Foundations of Modern Macroeconomics

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Problem set for Chapter 3

The questions with a star (★) are difficult.

Question 1

[*A variation on the Muth model*] Assume that the market for a particular commodity is described by the following model:

$$Q_t^D = a_0 - a_1 P_t + V_t, \qquad a_1 > 0, \tag{1}$$
$$Q_t^S = b_0 + b_1 P_t^e + U_t, \qquad b_1 > 0, \tag{2}$$
$$Q_t^D = Q_t^S \;\; [\equiv Q_t], \tag{3}$$

where Q_t^D is demand, P_t is the *actual* (market clearing) price, V_t is a stochastic shock term affecting demand, Q_t^S is supply, P_t^e is the *expected* price (i.e. the price that suppliers expect to hold in period t), U_t is a stochastic term affecting supply, and Q_t is the actual (market clearing) quantity traded in the market. It is assumed that the two stochastic shock terms, U_t and V_t, are independent and normally distributed white noise terms (there is no correlation between these terms and both terms display no autocorrelation): $U_t \sim N(0, \sigma_U^2)$ and $V_t \sim N(0, \sigma_V^2)$. Assume that expectations are formed according to the adaptive expectations hypothesis (AEH):

$$P_t^e = P_{t-1}^e + \lambda \left[P_{t-1} - P_{t-1}^e \right], \qquad \lambda > 0, \tag{4}$$

where λ regulates the speed at which expectations adjust.

(a) Interpret the equations of the model.

(b) Derive the stability condition for the model. Explain both formally and intuitively what you mean by stability in this model.

Next we replace the AEH assumption by the assumption of rational expectations (the REH). Instead of (4) we use:

$$P_t^e = E_{t-1}P_t, \tag{5}$$

where the expectations operator, E_{t-1}, denotes that agents form expectations using information dated period $t-1$ and earlier. The information set of the agents thus includes P_{t-1}, P_{t-2}, etc., Q_{t-1}, Q_{t-2}, etc., as well as knowledge about the structure and parameters of the model.

(c) Derive expressions for equilibrium output and the price level for this case. Is the model stable? Explain.

Question 2

Suppose that the stochastic process for Y_t is given by:

$$Y_t = \alpha_0 + \alpha_1 E_t Y_{t+1} + U_t, \qquad 0 < \alpha_1 < 1, \tag{1}$$

where α_0 and α_1 are constants, E_t is the conditional expectation (based on the period-t information set), and U_t is a stochastic shock term. We assume that this shock term features first-order autocorrelation:

$$U_t = \theta U_{t-1} + V_t, \qquad 0 < \theta < 1, \tag{2}$$

where θ is a constant and V_t is a white noise error term (with $EV_t = 0$ and $EV_t^2 = \sigma^2$).

(a) Can you think of an economic example for which an expression like (1) arises naturally?

(b) Compute the rational expectations solution for Y_t. *Hint*: use the method of undetermined coefficients by trying a candidate solution of the form $Y_t = \pi_0 + \pi_1 U_t$ and computing those values for π_0 and π_1 for which the candidate solution is the correct solution.

(c) Compute the asymptotic variance of Y_t. Show that it depends positively on the autocorrelation parameter θ.

Question 3

[*The Cagan (1956) model*] Phillip Cagan (one of Milton Friedman's friends) was very interested in the phenomenon of hyperinflation. He suggested that hyperinflation could be studied by looking at the demand for money equation. Assume that money demand, expressed in loglinear format, is given by:

$$m_t - p_t = \gamma - \alpha\left[\rho + E_t p_{t+1} - p_t\right] + u_t, \quad \alpha > 0, \tag{1}$$

where m_t is the money supply, p_t is the price level, u_t is a stochastic (white noise) error term, α and γ are constants, and ρ is the *real* interest rate (assumed to be constant). All variables are expressed in terms of logarithms. Assume that the money supply process is described by:

$$m_t = \mu_0 + \mu_1 m_{t-1} + e_t, \quad 0 < \mu_1 < 1, \tag{2}$$

where e_t is a (white noise) error term and μ_0 and μ_1 are parameters.

(a) Explain why we can interpret equation (1) as a money demand equation.

(b) Compute the rational expectations solution for the price level, p_t. Show that it can be written as a linear function of a constant, m_t and u_t.

