Aims of this chapter

- On who is the tax levied?
- Who ultimately bears the tax?
- Tax incidence
  - in partial equilibrium
  - in general equilibrium: the Harberger (1962) approach
  - applications: corporate tax, output tax
- Possible extensions:
  - endogenous factor supplies
  - intersectoral factor mobility
  - dynamics: see Chapter 8: Taxation and Growth
Types of Tax Incidence

- Agent upon whom the tax is levied may not actually bear the tax.

- As the tax may be shifted to other agents.

Different types of tax incidence analysis. Look at effects on:
- Producers, consumers, and suppliers of factor
- Main production factors (e.g. capital and labour)
- Personal income distribution
- Different regions
- Different generations
• Focus on incidence on factors

• Study tax effects in a “neutral fashion”
  – only one tax is changed at a time
  – revenue is recycled in a lump-sum fashion

**Tax Incidence in Partial Equilibrium**

• Classics had general equilibrium framework

• First half of 20th century: *Alfred Marshall*’s partial equilibrium analysis dominant
  [study markets in isolation]

• Example from Atkinson & Stiglitz (1980, p. 162):
  – crop (say “grapes”) produced with land (that cannot be used for anything else)
    and with labour, $L$
  – supply of labour perfectly elastic at wage $W$
production function is \( Y = F (L, \bar{K}) \) where \( Y \) is output (and \( F_L > 0 \geq F_{LL} \))

competitive demand for labour, \( L^D \):

\[
PF_L \left( L^D, \bar{K} \right) = W \quad \iff \quad L^D = \bar{K} l \left( \frac{W}{P} \right)
\]

where \( P \) is the producer price of the good

competitive supply curve:

\[
Y^S = F \left( L^D, \bar{K} \right) = \bar{K} S \left( \frac{W}{P} \right)
\]

demand curve:

\[
Y^D = D \left( P_D, Z \right)
\]

where \( P_D \) is the consumer price of the good and where we assume \( \frac{\partial D}{\partial P_D} < 0 \).

In Figure 6.1 we illustrate the situation on the grape market

initially no tax on grape consumption so \( P_D = P \)
– initial equilibrium is at $E_0$, equilibrium price is $P_0$ and equilibrium quantity is $Y_0$

– rents received by land owners is the area $BE_0P_0$:

\[
\Pi_0 \equiv P_0Y_0 - \int_0^{Y_0} MC \left( Y^S, W, \bar{K} \right) dY^S \\
= P_0Y_0 - \left[ TC \left( Y_0, W, \bar{K} \right) - TC \left( 0, W, \bar{K} \right) \right] \\
= P_0Y_0 - TC \left( Y_0, W, \bar{K} \right)
\]

where $TC$ is total cost and $MC$ is marginal cost:

\[
TC \left( Y^S, W, \bar{K} \right) \equiv WL^D \\
MC \left( Y^S, W, \bar{K} \right) \equiv \frac{\partial WL^D \left( Y^S \right)}{\partial Y^S}
\]

– when the tax is introduced, the demand price equals $P_D = P + t_G$ so that the demand curve shifts downwards.
– effects:

- producer price falls from $P_0$ to $P_1$
- demand price rises from $P_0$ to $P_{D1}$
- wage unchanged by assumption
- landowner rents decline from $BE_0 P_0$ to $BE_1 P_1$
- so tax is borne by both consumers and landowners [division of burden depends on the elasticities]!
• Limitations of partial equilibrium analysis:
  – extreme assumption on the *supply side*: factor supplies are either totally elastic [labour] or totally inelastic [land]
  – *demand side*: change in demand for grapes may affect other sectors and may thus affect factor demands
  – interaction demand-supply: change in factor incomes may affect demand patterns

• The classic analysis by Arnold Harberger (1962) made a strong case for using general equilibrium models.
Figure 6.1: Tax Incidence in Partial Equilibrium
Tax Incidence in General Equilibrium

- Key contributions by Harberger (1962 *JPE*) and Jones (1965 *JPE*)
- Also much used in the pure theory of international trade and in two-sector growth theory

Steps to be taken:
- outline basic model (without taxes) and study its key properties
- **digression** on duality theory: the cost function and input demands
- develop geometric illustration of the model
- introduce taxes into the model and study their effects
- discuss limitations/generalizations
Basic Two-Sector Model

- Minimum general equilibrium model has:
  - two sectors: the $X$-sector and the $Y$-sector (outputs $X$ and $Y$; prices $P_X$ and $P_Y$, respectively)
  - two factors of production: capital and labour ($K_i$ and $L_i$, $i \in (X, Y)$)

- Further features:
  - static model
  - fixed total supplies of capital and labour ($\bar{K}$ and $\bar{L}$)
  - perfect intersectoral mobility (common rental rates $W$ and $R$)
  - perfect competition in both sectors
  - constant returns to scale in both sectors
  - full employment of factors
Digression on the Cost Function

- Cost Function: yet another very useful tool from duality theory which will be used time and again [see the book for details]

- Cost function is analogous to the Expenditure Function (see Chapter 2).

- Focus on single-good, two-factor case for exposition purposes: $Z_1$ and $Z_2$ are the factors, $W_1$ and $W_2$ are their respective rental rates, and $Y$ is the output.

