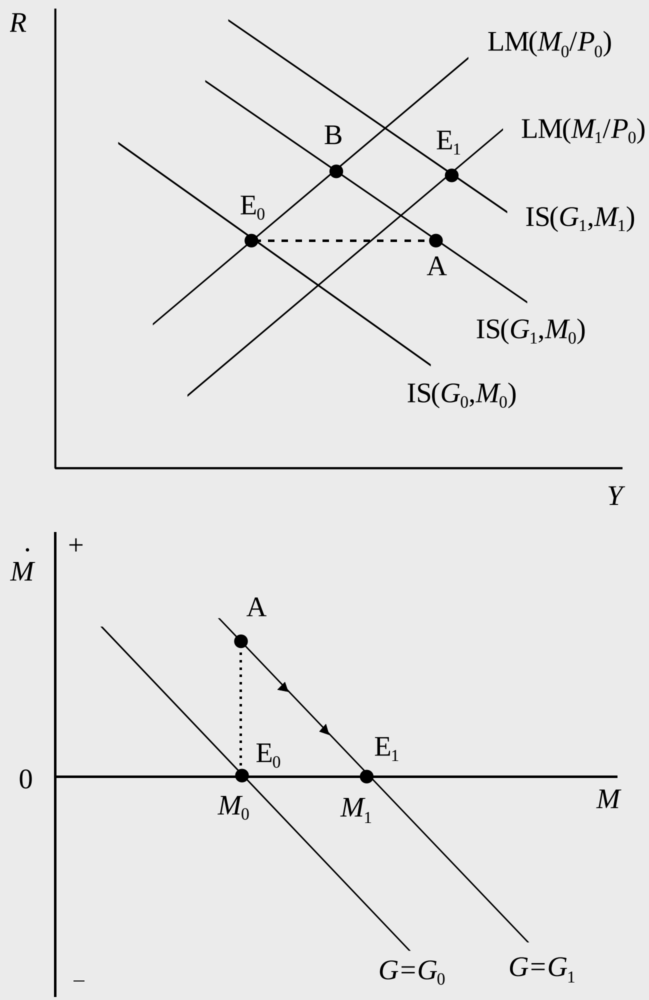

Figure 2.7: Fiscal Policy Under Money Financing

Pure bond financing $(\dot{M} = 0,\ \dot{B} \neq 0)$

- bond financing may be unstable:

$$\frac{\partial \dot{B}}{\partial B} = 1 - \underset{0<.<1}{T_{Y+B}}(1 + \underset{?}{AD_B}) \gtreqless 0$$

Economic intuition: AD_B is ambiguous because $B \uparrow$ shifts IS to the right (via consumption) but LM to the left (via money demand). Net effect ambiguous

- but Samuelsonian "correspondence principle" helps:

$$\begin{aligned} \frac{\partial \dot{B}}{\partial B} &< 0 \\ \Leftrightarrow\quad & 1 - T_{Y+B}(1 + AD_B) < 0 \\ \Leftrightarrow\quad & AD_B > \frac{1 - T_{Y+B}}{T_{Y+B}} > 0 \end{aligned}$$

- for the stable case the long-run multiplier again exceeds the short-run multiplier:

$$\underbrace{\left(\frac{dY}{dG}\right)^{LR}_{BF}}_{AD_G - AD_B\left(\frac{1-T_{Y+B}AD_G}{1-T_{Y+B}(1+AD_B)}\right)} > \underbrace{\left(\frac{dY}{dG}\right)^{SR}_{BF}}_{AD_G}$$

