Foundations of Modern Macroeconomics

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Problem set for Chapter 11

The questions with a star (★) are difficult.

Question 1

[*The Import Leakage*] Assuming that the economy is open to trade in goods and services, that prices are fixed, and that I, G, and T are all exogenous. The economy is described by the following equations:

$$Y = C + I + G + X \tag{1}$$
$$C = C_0 + c(Y - T), \qquad 0 < c < 1, \tag{2}$$
$$X = X_0 - mY, \qquad 0 < m < 1, \tag{3}$$

where X is net exports (exports minus imports), and m is the marginal propensity to import goods and services. The exogenous component of net exports is given by X_0.

(a) Solve the model by finding expressions for the endogenous variables (Y, C, and X) in terms of the exogenous variables (I, G, C_0, X_0, and T) and the parameters (c and m). These are the so-called *reduced-form* expressions for output, consumption, and net exports.

(b) Compute the output multiplier with respect to government consumption. Does the propensity to import increase or decrease this multiplier? Explain the intuition behind the import leakage.

(c) Compute the effect on output, consumption, and net exports of an increase in world trade (represented by an increase in X_0). Explain the intuition behind your results.

Question 2

[*The Mundell-Fleming Model*] Consider an open economy IS-LM model with perfect capital mobility. Assume that we extend the IS-LM model by introducing international trade. Assume furthermore that the price level is fixed (say $P = P_0$) and that domestic and foreign bonds are perfect substitutes. The extended model is given by:

$$\begin{aligned}
Y &= C + I + G + X && (1)\\
C &= C(Y - T) \qquad 0 < C_{Y-T} < 1 && (2)\\
I &= I(R), \qquad I_R < 0 && (3)\\
T &= T(Y), \qquad 0 < T_Y < 1 && (4)\\
M/P &= k(Y) + l(R) && (5)\\
X &\equiv EX(E) - IM(E,Y)E, \quad EX_E > 0,\ IM_E < 0,\ IM_Y > 0 && (6)\\
R &= R^* + \dot{E}/E && (7)
\end{aligned}$$

where Y, C, I, G, T, and R are, respectively, output, consumption, investment, government consumption, taxes, and the interest rate. Furthermore, R^* is the foreign interest rate, EX is exports, IM is imports, E is the exchange rate (guilders per unit of foreign currency), and X is net exports. Use this model to answer the following questions. Assume that the expectations regarding the exchange rate are perfectly inelastic (so that there is no speculation on the market for foreign exchange and the $\dot{E}/E$ term can be put equal to zero).

(a) Interpret the equations.

(b) Explain the so-called *Marshall-Lerner condition*.

(c) Why is there less scope for Keynesian countercyclical policy in an open economy with flexible exchange rate? How effective is monetary policy in such a situation?

(d) Can the government of an less-than-fully employed economy boost employment without putting pressure on the interest rate (and the foreign exchange rate)? Show how the government can engineer an appreciation of the currency without harming employment. Distinguish the two cases of fixed and flexible exchange rates.

(e) Explain why a small open economy with fixed exchange rates is extremely sensitive to shocks in world trade. Is it possible to use monetary or fiscal policy to counter the effects of world trade shocks?

Question 3

[*Dynamics of foreign reserves*] Consider the following model of a small open economy with fixed prices ($P = P_0 = 1$ for convenience) operating under a regime of fixed exchange rates.

$$\begin{aligned} Y &= C(Y) + I(R) + G + X(E, Y), && (1) \\ D + F &= l(Y, R), && (2) \\ \dot{F} &= X(E, Y) + KI\,(R - R^*)\,, && (3) \end{aligned}$$

where Y is output, C is consumption, I is investment, R is the domestic interest rate, G is government consumption, X is net exports, E is the exchange rate (domestic currency per unit of foreign currency), D is domestic credit (government bonds in the hands of the central bank), F is the stock of foreign exchange reserves (measured in units of the domestic currency), and KI is net capital inflows. As usual, a dot above a variable denotes that variable's time derivative, i.e. $\dot{F} \equiv dF/dt$. We make the usual assumptions regarding the partial derivatives of the various functions: $0 < C_Y < 1$, $X_Y < 0$, $I_R < 0$, $X_E > 0$, $l_Y > 0$, $l_R < 0$, and $KI_R > 0$.

