Problem Set Chapter 7: Tax Incidence in Imperfect Markets

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The questions with a star (★) are relatively difficult.

Question 1

[Holmlund and Lundborg (1990)] In this question we study tax policy analysis in a partially unionized small open economy. The economy produces two goods, X and Y. The country is small in world markets and faces given prices of the two goods. Technology is as given in the book, i.e. $X = F^x(L_X, K_X)$ and $Y = F^y(L_Y, K_Y)$, where $L_i$ and $K_i$ are, respectively, labour and capital used in the production of good $i$. The associated cost functions are given by $C^x = c^x(W_X, R_X)X$ and $C^y = c^y(W_Y, R_Y)Y$, where $W_i$ and $R_i$ are, respectively, the rental rates on labour and capital paid by producers of good $i$. There are $H$ identical agents who each have a homothetic utility function defined over both commodities (where $H$ is a large number). Each agent supplies one unit of labour to the labour market. A worker can be either employed in the secondary sector (at wage rate $W_Y$), employed in the primary sector (at wage $W_X$), or unemployed (in which case he/she received the unemployment benefit, $B$). The government budget constraint is $uHB = tKR\bar{K}$, where $u$ is the unemployment rate and $t_K$ is a capital income tax. The total supplies of capital and labour are fixed (at $\bar{L}$ and $\bar{K}$, respectively) and immobile internationally. The trade balance is in equilibrium. Capital is perfectly mobile across sectors, i.e. $R_X = R_Y = R$.

(a) Assume that capital ownership is equalized across households. Show that the indirect utility function for household $i$ can be written as:

$$V \equiv v(P_X, P_Y) \left[ Y_i + (1 - t_K) \frac{R\bar{K}}{H} \right],$$

(1)

where $v(\cdot)$ is a constant and $Y_i$ is non-capital income of the household:

$$Y_i = \begin{cases} W_X & \text{if employed in the (primary) X-sector} \\ B & \text{if unemployed} \\ W_Y & \text{if employed in the (secondary) Y-sector} \end{cases}$$

(2)
(b) Assume that there is a single monopoly union in the primary sector. This union has
a fixed number of members, \( N = \zeta H \) (with \( 0 < \zeta < 1 \)). The objective function of
the union is the expected (indirect) utility of its members:

\[
TU = v \left( \frac{L_X}{N} \left[ W_X + (1 - t_K) \frac{RK}{H} \right] + \left[ 1 - \frac{L_X}{N} \right] \left[ W_Y + (1 - t_K) \frac{RK}{H} \right] \right).
\]  

The union sets \( W_X \) in order to maximize \( TU (W_X, L_X) \) subject to the downward sloping
labour demand curve in the \( X \)-sector. It takes as given \( R, \bar{K}, t_K, \) and \( W_Y \). What do
we assume about union members who do not get a job in the primary sector? What is
the unemployment rate \( (u) \) in this economy? Derive an expression for the wage rate in
the \( X \)-sector.

(c) Write down the equations of the full model. Indicate which are the endogenous and
exogenous variables. Show that the number of endogenous variables is equal to the
number of independent equations.

(d) Using the model of part (c), derive the effects on all endogenous variable of an increase
in the capital endowment, \( \bar{K} \). Explain the intuition behind your results.

(e) \( (\star) \) Assume that union membership is endogenously determined. The worker either (i)
works (for sure) in the secondary sector or (ii) joins the queue in the primary sector
and gets a job with probability \( L_X/N \) or remains unemployed with probability \( 1 - L_X/N \). Once option (ii) is chosen the worker cannot change sectors within the period.
The worker evaluates the expected utility associated with the two options. Derive the
intersectoral equilibrium condition.

(f) \( (\star) \) Write down the equations of the full model of part (e). Indicate which are the
endogenous and exogenous variables. Show that the number of endogenous variables
is equal to the number of independent equations. Study the effects on all endogenous
variables of an increase in \( B \) financed by means of the capital income tax.

Question 2

[Brakman and Heijdra (2004)] In this question we conduct tax analysis in a simple monopo-
listic competition model. There are two sectors in the economy. The first sector produces a
homogeneous good \( Z \) under constant returns to scale and features perfect competition. The
second sector consists of a large group of monopolistically competitive firms who produce
under increasing returns to scale at firm level. The utility function of the representative
household is \( U = Z^\delta Y^{1-\delta} \), where \( 0 < \delta < 1 \) and \( Y \) is the consumption of a composite
differentiated good:

\[
Y \equiv \left[ \sum_{i=1}^{N} X_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad 1 < \sigma \ll \infty.
\] (1)

In equation (1), \(N\) is the existing number of different varieties, \(X_i\) is consumption of variety \(i\), and \(\sigma\) is the elasticity of substitution. The household inelastically supplies \(H\) units of labour. The household budget constraint is:

\[
\sum_{i=1}^{N} P_i X_i + P_Z Z = I,
\] (2)

where \(P_i\) is the price of variety \(i\), \(P_Z\) is the price of the homogeneous good, and \(I\) is household income. There is one factor of production, labour, which is perfectly mobile across sectors and across firms in the monopolistic sector. Firms in the homogeneous goods sector face the technology \(Z = L_Z/k_Z\), where \(L_Z\) is the amount of labour used and \(k_Z\) is the (exogenous) technology index. Production in the monopolistically competitive \(Y\)-sector is characterized by internal economies of scale. Each individual firm \(i\) uses labour to produce its product variety and faces the following technology:

\[
X_i = \begin{cases} 
0 & \text{if } L_i \leq F \\
(1/k_Y) [L_i - F] & \text{if } L_i \geq F 
\end{cases}
\] (3)

where \(X_i\) is the marketable output of firm \(i\), \(L_i\) is labour used by the firm, \(F\) is fixed cost in terms of units of labour, and \(k_Y\) is the (constant) marginal labour requirement. The profit of firm \(i\) is denoted by \(\Pi_i\):

\[
\Pi_i \equiv P_i X_i - W [k_Y X_i + F],
\] (4)

where \(W\) is the wage rate. Each firm \(i\) acts as a Cournot-Nash quantity setter aiming to maximize profit. There is free entry/exit of firms in the monopolistically competitive sector.

(a) Derive expressions for the representative household’s demands for \(Z\), \(Y\), and \(X_i\). Show that the demand for \(X_i\) depends negatively on the price \(P_i\) and compute the demand elasticity.

(b) Demonstrate that each firm \(i\) will set its own price according to a markup rule, \(P_i = \mu W k_Y\), where \(\mu > 1\). Show the relationship between the gross markup \(\mu\) and the elasticity of firm \(i\)’s demand curve.

(c) Compute the equilibrium firm size in the monopolistically competitive sector. Provide an expression for household income in the symmetric equilibrium.
(d) Write down the equations of the full model. Indicate which are the endogenous and exogenous variables. Show that the number of endogenous variables is equal to the number of independent equations.

(e) Introduce a payroll tax in the monopolistically competitive sector and assume that the revenue is recycled in a lump-sum fashion to the representative household. Derive the effects on all endogenous variable of an increase in the payroll tax, $t_{LX}$. Explain the intuition behind your results.

References
