Public Investment and Intergenerational Distribution Under Alternative Modes of Financing*

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Abstract

We study the effects of a gradual increase in the stock of public capital in a dynamic overlapping-generations model of a small open economy under alternative modes of financing: lump-sum taxation, a balanced-budget policy with a variable tax rate on labour income, and a once-and-for-all change in the tax on labour. The macroeconomic effects at impact as well as in the long run depend critically on the financing mode and on the initial tax rate on labour income. The same holds for the welfare effects on the different generations and thus for the political support among existing generations for increasing public investment. **JEL classification**: D62, E62, H23, H31, H54, **Keywords**: public investment, intergenerational welfare effects, taxation

1 Introduction

The empirical research by Aschauer (1989, 1990), which suggested that public capital has a powerful impact on the productivity of private capital, prompted a number of theoretical studies of the relation between public investment and private production. Several themes have dominated this literature so far. The first theme is public investment as a potential source of endogenous growth. The seminal paper in this area is Barro (1990) which demonstrates that endogenous growth emerges provided the private production function exhibits constant returns to scale in private and public capital and the government maintains a constant ratio

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between the two types of capital. Under these conditions, the marginal product of private capital is constant and growth is perpetual. Further notable contributions to this branch of literature are Glomm and Ravikumar (1994) and Turnovsky (1996, 2000).

The second theme that can be distinguished in the literature is congestion of public infrastructure. Key contributions are Uzawa (1988), Glomm and Ravikumar (1994), and Fisher and Turnovsky (1998). These papers typically assume that the services derived by individual firms from public capital are subject to congestion. Although the specific manner in which this congestion is modelled differs between studies, the common notion is that congestion introduces an additional externality which may be internalized by the policy maker, for example by means of a service charge on the users of public infrastructure (Uzawa, 1988, pp. 346-349).

The third theme that has dominated the literature is public investment as a potential source of macroeconomic fluctuations. Notable papers in this area are Barro (1981), Aschauer and Greenwood (1985), Aschauer (1988), Baxter and King (1993), Turnovsky and Fisher (1995), Lansing (1998), Cassou and Lansing (1998), and Chang (1999). The key insights that emerge from this research are the crucial importance of, on the one hand, the intertemporal substitution effect in labour supply and on the other hand, the financing mode employed by the government. Baxter and King (1993) illustrate both these insights by means of numerical simulations.

Our paper fits in the last theme as it emphasizes the importance of public financial decisions especially within the context of public investment policy. In particular, we study the macroeconomic and (intergenerational) welfare effects of public investment under alternative financing modes: lump-sum taxation, variable distorting taxes on labour income, and a once-and-for-all change in the labour tax. Our analysis deviates from most existing literature on public investment as a source of macroeconomic fluctuations in two respects. Firstly, we deny the validity of Ricardian equivalence and thus believe that the timing of taxes has real effects on the economy. Second, we maintain that, as a result of stock-flow interactions, the productivity-enhancing effects of a public investment shock only materialise gradually over time. The combination of these two elements significantly complicates the analysis.

In a Ramsey framework the uneven timing of costs and benefits does not complicate matters conceptually as all costs and benefits ultimately affect a single infinitely-lived representative agent. Put differently, since the representative agent constitutes a short-cut description of a dynasty of finitely-lived and altruistically linked generations, there is no intergenerational external effect to worry about. Indeed, altruistically linked generations view future generations as continuations of themselves. Consequently, there is Ricardian equivalence and, abstracting from tax distortions, the timing of costs and benefits is irrelevant.[1] Public investment policy is relatively uncomplicated in this framework since it only needs to determine the appropriate (efficient) amount of public capital. This effectively amounts to equalizing the rate of return on public capital to the costs of public resources, including possible tax distortions. As Gramlich (assuming lump-sum financing) puts it "[i]f the effective real rate of return exceeds the going real interest rate, the investment is worthwhile.." (1994, p. 1984).

In contrast, in a non-Ricardian world populated by overlapping but unlinked generations of finitely-lived agents the timing of cost and benefits becomes crucial as these costs and benefits no longer accrue to the same agent or dynasty, i.e. there are both *intertemporal* and *intergenerational* external effects in operation. Public investment policy thus must pay attention to both the efficiency question of "how much to invest" and the distributional question of "which generations pay for the public capital stock and how much." The results in this paper show that the public financial decisions cannot be ignored in such a setting as they critically affect the level of public support for infrastructural projects in a democratic society.

The analysis in this paper builds to a large extent on Heijdra and Meijdam (2002) (HM hereafter). To keep the analysis as simple as possible, we extend a small-open economy version of the Yaari (1965)-Blanchard (1985) overlapping generations model by including the stock of public capital in the production function. We focus on simple functional forms and abstract from endogenous growth and congestion. First we study a balanced-budget policy where the increase in public spending is financed by a lump-sum tax. This case is also studied in HM. In contrast to HM, we assume that labour supply is endogenous and that there is a pre-existing tax on labour income in the initial steady state. This enables us to study how this pre-existing distortion influences the macroeconomic and welfare effects of a lump-sum financed increase in public investment. Moreover, we study two alternative modes of financing that are not analysed in HM. The second financing mode is also a balanced-budget policy, but now it is assumed that the increase in public investment is financed by a time-varying change in the tax on labour income. The third policy we analyse is one where the government changes the tax on labour income once-and-for-all to such a level that the discounted value of the primary surpluses resulting from this policy equals zero.

