Foundations of Modern Macroeconomics

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December 2001

Problem set for Chapter 2

The questions with a star (★) are difficult.

Question 1

[*Stability of the Keynesian Cross model*] Consider the following Keynesian cross model for the closed economy:

$$Y = C + I + G, \tag{1}$$
$$C = C_0 + c(Y - T), \qquad 0 < c < 1, \tag{2}$$
$$I = I_0 + \dot{Z}, \tag{3}$$
$$\dot{Y} = -\gamma \dot{Z}, \qquad \gamma > 0, \tag{4}$$

where Y is output, C is consumption, I is *actual* investment, G is government consumption, T is taxes, I_0 is *planned* investment, and Z is the stock of inventories. A variable with a dot denotes that variables rate of change over time, i.e. $\dot{Z} \equiv dZ/dt$ and $\dot{Y} \equiv dY/dt$. C_0 and I_0 represent the exogenous parts of, respectively, consumption and investment, and c is the marginal propensity to consume. Assume that prices are fixed and that G and T are all exogenous.

(a) Interpret the equations of the model.

(b) Show that the model is stable. Illustrate your answer graphically by developing the phase diagram for the model.

(c) Show the effects over time on output, consumption, actual investment, and inventories, of a tax-financed increase in government consumption. Is the short-run output multiplier smaller or larger than the long-run output multiplier? Explain.

Question 2

[*Dynamics of foreign reserves*] Consider the following model of a small open economy with fixed prices ($P = P_0 = 1$ for convenience) operating under a regime of fixed exchange rates.

$$Y = C(Y) + I(R) + G + X(E, Y), \tag{1}$$

$$D + F = l(Y, R), \tag{2}$$

$$\dot{F} = X(E, Y) + KI(R - R^*), \tag{3}$$

where Y is output, C is consumption, I is investment, R is the domestic interest rate, G is government consumption, X is net exports, E is the exchange rate (domestic currency per unit of foreign currency), D is domestic credit (government bonds in the hands of the central bank), F is the stock of foreign exchange reserves (measured in units of the domestic currency), and KI is net capital inflows. As usual, a dot above a variable denotes that variable's time derivative, i.e. $\dot{F} \equiv dF/dt$. We make the usual assumptions regarding the partial derivatives of the various functions: $0 < C_Y < 1$, $X_Y < 0$, $I_R < 0$, $X_E > 0$, $l_Y > 0$, $l_R < 0$, and $KI_R > 0$.

(a) Interpret the equations of the model. What do we assume about the Marshall-Lerner condition? Which are the endogenous variables? Which are the exogenous variables?

(b) Derive the fundamental differential equation for the stock of foreign exchange reserves and show that the model is stable. Show that the speed of adjustment increases as the degree of capital mobility increases and illustrate your argument with the aid of a diagram.

(c) Derive the so-called BP curve, representing (R, Y) combinations for which the balance of payments is in equilibrium ($\dot{F} = 0$). Assume that the BP curve is flatter than the LM curve (when drawn in the usual diagram with R on the vertical axis and Y on the horizontal axis). Show that this is the case if the following condition holds: $X_Y l_R < l_Y KI_R$. Give an economic interpretation for this condition.

(d) Derive the impact, transitional, and long-run effects on the endogenous variables of an increase in government consumption. Assume that the condition mentioned in part (c) holds. Illustrate your answer with graphs and explain the economic intuition.

(e) Derive the impact, transitional, and long-run effects on the endogenous variables of monetary policy. Illustrate your answers with a graph and explain the intuition.