- see **Figure 2.8** for the graphical illustration

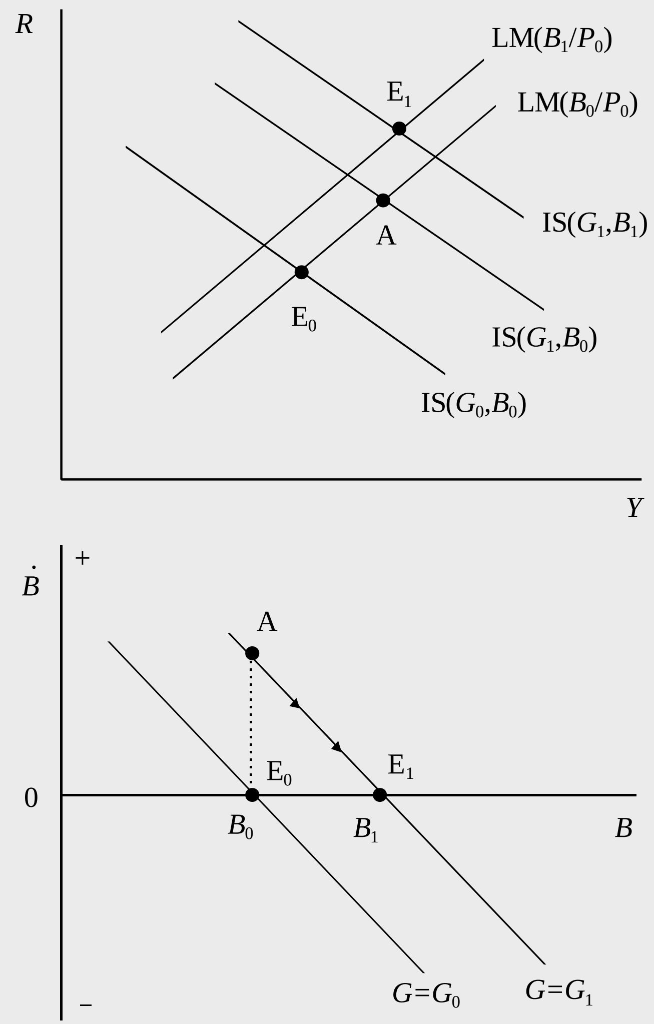

Figure 2.8: Fiscal Policy Under Stable Bond Financing

Comparison money financing and bond financing

- the long-run (stable) bond-financed multiplier exceeds the long-run money-finance multiplier:

$$\underbrace{\left(\frac{dY}{dG}\right)^{LR}_{BF}}_{\mathsf{AD}_G - \mathsf{AD}_B\left(\frac{1-T_{Y+B}\mathsf{AD}_G}{1-T_{Y+B}(1+\mathsf{AD}_B)}\right)} > \underbrace{\left(\frac{dY}{dG}\right)^{LR}_{MF}}_{\frac{1}{T_{Y+B}}}$$

- Economic intuition: under bond financing both increase in G and the additional interest payments (increase in B) must eventually be covered by higher tax receipts. Since $T = T(Y)$, it must be the case that Y rises by more.

- see **Figure 2.9** for the graphical illustration

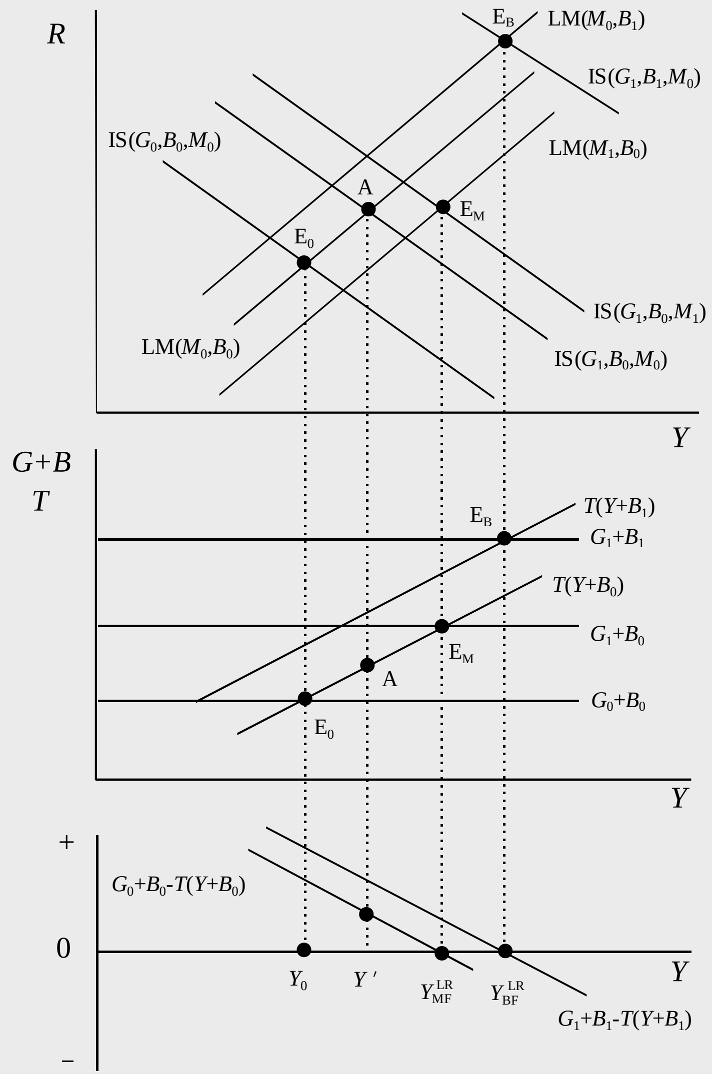

Figure 2.9: Comparison of Different Financing Modes