- **Cost function**: minimum level of factor costs needed to produce a given level of output, say $Y_0$, when faced with the rental rates $W_1$ and $W_2$:

$$C(W_1, W_2, Y_0) \equiv \min_{\{Z_1, Z_2\}} W_1 Z_1 + W_2 Z_2 \quad \text{subject to:} \quad F(Z_1, Z_2) = Y_0$$

- $F(Z_1, Z_2)$ is the production function
• Key properties of the cost function:

- $C(W_1, W_2, Y_0)$ is homogeneous of degree one in rental rates ($W_1$ and $W_2$)
- $C(W_1, W_2, Y_0)$ is concave in rental rates
- $C(W_1, W_2, Y_0)$ is increasing in each $W_i$
- $C(W_1, W_2, Y_0)$ is continuous in $W_1$ and $W_2$
- if $F(Z_1, Z_2)$ features CRTS then the cost function is linear in $Y_0$ and can be written as $C(\cdot) = c(W_1, W_2)Y_0$ (where $c(W_1, W_2)$ is unit-cost)
- the Allen-Uzawa substitution elasticity between factors $i$ and $j$ is defined as:

$$\sigma_{ij} \equiv \frac{C(\cdot)C_{ij}(\cdot)}{C_i(\cdot)C_j(\cdot)}, \quad i \neq j$$
• Conditional factor demand curves are given by Shephard’s Lemma:

\[ Z_i (W_1, W_2, Y_0) = \frac{\partial C (W_1, W_2, Y_0)}{\partial W_i} \]  

(SL1)

• Properties of the conditional factor demands:

– \( Z_i (W_1, W_2, Y_0) \) is decreasing in \( W_i \)

– \( Z_i (W_1, W_2, Y_0) \) is homogeneous of degree zero in rental rates \( (W_1 \text{ and } W_2) \)

– if \( F (Z_1, Z_2) \) features CRTS then, since \( C (\cdot) = c (W_1, W_2) Y_0 \), the conditional factor demand simplify to:

\[ Z_i (W_1, W_2, Y_0) = \frac{\partial c (W_1, W_2)}{\partial W_i} Y_0 \]  

(SL2)
Back to the Model

• Approach production sectors via the dual approach (i.e. via the cost function)

• Cost function in the sectors are linear in respective outputs:

\[ C^x \equiv c^x (W, R) X \]
\[ C^y \equiv c^y (W, R) Y \]

so marginal cost in sector \( i \) is \( c^i \)

• Note: \( c^i \) is also unit-cost, linear homogeneous in \( W \) and \( R \)
• Conditional factor demands:

\[ L_X = \frac{\partial c^x (W, R)}{\partial W} X \equiv c^x_W X \]

\[ K_X = \frac{\partial c^x (W, R)}{\partial R} X \equiv c^x_R X \]

\[ L_Y = \frac{\partial c^y (W, R)}{\partial W} Y \equiv c^y_W Y \]

\[ K_Y = \frac{\partial c^y (W, R)}{\partial R} Y \equiv c^y_R Y \]
• Full employment in both factor markets:

\[ c^x_W X + c^y_W Y = \bar{L} \]
\[ c^x_R X + c^y_R Y = \bar{K} \]

where the \( c^i_j \) \( i \in (x, y) \) and \( j \in (W, R) \) are input coefficients depending in general on \( W \) and \( R \).

• Perfectly competitive firms in both sectors equate price to marginal cost:

\[ P_X = c^x (W, R) \]
\[ P_Y = c^y (W, R) \]
Features of the goods demand side:

- representative agent model: single utility function, $U(X, Y)$, subject to aggregate budget restriction, $M = P_X X + P_Y Y$, where $M$ is aggregate income. Interesting first-order condition: $U_X / U_Y = P_X / P_Y$

- homothetic preferences: linear Engel curves so that Marshallian demand can be written as follows:

$$X = d^x (P_X, P_Y) M$$
$$Y = d^y (P_X, P_Y) M$$

where $d^x (\cdot)$ and $d^y (\cdot)$ are homogeneous of degree minus one in $P_X$ and $P_Y$

- definition of aggregate income:

$$M = W \bar{L} + R \bar{K}$$
Summary of the full general equilibrium model:

\[ \bar{L} = c^x_W (W, R) X + c^y_W (W, R) Y \]  
\[ \bar{K} = c^x_R (W, R) X + c^y_R (W, R) Y \]  
\[ P_X = c^x (W, R) \]  
\[ P_Y = c^y (W, R) \]  
\[ X = d^x (P_X, P_Y) [W \bar{L} + R \bar{K}] \]  
\[ Y = d^y (P_X, P_Y) [W \bar{L} + R \bar{K}] \]  

- endogenous: \( X, Y, P_X, P_Y, W, \) and \( R \)
- exogenous: \( \bar{K} \) and \( \bar{L} \)

- **Law of Walras**: when all but one markets are in equilibrium then so is the last market, i.e. one equation is redundant and we can only determine relative prices.
• Digression on Walras’ Law:

– (5) and (6) imply that:

\[ P_X X + P_Y Y = W \bar{L} + R \bar{K} \]

– insert (1) and (2):

\[
\begin{align*}
P_X X + P_Y Y &= W \left[ c_W^x X + c_W^y Y \right] + R \left[ c_R^x X + c_R^y Y \right] \\
&= \left[ W c_W^x + R c_R^x \right] X + \left[ W c_W^y + R c_R^y \right] Y
\end{align*}
\]

– but [by linear homogeneity] we have \( c^x = W c_W^x + R c_R^x \) and \( c^y = W c_W^y + R c_R^y \) so this means:

\[
\begin{align*}
P_X X + P_Y Y &= c^x X + c^y Y \\
(P_X - c^x) X &= -(P_Y - c^y) Y
\end{align*}
\]

– so if (3) holds so does (4) and vice versa. Q.E.D.
Qualitative Analysis of the Model

- The model in levels is given in (1)-(6)

- We adopt the usual strategy of log-linearizing the model.

