Foundations of Modern Macroeconomics

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Problem set for Chapter 4

The questions with a star (★) are difficult.

Question 1

Assume that there are two investment instruments: very *short-term bonds* and *perpetuities* (bonds with infinite term to maturity). The short-term bonds carry an interest rate of R_S whilst the perpetuities carry a coupon payment of unity and have an internal rate of return equal to R_L.

(a) Derive the condition for the efficient term structure of interest rates.

(b) Assume that the short-term interest rate is initially equal to R_S^0. Show the dynamic effects on the long-term interest rate, R_L, of an anticipated increase of the short-term interest rates in the future. As in the book, let t_A represent the time at which the news about the shock is received by the agents (the "announcement" time) and let t_I be the time at which the shock actually happens (the "implementation" time). The short-term interest rate is thus equal to R_S^0 for $t_A \leq t < t_I$ and equal to R_S^1 for $t > t_I$). Illustrate your answer with the aid of a phase diagram.

(c) Show what happens to the long-term interest rate when some time after t_A but before t_I, it becomes clear to the agents that the expected interest rate hike will not happen (and that the interest rate thus will remain equal to R_S^0 after t_I).

Question 2

Consider a closed economy featuring an efficient term structure of interest rates and slow price adjustment:

$$
\begin{aligned}
y &= -\sigma R_L + g, \qquad \sigma > 0, && (1)\\
m - p &= -\lambda R_S + \gamma y, \qquad \lambda, \gamma > 0, && (2)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (3)\\
R_L - \frac{\dot{R}_L}{R_L} &= R_S, && (4)
\end{aligned}
$$

where y is actual output, R_L is the long-term interest rate, g is an index of fiscal policy, m is the nominal money supply, p is the price level, R_S is the short-term interest rate, and $\bar{y}$ is full employment output. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{R}_L \equiv dR_L/dt$.

(a) Interpret the equations of the model.

(b) Show that an unanticipated and permanent budgetary expansion (rise in g) leads to an immediate increase in the long-term interest rate. Show what happens (at impact, during transition, and in the long run) to output, the price level, and the short-term interest rate. Illustrate your answers in an impulse-response diagram.

(c) Show that an anticipated and permanent budgetary expansion (a future increase in g) will cause a recession at first and will only stimulate the economy further into the future. Show what happens (at impact, during transition, and in the long run) to output, the price level, and the short-term interest rate. Illustrate your answers in an impulse-response diagram.

(d) What happens to the long-term interest rate if the anticipated budgetary expansion (studied in part (c)) does not take place? (At implementation time, t_I, the government announces that it will keep g unchanged). Illustrate your answers in an impulse-response diagram.

(e) ★ Assume that the *real* (rather than the nominal) long-term interest rate, $r_L \equiv R_L - \dot{p}$, features in equation (1). Assume furthermore that $0 < \sigma\phi < 1$. Study the (impact, transitional, and long-run) effects of an unanticipated and permanent technology shock (an increase in $\bar{y}$). Illustrate your answers in an impulse-response diagram.

Question 3

[*The Dornbusch model*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$
\begin{aligned}
y &= -\eta R + g + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta < 1, && (1)\\
m - p &= y - \lambda R, \qquad \lambda > 0, && (2)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (3)\\
R &= R^* + \dot{e}, && (4)
\end{aligned}
$$

where y is actual output, R is the domestic interest rate, g is an index for fiscal policy, e is the nominal exchange rate, p^* is the exogenous foreign price level, p is the domestic price level, m is the nominal money supply, $\bar{y}$ is full employment output, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$.

(a) Interpret the equations of the model.

(b) Suppose that the economy operates under a system of *fixed exchange rates* ($e = \bar{e}$). What are the endogenous variables? What is the coefficient of monetary accommodation (i.e. $\partial m/\partial p$) in this model? Derive the (impact, transitional, and long-term) effects of an expansionary fiscal policy (an increase in g).

(c) Now assume that the economy operates under a system of *flexible exchange rates*. Derive the model's phase diagram for the nominal exchange rate, e, and the domestic price level, p.

(d) Derive the (impact, transitional, and long-term) effects of an unanticipated and permanent expansionary fiscal policy.

(e) Show that under flexible exchange rates an unanticipated and permanent increase in the money supply leads to overshooting of the exchange rate in the short-term.

