The questions with a star (★) are relatively difficult.

**Question 1 (★)**

[Ramsey (1928), Intriligator (1971)] Consider the Ramsey growth model discussed in section 8.1.2 of the book. The representative household discounts future felicity by the pure rate of time preference, \( \rho \). In the original treatment of the problem of optimal economic growth, Frank Ramsey argued on the basis of ethical beliefs that there should be no discounting of future felicity, i.e. he thought that the social planner should aim for a social optimum based on the notion that \( \rho = 0 \). Since the welfare integral will then not generally converge, Ramsey suggested a different approach. He assumed that there is a finite maximum production level (imposed by technology) and thus an upper limit to felicity which he called bliss, \( B \):

\[
B \equiv \max_{\{C\}} U(C) = U(C_B),
\]

where \( U(\cdot) \) is the felicity function, \( C_B \) is the bliss consumption per worker which is assumed to be finite. He then postulated the following (undiscounted) social objective function that is to be minimized:

\[
SR \equiv \int_0^\infty [B - U(C(t))] \, dt,
\]

where \( SR \) is a measure of “social regret”, i.e. the social cost associated with deviating from the bliss point. The economy-wide resource constraint is:

\[
\{Y(t) \equiv \} \quad F(K(t), \bar{L}) = C(t) + \delta K(t) + \dot{K}(t),
\]

where \( F(\cdot) \) is the production function (featuring the usual properties), \( K(t) \) is the capital stock, and \( \delta \) is the depreciation rate. There is a single infinitely-lived representative agent who supplies \( \bar{L} \) units of labour in each period.
(a) Solve the Ramsey problem of minimizing social regret subject to the neoclassical growth model. Assume that there is no technological change and that the population is constant.

(b) Illustrate your answer to part (a) with the aid of a diagram and show that the social optimization model is saddle-point stable. Relate your answer to the so-called golden rule.

(c) In the decentralized economy, the representative agent does of course discount future felicity. Show that the social planner can use the corporate tax rate to make sure that the social optimum is replicated in the decentralized economy. Explain the intuition behind your results.

Question 2

Consider the capital-fundamentalist model based on the notion that capital and labour can be easily substituted:

$$f[k(t)] \equiv \left[1 - \varepsilon + \varepsilon k(t)^{(\sigma_{KL} - 1)/\sigma_{KL}}\right]^{\sigma_{KL}/(\sigma_{KL} - 1)},$$

where $f(\cdot)$ is the intensive-form production function, $k(t)$ is the capital-labour ratio, $0 < \varepsilon < 1$ and $\sigma_{KL} \gg 1$. Use the Solow-Swan model and assume that the saving rate $s$ depends negatively on the corporate tax, i.e. $s = s(t_C)$ with $s'(t_C) < 0$. Assume also that in the initial situation, the savings rate is such that $s\varepsilon^{\sigma_{KL}/(\sigma_{KL} - 1)} > \delta + n_L$.

(a) Show the effects of an increase of the corporate tax on both the level and growth rate of income per worker. Show the impact, transitional, and long-run effects.

(b) Is it possible for the tax increase to eliminate growth in income per worker altogether? Explain your reasoning.

Question 3

Consider the Lucas-Uzawa model discussed in section 8.2.2 of the book.

(a) Explain what happens to the steady-state values of $R^K$, $R^H$, $k_Y$, $k_E$, $g_C$ and $g_Y$ if the output tax levied on firms ($t_Y$) is increased. Explain the economic intuition.

(b) Introduce an ad valorem labour income tax, $t_L$, levied on the representative household. Re-derive the steady-state expressions for $R^K$, $R^H$, $k_Y$, $k_E$, $g_C$ and $g_Y$. Show what happens to these variables if $t_L$ is increased. Explain the economic intuition.
Question 4

Consider the endogenous technology model discussed in section 8.2.3 of the book.

(a) Show what happens to $L_X(t)$, $r(t)$, $\gamma_C(t)$, $\gamma_N(t)$, $\gamma_Y(t)$ if the government increases the wage subsidy in the R&D sector once and for all. Explain the economic intuition and illustrate your answers in an impulse-response diagram with time on the horizontal axis and the different variables on the vertical axis.

(b) Now assume that the R&D technology incorporates both diminishing returns (to giants’ shoulders) and a duplication externality (as suggested by Jones (1995)). Continue to assume that there is no population growth. Redo part (a) for the augmented model.

References