- Show some of the details of the derivation [which is non-trivial]

- Log-linearized demand equations [(5)-(6)].
  - recall key first-order condition for utility maximum:
    \[
    \frac{U_X}{U_Y} = \frac{P_X}{P_Y} \tag{A}
    \]
  - recall (from Chapter 2) that for homothetic preferences the elasticity of substitution is defined as:
    \[
    \sigma_D \equiv \frac{d \log \left( \frac{Y}{X} \right)}{d \log \left( \frac{U_X}{U_Y} \right)} > 0 \tag{B}
    \]
– it follows from (A) and (B) that:

\[ d \log \left( \frac{U_X}{U_Y} \right) = d \log \left( \frac{P_X}{P_Y} \right) = \frac{1}{\sigma_D} d \log \left( \frac{Y}{X} \right) \]  

(C)

or:

\[ \tilde{X} - \tilde{Y} = -\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y \right] \]  

(M1)

with:

\[ \tilde{X} \equiv \frac{dX}{X}, \quad \tilde{Y} \equiv \frac{dY}{Y} \]

\[ \tilde{P}_X \equiv \frac{dP_X}{P_X}, \quad \tilde{P}_Y \equiv \frac{dP_Y}{P_Y} \]

– **Note**: more general (non-homothetic) case done by A&K (p. 168).
• Log-linearized price equations [(3)-(4)].

– totally differentiate equation (3):

\[
\begin{align*}
\frac{dP_X}{P_X} &= \frac{Wc^x_w}{c^x} \frac{dW}{W} + \frac{Rc^x_R}{c^x} \frac{dR}{R} \\
\tilde{P}_X &= \theta_{LX} \tilde{W} + \theta_{KX} \tilde{R}
\end{align*}
\]

(A)

with:

\[
\begin{align*}
\tilde{W} & \equiv \frac{dW}{W}, \quad \tilde{R} \equiv \frac{dR}{R} \\
\theta_{LX} & \equiv \frac{Wc^x_w}{c^x}, \quad \theta_{KX} \equiv \frac{Rc^x_R}{c^x} = 1 - \theta_{LX}
\end{align*}
\]

where \( \theta_{LX} \) and \( \theta_{KX} \) are the factor shares of, respectively, labour and capital in the \( X \)-sector.
– totally differentiating equation (4) in a similar fashion yields:

\[ \tilde{P}_Y = \theta_{LY} \tilde{W} + \theta_{KY} \tilde{R} \]  

(B)

with:

\[ \theta_{LY} \equiv \frac{W_{cy}}{c_y}, \quad \theta_{KY} \equiv \frac{R_{cy}}{c_y} = 1 - \theta_{LY} \]

– deducting (B) from (A) yields:

\[
\tilde{P}_X - \tilde{P}_Y = \theta_{LX} \tilde{W} + \theta_{KX} \tilde{R} - \left[ \theta_{LY} \tilde{W} + \theta_{KY} \tilde{R} \right]
\]

\[
= \theta_{LX} \tilde{W} + (1 - \theta_{LX}) \tilde{R} - \left[ \theta_{LY} \tilde{W} + (1 - \theta_{LY}) \tilde{R} \right]
\]

\[
= \theta^* \left[ \tilde{W} - \tilde{R} \right]
\]

(M2)
with:

$$\theta^* \equiv \theta_{LX} - \theta_{LY}$$

$$= \theta_{KY} - \theta_{KX}$$

– **Note:** $\theta^*$ measures relative factor intensity in the two industries. According to (M2), if the $X$-industry is relatively labour intensive ($\theta_{LX} > \theta_{LY}$) then a rise in the relative price of labour ($W/R$) results in a rise in the relative price of good $X$ ($P_X/P_Y$)
• Log-linearized factor market clearing equations [(1)-(2)].

  – totally differentiate equation (1), taking into account that \( c^x_W \) and \( c^y_W \) depend on \( W/R \)

\[
\begin{align*}
  d\tilde{L} & = c^x_W dX + X dc^x_W + c^y_W dY + Y dc^y_W \\
  \frac{d\bar{L}}{\bar{L}} & = \frac{X c^x_W}{\bar{L}} \frac{dX}{X} + \frac{c^x_W X}{\bar{L}} \frac{dc^x_W}{c^x_W} + \frac{c^y_W Y}{\bar{L}} \frac{dY}{Y} + \frac{c^y_W Y}{\bar{L}} \frac{dc^y_W}{c^y_W} \\
  \tilde{L} & = \lambda_{LX} \left( \tilde{X} + \tilde{c}^x_W \right) + \lambda_{LY} \left( \tilde{Y} + \tilde{c}^y_W \right)
\end{align*}
\]

