Question 1

[Adapted from Sandmo (1985, p. 296)] Consider the gross taxation case studied in section 4.2.2 of the book. We are interested in the comparative static effect of the tax \( t_A \) on the amount of the risky asset bought \( \partial b/\partial t_A \). The conditions jointly determining \( \partial C_1/\partial t_A \) and \( \partial b/\partial t_A \) are stated in equations (4.36)-(4.38) in the book. Here we focus on the pure portfolio decision and assume that \( a + b = h_1 - C_1 \) is held constant (i.e. we set \( \partial C_1/\partial t_A = 0 \) and ignore (4.36)).

(a) Show that \( \partial b/\partial t_A \) can be written as:

\[
\frac{\partial b}{\partial t_A} = \frac{b}{1 - t_A} + \frac{r(a + b) E[U''(\tilde{x} - r)]}{1 - t_a} E[U''(\tilde{x} - r)^2].
\] (1)

(b) Compute the comparative static effect on \( b \) of an increase in \( a + b \). Show that it can be written as:

\[
\frac{\partial b}{\partial (a + b)} = -\frac{1 + r(1 - t_A) E[U''(\tilde{x} - r)]}{1 - t_A} E[U''(\tilde{x} - r)^2].
\] (2)

(c) Derive an expression relating \( \partial b/\partial t_A \) to the wealth elasticity of the risky asset, which we define as \( \varepsilon_B = \frac{a+b}{b} \frac{\partial b}{\partial (a+b)} \). Show that an increase in \( t_A \) has conflicting wealth and substitution effects. Which effect do you expect to dominate? Explain.

Question 2

[Sandmo (1970)] This question deals with the issue of income risk. The representative household has the following lifetime utility function:

\[
E(\tilde{\Lambda}) = U(C_1) + \left( \frac{1}{1 + \rho} \right) E[U(\tilde{C}_2)],
\] (1)
where $E(\cdot)$ is the expectations operator, $C_t$ is consumption in period $t$, $\rho$ is the constant rate of pure time preference ($\rho > 0$), and $U(\cdot)$ is the felicity function. This function features a positive but diminishing marginal felicity, i.e. $U''(\cdot) > 0 > U'''(\cdot)$. There is a single asset with a certain rate of return, $r$. The budget identities for the two periods are:

$$Y_1 = C_1 + S_1,$$
$$\tilde{C}_2 = \tilde{Y}_2 + (1 + r) S_1,$$

where $Y_t$ is income in period $t$ and $S_1$ is saving in the first period. First-period income is deterministic but future income is stochastic, i.e. the household faces income risk. The (known) probability density function for $\tilde{C}_2$ is denoted by $f(\tilde{C}_2)$.

(a) Derive the first-order necessary conditions for utility maximization. Also state the second-order condition.

(b) Derive the effects on current consumption and saving of an increase in current income, $Y_1$. Explain how you use the second-order condition to determine the sign of $\partial C_1 / \partial Y_1$.

(c) Assume that future income can be written as $\tilde{Y}_2 + \theta$, where $\theta = 0$ in the initial situation. Show the effect on current consumption and saving of a small increase in $\theta$. Explain what happens to the expected value (mean) of future income as a result of the shock.

(d) $(\star)$ Write future income as $\gamma \tilde{Y}_2 + \theta$ and study the effects on $C_1$ and $S_1$ of a mean-preserving increase in the uncertainty about future income. [Hint: increase $\gamma$ to increase riskiness but decrease $\theta$ to keep the mean constant.]

**Question 3**

[Eaton and Rosen (1980a), Block and Heineke (1973)] In this question we investigate the effect of wage uncertainty and taxation on labour supply by the representative household. The household has the following utility function:

$$U = U(C, \bar{L} - L),$$

where $U$ is utility, $C$ is consumption, $\bar{L}$ is the exogenous time endowment, and $L$ is labour supply ($\bar{L} - L$ is thus leisure). The utility function has the usual properties, i.e. $U_C > 0$, $U_{LL} > 0$, $U_{CC} < 0$, $U_{L-L,L-L} < 0$, and $U_{CC}U_{L-L,L-L} - U_{C,L-L}^2 > 0$. The budget constraint is given in real terms by:

$$\tilde{C} = m + \bar{w}(1 - t_L)L,$$
where \( m \) is real non-labour income, \( \tilde{w} \) is the stochastic real wage, and \( t_L \) is the labour income tax. Both \( m \) and \( t_L \) are non-stochastic. The household knows the probability density function for the gross wage \( (f(\tilde{W})) \) and chooses \( C \) and \( L \) in order to maximize expected utility.

(a) Derive the first-order necessary condition for labour supply. Also state the second-order condition for a maximum.

(b) Derive the effect on labour supply of an increase in non-labour income, \( m \). Prove that leisure is a normal good if the utility function is additively separable (i.e. \( U_{C,L-L} = 0 \)).

(c) (★) Assume that utility is additively separable and can be written as:

\[
U = \frac{1}{\gamma} [C^\gamma - 1] + V (\bar{L} - L),
\]

where \( 1 - \gamma \ (> 0) \) is the constant rate of relative risk-aversion, and \( V (\cdot) \) features the derivatives \( V'(\cdot) > 0 \) and \( V''(\cdot) < 0 \). Derive the effect on labour supply of an increase in the labour income tax, \( t_L \). Prove that the effect will be negative if \( 1 - \gamma \approx 0 \) but positive if \( 1 - \gamma \) is sufficiently high.

**Question 4**

[Allingham and Sandmo (1972)] This question investigates a simple static model of income tax evasion. A risk-averse household’s actual before-tax income, \( W \), is exogenously given and known to that household but not to the tax collection agency. The household declares income \( X \) to the tax agency and pays a proportional tax, \( t_Y \), on declared income. With probability \( \pi \) the household is investigated by the tax agency who will then observe \( W \). If \( W \) exceeds \( X \) then the household pays tax on the undeclared income, \( W - X \), and the rate \( t_P \) which exceeds \( t_Y \). With probability \( 1 - \pi \) the household is not investigated. The indirect utility function is written as \( \tilde{U} = V (\tilde{Y}) \) where \( \tilde{U} \) is stochastic utility, \( \tilde{Y} \) in stochastic after-tax income, and the indirect utility function satisfies \( V'(\tilde{Y}) > 0 \) and \( V''(\tilde{Y}) < 0 \) for all realizations of \( \tilde{Y} \).

(a) Derive the expression for \( \tilde{Y} \).

(b) Assume that the household chooses declared income \( X \) in order to maximize its expected utility. Derive the first- and second order conditions for a maximum.

(c) Derive the conditions under which the household will rationally choose to under-report income (by choosing \( X < W \)). Explain.

(d) Show that for an initially tax-evading household, an increase in the penalty rate \( (t_P) \) or the probability of detection \( (\pi) \) will lead to an increase in declared income.
(e) Define the relative risk aversion function as \( R(\tilde{Y}) \equiv -\tilde{Y}U''(\tilde{Y})/U''(\tilde{Y}) \). Derive the comparative static result of an increase in before-tax income \((W)\) on the proportion of income that is declared \((X/W)\). Relate your expression to the relative risk aversion function.

References


