Foundations of Modern Macroeconomics

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January 2001

# Problem set for Chapter 6

The questions with a star (★) are difficult.

## Question 1

Use the theory of "tax smoothing" to answer the following questions.

(a) Explain what we mean by the Golden Rule of Public Finance. Explain the pros and cons of that rule.

(b) Explain how a temporary increase in government spending must be financed.

(c) Explain how a permanent increase in public spending should be financed.

(d) Use the model to demonstrate what happens to the time paths of taxes and public debt if the government consists of myopic ("short-sighted") politicians. (*Hint*: analyse the effects of a political rate of time preference, $\rho_G$, that is higher than the market rate of interest, $r$.

## Question 2

Carefully explain why Ricardian Equivalence Theorem is invalid if any of the following assumptions are made. (Be precise and concise and use figures if they facilitate your argument.)

(a) Labour supply is endogenous.

(b) Households have only limited access to capital markets.

(c) New generations of households are born.

(d) Households are risk averse, there is a tax on income, and future income is uncertain.

## Question 3

Use the two-period model of the representative household discussed extensively in the book.

$$V = U(C_1) + \beta U(C_2), \tag{1}$$

$$A_1 = (1+r)A_0 + (1-t_1)Y_1 - C_1, \tag{2}$$

$$A_2 = (1+r)A_1 + (1-t_2)Y_2 - C_2, \tag{3}$$

where $V$ is life-time utility, $\beta \equiv 1/(1+\rho)$ is the rate of felicity discounting due to time preference, $\rho$ is the rate of time preference, $C_t$ is consumption in period $t$ $(= 1, 2)$ of the agent's life, $Y_t$ is the (exogenous) income in period $t$, and $A_t$ is financial assets possessed by the household. Assume that the household saves in the first period of life in order to enjoy a pleasant retirement in the second period of life. Assume furthermore that the periodic utility (or "felicity") function, $U(.)$, takes the following iso-elastic form:

$$U(C_t) \equiv \frac{C_t^{1-1/\sigma} - 1}{1 - 1/\sigma}, \qquad \sigma > 0,\ \sigma \neq 1. \tag{4}$$

(a) Interpret the model and derive the lifetime budget equation, Explain what you assume about $A_2$.

(b) Introduce the government and demonstrate Ricardian equivalence.

(c) Compute the expressions for optimal consumption and savings plans (i.e. $C_1$, $C_2$, and $S_1 \equiv A_1 - A_0$). Show that your expressions are the same as the ones in the book if and only if $\sigma$ is equal to unity.

(d) Assume that there is a broad income tax (which also taxes interest income). Show that Ricardian equivalence fails to hold in this case. Why is this the case?

(e) Redo part (c), taking into account the broad income tax of part (d). Show how consumption and saving depend on the income tax rate. Decompose the results for consumption in terms of the income effect, substitution effect, and the human wealth effect.