(M3)

with:

\[
\begin{align*}
  \tilde{c}^x_W & \equiv \frac{dc^x_W}{c^x_W}, \quad \tilde{c}^y_W \equiv \frac{dc^y_W}{c^y_W}, \quad \tilde{L} \equiv \frac{d\bar{L}}{\bar{L}} \\
  \lambda_{LX} & \equiv \frac{L_X}{\bar{L}} = \frac{X c^x_W}{\bar{L}}, \quad \lambda_{LY} \equiv \frac{L_Y}{\bar{L}} = \frac{Y c^y_W}{\bar{L}} = 1 - \lambda_{LX}
\end{align*}
\]

where \( \lambda_{LX} \) and \( \lambda_{LY} \) are the shares of the labour force employed in, respectively, the \( X \) and the \( Y \) sector.
– total differentiation of (2) yields in a similar fashion:

\[
\tilde{K} = \lambda_{KX} \left( \tilde{X} + \tilde{c}^x_R \right) + \lambda_{KY} \left( \tilde{Y} + \tilde{c}^y_R \right) \tag{M4}
\]

with:

\[
\tilde{c}^x_R \equiv \frac{d c^x_R}{c^x_R}, \quad \tilde{c}^y_R \equiv \frac{d c^y_R}{c^y_R}, \quad \tilde{K} \equiv \frac{d \bar{K}}{\bar{K}}
\]

\[
\lambda_{KX} \equiv \frac{K_X}{\bar{K}} = \frac{X c^x_R}{\bar{K}}, \quad \lambda_{KY} \equiv \frac{K_Y}{\bar{K}} = \frac{Y c^y_R}{\bar{K}} = 1 - \lambda_{KX}
\]

– **Note:** for the special case of *Leontief technologies* (zero substitutability in production) all \( c^i_j \) coefficients would be fixed. The log-linearized model is then given by (M1)-(M4). We occasionally look at the Leontief case to build intuition. In the general case we must look at......
• Log-linearized production coefficients \([c^i_j \text{ for } i \in (x, y) \text{ and } j \in (W, R)]\)
  
  – totally differentiate \(c^x_W (W, R):\)

  \[
  \begin{align*}
  dc^x_W &= c^x_{WW} dW + c^x_{WR} dR \\
  \frac{dc^x_W}{c^x_W} &= \frac{W c^x_{WW}}{c^x_W} \frac{dW}{W} + \frac{R c^x_{WR}}{c^x_W} \frac{dR}{R}
  \end{align*}
  \]

  (A)

  – recall that \(c^x_W (W, R)\) is homogeneous of degree zero in \(W\) and \(R\) so that [by Euler’s Theorem]:

  \[
  0 \times c^x_W = W c^x_{WW} + R c^x_{WR}
  \]

  (B)

  – use (B) in (A):

  \[
  \tilde{c}^x_W = - \frac{R c^x_{WR}}{c^x_W} \left[ \tilde{W} - \tilde{R} \right]
  \]

  (C)
the [Allen-Uzawa] substitution elasticity in technology in the $X$-sector is defined (via the cost function) as [see Sydsæter, Strom, and Berck (2000, p. 155)]:

$$\sigma_X \equiv \frac{C^x C'_{WR}}{C'_{R} C_{W}^x} = \frac{c^x c^x_{WR}}{c^x_{R} c^x_{W}} \geq 0$$

(D)

using (D) in (C) we get after some steps:

$$\tilde{c}_W^x = - \frac{c^x_{R} c^x_{R}}{c^x_{R}} \frac{R c^x_{WR}}{c^x_{W}} \left[ \tilde{W} - \tilde{R} \right]$$

$$= - \left( \frac{c^x_{R} R}{c^x_{R}} \right) \left( \frac{c^x_{W} c^x_{WR}}{c^x_{R} c^x_{W}} \right) \left[ \tilde{W} - \tilde{R} \right]$$

$$= - \theta_{KX} \sigma_X \left[ \tilde{W} - \tilde{R} \right]$$

(M5)
– using the same approach we find:

\[ \tilde{c}_W^y = -\theta_{KY} \sigma_Y \left[ \tilde{W} - \tilde{R} \right] \]  \hspace{1cm} (M6)

\[ \tilde{c}_R^x = \theta_{LX} \sigma_X \left[ \tilde{W} - \tilde{R} \right] \]  \hspace{1cm} (M7)

\[ \tilde{c}_R^y = \theta_{LY} \sigma_Y \left[ \tilde{W} - \tilde{R} \right] \]  \hspace{1cm} (M8)

where \( \sigma_Y \) is the substitution elasticity in the \( Y \)-sector:

\[ \sigma_Y \equiv \frac{c_y^y c_{WR}^y}{c_R^y c_W^y} \geq 0 \]
• By substituting (M5)-(M8) in the relevant places in (M3) and (M4) we obtain the final expressions for the factor market equilibrium loci:

\[ \tilde{L} = \lambda_{LX} \left( \tilde{X} + \tilde{c}_W^x \right) + \lambda_{LY} \left( \tilde{Y} + \tilde{c}_W^y \right) \]

