Final Exam, Public Economics
Faculty of Economics
University of Groningen

31 January 2005 (9.00 - 12.00)

Instructions

1. Put your name and student number on all your answer sheets.

2. The exam consists of five (5) questions. All questions should be answered.

3. This is an “open book” exam. You are allowed to consult the Lecture Notes of Public Economics, the assigned readings (the book by Salanié and the articles), and the slides for the course.

4. The maximum number of points you can obtain is 180. This means that your time budget (180 minutes) indicates an exchange rate of 1 point per minute.
Question 1 [60 points]

This question deals with a number of key concepts used in the field of public economics. Give intuitive answers/reactions to the following questions/propositions. Try to use no more than half a page per answer. Do not just cite the book. Explain in your own words. [Each item is worth 6 points]

(a) “In the standard Ramsey growth model, the corporate income tax cannot affect economic growth at all.” True or false. Explain.

(b) What do we mean by rivalry of a good? Give examples of rival and non-rival goods. To what extent is rivalry a crucial aspect of a public good?

(c) “The labour income tax has a stronger effect on labour supply of the poor than of the rich”. True or false? Explain in the context of a standard static labour supply model, and make use of the Slutsky decomposition.

(d) Give an example of how a system of means-tested benefits can make the effective marginal tax rate higher for low income households than for high income households. Explain the intuition.

(e) Explain the key difference between positive economics and normative economics. Give an example of both types of analysis.

(f) Consider the Uzawa-Lucas model studied in Chapter 8 of the lecture notes. Derive the long-run effect on the economic growth rate of an unanticipated and permanent increase in the output tax, \( tY \). Explain the intuition behind your results.

(g) Explain what we mean by income risk and by capital risk. Give a simple example of both types of risk.

(h) Consider an economy with a defined-benefit PAYG pension system and assume that there is a new baby boom. Show what happens to the economy at impact, over time, and in the long run as a result of this baby boom. Explain the intuition.

(i) “The Pareto Principle is useless for policy making because it is biased to maintain the status quo.” True or false? Explain.

(j) “A firm determining its optimal financial policy will never want to repurchase its own shares, even if capital gains accruing to investors are not taxed at all.” True or false? Explain the role of taxes.
Question 2 [30 points]

Assume that the household’s (direct) utility function is of the Cobb-Douglas form:

\[ U = C^\alpha (L_0 - L)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1} \]

where \( U \) is utility, \( C \) is consumption, \( L_0 \) is the exogenous time endowment, \( L \) is labour supply (so that \( L_0 - L \) is leisure). The household budget constraint is:

\[ PC = M + WL, \tag{2} \]

where \( P \) is the price of the consumption good, \( M \) is exogenous non-labour income, and \( W \) is the wage rate.

(a) Show that (1) represents homothetic preferences. Derive the Engel curve and show that it is linear. [5 points]

(b) Derive the expression for the Expenditure Function associated with (1)-(2). [10 points]

(c) Derive the expressions for the Hicksian (or compensated) consumption demand and labour supply equations. Show that the Hicksian consumption demand is downward sloping in \( P \) and that Hicksian labour supply is upward sloping in \( W \). [10 points]

(d) Illustrate your answer to item (c) with the aid of a figure [e.g. like Figure 2.3 in the lecture notes]. [5 points]

Question 3 [20 points]

Consider the standard Harberger general equilibrium tax incidence model discussed in Chapter 6 (section 4) of the lecture notes. Assume, however, that the \( X \)-sector is relatively capital-intensive according to both intensity measures. Derive the effects of an increase in the tax on capital in the \( Y \)-sector (i.e. an increase in \( t_{KY} \)). All other taxes remain constant. Illustrate your answer with two diagrams: the first is based on the assumption that technology in the \( Y \)-sector is Leontief (\( \sigma_Y = 0 \)). The second is the general case (\( \sigma_Y > 0 \)). Explain the key economic mechanisms behind your results.
Question 4 [40 points]

The market for commodity $X_g$ is described by:

$$X_g = \alpha_g [P_g (1 + t_g)]^{-\beta_g}, \quad \alpha_g, \beta_g > 0,$$

(1)

$$P_g = \bar{P}_g,$$

(2)

where $X_g$ is demand for good $g$, $P_g$ is the producer price of good $g$, $t_g$ is the ad valorem tax on good $g$, and $\bar{P}_g$ is a constant. There is equilibrium in the markets for the goods. There are $G$ goods in total and the revenue requirement constraint of the government is:

$$R_0 = \sum_{g=1}^{G} t_g P_g X_g$$

(3)

where $R_0$ is exogenous ($R_0 > 0$).

