Foundations of Modern Macroeconomics

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Problem set for Chapter 10

The questions with a star (★) are difficult.

Question 1

[*Choice of policy instrument, Poole (1970)*] Assume that the closed economy is described by the simple (log-linear) IS-LM model:

$$y_t = \alpha_0 - \alpha_1 R_t + U_t, \qquad \alpha_1 > 0, \tag{1}$$

$$m_t - p_t = \beta_0 + \beta_1 y_t - \beta_2 R_t + V_t, \qquad \beta_1 > 0,\ \beta_2 > 0, \tag{2}$$

where y_t is output, R_t is the nominal interest rate, U_t is a stochastic shock term affecting the IS curve, m_t is the nominal money supply, p_t is the price level, and U_t is a stochastic term affecting the LM curve. It is assumed that the two stochastic shock terms, U_t and V_t, are pure independent and normally distributed white noise terms (there is no correlation between these terms and both terms display no autocorrelation): $U_t \sim N(0, \sigma_U^2)$ and $V_t \sim N(0, \sigma_V^2)$. All variables except the interest rate are measured in logarithms. Assume furthermore that the price level is fixed and can be normalized to unity (so that $p_t \equiv \log P_t = 0$ in (2)).

The policy maker is concerned about stabilizing the economy and wishes to use monetary policy in order to minimize the following loss function:

$$\Omega \equiv E\left[y_t - y^*\right]^2, \tag{3}$$

where Ω is social loss and y^* is the (fixed) target level of output, e.g. full employment output. The policy maker can choose one of two instruments of monetary policy. It can control the money supply and let the interest rate settle at its equilibrium level determined in the economy. Alternatively, it can peg the interest rate and let the money supply settle at the equilibrium level determined in the market.

(a) Interpret the equations of the model.

(b) Show that in the *deterministic case*, with both U_t and V_t identically equal to zero in all periods, the two instruments of monetary policy are completely equivalent, i.e. it does not matter which one is used.

(c) Now assume that the IS curve is subject to stochastic shocks but the LM curve is deterministic (i.e. $U_t \sim N(0, \sigma_U^2)$ and $V_t \equiv 0$ for all periods). Show that the rational policy maker will choose the money supply instrument. Illustrate your answer both formally and with the aid of a diagram.

(d) Now assume that the LM curve is subject to stochastic shocks but the IS curve is deterministic (i.e. now $U_t \equiv 0$ for all periods and $V_t \sim N(0, \sigma_V^2)$). Show that the rational policy maker will now choose the interest rate instrument. Illustrate your answer both formally and with the aid of a diagram.

(e) Use the general stochastic model, with both U_t and V_t non-zero, and derive the value of the loss function (3) under the two monetary instruments. Show that the money supply instrument is preferred to the interest instruments if the following condition holds:

$$\frac{\alpha_1^2\sigma_V^2 + \beta_2^2\sigma_U^2}{(\alpha_1\beta_1 + \beta_2)^2} < \sigma_U^2.$$

Explain your result intuitively.

Question 2

The supply of goods is determined by:

$$y = \bar{y} + \alpha(\pi - \pi^e) + \epsilon, \tag{1}$$

where y is output, $\bar{y}$ is full employment output, π is inflation, π^e is expected inflation, and ϵ is a stochastic disturbance term with zero mean ($E\epsilon = 0$) and constant variance ($E\epsilon^2 = \sigma^2$). The preferences of citizen i are represented by:

$$\Omega_i = -\frac{1}{2}(y - y^*)^2 - \frac{\beta_i}{2}\pi^2, \tag{2}$$

where y^* is the optimal output level (from the perspective of all citizens). Each citizen tries to attain a minimum level of Ω_i, so equation (2) can be interpreted as a "regret function" stating the welfare costs associated with being away from the optimum ($y = y^*, \pi = 0$).

(a) Interpret these two equations.

(b) "Just like Ulysses, Wim Duisenberg should tie himself to the mast of a zero-inflation rule." Discuss this proposition in the light of the literature on "rules versus discretion." Assume in this part of the question that the preferences of the population are homogeneous (so that $\beta_i = \beta$). Explain the importance of the quantity $\bar{y} - y^*$ to your conclusion.

(c) Now assume that the population features heterogeneous preferences. Assume furthermore that there is asymmetric information. In particular, when the wages are set, the realization of the supply shock, ϵ, is unknown. The central banker, on the other hand, is assumed to know this realization when he/she sets monetary policy and determines the inflation rate. Explain why a majority decision will lead to the appointment of a central banker who is more "right-wing" than the population itself.

Question 3

[Monetary policy of the ECB] Assume that the economy is described by the following model:

$$y_t = \bar{y} + \alpha(\pi_t - E_{t-1}\pi_t) + e_t, \tag{1}$$

$$y_t = \bar{y} - \beta(R_t - E_t\pi_{t+1}) + u_t, \tag{2}$$

$$m_t - p_t = y_t - \gamma R_t + v_t, \tag{3}$$

where y_t is production, $\bar{y}$ is full-employment output (constant), m_t is the nominal money supply, p_t is the price level, π_t is the inflation rate, R_t is the nominal interest rate, and e_t, u_t and v_t are independent white noise stochastic processes. As usual, E_t denotes the conditional expectations operator based on period-t information. Assume that the loss function of the European central bank (ECB) is given by:

$$\Omega_t \equiv E_t\left[\lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2\right], \tag{4}$$

where $\bar{\pi}$ is an exogenously given target inflation rate ($\bar{\pi} > 0$). **CHECK Walsh.**

(a) Is the policy ineffectiveness proposition (PIP) valid in this model. If so, why? If not, why not?

(b) Suppose that the ECB wants to conduct monetary policy by controlling the money supply. Formulate the money supply rule in an explicit equation. State all the assumptions that you make.

(c) Compute the expected loss under both a pure interest rate rule and under a money supply rule. Under which circumstances is the one rule preferable to the other?

(d) Suppose that the ECB uses the general policy rule $\beta_t = \mu R_t$. Compute the optimal value of μ. Explain and interpret your results.

Question 4

[Sargent (1999, p. 23)] Formulate question on Kydland-Prescott under AEH. See also page 51, Tinbergen-Theil approach.

(a) ???

(b) ???.