The remainder of the paper is structured as follows. Section 2 sets out the model. The household sector comprises a large number of cohorts which differ with respect to age and thus the level and composition of their wealth. Firms are perfectly competitive and face standard convex adjustment costs of investment. The government levies taxes, engages in bond policy,

[1] If tax distortions are taken into account the optimal policy for the timing of costs is the well-known tax smoothing policy (Barro, 1979).

and invests in public capital. After log-linearizing, the model can be solved recursively and the welfare effects for different generations can be assessed. Moreover, a suitable egalitarian bond policy can be designed which ensures that all generations gain to the same extent and thus neutralizes the intergenerational externality.

Section 3 constitutes the core of our paper and studies the effects of a permanent increase in public investment on the macroeconomy and the intergenerational welfare distribution under the alternative financing modes. First, lump-sum taxation is assumed. In the absence of bond policy the welfare effects are very unevenly distributed over generations. Old existing generations always benefit as they possess a large amount of financial assets which appreciate in value as a result of the shock. The effect on generations born close to the time of the shock is ambiguous, but is positively influenced by the initial tax on labour income. Consequently, the political support among existing generations for an increase in public investment is higher if the initial tax distortion is higher. Generations born in the new steady state are always better off than newborns because the path of full income is monotonically increasing over time. Whether they benefit from the shock depends not only on the initial share of public investment and the productivity of public capital (as in HM) but also on the elasticity of labour supply and the initial tax rate on labour. The higher the initial tax rate and the more elastic the labour supply, the better is the outcome for generations born in the far future. Finally, we isolate the pure efficiency effects of public investment by employing a so-called egalitarian bond policy which neutralizes the intergenerational externalities. We show that the resulting common welfare gain (as well as the optimal share of public investment) is a positive function of the initial distortionary tax rate. Our analysis also shows, however, that the policy package required to realize this optimum is quite complex so that it may not be attainable in actual economies.

Second, we assume a balanced-budget policy where the increase in public investment is financed by a time-varying change in the tax rate on labour. In this case, if public capital is relatively abundant, existing old generations gain less than under lump-sum finance as financial assets do not appreciate as much (or may even depreciate). On the other hand, however, they may also gain more than in case of lump-sum financing. This happens if the initial share of public investment is quite low. In that case, the tax on labour income, which rises initially, falls in the long run, leading to a stronger impulse to the economy than under lump-sum finance. Consequently, generations born in the new steady state are better off with distortionary taxation. This is not necessarily true for generations born close to the time of the shock, however. Their welfare will be lower than under non-distortionary taxation if the initial tax rate is relatively high, even when the initial share of public investment is very low.

Next, we analyse a policy where the distortionary tax on labour is changed once-and-for-all. We show that the tax rate can be decreased permanently if the share of public investment is relatively low and initial tax rate and the elasticity of labour supply are relatively high. In that case, all generations gain more with distortionary taxation. If the tax rate rises once-and-for-all, old existing generations benefit less than under lump-sum finance. Moreover, the pure efficiency effect of the increase in public investment is smaller in that case. Yet, young existing and future generations may be better off than under lump-sum finance. So, distortionary labour taxation redistributes welfare from the old to the young at the cost of lower efficiency.

We complete Section 3 by comparing the three modes of financing in the special case that there is no distortionary tax in the initial steady state. In Section 4 we present some numerical simulations with a plausibly calibrated version of the model. Conclusions are briefly outlined in Section 5.

2 The model

2.1 Households

The utility functional at time t of the representative agent born at time v is denoted by $\Lambda(v,t)$ and has the following form:

$$\Lambda(v,t) = \int_t^{\infty} \log X(v,\tau) e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{2.1}$$

where $\alpha > 0$ is the pure rate of time preference, $\beta \geq 0$ is the probability of death and $X(v,\tau)$ is full consumption which depends on labour supply, $L(v,\tau)$, and goods consumption, $C(v,\tau)$:

$$X(v,\tau) = C(v,\tau) - \frac{L(v,\tau)^{1+1/\sigma_L}}{1+1/\sigma_L}, \tag{2.2}$$

with $\sigma_L > 0$. This specification of full consumption was suggested by Greenwood et al. (1988) and eliminates the intertemporal substitution effect in labour supply. This not only simplifies the analysis substantially but also appears to be more empirically relevant.[2] The household faces the following budget identity (in terms of the world price of the good):

$$\dot{A}(v,\tau) \equiv (r+\beta)A(v,\tau) + W(\tau)[1-t_L(\tau)]L(v,\tau) - T(\tau) - C(v,\tau), \tag{2.3}$$

where $A(v,\tau)$ are real tangible assets ($\dot{A}(v,\tau) \equiv \partial A(v,\tau)/\partial\tau$), r is the constant world interest rate, $W(\tau)$ is the gross wage rate (assumed age-independent for convenience), $t_L(\tau)$ is the

[2] Indeed, to the present day, the empirical literature has not been able to demonstrate a strong intertemporal substitution effect in labour supply. See Card (1994).

uniform labour tax, and $T(\tau)$ is a lump-sum tax. The three types of tangible assets are perfect substitutes:

$$A(v,\tau) = V(v,\tau) + B(v,\tau) + F(v,\tau), \tag{2.4}$$

where $V(v,\tau)$, $B(v,\tau)$, and $F(v,\tau)$ denote, respectively, shares in domestic companies, government bonds, and net foreign assets.