(f) Derive the (impact, transitional, and long-term) effects of an anticipated and permanent increase in the money supply.

(g) Show how your answer to parts (e) and (f) change if domestic prices are perfectly flexible, i.e. if $\phi \to \infty$ in equation (3).

Question 4

[*Turnovsky (1979)*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$
\begin{aligned}
y &= -\eta R + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta < 1, && (1)\\
m - p &= y - \lambda R, \qquad \lambda > 0, && (2)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (3)\\
R &= R^* + \dot{e}, && (4)
\end{aligned}
$$

where y is actual output, R is the domestic interest rate, e is the nominal exchange rate, p^* is the exogenous foreign price level, p is the domestic price level, m is the nominal money supply, $\bar{y}$ is (exogenous) full employment output, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$. Assume that the policy maker adopts the following policy rule for the nominal money supply:

$$
m - \bar{m} = -\mu(e - \bar{e}), \qquad \mu \gtreqless 0, \tag{5}
$$

where $\bar{m}$ is the exogenous component of money supply, $\bar{e}$ is the equilibrium exchange rate, and μ is a policy parameter.

(a) Interpret the equations of the model. Which are the endogenous and which are the exogenous variables? Explain why the policy rule embodies "leaning against the wind" if $\mu > 0$. What do we mean by "leaning with the wind"?

(b) Show that the model is saddle-point stable provided $1 + \mu \geq 0$. Illustrate the phase diagram of the model for the case where the policy maker engages strongly in "leaning against the wind" (so that $\eta\mu > \lambda\delta$).

(c) Assume that $\eta\mu = \lambda\delta$. Derive the (impact, transitional, and long-term) effects of an unanticipated and permanent increase in the foreign price level, p^*. Why is there no transitional dynamics in this case?

Question 5

[*Buiter & Miller (1981, 1982)*] Consider the following model of a small open economy featuring perfect capital mobility and sluggish price adjustment.

$$
\begin{aligned}
y &= -\eta\,[R - \dot{p}_C] + \delta(e + p^* - p), \qquad \eta > 0, \quad 0 < \delta, \gamma < 1, && (1)\\
m - p_C &= y - \lambda R, \qquad \lambda > 0, && (2)\\
p_C &\equiv \alpha p + (1-\alpha)(e + p^*), \qquad 0 < \alpha < 1, && (3)\\
\dot{p} &= \phi(y - \bar{y}), \qquad \phi > 0, && (4)\\
R &= R^* + \dot{e}, && (5)
\end{aligned}
$$

where y is actual output, R is the domestic interest rate, p_C is the price index for goods used in the domestic economy, e is the nominal exchange rate, p^* is the exogenous (and constant) foreign price level, p is the price of domestically produced goods, $\bar{y}$ is full employment output, m is the (constant) nominal money supply, and R^* is the exogenous world interest rate. All variables, except the two interest rates, are measured in logarithms. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{p} \equiv dp/dt$ and $\dot{e} \equiv de/dt$. We define the auxiliary variables $l \equiv m - p$ (measure of "liquidity") and $c \equiv e + p^* - p$ (index for "competitiveness").

(a) Interpret the equations of the model. Which are the endogenous and which are the exogenous variables?

(b) Derive the dynamical system for this model in terms of l and c. Show that the model is saddle-point stable provided $\lambda + \alpha\eta(1 - \lambda\phi) > 0$. Which is the predetermined variable? Which is the jumping variable?

(c) Construct the phase diagram for the model. (Many saddle-point stable slope configurations are possible. Assume that the $\dot{c} = 0$ line is upward sloping and $\dot{l} = 0$ line is downward sloping. State the corresponding parameter assumptions.)

(d) Derive the (impact, transitional, and long-term) effects on c and l of an unanticipated and permanent increase in the money supply. Does overshooting of the exchange rate occur in this model.

Question 6

[*Gertler (1979a-b)*]

(a)

(b)

(c)

(d)

(e)

(f)

Question 7

[*Murphy (1989)*]

(a)

(b)

(c)

(d)

(e)

(f)

Question 8

[*Sachs (1980)*]

(a)

(b)

(c)

(d)

(e)

(f)

Question 9

[*Nickell (1986)*] Adjustment costs on labour.

(a)

(b)

(c)

(d)

(e)

(f)

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