\[ = \lambda_{LX} \left( \tilde{X} - \theta_{KX} \sigma_X \left[ \tilde{W} - \tilde{R} \right] \right) + \lambda_{LY} \left( \tilde{Y} - \theta_{KY} \sigma_Y \left[ \tilde{W} - \tilde{R} \right] \right) \]

\[ = \lambda_{LX} \tilde{X} + \lambda_{LY} \tilde{Y} - \left[ \lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] \quad (M3') \]

\[ \tilde{K} = \lambda_{KX} \left( \tilde{X} + \tilde{c}_R^x \right) + \lambda_{KY} \left( \tilde{Y} + \tilde{c}_R^y \right) \]

\[ = \lambda_{KX} \left( \tilde{X} + \theta_{LX} \sigma_X \left[ \tilde{W} - \tilde{R} \right] \right) + \lambda_{KY} \left( \tilde{Y} + \theta_{LY} \sigma_Y \left[ \tilde{W} - \tilde{R} \right] \right) \]

\[ = \lambda_{KX} \tilde{X} + \lambda_{KY} \tilde{Y} + \left[ \lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] \quad (M4') \]
In the final step we deduct \((M4')\) from \((M3')\) to get:

\[
\tilde{L} - \tilde{K} = (\lambda_{LX} - \lambda_{KX}) \tilde{X} + (\lambda_{LY} - \lambda_{KY}) \tilde{Y} - [\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y + \lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y] [\tilde{W} - \tilde{R}]
\]

or [after simplification, using \(\lambda_{LY} = 1 - \lambda_{LX}, \lambda_{KY} = 1 - \lambda_{KX}\), and gathering terms]:

\[
\lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ \tilde{L} - \tilde{K} \right] + \left[ a_X \sigma_X + a_Y \sigma_Y \right] [\tilde{W} - \tilde{R}] \tag{M9}
\]

with:

\[
\lambda^* \equiv \lambda_{LX} - \lambda_{KX}
\]

\[
a_X \equiv \lambda_{LX} \theta_{KX} + \lambda_{KX} \theta_{LX} > 0
\]

\[
a_Y \equiv \lambda_{LY} \theta_{KY} + \lambda_{KY} \theta_{LY} > 0
\]
• **Note:** $\lambda^*$ is again a measure of relative factor intensity but now in terms of physical units [$\theta^*$ is in terms of factor shares]

– if $X$ is relatively labour intensive ($\lambda^* > 0$) then an increase in $X/Y$ is associated with a rise in $W/R$.

– in the absence of distortions and/or taxes $\lambda^*$ and $\theta^*$ always have the same sign:

\[
\lambda^* = 0 \iff \theta^* = 0
\]

**Proof:** see A&S (1980, pp. 169-170)

• We now have all the ingredients to characterize the general equilibrium in this model.
The key equations are:

\[ \tilde{X} - \tilde{Y} = -\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y \right] \]  \hspace{1cm} (M1)

\[ \tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] \]  \hspace{1cm} (M2)

\[ \lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ \tilde{L} - \tilde{K} \right] + \left[ a_X \sigma_X + a_Y \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] \]  \hspace{1cm} (M9)

- (M1) is the relative demand equation relating \( X/Y \) to \( P_X/P_Y \)
- (M2) is the competitive (relative) pricing relationship relating \( P_X/P_Y \) to \( W/R \)
- (M9) represents the factor market equilibrium conditions relating \( W/R \) to \( X/Y \) and \( \bar{L}/\bar{K} \)

- In Figure 6.2 we illustrate the determination of the general equilibrium under the assumption that the \( X \)-sector is relatively labour intensive (so that \( \lambda^* > 0 \) and \( \theta^* > 0 \))
- The D curve in the top right-hand panel is the demand equation (M1). It slopes down because $\sigma_D > 0$

- The FME curve in the top left-hand panel is the factor markets equilibrium locus (M9), holding constant $\bar{L}/\bar{K}$. In view of the assumption that $\lambda^* > 0$ it is an upward sloping line.

- In the bottom left-hand panel we turn the corner.

- The CPR curve in the bottom right-hand panel represents the competitive pricing relationship equation (M2). The assumption that $\theta^* > 0$ implies that this curve is upward sloping.

- Together, CPR and FME characterize the supply side of the model. The supply curve, S, in the top right-hand panel is constructed graphically by “completing the boxes” for different relative price levels (see ABCD and FGHI). The thus constructed supply curve is upward sloping.

- General equilibrium occurs at point $E_0$ where demand equals supply.
Figure 6.2: Two-Sector General Equilibrium (X labour-intensive)
Before turning to the tax analysis we first study two applications:

- increase in $\frac{L}{K}$
- special Leontief-case with zero substitution in production ($\sigma_Y = \sigma_X = 0$)

The effect of an increase in $\frac{L}{K}$

- In Figure 6.3 the only curve affected is the FME curve (which shifts up)
- as a result the supply curve shifts up (from $S_0$ to $S_1$) and the equilibrium shifts from $E_0$ to $E_1$
- $P_X/P_Y$ falls, $X/Y$ rises, and $W/R$ falls. [Result related to Rybczynski Theorem: at unchanged commodity prices, an expansion in one factor results in an absolute decline in the commodity intensive in the use of the other factor]
Figure 6.3: Increase in Labour Endowment (X labour-intensive)
The Leontief case