(a) What do we assume about the supply curve in these markets? Does the system (1)-(2) represent a general equilibrium model? Explain. [7 points]

(b) Assume that the government sets the tax rates $t_g$ (for $g = 1, \ldots, G$) such that the overall excess burden of the indirect tax system is minimized. Derive expressions for the optimal tax rates. Explain the intuition behind your results. [25 points]

(c) Do the constants of this model, like $\alpha_g$ and $\bar{P}_g$, influence the optimal tax rates? Explain. [8 points]

Question 5 [30 points]

Consider the private subscription model of public goods provision discussed in Chapter 12 of the lectures notes.

(a) Show that the first-best optimum can be decentralized (replicated in a market setting) provided there is a Pigouvian subsidy plus a lump-sum tax. Derive the explicit expressions for the subsidy and the lump-sum tax. [20 points]

(b) Are there any alternative ways for the government to ensure that the first-best equilibrium is produced in the market equilibrium? Explain. [10 points]
Model Solutions

Question 1

(a) FALSE. The long-run growth rate is unaffected but the transitional growth rate is affected. The Ramsey growth model is treated in Chapter 8.

- It is summarized by the following equations:

\[ U'[C(t)] = \lambda(t) \] (M1)

\[ \frac{\dot{\lambda}(t)}{\lambda(t)} = \rho + \delta - (1 - t_C) F_K(K(t), \bar{L}) \] (M2)

\[ \dot{K}(t) = F(K(t), \bar{L}) - C(t) - \delta K(t) \] (M3)

\[ W(t) = F(K(t), \bar{L}) - K(t) F_K(K(t), \bar{L}) \] (M4)

- phase diagram is found in Figure 1 (adapted from Figure 8.4 in the book)

- a reduction in a pre-existing corporate tax shifts the \( \dot{\lambda} = 0 \) line to the right. The equilibrium shifts from \( E_0 \) to \( E_1 \) in the long run. Consumption and capital increase.

- at impact there is a jump from \( E_0 \) to \( A \) : \( C \) unchanged, \( r \) rises, \( \dot{\lambda} < 0 \) and \( \dot{k} > 0 \) (savings response)

- during transition gradual move from \( A \) to \( E_1 \) : \( C \) and \( W \) rise, \( r \) falls as \( k \) rises.

- long-run effect: \( (1 - t_C) F_K = \rho + \delta \), i.e. after-tax reward to capital owners is restored. Wage rate rises:

\[ \frac{dW(\infty)}{dt_C} = -KF_{KK} \frac{dK(\infty)}{dt_C} > 0 \]

because there is more capital to cooperate with labour.

(b) Rivalry. A good is rival in consumption if one agent’s consumption of it reduces the amount available to the other agents. (A similar definition can be formulated for rival inputs.) Pure public goods are both non-rival and non-excludable. Especially the non-rivalry is crucial for pure public goods. Non-excludability just means that efficient provision mechanisms may not be feasible. Examples of (non-rival) goods:

- rival good: sandwich, glass of beer, pizza.
- non-rival good: TV broadcast, national defense.

(c) FALSE: for the poor both substitution and income effects are relevant and the labour supply curve may even slope downwards. In contrast, for rich people, the income effect
is negligible and the labour supply curve slopes up. The standard labour supply model is treated in Chapter 2. The Slutsky equation is given by:

\[
\frac{\partial L}{\partial w^*} = \left[ \frac{\partial L}{\partial w^*} \right]_{U=U_0} + L \frac{\partial L}{\partial Y_0},
\]

where \( L \) is labour supply, \( w^* \equiv \frac{W(1-t_L)}{P(1+t_C)} \) is the after-tax real wage rate, and \( Y_0 \) is full income:

\[
Y_0 = \frac{M - T_0}{P(1+t_C)} + w^* L_0,
\]

where \( L_0 \) is the time endowment. We know that \([\partial L/\partial w^*]_{U=U_0}\) is positive (pure substitution effect) and that \( \partial L/\partial Y_0 \) is negative provided leisure is a normal good (the usual assumption). Ceteris paribus, \( L \) is lower for rich than for poor people. Hence, the second term in (1) is negligible for rich people. For this reason, the labour supply curve for rich people is not likely to slope downward.