Question 3

[*The Blinder-Solow Model*] Consider a closed economy with fixed prices ($P = P_0 = 1$ for convenience), a given stock of capital ($K = \bar{K}$) and wealth effects in the demand for money and the consumption function:

$$\begin{aligned}
Y &= C + I + G, && (1)\\
C &= C(Y + B - T, A) \qquad 0 < C_{Y+B-T} < 1, \quad C_A > 0, && (2)\\
I &= I(R), \qquad I_R < 0, && (3)\\
T &= T_0 + t(Y + B), \qquad 0 < t < 1, && (4)\\
M &= l(Y, R, A), \qquad l_Y > 0, \quad l_R < 0, \quad 0 < l_A < 1, && (5)\\
A &\equiv \bar{K} + M + B/R, && (6)\\
G + B &= T + \dot{M} + (1/R)\dot{B}, && (7)
\end{aligned}$$

where Y is output, C is consumption, I is investment, G is government consumption, B is government debt, T is taxes, A is private wealth, R is the rate of interest, T_0 is the exogenous part of taxes, t is the marginal tax rate, $\bar{K}$ is the capital stock, and M is the money supply. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{M} \equiv dM/dt$ and $\dot{B} \equiv dB/dt$.

(a) Interpret the equations of the model.

(b) In the book we derive reduced-form expressions for output and the nominal interest rate which we write here in short-hand notation as $Y = AD(B, M, G)$ and $R = H(B, M, G)$. Draw IS-LM style diagrams to motivate the signs of AD_B, AD_M, AD_G, H_B, H_M and H_G. Explain the intuition behind your results.

(c) Compute the "balanced-budget" output multiplier for the case in which the additional government consumption is financed by means of additional taxes. Assume that the government adjusts T_0 in order to ensure that $dG = dT$. Show that the required change in T_0 satisfies $0 < dT_0/dG < 1$. Explain your results graphically.

(d) Is the multiplier obtained in part (c) larger or smaller than the Haavelmo multiplier derived in question 1(c) of Chapter 1. Explain any differences.

Question 4

Consider the following short-run dynamics in the closed-economy IS-LM model. It is assumed that the price level is fixed and (for convenience) has been normalized to unity ($P = 1$):

$$\begin{aligned}
\dot{R} &= \phi_1 \left[l(Y, R) - M \right], \ \ \phi_1 > 0, && (1)\\
\dot{Y} &= \phi_2 \left[C(Y - T) + I(R) + G - Y \right], \ \ \phi_2 > 0, && (2)
\end{aligned}$$

where Y is output, R is the interest rate, M is the money stock, C is consumption, T is taxes, I is investment, and G is government consumption. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{R} \equiv dR/dt$ and $\dot{Y} \equiv dY/dt$.

(a) Interpret these equations.

(b) Can you say something about the relative speeds of adjustment in the goods and financial markets?

(c) Use a two-dimensional phase diagram in order to derive the stability properties of this model and the qualitative nature of the transient adjustment paths for output and the interest rate associated with a fiscal expansion.

(d) ★ Derive the stability condition for this model mathematically.

Question 5

Some economists stress the importance of Ricardian equivalence, i.e. that bond finance or tax finance of a given stream of public spending is irrelevant for private consumption. The idea is that bond finance is perceived as postponed taxation, so that households save in order to provide for future taxation. A simple way to allow for this idea is to modify the Blinder-Solow model as follows:

$$C = C(Y^D, A), \quad 0 < C_{Y^D} < 1, \; C_A > 0, \tag{1}$$

$$Y^D \equiv Y + B - T - \dot{B}/R, \tag{2}$$

$$M/P = l(Y, R, A), \quad l_Y > 0, \; l_R < 0, \; 0 < l_A < 1, \tag{3}$$

$$A \equiv \bar{K} + M/P, \tag{4}$$

$$\dot{M} + \dot{B}/R = G + B - T, \tag{5}$$

$$T = T_0 + t(Y + B - \dot{B}/R), \quad 0 < t < 1, \tag{6}$$

$$Y = C + I(R) + G, \quad I_R < 0, \tag{7}$$

where C is consumption, Y^D is disposable income, A is household wealth, Y is output, B is government debt, T is taxes, R is the interest rate, M is the money stock, P is the fixed price level, $\bar{K}$ is the capital stock, G is government consumption, T_0 is the lump-sum tax, t is the marginal tax, and I is investment. As usual, a dot above a variables denotes that variable's time rate of change, i.e. $\dot{M} \equiv dM/dt$ and $\dot{B} \equiv dB/dt$.

(a) Interpret the consumption function and the definition of private wealth, A. Why does this modification not alter the conclusions regarding money financing?