- In Figure 6.4 we illustrate the Leontief case of the model.
- Since $\sigma_X = \sigma_Y = 0$ it follows from (M9) that FME is horizontal (see $\text{FME}_0$).
- As a result, the supply curve is also horizontal (see $\text{S}_0$).
- Equilibrium is at point $E_0$. Equality between $D$ and $\text{S}_0$ determines the relative goods price ($P_X/P_Y$) which, via the pricing relationship $\text{CPR}$, determines the wage-rental rate ($W/R$).
- An increase in $\bar{L}$ leads to an upward shift in the FME curve (from $\text{FME}_0$ to $\text{FME}_1$), an upward shift in supply (from $\text{S}_0$ to $\text{S}_1$), a decrease in $P_X/P_Y$, and a decrease in $W/R$. 
Figure 6.4: Leontief Technology (X labour-intensive)
Adding Taxes to the Model

• Following A & S (pp. 173-183) we consider the following range of *ad valorem* taxes:
  
  – taxes on factor prices in both sectors: \( t_{KX}, t_{KY}, t_{LX}, \) and \( t_{LY} \) (sometimes we consider common factor taxes \( t_{KX} = t_{KY} \) and \( t_{LX} = t_{LY} \) and sometimes common sector taxes \( t_{KX} = t_{LX} \) and \( t_{KY} = t_{LY} \))
  
  – taxes on the outputs: \( t_X \) and \( t_Y \)
  
  – **Note**: all revenue recycled to households in a lump-sum fashion and we continue to focus on the homothetic case

• The model is affected as follows:
  
  – the factor market equilibrium conditions are now:
    
    \[
    \bar{L} = c_x^w \left[ W (1 + t_{LX}) , R (1 + t_{KX}) \right] X + c_y^w \left[ W (1 + t_{LY}) , R (1 + t_{KY}) \right] Y \tag{1}
    \]
    
    \[
    \bar{K} = c_x^r \left[ W (1 + t_{LX}) , R (1 + t_{KX}) \right] X + c_y^r \left[ W (1 + t_{LY}) , R (1 + t_{KY}) \right] Y \tag{2}
    \]
the (producer) price equations are:

\[ P_X = c^x [W (1 + t_{LX}) , R (1 + t_{KX})] \] \hspace{1cm} (3)
\[ P_Y = c^y [W (1 + t_{LY}) , R (1 + t_{KY})] \] \hspace{1cm} (4)

the demand equations are:

\[ X = d^x [P_X (1 + t_X) , P_Y (1 + t_Y)] W \bar{L} + R \bar{K} + T \] \hspace{1cm} (5)
\[ Y = d^y [P_X (1 + t_X) , P_Y (1 + t_Y)] W \bar{L} + R \bar{K} + T \] \hspace{1cm} (6)

the tax revenue is:

\[ T = W [t_{LX} L_X + t_{LY} L_Y] + R [t_{KX} K_X + t_{KY} K_Y] \]
\[ + t_X P_X X + t_Y P_Y Y \] \hspace{1cm} (7)

endogenous: \(X, Y, P_X, P_Y, W, R,\) and \(T\)

exogenous: \(\bar{K}, \bar{L}, t_{LX}, t_{LY}, t_{KX}, t_{KY}, t_X, t_Y\)
Following exactly the same steps as before we find the log-linearized model [see also Slide 34]

\[ 
\tilde{X} - \tilde{Y} = -\sigma_D \left[ \left( \tilde{P}_X - \tilde{P}_Y \right) + (\tilde{t}_X - \tilde{t}_Y) \right] \quad (M1) 
\]

\[ 
\tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] + \theta_{LX} \tilde{t}_{LX} - \theta_{LY} \tilde{t}_{LY} + \theta_{KX} \tilde{t}_{KX} - \theta_{KY} \tilde{t}_{KY} \quad (M2) 
\]

\[ 
\lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ a_X \sigma_X + a_Y \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] 
+ a_X \sigma_X \left[ \tilde{t}_{LX} - \tilde{t}_{KX} \right] + a_Y \sigma_Y \left[ \tilde{t}_{LY} - \tilde{t}_{KY} \right] \quad (M3) 
\]

- (M1) is the relative demand equation relating \( X/Y \) to \( P_X/P_Y \) and \( t_X/t_Y \)

- (M2) is the competitive (relative) pricing relationship relating \( P_X/P_Y \) to \( W/R \) and the various tax rates

- (M3) represents the factor market equilibrium conditions relating \( W/R \) to \( X/Y \) and the various tax rates
• **Remark 1**: tax equivalencies

- A tax on capital income and labour income at the same rate \( t_{KX} = t_{KY} = t_K \), \( t_{LX} = t_{LY} = t_L \), and \( t_L = t_K = t \) has the same effect as a tax on both products at the same rate \( t_X = t_Y \). No effect on \( X/Y \), \( P_X/P_Y \), or \( W/R \); taxes drop out of (M3) and (M2) simplifies to:

\[
\tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] + (\theta_{LX} - \theta_{LY}) \tilde{t}_L + (\theta_{KX} - \theta_{KY}) \tilde{t}_K \\
= \theta^* \left[ \tilde{W} - \tilde{R} \right] + \theta^* (\tilde{t}_L - \tilde{t}_K) \tag{M2'}
\]