(d) See Chapter 2. In a means-tested benefit program, the means-testing parameter \((t_Z)\) acts as an effective tax on labour supply. The recipient loses benefits by working harder and earning a higher income. As a result the choice set is non-convex and the household may be trapped in poverty.
(e) Positive economics: concerns with “what is” whilst normative economics concerns with “what should be.” Examples:

- positive: effects of taxes on labour supply, risk taking, etcetera
- normative: optimal commodity and income taxation, optimal public goods provision, etcetera

(f) In Chapter 8 we derive the long-run growth rate for the Lucas-Uzawa model:

\[ g^*_C = \sigma \left[ \Psi [(1 - t_Y) A_Y]^{\theta} \left( A_H \bar{L} \right)^{1-\theta} - \delta - \rho \right], \tag{3} \]

where \( t_Y \) is the output tax. By differentiating (3) with respect to \( t_Y \) we find:

\[ \frac{\partial g^*_C}{\partial t_Y} = -\sigma \theta A_Y \Psi [(1 - t_Y) A_Y]^{\theta-1} \left( A_H \bar{L} \right)^{1-\theta} < 0, \]

i.e. the growth rate falls. Intuition: the capital-labour ratio falls in both sectors. The private sector substitutes away from the production factor whose production is taxed more heavily (i.e. away from capital). This hurts the long-run growth rate.

(g) See Chapter 4.

- Income risk: current and/or future income is stochastic. Example: if you don’t know the future wage rate (and you plan to work in the future) then your future labour income is risky.
- Capital risk: the rate of return on one (or more) asset(s) is stochastic. Example: risky assets such as shares or bonds.

(h) In Chapter 13, Figure 13.4, we study a reduction in the fertility rate. An increases in the fertility rate is shown in Figure 2. The pre-shock path is from A to E_0. The post-shock path is from B to E_1. The capital stock per worker falls. There are more young people (who own no assets) and fewer old people.

(i) Pareto optimality is a rather weak criterion for policy making. It characterizes the set of efficient equilibria but does not pick any particular equilibrium as the best. The distributional issue cannot be tackled with it. Redistribution takes resources from some and give them to someone else. The social welfare function approach allows one to identify first-best or second-best (constrained) social optima.

(j) Consider equation (5.30) in the book:

\[ V_t = \sum_{s=t}^{\infty} \left[ \prod_{z=t}^{s} \left( \frac{1}{1 + \frac{\rho_s}{1-t_G}} \right) \right] \left[ \left( \frac{1-t_D}{1-t_G} \right) (D_s - V^{N}_{s+1}) - \left( \frac{t_D - t_G}{1-t_G} \right) V^{N}_{s+1} \right]. \tag{(5.30)} \]
Figure 2: Baby Boom and Capital Accumulation
In the conventional case, with $t_D > t_G$, it is attractive for the firm to buy back its own shares ($V_{s+1}^N < 0$) and thus increase its market value. This is typically forbidden by law. If there are no taxes, then only $D_s - V_{s+1}^N$ matters and buying back own shares does not affect the market value of the firm (see also (5.27) in the book).

**Question 2**

(a) Homothetic preferences: utility function can be written as $G[V(C, L_0 - L)]$, where $G[]$ is strictly increasing and $V(C, L_0 - L)$ is linear homogenous. Equation (1) is itself linear homogeneous, so we can write $G[] = 1$ and deduce that $U$ is homothetic. To prove that (1) is linear homogeneous we write (for any $\zeta > 0$):

$$U(\zeta C, \zeta [L_0 - L]) = (\zeta C)^\alpha (\zeta [L_0 - L])^{1-\alpha} = \zeta^{\alpha+1-\alpha} C^\alpha L_0 - L^{1-\alpha} = \zeta U(C, L_0 - L).$$

The Engel curve is derived below—see equation (1).

(b) Expenditure function:

$$E(P, W, U_0) \equiv \min_{C,L_0-L} PC + W(L_0 - L) \text{ subject to } C^\alpha (L_0 - L)^{1-\alpha} = U_0,$$

where $U_0$ is held constant. The Lagrangian is:

$$\mathcal{L} \equiv PC + W(L_0 - L) + \lambda \left[U_0 - C^\alpha (L_0 - L)^{1-\alpha}\right]$$

- first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial C} = P - \alpha \lambda C^{\alpha-1} (L_0 - L)^{1-\alpha} = 0$$

$$\frac{\partial \mathcal{L}}{\partial (L_0 - L)} = W - (1 - \alpha) \lambda C^\alpha (L_0 - L)^{-\alpha} = 0$$