Question 4

[*Fiscal policy under rational expectations*] Consider the following log-linear model of a closed economy featuring rational expectations.

$$y_t = a_0 - a_1\left[R_t - E_{t-1}(p_{t+1} - p_t)\right] + a_2 g_t + v_{1t}, \tag{1}$$

$$m_t - p_t = c_0 + c_1 y_t - c_2 R_t + v_{2t}, \tag{2}$$

$$y_t = \alpha_0 + \alpha_1(p_t - E_{t-1}p_t) + \alpha_2 y_{t-1} + u_t, \tag{3}$$

$$g_t = \gamma_0 + \gamma_1 g_{t-1} + \gamma_2 y_{t-1} + e_t, \tag{4}$$

$$m_t = \mu_0 + m_{t-1}, \tag{5}$$

where y_t is output, R_t is the nominal interest rate, p_t is the price level, g_t is an index for fiscal policy, and m_t is the money supply. Furthermore, v_{1t}, v_{2t}, u_t, and e_t are stochastic (white noise) shock terms affecting the various equations of the model. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $v_{1t} \sim N\left(0, \sigma_{v_1}^2\right)$, $v_{2t} \sim N\left(0, \sigma_{v_2}^2\right)$, $u_t \sim N\left(0, \sigma_u^2\right)$, and $e_t \sim N\left(0, \sigma_e^2\right)$. All variables, except the nominal interest rate R_t, are measured in logarithms. The parameters of the model satisfy: $a_1 > 0$, $a_2 > 0$, $c_1 > 0$, $c_2 > 0$, $\alpha_1 > 0$, $0 < \alpha_2 < 1$, $0 < \gamma_1 < 1$, and $\mu_0 > 0$.

(a) Interpret the equations of the model.

(b) Derive the expression for the AD curve for this model. Denote the coefficients for the AD curve by β_0, β_1, etcetera, and define the (composite) shock term entering this equation by v_t. State the stochastic properties of v_t.

(c) Find the rational expectations solution for output.

(d) Is fiscal policy ineffective in this model? What does your conclusion imply about the validity of the policy ineffectiveness proposition (PIP)?

(e) ★ Derive the rational expectations solution for the equilibrium price, p_t. Show that the price level moves one-for-one with the money stock.

Question 5

Assume that the labour market is described by the following loglinear model:

$$n_t^D = \alpha_0 - \alpha_1(w_t - p_t) + \alpha_2 n_{t-1} + u_{1t}, \qquad \alpha_1 > 0,\ 0 < \alpha_2 < 1, \tag{1}$$

$$n_t^S = \beta_0 + \beta_1(w_t - E_{t-1}p_t) + u_{2t}, \qquad \beta_1 > 0, \tag{2}$$

$$n_t^D = n_t^S \ [= n_t], \tag{3}$$

where n_t^D is the demand for labour, w_t is the nominal wage rate, p_t is the price level, n_t^S is the supply of labour, and n_t is equilibrium employment. The shock terms in labour demand and supply are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $u_{1t} \sim N\left(0, \sigma_{u_1}^2\right)$ and $u_{2t} \sim N\left(0, \sigma_{u_2}^2\right)$. All variables are measured in logarithms. Expectations are formed according to the rational expectations hypothesis and E_{t-1} represents the objective expectation conditional upon the information set available in period $t-1$.

(a) Interpret the equations of the model. Why does the lagged employment term (n_{t-1}) feature in the labour demand equation? What do we assume about the income and substitution effects in labour supply?

(b) Assume that the short-run production function can be written, in loglinear terms, as $y_t = \gamma_0 + \gamma_1 n_t$, where y_t is aggregate output and $0 < \gamma_1 < 1$. Show that the labour market model (1)-(3) in combination with the production function gives rise to the Lucas supply curve (LSC). State and explain any stability conditions that may be required.

(c) Derive the stochastic properties of the shock term of the LSC determined in part (c).

Question 6

[*Liquidity trap, McCallum (1983)*] Consider the following log-linear model of a closed economy featuring rational expectations.

$$y_t - \bar{y} = \alpha_1(p_t - E_{t-1}p_t) + \alpha_2(y_{t-1} - \bar{y}) + u_t, \tag{1}$$

$$y_t = a_0 - a_1\left[R_t - E_t(p_{t+1} - p_t)\right] + v_{1t}, \tag{2}$$

$$m_t - p_t = y_t - c_1\left[R_t - R^{MIN}\right] + v_{2t}, \tag{3}$$

where y_t is actual output, $\bar{y}$ is full employment output (assumed to be constant), p_t is the price level, R_t is the nominal interest rate, m_t is the money supply, R^{MIN} is minimum interest rate (which is attained in the liquidity trap), and u_t, v_{1t}, and v_{2t} are stochastic (white noise) shock terms. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $u_t \sim N\left(0, \sigma_u^2\right)$, $v_{1t} \sim N\left(0, \sigma_{v_1}^2\right)$, and $v_{2t} \sim N\left(0, \sigma_{v_2}^2\right)$. All variables except R and R^{MIN} are measured in logarithms. To keep the model as simple as possible we assume that the money supply is constant over time, i.e. $m_t = m$. The parameters of the model satisfy: $\alpha_1 > 0$, $0 < \alpha_2 < 1$, $a_1 > 0$, and $c_1 > 0$.