The household chooses the profile of consumption and labour supply in order to maximize utility, $\Lambda(v,t)$, subject to the budget identity (2.3) and an no-Ponzi-game solvency condition. It takes as given the initial level of assets $A(v,t)$. The optimal solution for this problem is:[3]

$$\frac{\dot{X}(v,\tau)}{X(v,\tau)} = r - \alpha, \qquad \tau \geq t, \tag{2.5}$$

$$X(v,t) = (\alpha + \beta)[A(v,t) + H(t)], \tag{2.6}$$

$$L(v,\tau) = \Big(W(\tau)[1 - t_L(\tau)]\Big)^{\sigma_L}, \qquad \tau \geq t, \tag{2.7}$$

where $H(\tau)$ is human wealth, i.e. the present value of full income where the risk-of-death adjusted interest rate $r + \beta$ is used as the discount factor:

$$H(t) \equiv \int_t^\infty Y_F(\tau) e^{-(r+\beta)(\tau - t)} d\tau, \tag{2.8}$$

and full income is defined as the after-tax labour income minus the utility costs of supplying the optimal amount of labour:

$$Y_F(\tau) \equiv \frac{\left(W(\tau)\,[1 - t_L(\tau)]\right)^{1+\sigma_L}}{1 + \sigma_L} - T(\tau). \tag{2.9}$$

Since wages and taxes are age-independent, human wealth is the same for agents of all vintages. Equation (2.5) is the household's Euler equation, relating the optimal time profile for full consumption to the difference between the interest rate and the rate of time preference. Equation (2.6) shows that full consumption in the decision period t is a constant proportion of total wealth of the household, comprising financial wealth and human wealth. Equation (2.7) describes optimal labour supply as a function of the net wage rate.

A crucial feature of the Blanchard (1985) model is the simple demographic structure, which enables the aggregation over all currently alive households. Assuming that at each instance a large cohort of size βN is born and that βN agents die, the size of the population is constant and can be normalized to unity. Aggregate variables can then be calculated as the weighted sum of the values for the different generations. For example, aggregate financial wealth is calculated as $A(\tau) = \int_{-\infty}^{\tau} \beta A(v,\tau) e^{\beta(v-\tau)} dv$. Other aggregate variables can be

[3] Details of all derivations are found in Heijdra, van der Horst, and Meijdam (2002).

computed in the same fashion. The main equations describing the behaviour of the aggregate household sector are:

$$\frac{\dot{X}(\tau)}{X(\tau)} = r - \alpha - \beta(\alpha+\beta)\left(\frac{A(\tau)}{X(\tau)}\right), \tag{2.10}$$

$$X(\tau) = (\alpha+\beta)[A(\tau)+H(\tau)], \tag{2.11}$$

$$L(\tau) = (W(\tau)[1-t_L(\tau)])^{\sigma_L}. \tag{2.12}$$

On the right-hand side of (2.10), $A(\tau) \equiv V(\tau)+F(\tau)+B(\tau)$ represents aggregate financial wealth. Throughout the paper we analyse the case in which *initially* both the government debt and the stock of foreign assets are zero ($B = F = 0$). This ensures that in the initial steady state the trade balance is zero, the capital stock is fully owned by domestic households, and financial wealth is strictly positive ($A = V > 0$). Equation (2.10) shows that this is consistent with a steady state for aggregate full consumption provided the country's households are relatively patient, in that their rate of pure time preference falls short of the world interest rate ($r > \alpha$). The rising consumption profile that this implies for individual households ($\dot{X}(v,\tau) > 0$) ensures that even in the steady state financial wealth is transferred (via the insurance market) from generations that pass away to young generations (who save early on in life) (see Blanchard (1985, p. 233)).

2.2 Firms

Following Baxter and King (1993, p. 317) we assume that the representative, perfectly competitive firm has a Cobb-Douglas production function which is linearly homogeneous in the two private factors of production, private capital, $K(\tau)$, and labour, $L(\tau)$:

$$Y(\tau) = L(\tau)^{\epsilon}K(\tau)^{1-\epsilon}K_G(\tau)^{\eta}, \quad 0 \le \eta < \epsilon < 1, \tag{2.13}$$

where $Y(t)$ is gross output and $K_G(t)$ is the *stock* of public capital. The restriction $\eta < \epsilon$ ensures diminishing returns to broadly defined capital and excludes the possibility of endogenous growth. The firm faces convex adjustment costs defined on gross investment. We follow Uzawa (1969) by postulating a concave accumulation function, $\Phi(\cdot)$, which provides the link between gross investment and net capital accumulation:

$$\frac{\dot{K}(\tau)}{K(\tau)} = \Phi\left(\frac{I(\tau)}{K(\tau)}\right) - \delta, \qquad \Phi(0) = 0,\ \Phi'(\cdot) > 0,\ \Phi''(\cdot) < 0, \tag{2.14}$$

where $I(\tau)$ is gross investment and $\delta > 0$ is the (constant) depreciation rate. The firm maximizes the present value of its future cash flow, $V(t)$, discounted at the world interest rate:

$$V(t) = \int_t^{\infty} \Big[Y(\tau) - W(\tau)L(\tau) - I(\tau)\Big]e^{-r(t-\tau)}d\tau, \tag{2.15}$$

subject to the production function (2.13) and the capital accumulation function (2.14), taking as given its initial capital stock, $K(t)$, and the perceived time path of public capital ($K_G(\tau)$, $\tau \geq t$). The resulting optimality conditions yield expressions for, respectively, labour demand, investment demand, and the growth rate of the shadow price of installed capital, $q(\tau)$:

$$W(\tau) = \epsilon \left(\frac{Y(\tau)}{L(\tau)} \right), \tag{2.16}$$

$$1 = q(\tau)\Phi' \left(\frac{I(\tau)}{K(\tau)} \right), \tag{2.17}$$

$$\frac{\dot{q}(\tau)}{q(\tau)} = r + \delta - \Phi \left(\frac{I(\tau)}{K(\tau)} \right) - \frac{(1-\epsilon)Y(\tau)}{q(\tau)K(\tau)} + \frac{I(\tau)}{q(\tau)K(\tau)}. \tag{2.18}$$