(a) Interpret the equations of the model. What do we assume about the Marshall-Lerner condition? Which are the endogenous variables? Which are the exogenous variables?

(b) Derive the fundamental differential equation for the stock of foreign exchange reserves and show that the model is stable. Show that the speed of adjustment increases as the degree of capital mobility increases and illustrate your argument with the aid of a diagram.

(c) Derive the so-called BP curve, representing (R, Y) combinations for which the balance of payments is in equilibrium ($\dot{F} = 0$). Assume that the BP curve is flatter than the LM curve (when drawn in the usual diagram with R on the vertical axis and Y on the horizontal axis). Show that this is the case if the following condition holds: $X_Y l_R < l_Y KI_R$. Give an economic interpretation for this condition.

(d) Derive the impact, transitional, and long-run effects on the endogenous variables of an increase in government consumption. Assume that the condition mentioned in part (c) holds. Illustrate your answer with graphs and explain the economic intuition.

(e) Derive the impact, transitional, and long-run effects on the endogenous variables of monetary policy. Illustrate your answers with a graph and explain the intuition.

Question 4

[*The Dornbusch model*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$y = -\eta R + g + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta < 1, \tag{1}$$

$$m - p = y - \lambda R, \qquad \lambda > 0, \tag{2}$$

$$\dot{p} = \phi(y - \bar{y}), \qquad \phi > 0, \tag{3}$$

$$R = R^* + \dot{e}, \tag{4}$$

where y is actual output, R is the domestic interest rate, g is an index for fiscal policy, e is the nominal exchange rate, p^* is the exogenous foreign price level, p is the domestic price level, m is the nominal money supply, $\bar{y}$ is full employment output, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$.

(a) Interpret the equations of the model.

(b) Suppose that the economy operates under a system of *fixed exchange rates* ($e = \bar{e}$). What are the endogenous variables? What is the coefficient of monetary accommodation (i.e. $\partial m/\partial p$) in this model? Derive the (impact, transitional, and long-term) effects of an expansionary fiscal policy (an increase in g).

(c) Now assume that the economy operates under a system of *flexible exchange rates*. Derive the model's phase diagram for the nominal exchange rate, e, and the domestic price level, p.

(d) Derive the (impact, transitional, and long-term) effects of an unanticipated and permanent expansionary fiscal policy.

(e) Show that under flexible exchange rates an unanticipated and permanent increase in the money supply leads to overshooting of the exchange rate in the short-term.

(f) Derive the (impact, transitional, and long-term) effects of an anticipated and permanent increase in the money supply.

(g) Show how your answer to parts (e) and (f) change if domestic prices are perfectly flexible, i.e. if $\phi \to \infty$ in equation (3).

Question 5

[*Turnovsky (1979)*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$
\begin{aligned}
y &= -\eta R + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta < 1, && (1)\\
m - p &= y - \lambda R, \qquad \lambda > 0, && (2)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (3)\\
R &= R^* + \dot{e}, && (4)
\end{aligned}
$$

where y is actual output, R is the domestic interest rate, e is the nominal exchange rate, p^* is the exogenous foreign price level, p is the domestic price level, m is the nominal money supply, $\bar{y}$ is (exogenous) full employment output, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$. Assume that the policy maker adopts the following policy rule for the nominal money supply:

$$
m - \bar{m} = -\mu(e - \bar{e}), \qquad \mu \gtreqless 0, \tag{5}
$$

where $\bar{m}$ is the exogenous component of money supply, $\bar{e}$ is the equilibrium exchange rate, and μ is a policy parameter.

(a) Interpret the equations of the model. Which are the endogenous and which are the exogenous variables? Explain why the policy rule embodies "leaning against the wind" if $\mu > 0$. What do we mean by "leaning with the wind"?

(b) Show that the model is saddle-point stable provided $1 + \mu \geq 0$. Illustrate the phase diagram of the model for the case where the policy maker engages strongly in "leaning against the wind" (so that $\eta\mu > \lambda\delta$).

(c) Assume that $\eta\mu = \lambda\delta$. Derive the (impact, transitional, and long-term) effects of an unanticipated and permanent increase in the foreign price level, p^*. Why is there no transitional dynamics in this case?