(a) Interpret the equations of the model.

(b) Derive the expression for the AD curve and denote its coefficients by β_0, β_1, and β_2 and its shock term by v_t. Explain what happens to the AD curve if the economy finds itself in a Keynesian liquidity trap.

(c) Compute the rational expectations solution for output and the price level. Hint: use the method of undetermined coefficients and use trial solutions of the form $y_t = \pi_0 + \pi_1 y_{t-1} + \pi_2 u_t + \pi_3 v_t$ and $p_t = \omega_0 + \omega_1 y_{t-1} + \omega_2 u_t + \omega_3 v_t$.

(d) Show that the model is internally inconsistent, and the price level is indeterminate, if the economy is in the liquidity trap. Demonstrate that Pigou's suggestion, that consumption depends on real money balances, leads to a consistent model again.

Question 7

Consider the following log-linear model of a closed economy featuring rational expectations.

$$y_t = a_0 - a_1 r_t + a_2(m_t - p_t) + v_{1t}, \tag{1}$$

$$m_t - p_t = c_0 + c_1 y_t - c_2 R_t + v_{2t}, \tag{2}$$

$$y_t = \alpha_0 + \alpha_1(p_t - E_{t-1}p_t) + \alpha_2 y_{t-1} + \alpha_3 k_t + u_t, \tag{3}$$

$$k_{t+1} = \gamma_1 k_t + \gamma_2 r_t, \tag{4}$$

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t, \tag{5}$$

$$r_t \equiv R_t - E_{t-1}(p_{t+1} - p_t), \tag{6}$$

where y_t is output, r_t is the *real* interest rate, m_t is the money supply, p_t is the price level, R_t is the nominal interest rate, and k_t is the capital stock. Furthermore, v_{1t}, v_{2t}, u_t and e_t are stochastic (white noise) shock terms affecting the various equations of the model. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $v_{1t} \sim N\left(0, \sigma_{v_1}^2\right)$, $v_{2t} \sim N\left(0, \sigma_{v_2}^2\right)$, $u_t \sim N\left(0, \sigma_u^2\right)$, and $e_t \sim N\left(0, \sigma_e^2\right)$. All variables, except the various interest rates (r_t and

R_t) are measured in logarithms. The parameters of the model satisfy: $a_1 > 0$, $a_2 \geq 0$, $c_1 > 0$, $c_2 > 0$, $\alpha_1 > 0$, $0 < \alpha_2 < 1$, $\alpha_3 > 0$, $0 < \gamma_1 < 0$, $\gamma_2 < 0$, and $0 < \mu_1 \leq 1$.

(a) Interpret the equations of the model.

(b) Derive the expression for the AD curve and denote its coefficients by β_0, β_1, and β_2 and its shock term by v_t.

(c) Show that the expectational gap, $p_t - E_{t-1}p_t$, only depends on the shock terms of AD, AS, and the money supply rule (i.e. v_t, u_t, and e_t) but not on the parameters of the money supply rule (i.e. μ_0, μ_1, and μ_2).

(d) ★ Characterize the rational expectations solution for the model and show that the *policy ineffectiveness proposition* (PIP) does not hold, unless the real balance effect in the IS curve is absent.

McCallum (1980) argues that stabilization policy is not about stabilizing actual output (y_t) itself but rather about stabilizing the deviation of output relative to *capacity* output ($y_t - \bar{y}_t$). He suggests that in the context of the present model, capacity output should be measured as follows:

$$\bar{y}_t = \alpha_0 + \alpha_2 \bar{y}_{t-1} + \alpha_3 k_t + u_t. \tag{7}$$

(e) Can you give a rationale for the expression in (7)?

(f) Show that the reinterpreted PIP holds in the model because the path for $y_t - \bar{y}_t$ does not depend on the parameters of the money supply rule.

Question 8

Consider the following log-linear model of a closed economy featuring rational expectations.

$$y_t = a_0 - a_1 r_t + a_2(m_t - p_t) + v_{1t}, \tag{1}$$

$$m_t - p_t = c_0 + c_1 y_t - c_2 R_t + v_{2t}, \tag{2}$$

$$y_t = \alpha_0 + \alpha_1(p_t - E_{t-1}p_t) + \alpha_2 y_{t-1} + \alpha_3 r_t + u_t, \tag{3}$$

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t, \tag{4}$$

$$r_t \equiv R_t - E_{t-1}(p_{t+1} - p_t), \tag{5}$$

where y_t is output, r_t is the *real* interest rate, m_t is the money supply, p_t is the price level, and R_t is the nominal interest rate. Furthermore, v_{1t}, v_{2t}, u_t and e_t are stochastic (white noise) shock terms affecting the various equations of the model. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $v_{1t} \sim N\left(0, \sigma_{v1}^2\right)$, $v_{2t} \sim N\left(0, \sigma_{v2}^2\right)$, $u_t \sim N\left(0, \sigma_u^2\right)$, and $e_t \sim N\left(0, \sigma_e^2\right)$. All variables, except the various interest rates (r_t and R_t) are measured in logarithms. The parameters of the model satisfy: $a_1 > 0$, $a_2 \geq 0$, $c_1 > 0$, $c_2 > 0$, $\alpha_1 > 0$, $0 < \alpha_2 < 1$, and $\alpha_3 > 0$.