- combining the FONCs to eliminate $\lambda$:

$$\frac{W}{P} = \frac{(1 - \alpha) \lambda C^\alpha (L_0 - L)^{-\alpha}}{\alpha \lambda C^{\alpha-1} (L_0 - L)^{1-\alpha}} = \frac{(1 - \alpha) C}{\alpha (L_0 - L)} \quad \Leftrightarrow$$

$$C = \frac{\alpha}{1 - \alpha} \frac{W}{P} (L_0 - L). \quad \text{(1)}$$

- substitute (1) into the utility function:

$$U_0 = \left[\frac{\alpha}{1 - \alpha} \frac{W}{P} (L_0 - L)\right]^\alpha (L_0 - L)^{1-\alpha} = \alpha^\alpha (1 - \alpha)^{-\alpha} W^\alpha P^{-\alpha} (L_0 - L) \quad \Leftrightarrow$$

$$W(L_0 - L) = \alpha^{-\alpha} (1 - \alpha)^\alpha W^{1-\alpha} P^\alpha U_0. \quad \text{(2)}$$
- it follows from (1) and (2) that:
\[ PC = \frac{\alpha}{1 - \alpha} W (L_0 - L) \]
\[ = \alpha^{1 - \alpha} (1 - \alpha)^{\alpha - 1} W^{1 - \alpha} P^\alpha U_0. \]  

(3)

- By substituting (2) and (3) into the budget equation we get:
\[ PC + W (L_0 - L) = \alpha^{1 - \alpha} (1 - \alpha)^{\alpha - 1} W^{1 - \alpha} P^\alpha U_0 + \alpha^{-\alpha} (1 - \alpha)\alpha W^{1 - \alpha} P^\alpha U_0 \]
\[ = \left[ \alpha^{1 - \alpha} (1 - \alpha)^{\alpha - 1} + \alpha^{-\alpha} (1 - \alpha)\alpha \right] W^{1 - \alpha} P^\alpha U_0 \]
\[ = \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1} [\alpha + 1 - \alpha] W^{1 - \alpha} P^\alpha U_0 \]
\[ = \left( W \frac{1}{1 - \alpha} \right)^{1 - \alpha} \left( \frac{P}{\alpha} \right)^\alpha U_0 \equiv E \left( P, W, U_0 \right). \]  

(4)

(c) The differentiating \( E \left( P, W, U_0 \right) \) we obtain:
\[ C^H = \frac{\partial E \left( P, W, U_0 \right)}{\partial P} = \left( W \frac{1}{1 - \alpha} \right)^{1 - \alpha} \left( \frac{P}{\alpha} \right)^{\alpha - 1} U_0, \]  

(5a)

\[ (L_0 - L)^H = \frac{\partial E \left( P, W, U_0 \right)}{\partial W} = \left( W \frac{1}{1 - \alpha} \right)^{-\alpha} \left( \frac{P}{\alpha} \right)^\alpha U_0. \]  

(5b)

Clearly, \( C^H \) is downward sloping in \( P \) (as \( 0 < \alpha < 1 \)) whilst \( (L_0 - L)^H \) is downward sloping in \( W \). It follows that \( L^H \) is upward sloping in \( W \).

(d) See Figure 3. The budget line is:
\[ PC + W (L_0 - L) = M + WL_0 \]  

(6)

At point B, \( L = 0 \) and \( C = M/P \). A decrease in \( W \) rotates the budget line around point B in a counterclockwise fashion. Substitution effect: \( E_0 \) to \( E' \). Income effect: \( E' \) to \( E_1 \). Leisure demand rises, and labour supply falls. An increase in \( P \) (i) rotates the budget line counterclockwise and (ii) shifts point B downward. A decomposition can again be done.

**Question 3**

We use the log-linearized model from Chapter 6:
\[ \tilde{X} - \tilde{Y} = -\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y \right], \]  

(M1)

\[ \tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] - \theta_{KY} \tilde{t}_{KY}, \]  

(M2)

\[ \lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ a_X \sigma_X + a_Y \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] - a_Y \sigma_Y \tilde{t}_{KY}, \]  

(M3)
Figure 3: Decrease in the wage rate
where we keep all taxes other than \( t_{KY} \) constant. Note that, in contrast to the book, we now assume that the \( X \)-sector is relatively capital intensive, i.e.:

\[
\lambda^* < 0 \text{ and } \theta^* < 0.
\] (1)