Question 6

[*Buiter & Miller (1981, 1982)*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$
\begin{aligned}
y &= -\eta\,[R - \dot{p}_C] + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta, \gamma < 1, && (1)\\
m - p_C &= y - \lambda R, \qquad \lambda > 0, && (2)\\
p_C &\equiv \alpha p + (1-\alpha)(e + p^*), \qquad 0 < \alpha < 1, && (3)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (4)\\
R &= R^* + \dot{e}, && (5)
\end{aligned}
$$

where y is actual output, R is the domestic interest rate, p_C is the price index for goods used in the domestic economy, e is the nominal exchange rate, p^* is the exogenous (and constant) foreign price level, p is the price of domestically produced goods, $\bar{y}$ is full employment output, m is the (constant) nominal money supply, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$. We define the auxiliary variables $l \equiv m - p$ (measure of "liquidity") and $c \equiv e + p^* - p$ (index for "competitiveness").

(a) Interpret the equations of the model. Which are the endogenous and which are the exogenous variables?

(b) Derive the dynamical system for this model in terms of l and c. Show that the model is saddle-point stable provided $\lambda + \alpha\eta(1 - \lambda\phi) > 0$. Which is the predetermined variable? Which is the jumping variable?

(c) Construct the phase diagram for the model. (Many saddle-point stable slope configurations are possible. Assume that the $\dot{c} = 0$ line is upward sloping and $\dot{l} = 0$ line is downward sloping. State the corresponding parameter assumptions.)

(d) Derive the (impact, transitional, and long-term) effects on c and l of an unanticipated and permanent increase in the money supply. Does overshooting of the exchange rate occur in this model.

Question 7 (★)

[*Obstfeld (1986)*] Consider the following Mundell-Fleming type model of a small open economy:

$$
\begin{aligned}
m_t - p_t &= y_t - \alpha R_t, \quad \alpha > 0, && (1)\\
y_t &= -\beta\left[R_t - (E_t p_{t+1} - p_t)\right] + \gamma(e_t + p^* - p_t) + u_t, \quad \beta > 0,\ \gamma > 0, && (2)\\
R_t &= R^* + E_t e_{t+1} - e_t, && (3)\\
p_t &= \bar{p}_t - (1-\delta)(\bar{p}_t - E_{t-1}\bar{p}_t), \quad 0 \leq \delta \leq 1, && (4)\\
m_t &= \mu_0 + m_{t-1} + v_t, && (5)\\
\bar{y}_t &= \zeta_0 + \bar{y}_{t-1}, && (6)
\end{aligned}
$$

where m_t is the nominal money supply, p_t is the actual domestic price level, y_t is output, R_t is the domestic (nominal) interest rate, e_t is the nominal exchange rate, p^* is the world price level, R^* is the world (nominal) interest rate, and $\bar{p}_t$ is the *equilibrium* domestic price level, μ_0 is the constant money growth rate, $\bar{y}_t$ is the full employment level of output, and ζ_0 is the constant growth rate in full employment output. In equation (4), $\bar{p}_t$ is the price for which actual output, y_t, equals its exogenously given full employment level, $\bar{y}_t$. All variables except R_t and R^* are in logarithms. E_t and E_{t-1} denote conditional expectations based on, respectively, period-t and period-$t-1$ information. It is assumed that agents hold rational expectations. The shock terms, u_t and v_t, are independent from each other and are both normally distributed with mean zero and constant variance, i.e. $u_t \sim N\left(0, \sigma_u^2\right)$ and $v_t \sim N\left(0, \sigma_v^2\right)$. They also features no serial correlation.

(a) Provide a brief interpretation of these equations.

(b) Consider the special case of the model for which $\delta = 1$. Define the *real* exchange rate as $q_t \equiv e_t + p^* - p_t$. Solve for the rational expectations solutions for the real exchange rate, the domestic price, and the nominal exchange rate. Denote these solutions by, respectively, $\bar{q}_t$, $\bar{p}_t$, and $\bar{e}_t$.

(c) Show that the money supply does not affect the equilibrium real exchange rate, $\bar{q}_t$. Explain intuitively why this is the case.

(d) Now use the general case of the model, with $0 < \delta < 1$, and solve for the rational expectations solution for the real exchange rate, the domestic price, the nominal exchange rate, and output.

(e) Does the classical dichotomy still hold for the sticky-price model solved in part (d). Explain.

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