(b) Show that under bond financing disposable income is simply income minus government spending and thus demonstrate that private consumption rises only with income minus government spending.

(c) Show that a debt-financed and a tax-financed rise in public spending have identical effects on output, consumption and the interest rate and, furthermore, that the short-run and long-run effects coincide.

(d) Show that the government debt explodes unless taxes rise strongly enough with government debt (or public spending is cut back severely enough as government debt explodes).

Question 6

Consider a Blinder-Solow model of a small open economy with an integrated capital market. Assume that the price level is fixed and has been normalized to unity ($P = 1$):

$$\begin{aligned}
C &= C(Y^D, A), \qquad 0 < C_{Y^D} < 1,\ C_A > 0, && (1)\\
Y^D &\equiv Y + B + F - T, && (2)\\
T &= T_0 + t(Y + B + F), \qquad 0 < t < 1, && (3)\\
M &= l(Y, R, A), \qquad l_Y > 0,\ l_R < 0,\ 0 < l_A < 1, && (4)\\
A &\equiv \bar{K} + M + (B + F)/R, && (5)\\
R &= R^*, && (6)\\
\dot{B}/R &= G + B - T - \dot{M}, && (7)\\
\dot{F}/R &= F + [Y - C - I(R) - G], \qquad I_R < 0, && (8)
\end{aligned}$$

where C is consumption, Y^D is disposable income, A is wealth, Y is output, B is government bonds, F denotes net foreign asset holdings of the country, T is taxes, t is the marginal tax rate, M is the money supply, R is the domestic interest rate, $\bar{K}$ is the fixed capital stock, R^* is the world interest rate, G is government consumption, and I is investment.

(a) Interpret the equations of this model. What are the endogenous variables? What are the exogenous variables?

(b) Can you say something about the effectiveness of fiscal policy and stability under money finance? Show that $l_A > R^* l_Y$ is a *sufficient* stability condition.

(c) Can you say something about the effectiveness of fiscal policy and stability under bond finance?

Question 7

Consider the following monetarist model with adaptive expectations:

$$\begin{aligned}
Mv &= PY, \qquad v > 0, && (1)\\
\pi &= \phi(Y - Y^*) + \pi^e, \qquad \phi > 0, && (2)\\
\dot{\pi}^e &= \zeta(\pi - \pi^e), \qquad \zeta > 0, && (3)\\
\mu &\equiv \dot{M}/M, && (4)
\end{aligned}$$

where M is the money supply, v is the velocity of circulation of the money supply (a constant), P is the price level, Y is output, π is the actual inflation rate ($\pi \equiv \dot{P}/P$), Y^* is full employment output, π^e is expected inflation, and μ is the growth rate in the money supply.

(a) Interpret these equations. What are the exogenous and what are the endogenous variables?

(b) Show that the reduced form of this model is given by:

$$\dot{Y}/Y = \mu - \pi^e - \phi(Y - Y^*), \tag{5}$$
$$\dot{\pi}^e = \zeta\phi(Y - Y^*). \tag{6}$$

(c) Demonstrate mathematically the stability of this model (i.e. prove that the two eigenvalues have negative real parts).

(d) Use a phase diagram to derive the transient and steady-state effects on output and inflation of a monetary disinflation (a cut in the monetary growth rate μ).

Question 8

Consider the following log-linear macroeconomic model of a closed economy featuring adaptive expectations:

$$y = \theta(m - p) + \psi\pi^e + \zeta g, \quad \theta > 0,\ \psi > 0,\ \zeta > 0, \tag{1}$$
$$\pi = \gamma(y - y^*) + \pi^e, \quad \gamma > 0, \tag{2}$$
$$\dot{\pi}^e = \lambda(\pi - \pi^e), \quad \lambda > 0, \tag{3}$$

where y is output, m is the money supply, p is the price level, π^e is expected inflation, g is an index for fiscal policy, $\pi \equiv \dot{p}$ is actual inflation, and y^* is full employment output. All variables are measured in logarithms and a dot above a variable denotes that variable's time rate of change. The endogenous variables are y, p, and π^e. The exogenous variables are g, m, and y^*. All parameters as well as y^* are assumed to be constant over time. The rate of nominal money growth is defined as $\mu \equiv \dot{m}$.