- A tax on both factors in the same industry (e.g. \( t_{LX} = t_{KX} \)) has no substitution effect and is equivalent to an excise tax [specific tax on good \( X \)]

- A tax on capital in both sectors at the same rate \( t_{KX} = t_{KY} \) is simply a tax on the fixed factor \( \bar{K} \)
– implication: if we know the effect of the general tax $t$ then we only need to study three (out of the eight) taxes set out in the model, provided they are independent. E.g. if we know the respective effects of $t_{KX}$, $t_X$, and $t_K$ then we also know the effects of the remaining taxes. A&K present the following equivalency table:

\[
\begin{align*}
    t_{KX} + t_{LX} &= t_X \\
    t_{KY} + t_{LY} &= t_Y \\
    t_K + t_L &= t
\end{align*}
\]

Table 1: Tax Equivalencies
• **Remark 2** on factor intensities

  – in presence of taxes [or other distortions] the ranking according to physical factor intensity \((\lambda^*)\) may not be the same as the ranking according to factor shares \((\theta^*)\).


  – For now we simply assume that the rankings remain the same.
General Equilibrium Tax Effects

- The effect on a marginal change in the **output tax** in sector $X$ is illustrated in Figure 6.5.
  - $\tilde{t}_X > 0$; all other taxes unchanged: $\tilde{t}_Y = \tilde{t}_{LX} = \tilde{t}_{LY} = \tilde{t}_{KX} = \tilde{t}_{KX} = 0$
  - $X$ relatively labour-intensive, i.e. we continue to assume that $\theta^* > 0$ and $\lambda^* > 0$
  - model reduces to:

$$\tilde{X} - \tilde{Y} = -\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y + \tilde{t}_X \right] \quad (M1)$$

$$\tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] \quad (M2)$$

$$\lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ a_X \sigma_X + a_Y \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] \quad (M3)$$

- demand shifts down from $D_0$ to $D_1$
- equilibrium shifts from $E_0$ to $E_1$, $P_X/P_Y$ falls, $X/Y$ falls, and $W/R$ falls.
there is only an output (or volume) effect: pattern of demand changes in favour of good Y. Since $X/Y$ falls and since $X$ is relatively labour intensive, the wage-rental ratio falls. This effect holds regardless of the magnitude of the substitution elasticity in the $X$-sector.

mathematically we get by solving the model (after eliminating $\tilde{W} - \tilde{R}$):

$$
\begin{bmatrix}
1 & \sigma_D \\
-\lambda^*\theta^* & a_X\sigma_X + a_Y\sigma_Y
\end{bmatrix}
\begin{bmatrix}
\tilde{X} - \tilde{Y} \\
\tilde{P}_X - \tilde{P}_Y
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
0
\end{bmatrix}
\sigma_D\tilde{t}_X
$$
or:

\[
\begin{bmatrix}
\tilde{X} - \tilde{Y} \\
\tilde{P}_X - \tilde{P}_Y
\end{bmatrix}
= \frac{1}{|\Delta|}
\begin{bmatrix}
a_X \sigma_X + a_Y \sigma_Y & -\sigma_D \\
\lambda^* \theta^* & 1
\end{bmatrix}
\begin{bmatrix}
-1 \\
0
\end{bmatrix}
\sigma_D \tilde{t}_X
\]

\[
= -\frac{1}{|\Delta|}
\begin{bmatrix}
a_X \sigma_X + a_Y \sigma_Y \\
\lambda^* \theta^*
\end{bmatrix}
\sigma_D \tilde{t}_X
\]

where \(|\Delta| \equiv a_X \sigma_X + a_Y \sigma_Y + \lambda^* \theta^* \sigma_D > 0\)
Figure 6.5: Increase in the Output Tax $t_X$ (X labour-intensive)
The effect on a marginal change in the capital tax in sector $X$ is illustrated in Figures 6.6 and 6.7.

- $\tilde{t}_{KX} > 0$; all other taxes unchanged: $\tilde{t}_X = \tilde{t}_Y = \tilde{t}_{LX} = \tilde{t}_{LY} = \tilde{t}_{KY} = 0$
- $X$ relatively labour-intensive, i.e. we continue to assume that $\theta^* > 0$ and $\lambda^* > 0$
- model reduces to:

\[
\tilde{X} - \tilde{Y} = -\sigma_D \left[ \tilde{P}_X - \tilde{P}_Y \right] \tag{M1}
\]
\[
\tilde{P}_X - \tilde{P}_Y = \theta^* \left[ \tilde{W} - \tilde{R} \right] + \theta_{KX} \tilde{t}_{KX} \tag{M2}
\]
\[
\lambda^* \left[ \tilde{X} - \tilde{Y} \right] = \left[ a_X \sigma_X + a_Y \sigma_Y \right] \left[ \tilde{W} - \tilde{R} \right] - a_X \sigma_X \tilde{t}_{KX} \tag{M3}
\]

- in general both FME and CPR are affected: both output effect and factor substitution effect
In **Figure 6.6** we assume the substitution elasticity in the $X$-sector is zero ($\sigma_X = 0$)

- FME is not affected by the tax change
- CPR shifts to the right from CPR$_0$ to CPR$_1$
- supply shifts to the right from S$_0$ to S$_1$
- equilibrium shifts from E$_0$ to E$_1$, $P_X/P_Y$ rises, $X/Y$ falls, and $W/R$ falls.
  
