Foundations of Modern Macroeconomics

Ben J. Heijdra
University of Groningen

February 2002

Problem set for Chapter 7

Question 1

In Chapter 1 we took an informal look at the way in which the representative household chooses its optimal supply of labour. In this question we look more precisely at this matter. In particular, we focus on the interaction between a progressive tax system and the labour supply decision. The representative household has a utility function which depends on consumption, C, and leisure, $1-N$, according to:

$$U = U(C, 1-N), \tag{1}$$

where the time endowment is unity and where N is labour supply. We assume that the utility function is homothetic. The budget restriction is:

$$PC = WN - T, \tag{2}$$

where T is the amount of tax paid by the household. Tax payments are assumed to depend on wage income according to:

$$T = t_M WN - \theta_0 P, \quad \theta_0 > 0, \quad 0 < t_M < 1. \tag{3}$$

According to (3), the household receives $\theta_0 P$ from the government (regardless of whether it works or not) but must pay taxes over its wage income equal to $t_M WN$.

(a) Compute the average tax rate ($t_A \equiv T/(WN)$) and the marginal tax rate ($dT/d(WN)$). Show that the tax system is indeed a progressive one.

(b) Show that in the optimal point, the marginal rate of substitution between leisure and consumption can be written as follows:

$$\frac{U_{1-N}}{U_C} = \frac{W(1-t_M)}{P}. \tag{4}$$

Explain intuitively why the *marginal* (rather than the *average*) tax rate features in this expression.

The substitution elasticity between consumption and leisure is formally defined as follows:

$$\sigma_{CM} = \frac{\%\text{ge change in } C/(1-N)}{\%\text{ge change in } U_{1-N}/U_C} \equiv \frac{d\log(C/(1-N))}{d\log(U_{1-N}/U_C)} > 0. \tag{5}$$

This coefficient measures the degree of substitutability of consumption and leisure in the utility function. If σ_{CM} is very high then substitution is quite easy, whereas substitution is difficult if σ_{CM} is low.

(c) Draw the indifference curves for the following three cases: $\sigma_{CM} = 0$, $\sigma_{CM} = 1$, and $\sigma_{CM} \to \infty$.

(d) Show that the two first-order conditions for utility maximization can be loglinearized as follows:

$$\tilde{C} + \left(\frac{N}{1-N}\right)\tilde{N} = \sigma_{CM}(\tilde{W} - \tilde{P} - \tilde{t}_M), \tag{6}$$

$$\tilde{P} + \tilde{C} = \tilde{W} - \tilde{t}_A + \tilde{N}, \tag{7}$$

where $\tilde{C} \equiv dC/C$, $\tilde{N} \equiv dN/N$, $\tilde{W} \equiv dW/W$, $\tilde{P} \equiv dP/P$, $\tilde{t}_M \equiv dt_M/(1-t_M)$, and $\tilde{t}_A \equiv dt_A/(1-t_A)$. Derive the loglinearized expressions for labour supply and consumption.

(e) Show (for the two cases $\sigma_{CM} = 0$ and $\sigma_{CM} = 1$) what happens to consumption and labour supply if the tax system is made more progressive. Assume that the average tax rate (evaluated at the initial optimum) remains unchanged. Explain your answers with the aid of diagrams.

Question 2

[*Indivisible labour*] Assume that jobs come in a fixed number of hours per day. In this setting, a household either has no work at all ($N = 0$) or works an exogenously determined number of hours per day ($N = \bar{N} < 1$). If the household does not work, it receives an unemployment benefit equal to B. Unemployment benefits are not taxed. The budget restriction of an unemployed household is then:

$$PC = B, \tag{1}$$

where P is the price level and C is consumption. An employed household has the usual budget restriction:

$$PC = W\bar{N}(1-t), \tag{2}$$

where t is the (constant) tax rate and W is the nominal wage. Assume that there is a linkage between the unemployment benefit is proportional to the *after-tax* income of the employed households.

$$B = \gamma W \bar{N}(1-t), \tag{3}$$

where γ is the so-called *replacement rate* $(0 < \gamma < 1)$. The utility function of household i is given by:

$$U^i(C, 1-N) \equiv C^\alpha (1-N)^{\beta_i}, \tag{4}$$

where $\alpha > 0$ and $\beta_i \geq 0$.

(a) Derive the labour supply decision for household i. Show that it depends on the magnitude of β_i. (*Hint*: do not differentiate anything.)

(b) Assume that the population size is Z and that the β_i's are distributed uniformly over the interval $[0, \bar{\beta}]$. Show that the replacement rate exerts a negative influence on aggregate labour supply in this economy.

(c) Assume now that the unemployment benefit is proportional to *gross* wage income, i.e. $B = \gamma W \bar{N}$. Assume furthermore that $\gamma < 1 - t$. Redo part (a) and derive the effect on aggregate labour supply of a higher tax rate.

Question 3

(a) Provide three reasons why it may be advantageous for a firm to pay its workers a wage in excess of the market clearing wage.

(b) Explain why unemployment is a "necessary evil" for firms to get a well-disciplined labour force in the Shapiro-Stiglitz (1984) model. Is unemployment voluntary or involuntary in this model?

References

Shapiro, C. and Stiglitz, J. E. (1984). Equilibrium unemployment as a worker discipline device. *American Economic Review*, 74:433–444.