(a) Interpret the equations of the model.

(b) Show that in the short run (for a given expected inflation rate) the model has Keynesian features whilst it has classical features in the long run (with a variable expected inflation rate).

(c) Investigate the stability properties of the model by deriving a system of differential equations in π^e and y. Why is the model not automatically stable for all parameter values, as was the case for the monetarist model of question 8?

(d) Derive the impact, transitional, and long-run effects on output, the price *level*, actual inflation, and expected inflation, of an increase in the money growth rate. Assume that the parameters satisfy $\gamma(\theta-\psi\lambda)^2 > 4\lambda\theta$.

(e) Derive the impact, transitional, and long-run effects on output, the price *level*, actual inflation, and expected inflation, of an increase in the index of fiscal policy. *Hint*: do not forget that the fiscal impulse causes a positive impact effect on output! Make the same assumption as in part (d).

Question 9 ★

[*Samuelson (1939)*] Consider the Samuelson-Hansen discrete-time variant of the multiplier-accelerator model:

$$Y_t = C_t + I_t + G, \tag{1}$$

$$C_t = cY_{t-1}, \qquad 0 < c < 1, \tag{2}$$

$$I_t = v(C_t - C_{t-1}), \qquad v > 0, \tag{3}$$

where Y_t is output, C_t is consumption, I_t is investment, G is (time-invariant) government consumption, c is the marginal propensity to consume, and v is the investment acceleration coefficient (vc is thus the desired capital-output ratio).

(a) Interpret the equations of the model.

(b) Define the marginal propensity to save as $s \equiv (1-c)$. Show that the adjustment path in output after, say, a rise in public spending can be characterized by:

I	$v \le \frac{1-\sqrt{s}}{1+\sqrt{s}}$	stable monotonic adjustment
II	$\frac{1-\sqrt{s}}{1+\sqrt{s}} < v < \frac{1}{1-s}$	stable cyclical adjustment
III	$\frac{1}{1-s} < v < \frac{1+\sqrt{s}}{1-\sqrt{s}}$	explosive oscillations
IV	$v \ge \frac{1+\sqrt{s}}{1-\sqrt{s}}$	steady, monotonic explosion

Illustrate your answers by constructing a diagram with v on the horizontal axis and c on the vertical axis displaying the various qualitative modes of dynamic adjustment.

Question 10 ★

[*Automatic stabilizer. Based on Scarth (1988, p. 190)*] Consider the following Keynesian Cross model formulated in discrete time.

$$Y_t = C_t + I_t + \bar{G}, \tag{1}$$

$$C_t = c(1-t)Y_t, \qquad 0 < c < 1,\ 0 < t < 1, \tag{2}$$

$$I_t = \bar{I} + vY_{t-1} + U_t, \qquad 0 < v < 1 - c(1-t), \tag{3}$$

where Y_t is output, C_t is consumption, I_t is investment, $\bar{G}$ is exogenous government consumption, c is the marginal propensity to consume, t is the tax rate, $\bar{I}$ is the exogenous part of investment, and U_t is a stochastic term. It is assumed that U_t is distributed normally with mean zero and variance σ^2 (i.e. $U_t \sim N(0, \sigma^2)$ in the notation of Chapter 3) and that it features no autocorrelation.

(a) Interpret the equations of the model.

(b) Compute the asymptotic variances of output, consumption, and investment. (See the Intermezzo on the asymptotic variance in Chapter 3 if you are unfamiliar with the concept). Denote these asymptotic variances by, respectively, σ_Y^2, σ_C^2, and σ_I^2.

(c) Does the tax system act as an automatic stabilizer in this model? Explain both formally and intuitively.

References

Samuelson, P. A. (1939). Interactions between the multiplier analysis and the principle of acceleration. *Review of Economics and Statistics*, 21:75–78.

Scarth, W. M. (1988). *Macroeconomics: An Introduction to Advanced Methods.* Harcourt Brace Jovanovich, Toronto.