(a) Interpret the equations of the model.

(b) Show that the expectational gap, $p_t - E_{t-1}p_t$, only depends on the shock terms of IS, LM, AS, and the money supply rule (i.e. v_{1t}, v_{2t}, u_t, and e_t) but *not* on the parameters of the money supply rule (i.e. μ_0, μ_1, and μ_2).

(c) ★ Characterize the rational expectations solution for the model and show that the *policy ineffectiveness proposition* (PIP) does not hold, unless the real balance effect in the IS curve is absent.

(d) Define capacity output as $\bar{y}_t = \alpha_0 + \alpha_2 \bar{y}_{t-1} + \alpha_3 r_t + u_t$ and show that monetary policy cannot be used to stabilize the output gap, $y_t - \bar{y}_t$.

Question 9 ★

[*Contemporaneous information*] Consider the following model of a closed economy featuring rational expectations.

$$y_t = \alpha_0 + \alpha_1(p_t - E_{t-1}p_t) + u_t, \tag{1}$$

$$y_t = \beta_0 + \beta_1(m_t - p_t) + \beta_2 E_t(p_{t+1} - p_t) + v_t, \tag{2}$$

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t, \tag{3}$$

where y_t is output, p_t is the price level, m_t is the money supply, and u_t, v_t and e_t are stochastic (white noise) shock terms affecting the various equations of the model. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $u_t \sim N\left(0, \sigma_u^2\right)$, $v_t \sim N\left(0, \sigma_v^2\right)$, and $e_t \sim N\left(0, \sigma_e^2\right)$. All variables are measured in logarithms. The parameters of the model satisfy: $\alpha_1 > 0$, $\beta_1 > 0$, $\beta_2 > 0$, and $0 < \mu_1 < 1$. The key thing to note is that agents are assumed to possess current aggregate information when estimating the future inflation rate, i.e. E_t (rather than E_{t-1}) features in equation (2).

(a) Interpret the equations of the model.

(b) Find the rational expectations solution for output and show that the *policy ineffectiveness proposition* (PIP) does not hold. Explain why this is the case.

(c) Compute the asymptotic variance of output. Should the government pursue a countercyclical monetary policy? Explain the intuition behind your results.

Question 10 ★

[*Sticky prices*] Consider the following model of a closed economy featuring rational expectations.

$$p_t = \bar{p}_t - (1-\delta)(\bar{p}_t - E_{t-1}\bar{p}_t), \tag{1}$$

$$y_t = \beta_0 + \beta_1(m_t - p_t) + \beta_2 E_{t-1}(p_{t+1} - p_t) + v_t, \tag{2}$$

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t, \tag{3}$$

$$\bar{y}_t = \zeta_0 + \bar{y}_{t-1} + u_t, \tag{4}$$

where p_t is the actual price level, $\bar{p}_t$ is the *equilibrium* price level (see below), y_t is actual output, m_t is the money supply, and $\bar{y}_t$ is full employment output. In equation (1), $\bar{p}_t$ is the price for which actual output, y_t, equals its exogenously given full employment level, $\bar{y}_t$. As usual, v_t, u_t and e_t are stochastic (white noise) shock terms affecting the various equations of the model. These terms are independent from each other, feature no autocorrelation, and are normally distributed with mean zero and constant variance, i.e. $v_t \sim N\left(0, \sigma_v^2\right)$, $e_t \sim N\left(0, \sigma_e^2\right)$, and $u_t \sim N\left(0, \sigma_u^2\right)$. All variables are measured in logarithms. The parameters of the model satisfy: $0 \leq \delta \leq 1$, $\beta_1 > 0$, $\beta_2 > 0$, and $0 < \mu_1 < 1$.

(a) Provide a brief interpretation of these equations.

(b) Consider the special case of the model for which $\delta = 1$. Compute the rational expectations solutions for output and the price level. Show that the policy ineffectiveness proposition holds.

(c) Now use the general case of the model, with $0 < \delta < 1$, and solve for the rational expectations solution for output and the price level. Can the government pursue a countercyclical monetary policy? Explain the intuition behind your results.

(d) Compute the asymptotic variance of the output gap, $y_t - \bar{y}_t$. Does the degree of price stickiness, as parameterized by δ, increase or decrease output fluctuations in the economy? Explain.