To study the effect of the tax increase (\( \tilde{t}_{KY} > 0 \)) we first obtain an analytical expression for the supply curve [S in the figures below]. By solving (M2) for \( \tilde{W} - \tilde{R} \) and substituting the result into (M3) we obtain:

\[
\tilde{X} - \tilde{Y} = \frac{a_X \sigma_X + a_Y \sigma_Y}{\lambda^* \theta^*} \left[ \tilde{P}_X - \tilde{P}_Y + \theta_{KY} \tilde{t}_{KY} \right] - \frac{a_Y \sigma_Y}{\lambda^*} \tilde{t}_{KY}.
\] (2)

The slope of the supply curve is positive (as \( \lambda^* \theta^* > 0 \)).

(a) **Case 1:** Leontief technology in the \( Y \)-sector (\( \sigma_Y = 0 \)). The S-curve reduces to:

\[
\tilde{X} - \tilde{Y} = \frac{a_X \sigma_X}{\lambda^* \theta^*} \left[ \tilde{P}_X - \tilde{P}_Y + \theta_{KY} \tilde{t}_{KY} \right]
\] (5)

See Figure 4.

– The CPR curve (M2) is downward sloping because \( \theta^* \) is negative.
– The FME curve (M3) is downward sloping because \( \lambda^* \) is negative.
– The CPR curve shifts to the left, from CPR\(_0\) to CPR\(_1\). As a result, the S curve shifts to the left also. There is only an *output effect* which ensures that \( X/Y \) increases. Capital flows out of the \( Y \)-sector into the \( X \)-sector. Producers in the \( Y \)-sector cannot substitute towards labour because technology is Leontief. As a result, output in that sector must fall.

(b) **Case 2:** general technology in the \( Y \)-sector (\( \sigma_Y > 0 \)). See Figure 5.

– Now both the CPR and FME curves shift.
– FME shifts to the left, from FME\(_0\) to FME\(_1\)
– Solving (M1) and (2) we get:

\[
-\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y \right] = \frac{a_X \sigma_X + a_Y \sigma_Y}{\lambda^* \theta^*} \left[ \tilde{P}_X - \tilde{P}_Y + \theta_{KY} \tilde{t}_{KY} \right] - \frac{a_Y \sigma_Y}{\lambda^*} \tilde{t}_{KY}
\]

\[
\left[ \sigma_D + \frac{a_X \sigma_X + a_Y \sigma_Y}{\lambda^* \theta^*} \right] \left[ \tilde{P}_X - \tilde{P}_Y \right] = \frac{-\theta_{KY} a_X \sigma_X + a_Y \sigma_Y}{\lambda^* \theta^*} \left[ \tilde{P}_X - \tilde{P}_Y + \theta_{KY} \tilde{t}_{KY} \right] + \frac{a_Y \sigma_Y}{\lambda^*} \tilde{t}_{KY}
\]

\[
= \frac{-a_X \sigma_X \theta_{KY} + (\theta^* - \theta_{KY}) a_Y \sigma_Y}{\lambda^* \theta^*} \tilde{t}_{KY}
\]

\[
= - \frac{a_X \sigma_X \theta_{KY} + \theta_{KY} a_Y \sigma_Y}{\lambda^* \theta^*} \tilde{t}_{KY} < 0,
\] (6)

where we have used \( \theta^* \equiv \theta_{KY} - \theta_{KX} \) to get the final expression.

– The output effect cannot be dominated by the *factor substitution effect*. This case has been drawn in Figure 5.
Figure 4: Increasing $t_{KY}$: The Output Effect
Figure 5: Increasing $t_{KY}$: The Output and Factor Substitution Effects
Question 4

(a) It is assumed that the supply curve is horizontal. The sector has no effect on the rest of the economy. If this sector expands then the additional demand for production factors does not drive up factor prices. The supply price remains unchanged. Typical partial equilibrium assumptions.