Since the accumulation function, $\Phi(\cdot)$, is homogeneous of degree zero in $I(\tau)$ and $K(\tau)$ and the production function is linearly homogeneous in the private factors of production, Tobin's average and marginal q coincide, and the stock market value of the firm equals $V(t) = q(t)K(t)$ (see Hayashi (1982)).

2.3 The government and the foreign sector

The government taxes households and uses the revenue to invest in public capital. Just like the firm, the government faces convex adjustment costs defined on gross investment, $I_G(\tau)$. The stock of public capital evolves according to:

$$\frac{\dot{K}_G(\tau)}{K_G(\tau)} = \Phi_G \left(\frac{I_G(\tau)}{K_G(\tau)} \right) - \delta_G, \quad \Phi_G(0) = 0, \; \Phi_G'(\cdot) > 0, \; \Phi_G''(\cdot) < 0, \tag{2.19}$$

where δ_G represents the (constant) rate of depreciation of public capital. The periodic budget identity of the government is:

$$\dot{B}(\tau) \equiv rB(\tau) + I_G(\tau) - T(\tau) - t_L(\tau)W(\tau)L(\tau). \tag{2.20}$$

The government is expected to remain solvent. By combining the government's no-Ponzi-game condition, $\lim_{\tau\to\infty} B(\tau)e^{r(t-\tau)} = 0$, with its flow budget identity (2.20), we obtain the intertemporal budget restriction of the government:

$$B(t) = \int_t^\infty [T(\tau) + t_L(\tau)W(\tau)L(\tau) - I_G(\tau)]e^{r(t-\tau)}d\tau. \tag{2.21}$$

To the extent that there is a pre-existing debt (positive left-hand side), solvency requires it to be equal to the present value of current and future primary surpluses (positive right-hand side).

The foreign sector of the model is represented by the current account, giving the evolution of net foreign assets:

$$\dot{F}(\tau) = rF(\tau) + \Big[Y(\tau) - C(\tau) - I(\tau) - I_G(\tau) \Big], \tag{2.22}$$

where the term in square brackets on the right-hand side is the trade balance, i.e. the difference between domestic production and absorption.

2.4 Allocational effects

We study the effects of an unanticipated and permanent increase in the level of public investment (assuming that the economy is initially in a steady state) and normalize the time at which the policy shock occurs to zero ($\tilde{I}_G > 0$ for $t \geq 0$). By log-linearizing the model around the initial steady state, analytical expressions for the impact, transitional, and long-run effects can be computed. The following notation is used. A tilde ('$\sim$') above a variable denotes its rate of change around the initial steady state, e.g. $\tilde{x}(t) \equiv dx(t)/x$. A variable with a tilde and a dot is (the change in) the time derivative expressed in terms of the initial steady state, for example $\dot{\tilde{x}}(t) \equiv d\dot{x}(t)/x$. The only exceptions to that convention refer to the various wealth components, full income, and taxes, i.e. $\tilde{x}(t) \equiv rdx(t)/Y$ and $\dot{\tilde{x}}(t) \equiv rd\dot{x}(t)/Y$ for $x \in (A, H, B, F)$, $\tilde{T}(t) \equiv dT(t)/Y$, $\tilde{Y}_F(t) \equiv dY_F(t)/Y$ and $\tilde{t}_L(t) \equiv dt_L(t)/(1-t_L)$. The main equations of the log-linearized model can be found in Table 1.

The log-linearized model can be solved recursively. The first step is the determination of the change in public capital associated with a given change in public investment. By assumption, the level of public capital does not change at impact ($\tilde{K}_G(0) = 0$) but its growth rate becomes positive because gross public investment becomes larger than the depreciation of public capital. From equation (2.23) it follows that its long-run change is equal to the change in public investment and that the transition path is monotonically increasing:

$$\tilde{K}_G(t) = \left[1 - e^{-\sigma_G t}\right]\tilde{I}_G, \tag{2.23}$$

where the speed of adjustment is dictated by the (finite) elasticity of the public installation function, σ_G ($\equiv I_G\Phi'(\cdot)/K_G > 0$).