  Despite the fact that capital is taxed, the rental rate on capital rises relative to wages.

- there is only an **output** (or **volume**) **effect**: $P_X/P_Y$ falls and demand shifts toward good $Y$ ($X/Y$ falls). Since $X$ is labour-intensive, labour demand drops off whilst capital demand is boosted. In equilibrium $W/R$ has to fall.
Figure 6.6: Increase in the Corporate Tax $t_{KX}$ ($X$ labour-intensive, $\sigma_X = 0$)
In Figure 6.7 we consider the general case (i.e. $\sigma_X > 0$)

- now FME shifts down from $FME_0$ to $FME_1$
- as before CPR shifts to the right from $CPR_0$ to $CPR_1$
- supply shifts to the right from $S_0$ to $S_1$
- equilibrium shifts from $E_0$ to $E_1$, $P_X/P_Y$ rises, $X/Y$ falls, and $W/R$ rises. Now the rental rate on capital falls relative to wages.

- we can write the model is one matrix equation as:

$$
\begin{bmatrix}
1 & \sigma_D \\
-\lambda^*\theta^* & a_X\sigma_X + a_Y\sigma_Y
\end{bmatrix}
\begin{bmatrix}
\tilde{X} - \tilde{Y} \\
\tilde{P}_X - \tilde{P}_Y
\end{bmatrix}
= \begin{bmatrix}
0 \\
\Gamma
\end{bmatrix}
\tilde{t}_{KX}
$$

where $\Gamma$ is:

$$
\Gamma \equiv a_X\sigma_X (\theta_{KX} + \theta^*) + a_Y\sigma_Y \theta_{KX}
= a_X\sigma_X \theta_{KY} + a_Y\sigma_Y \theta_{KX} > 0
$$
$$\begin{bmatrix}
\tilde{X} - \tilde{Y} \\
\tilde{P}_X - \tilde{P}_Y
\end{bmatrix} = \frac{1}{|\Delta|} \begin{bmatrix}
a_X \sigma_X + a_Y \sigma_Y & -\sigma_D \\
\lambda^* \theta^* & 1
\end{bmatrix} \begin{bmatrix}
0 \\
1
\end{bmatrix} \Gamma\tilde{t}_{KX}
$$

$$= \frac{1}{|\Delta|} \begin{bmatrix}
-\sigma_D \\
1
\end{bmatrix} \Gamma\tilde{t}_{KX}$$

where $|\Delta| \equiv a_X \sigma_X + a_Y \sigma_Y + \lambda^* \theta^* \sigma_D > 0$

- the effect on the wage-rental ratio, $W/R$, can be found by substituting the solution for $\tilde{P}_X - \tilde{P}_Y$ into \( (M2) \):

$$\theta^* \left[ \tilde{W} - \tilde{R} \right] = \left[ \tilde{P}_X - \tilde{P}_Y \right] - \theta_{KX} \tilde{t}_{KX}$$

$$= \frac{a_X \sigma_X \theta_{KY} + a_Y \sigma_Y \theta_{KX}}{a_X \sigma_X + a_Y \sigma_Y + \lambda^* \theta^* \sigma_D} \tilde{t}_{KX} - \theta_{KX} \tilde{t}_{KX}$$
or (after some steps):

\[ \theta^\ast \left[ \tilde{W} - \tilde{R} \right] = \left[ \frac{a_X \sigma_X - \lambda^\ast \theta_{KX} \sigma_D}{a_X \sigma_X + a_Y \sigma_Y + \lambda^\ast \theta^\ast \sigma_D} \right] \tilde{t}_{KX} \]  

\text{(FP)}

**Notes:**

- **diamond** denominator is positive (as \( a_X > 0, a_Y > 0, \sigma_X \geq 0, \sigma_Y \geq 0, \sigma_D > 0, \) and \( \lambda^\ast \theta^\ast > 0 \)) so sign of the effect on \( W/R \) is determined by numerator

- **diamond** numerator has two terms: (a) factor substitution effect (represented by \( a_X \sigma_X \)) and (b) output effect (represented by \( -\lambda^\ast \theta_{KX} \sigma_D \))

- **diamond** output effect is negative (positive) if the \( X \)-sector is labour (capital) intensive
Figure 6.7: Increase in the Corporate Tax $t_{KX}$ (X labour-intensive)
Extensions to the Basic Model

The basic two-by-two model can easily be extended.

- factor supplies can be made endogenous
  - [static] endogenous labour supply [add leisure to household utility function]
  - [dynamic] saving and capital accumulation [studied in Chapter 8]
- intersectoral mobility assumption can be augmented:
  - [static] Mussa-Neary: labour mobile but capital sector-specific
  - [static] McLure: capital mobile but labour sector-specific
  - [dynamic] adjustment costs on capital and/or labour
- representative agent model can be replaced by heterogeneous agent model
  - now we can also study distribution issues [how do taxes affect different households etc.]
• open economy version of the model

• allow for imperfections on the goods and or labour market [see Chapter 7]

• using computers we can formulate, calibrate, and run simulations with highly detailed/complex computable general equilibrium models
  – sky is the limit
  – information on key elasticities shaky
  – scenario analyses