(b) Define the consumer price as \( Q_g = P_g (1 + t_g) \). The inverse demand function can then be written as:

\[
Q_g = \left(\frac{X_g}{\alpha_g}\right)^{-1/\beta_g} 
\]  

(1)

– The excess burden is defined as follows (see Chapter 10 in the book):

\[
EB_g \equiv \int_{X_0^g}^{X_1^g} \left(\frac{X_g}{\alpha_g}\right)^{-1/\beta_g} dX_g - \bar{P}_g [X_0^g - X_1^g] 
\]

\[
= \alpha_g^{1/\beta_g} \int_{X_0^g}^{X_1^g} \alpha_g^{1/\beta_g} X_g^{-1/\beta_g} dX_g - \bar{P}_g [X_0^g - X_1^g] 
\]

\[
= \alpha_g^{1/\beta_g} \left[ X_g^{1-1/\beta_g} \right]_{X_0^g}^{X_1^g} - \bar{P}_g [X_0^g - X_1^g], 
\]  

(2)

where \( X_0^g \) and \( X_1^g \) are defined as follows:

\[
X_0^g = \alpha_g \bar{P}_g^{-\beta_g}, 
\]  

(3a)

\[
X_1^g = \alpha_g [\bar{P}_g (1 + t_g)]^{-\beta_g}. 
\]  

(3b)

We easily find that:

\[
\bar{P}_g [X_0^g - X_1^g] = \bar{P}_g \left[ \alpha_g \bar{P}_g^{-\beta_g} - \alpha_g [\bar{P}_g (1 + t_g)]^{-\beta_g} \right] 
\]

\[
= \alpha_g \bar{P}_g^{1-\beta_g} \left[ 1 - (1 + t_g)^{-\beta_g} \right] 
\]  

(4a)

Using expressions (3a)-(3b) we find:

\[
\alpha_g^{1/\beta_g} \left[ X_g^{1-1/\beta_g} \right]_{X_0^g}^{X_1^g} = \alpha_g^{1/\beta_g} \left[ \alpha_g \bar{P}_g^{-\beta_g} \right]^{(\beta_g-1)/\beta_g} - \left[ \alpha_g \bar{P}_g^{-\beta_g} (1 + t_g)^{-\beta_g} \right]^{(\beta_g-1)/\beta_g} 
\]

\[
= \alpha_g^{1/\beta_g} \left[ (\alpha_g \bar{P}_g^{-\beta_g})^{(\beta_g-1)/\beta_g} - \alpha_g \bar{P}_g^{-\beta_g} (1 + t_g)^{-\beta_g} \right]^{(\beta_g-1)/\beta_g} 
\]

\[
= \alpha_g^{1/\beta_g} \alpha_g^{(\beta_g-1)/\beta_g} \bar{P}_g^{1-\beta_g} \left[ 1 - (1 + t_g)^{1-\beta_g} \right] 
\]

\[
= \alpha_g \bar{P}_g^{1-\beta_g} \left[ 1 - (1 + t_g)^{1-\beta_g} \right] 
\]  

(4b)

Finally, using (4a) and (4b) in (2) we obtain the desired expression for the excess burden:

\[
EB_g = \alpha_g \bar{P}_g^{1-\beta_g} \left[ \left( \frac{1 - (1 + t_g)^{1-\beta_g}}{1 - 1/\beta_g} \right) - \left[ 1 - (1 + t_g)^{-\beta_g} \right] \right] 
\]  

(5)
Figure 10.1 in the book can be used to illustrate this welfare measure.

- The optimal tax rate is determined by using equation (10.8) in the book:

\[
\frac{t_g}{1 + t_g} = \frac{\theta}{\varepsilon_g^D},
\]

where \( \varepsilon_g^D \) is the demand elasticity:

\[
\varepsilon_g^D \equiv -\frac{Q_g}{X_g} \frac{\partial X_g}{\partial Q_g} = \beta_g
\]

(c) As is clear from (7) the constants can only affect the optimal tax rates via the common term \( \theta \equiv \lambda / (1 + \lambda) \). Note that \( \lambda \) is determined implicitly by the (6) and the government budget constraint.

**Question 5**

(a) In the first-best optimum we must have:

\[
H \frac{\partial U[C^h,G]}{\partial G} \frac{k_G}{k_Y} = (1 - s_G) k_G
\]

If the government subsidizes the public good, the budget equation becomes:

\[
PC^h = W - P_G (1 - s_G) G^h - T,
\]

where \( s_G \) is the subsidy and \( T \) is the lump-sum tax. In the decentralized setting, households choose:

\[
\frac{\partial U[C^h,G]}{\partial C^h} \frac{1 - s_G}{k_Y} = \frac{k_G}{k_Y}
\]

Matching (1) and (3) yields:

\[
\frac{1}{H} = 1 - s_G \iff s_G = 1 - \frac{1}{H}.
\]

The lump-sum tax is needed to raise the revenue for the subsidy.

(b) The government could just provide the correct level of public goods itself. Provided it can use lump-sum taxes, the first-best optimum can be achieved.