Next, imposing full employment on the labour market the *investment sub-system*, consisting of the dynamic equations in the stock of capital and Tobin's q can be solved. We can then use this information to derive time paths for output, employment, wages, the labour tax rate, and full income. The investment sub-system can be written in a single matrix expression:

$$\begin{bmatrix} \dot{\tilde{K}}^i(t) \\ \dot{\tilde{q}}^i(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{r\omega_I}{\sigma_A\omega_A} \\ \delta_{21}^i & r \end{bmatrix} \begin{bmatrix} \tilde{K}^i(t) \\ \tilde{q}^i(t) \end{bmatrix} - \begin{bmatrix} 0 \\ \gamma_I^i(t) \end{bmatrix}, \tag{2.24}$$

where the superscript i ($i \in \{N, D, S\}$) refers to the financing policy used (see below) and the element δ_{21}^i is a complicated function of the parameters and shares (see below). The shock to the investment system under the alternative financing modes, γ_I^i, is defined in Section 3. We assume that $t_L < \frac{1}{1+\sigma_L}$ which guarantees that Tobin's q is a decreasing function of the stock

Table 1: Summary of the linearized model

(a) Investment sub-system:

$$\dot{\tilde{K}}(t) = \frac{r\omega_I}{\omega_A}[\tilde{I}(t) - \tilde{K}(t)] \tag{T.1}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - \frac{r(1-\epsilon)}{\omega_A}[\tilde{Y}(t) - \tilde{K}(t)] \tag{T.2}$$

$$\tilde{q}(t) = \sigma_A[\tilde{I}(t) - \tilde{K}(t)], \qquad \sigma_A \equiv -\frac{I\Phi''(\cdot)}{K\Phi'(\cdot)} \tag{T.3}$$

$$\tilde{L}(t) = \sigma_L[\tilde{W}(t) - \tilde{t}_L(t)] \tag{T.4}$$

$$\tilde{W}(t) = \tilde{Y}(t) - \tilde{L}(t) \tag{T.5}$$

$$\tilde{Y}(t) = \epsilon\tilde{L}(t) + (1-\epsilon)\tilde{K}(t) + \eta\tilde{K}_G(t) \tag{T.6}$$

$$\dot{\tilde{K}}_G(t) = \sigma_G[\tilde{I}_G - \tilde{K}_G(t)], \qquad \sigma_G \equiv \frac{I_G\Phi'_G(\cdot)}{K_G} \tag{T.7}$$

(b) Full income:

$$\tilde{Y}_F(t) = (1-t_L)\epsilon[\tilde{W}(t) - \tilde{t}_L(t)] - \tilde{T}(t) \tag{T.8}$$

$$\tilde{B}(0) = r\left[\epsilon(1-t_L)\mathcal{L}\{\tilde{t}_L, r\} + \mathcal{L}\{\tilde{T}, r\} + \epsilon t_L\mathcal{L}\{\tilde{Y}, r\} - \omega_G\tilde{I}_G\right] \tag{T.9}$$

(c) Saving sub-system:

$$\dot{\tilde{H}}(t) = (r+\beta)\tilde{H}(t) - r\tilde{Y}_F(t) \tag{T.10}$$

$$\dot{\tilde{A}}(t) = -(\alpha+\beta-r)\tilde{A}(t) - (\alpha+\beta)\tilde{H}(t) + r\tilde{Y}_F(t) \tag{T.11}$$

(d) Net foreign assets and (full) consumption:

$$\tilde{X}(t) = \left(\frac{\alpha+\beta}{r\omega_X}\right)[\tilde{A}(t) + \tilde{H}(t)] \tag{T.12}$$

$$\omega_C\tilde{C}(t) = \omega_X\tilde{X}(t) + \sigma_L[\tilde{Y}_F(t) + \tilde{T}(t)] \tag{T.13}$$

$$\tilde{F}(t) = \tilde{A}(t) - \omega_A[\tilde{q}(t) + \tilde{K}(t)] - \tilde{B}(t) \tag{T.14}$$

Definitions: $\omega_C \equiv C/Y$: output share of goods consumption; $\omega_I \equiv I/Y$: output share of firm investment; $\omega_G \equiv I_G/Y$: output share of public investment; $\omega_X \equiv X/Y$: output share of full consumption; $\omega_A \equiv rA/Y$: output share of financial wealth income. **Initial relationships** between shares: $\omega_C + \omega_I + \omega_G \equiv 1$ and $\omega_A + \epsilon + \omega_C \equiv \omega_G$. **Elasticities**: σ_A: elasticity of the marginal private installation function; σ_G: elasticity of the public installation function.

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of private capital and thus that the investment sub-system is saddle-point stable for all three policies. The unstable (positive) root of this sub-system is denoted by r_I^i and the stable root by $-h_I^i < 0$ ($i \in \{N, D, S\}$).

The long-run changes in Tobin's q and in the private capital stock are obtained by setting $\dot{\tilde{K}}^i(t) = \dot{\tilde{q}}^i(t) = 0$ in (2.24) and inverting the Jacobian matrix. It follows that:

$$\tilde{q}^i(\infty) = 0, \qquad \tilde{K}^i(\infty) = \frac{\gamma_I^i(\infty)}{\delta_{21}^i}. \tag{2.25}$$

The impact effect on Tobin's q is determined by the requirement that the economy must be on a stable trajectory leading to the steady-state equilibrium in combination with the fact that the private capital stock is predetermined (so that $\tilde{K}^i(0) = 0$). Given the jump in Tobin's q at impact ($\tilde{q}^i(0)$) and the new steady-state level of private capital ($\tilde{K}^i(\infty)$), the transition paths for $\tilde{K}$ and $\tilde{q}$ can easily be derived:

$$\tilde{K}^i(t) = \tilde{K}^i(\infty)\big[1 - e^{-h_I^i t}\big] - \left(\frac{r\omega_I[\gamma_I^i(\infty) - \gamma_I^i(0)]}{\sigma_A \omega_A \left(r_I^i + \sigma_G\right)}\right) \mathrm{T}(h_I^i, \sigma_G, t), \tag{2.26}$$

$$\tilde{q}^i(t) = \tilde{q}^i(0) e^{-h_I^i t} - \left(\frac{\sigma_G[\gamma_I^i(\infty) - \gamma_I^i(0)]}{r_I^i + \sigma_G}\right) \mathrm{T}(h_I^i, \sigma_G, t), \tag{2.27}$$

where $\mathrm{T}(h_I^i, \sigma_G, t)$ is a bell-shaped transition term, which is zero at impact and in the long run and positive during transition (see HM, p. 718).

Finally, the evolution of human and financial wealth is derived from the *saving sub-system:*

$$\begin{bmatrix} \dot{\tilde{H}}^i(t) \\ \dot{\tilde{A}}^i(t) \end{bmatrix} = \begin{bmatrix} r+\beta & 0 \\ -(\alpha+\beta) & r-\alpha-\beta \end{bmatrix} \begin{bmatrix} \tilde{H}^i(t) \\ \tilde{A}^i(t) \end{bmatrix} + \begin{bmatrix} -r\tilde{Y}_F^i(t) \\ r\tilde{Y}_F^i(t) \end{bmatrix}, \tag{2.28}$$

where:

$$\tilde{Y}_F^i(t) = (1 - t_L)\epsilon[\tilde{W}^i(t) - \tilde{t}_L^i(t)] - \tilde{T}^i(t). \tag{2.29}$$

This system is saddle-point stable under the condition that $r < \alpha + \beta$ (Blanchard (1985, p. 230)). Financial wealth is predetermined, but its value may change at impact because of capital gains or losses: $\tilde{A}^i(0) = \omega_A \tilde{q}^i(0)$. The forcing variable affecting the saving system is full income. As is shown in equation (2.8), human wealth is the present value of current and future full income. So it follows that:

$$\tilde{H}^i(0) = r \int_0^\infty \tilde{Y}_F^i(t) e^{-(r+\beta)t} dt, \quad \tilde{H}^i(\infty) = \left(\frac{r}{r+\beta}\right) \tilde{Y}_F^i(\infty). \tag{2.30}$$

2.5 Welfare effects

Because there exists a considerable amount of transitional dynamics in the model, as both private and public capital accumulate only gradually, generations born at different dates will generally be affected differently by the public investment shock. By using the Laplace transform techniques pioneered by Judd (1982) and Bovenberg (1993, 1994), it is nevertheless possible to evaluate the entire welfare profile across time (for future generations) and across ages (for generations alive at the time of the shock).

The welfare effect on generations that exist at the time of the shock ($t = 0$) is denoted by $d\Lambda^i(v, 0)$ ($v \le 0$) and can be written as follows:

$$(\alpha + \beta) d\Lambda^i(v, 0) = \left(\frac{\tilde{A}^i(0)}{\omega_A} \right) \left[1 - e^{(r-\alpha)v} \right] + \left(\frac{\tilde{H}^i(0)}{\omega_H} \right) e^{(r-\alpha)v}, \quad v \le 0, \tag{2.31}$$

where $\omega_A \equiv rA/Y$ and $\omega_H \equiv rH/Y$. Notice that extremely old generations possess a large amount of financial assets so that what happens to their human wealth does not matter in the limit. Hence, the welfare effect on these generations consists of only the capital-gain (or loss) effect on their financial wealth. The welfare effect on newly-born generations at the time of the shock, $d\Lambda^i(0, 0)$, is fully explained by the impact effect on human wealth as this is the only kind of wealth these generations possess.

The change in wealth experienced by future generations is evaluated at birth, i.e. the relevant indicator is $d\Lambda^i(t, t)$, for $t \ge 0$. This welfare indicator is proportional to human wealth:

$$(\alpha + \beta) d\Lambda^i(t, t) = \frac{\tilde{H}^i(t)}{\omega_H}, \quad t \ge 0. \tag{2.32}$$

3 The effects of an increase in public investment

In this section we study the macroeconomic and welfare effects of an unanticipated and permanent increase in the level of public investment in an economy featuring a positive pre-existing labour income tax. Three different modes of financing are considered.

First we study a balanced-budget policy where the increase in public outlays is financed by adjusting the lump-sum tax. This case is also studied by HM. We generalize their analysis by endogenizing labour supply and by assuming a positive initial tax on labour income. In the sequel, all variables referring to this policy are indicated by the superscript 'N' (for **n**on-distorting).

The second policy studied is also a balanced-budget policy, but now we assume that the increase in public spending is financed by adjusting the tax on labour income on a period-by-period basis. Since the tax base features transitional dynamics, it follows that for this financing mode the tax on labour income varies over time. We denote all variables referring to this case by the superscript 'D' (for **d**istorting).

The third policy we analyse is one where the government adjusts the tax on labour income once-and-for-all. Of course, this is not a balanced-budget policy so the government runs deficits or surpluses during transition. It is assumed, however, that the government sets the tax rate at such a level that the discounted value of the primary surpluses resulting from this policy equals zero. A superscript 'S' is used to indicate this policy (for tax **s**moothing).

For each of the three policies we map out the macroeconomic effects as well as the effects on the welfare of different generations. Moreover, we bring out the pure efficiency effect of the policy with a lump-sum tax and the policy with a once-and-for-all increase in the tax on labour income by deriving the common welfare gain that results when the government uses bond policy to neutralize the intergenerational welfare effects.[4]

3.1 Lump-sum taxation

In this section we analyse the case where the additional revenues needed to finance the increase in public investment are generated by raising the lump sum tax. That is, the government budget constraint is:

$$\tilde{T}^N(t) + \epsilon t_L \tilde{Y}^N(t) = \omega_G \tilde{I}_G. \tag{3.1}$$

The investment sub-system In this case the shock to the investment sub-system is:

$$\gamma_I^N(t) = \delta_{21}^N \left(\frac{\eta(1+\sigma_L)}{\epsilon} \right) \tilde{K}_G(t) \geq 0, \tag{3.2}$$

where $\delta_{21}^N > 0$.[5] Because public capital is a stock, the shock to the system is zero at impact ($\gamma_I^N(0) = 0$).

As was noted above, the long-run change in Tobin's q is zero ($\tilde{q}^N(\infty) = 0$) while the long-run effect on the private capital stock follows from (2.25) and (3.2):

$$\tilde{K}^N(\infty) = \left(\frac{\eta(1+\sigma_L)}{\epsilon} \right) \tilde{I}_G > 0. \tag{3.3}$$

The increase in the public capital stock boosts the marginal productivity of private capital, increases private investment, and ultimately results in a larger stock of private capital. The long-run changes in output, wages, and employment are:

$$\tilde{Y}^N(\infty) = \tilde{K}^N(\infty) > 0, \quad \tilde{W}^N(\infty) = \left(\frac{\epsilon}{\eta} \right) \tilde{I}_G > 0, \quad \tilde{L}^N(\infty) = \left(\frac{\sigma_L \eta}{\epsilon} \right) \tilde{I}_G > 0. \tag{3.4}$$

[4] In Section 4 we also compute the effects of a time-varying labour tax in combination with a bond policy that neutralizes the intergenerational welfare effects.

[5] The parameter δ_{21}^N is defined as follows:

$$\delta_{21}^N \equiv \frac{r(1-\epsilon)\epsilon}{\omega_A \left[1 + \sigma_L(1-\epsilon)\right]} > 0.$$

The larger stock of public capital increases labour productivity which is reflected in the wage increase. The pure effect of an increase in public investment is an impetus to the economy; output, employment, wages and the capital stock are larger in the new steady state.

At impact, output, employment and the wage rate are all predetermined (i.e. $\tilde{Y}^N(0) = \tilde{L}^N(0) = \tilde{W}^N(0) = 0$) but Tobin's q and thus investment rises unambiguously as a result of the boost in public investment:

$$\tilde{q}^N(0) = \sigma_A \tilde{I}^N(0) = \delta_{21}^N \left(\frac{1+\sigma_L}{\epsilon} \right) \left(\frac{\sigma_G \eta}{r_I^N (r_I^N + \sigma_G)} \right) \tilde{I}_G > 0. \tag{3.5}$$

Public investment thus causes an immediate boom in private investment which gives rise to a gradual monotonic increase in private capital. This in turn affects the marginal product of capital during transition and thus also Tobin's q. In contrast to the time path of the private capital stock, the time profile of the adjustment path in Tobin's q is ambiguous and depends on σ_G (see HM, p. 718). For future reference, we note that the results for the investment sub-system are independent of the initial level of the labour tax t_L.

The saving sub-system In contrast to the investment sub-system, the saving sub-system–describing the evolution of human and financial wealth–is critically affected by the initial tax rate t_L. The shock affecting the saving system is the rate of change in full income which consists of two parts: real net labour income, $(1-t_L)L(t)W(t) - T(t)$, and the costs of leisure, $\frac{L(t)^{1+\sigma_L}}{1+\sigma_L} = \frac{[(1-t_L)W(t)]^{1+\sigma_L}}{1+\sigma_L}$. The first part is not affected by t_L. This becomes clear if we rewrite this part of full income (by using the government budget constraint) as $L(t)W(t) - I_G$. Because the evolution of the investment sub-system and thus of labour supply and wages is independent of t_L, the effect of the initial labour tax on full income must come from the second part. To be more precise, it results from the change in the price of leisure, i.e. the change in the net wage rate. Following an increase in public investment, after-tax wages gradually rise over time making leisure more expensive. But this increase is smaller, the larger is the initial tax rate on labour income as this tax drives a wedge between the marginal product of labour and the price of leisure. Consequently, the higher the initial tax rate on labour income, the larger the increase of full income over time. This is compactly expressed by the following equation for the rate of change in full income:

$$\tilde{Y}_F^N(t) = \left[\tilde{Y}_F^N(t) \right]_{t_L=0} + \epsilon \sigma_L t_L \tilde{W}^N(t). \tag{3.6}$$

It follows that the initial effect on full income is independent of t_L (because $\tilde{W}^N(0) = 0$) and unambiguously negative while the long-run effect depends on the parameter values:

$$\tilde{Y}_F^N(0) = -\omega_G \tilde{I}_G < 0, \qquad \tilde{Y}_F^N(\infty) = [\eta(1+\sigma_L t_L) - \omega_G] \tilde{I}_G. \tag{3.7}$$

The long-run effect on full income is determined by the interplay of three separate influences. Firstly, an increase in public investment raises the marginal product of labour and thus the before-tax wage. This is represented by the positive term involving η. Secondly, this positive effect is intensified by a factor $(1+\sigma_L t_L)$ due to the *tax-wedge effect* described above. Finally, the additional public investment leads to higher taxes which reduces after-tax income. On balance, if public capital is relatively abundant ($\omega_G > \eta(1+\sigma_L t_L)$) the positive effects are dominated by the tax effect, so that full income falls in the long run. Conversely, if public capital is relatively scarce ($\omega_G < \eta(1+\sigma_L t_L)$) the positive effects are dominant and full income rises in the long run.

Since full-income rises monotonically during transition, the same holds for human capital (see equation (2.30)). It follows that $\tilde{H}^N(0) < \tilde{H}^N(\infty)$. Since $\tilde{H}^N(0)$ depends on the entire time path of full income its sign is ambiguous. The impact reduction of full income gives it a negative impulse which may or may not be offset by the long-run effect on full income.

Intergenerational welfare Financial assets jump at impact due to the jump in Tobin's q, i.e. there is a capital gain to the owners of domestic capital. Indeed, it follows from (2.31) and (3.5) that extremely old generations gain as a result of the increase in public investment, i.e. $d\Lambda^N(-\infty, 0) > 0$. Moreover, this gain is independent of the initial tax rate.

The effect on the welfare of newly-born generations is determined by $\tilde{H}^N(0)$ and is thus ambiguous. It follows from (3.7), however, that the increase in welfare of newly-born generations is larger (or the decrease is smaller) if the initial tax rate on labour is higher. So we reach the surprising conclusion that the political support among existing generations for a marginal increase in public investment will be larger, the higher is the initial labour income tax rate.

As human wealth rises over time the same holds for the welfare effect on future generations, implying that steady-state generations are better off than newly-born generations. Furthermore, just as for newly-born generations, the initial tax on labour income has a positive influence on the welfare effect for steady-state generations (see equations (2.30) and (3.7)). Finally, steady-state generations are better off in absolute terms if (and only if) public capital is relatively scarce, $\omega_G < \eta(1+\sigma_L t_L)$.

Pure efficiency effect Suppose the government uses bond policy to neutralize the intergenerational welfare effects. That is, the government redistributes the gains and losses in such a way that the marginal benefit of public investment is equal for all generations, i.e. $d\Lambda^N(v, 0) = d\Lambda^N(t, t) = \pi^N$ for $v \leq 0$ and $t \geq 0$. This leads to a common welfare gain equal to:

$$(\alpha+\beta)\pi^N = \frac{1}{\omega_X}\left[\left(\frac{\eta\sigma_G}{r+\sigma_G}\right)(1+\sigma_L t_L \psi_N) - \omega_G\right]\tilde{I}_G, \tag{3.8}$$

where $0 < \psi_N < 1$.[6] Since ψ_N measures the effect of the change in government investment on wages, $\sigma_L t_L \psi_N$ can be interpreted as the tax-wedge effect on the common welfare gain. Notice that it follows from (3.8) that, due to the tax-wedge effect, the common welfare gain is increasing in t_L. The sign of the common welfare gain is fully determined by the term in square brackets on the right-hand side of equation (3.8). If public capital is relatively scarce (ω_G small) this term is positive and all generations can be made better off by increasing the level of public investment. By setting ω_G such that all present and future generations are unaffected by a marginal increase in the level of public investment (i.e. $\pi^N = 0$) the expression for optimal public investment is obtained:

$$\omega_G^{MGR} = \left(\frac{\eta\sigma_G}{r+\sigma_G}\right)(1+\sigma_L t_L \psi_N). \tag{3.9}$$

Equation (3.9) is a simple "modified golden rule" (MGR) of public investment in a dynamic economy featuring a pre-existing tax distortion. It demonstrates the crucial interaction between the efficiency and durability of public capital (as parameterized by η and σ_G), the world interest rate, and the tax wedge (as parameterized by $\sigma_L t_L \psi_N$). Notice that the optimal share of public investment is higher when the initial tax rate on labour is higher. Proposition 1 summarizes the most important effects of a lump-sum financed increase in public investment.

Proposition 1 *(i) At impact Tobin's q, financial assets and private investment rise, but full income falls. These effects are independent of the initial tax rate on labour. (ii) In the long run the private capital stock, output, employment, and wages rise. These effects are independent of the initial tax rate on labour. (iii) In the long run full income and human wealth rise iff* $\eta(1+\sigma_L t_L) > \omega_G$. *(iv) The impact effect on human wealth, and thus on the welfare of newly-born generations is ambiguous, but is positively influenced by the initial tax rate on labour. Consequently, the political support among existing generations for an increase in public investment is higher if the initial tax distortion is higher. (v) If the government is able to neutralize all intergenerational redistributional effects on welfare, the optimal egalitarian share of public investment is* $\frac{\eta\sigma_G}{r+\sigma_G}(1+\sigma_L t_L \psi_N)$, *with* $0 < \psi_N < 1$. *This optimal share is larger, the higher is the initial tax on labour.*

[6] The composite parameter ψ_N is defined as follows:

$$\psi_N \equiv 1 - \frac{r(r+\sigma_G)(1+\sigma_L)(1-\varepsilon)}{r_I^N(r_I^N+\sigma_G)\left[1+\sigma_L(1-\varepsilon)\right]}, \qquad 0 < \psi_N < 1.$$

3.2 A time-varying tax on labour income

In this subsection we assume that the government is unable to use lump-sum taxes and instead varies the tax on labour income to keep the budget balanced. The government budget constraint is thus given by:

$$\tilde{t}_L^D(t) = \frac{\omega_G \tilde{I}_G - \epsilon_L t_L \tilde{Y}^D(t)}{\epsilon(1-t_L)}. \tag{3.10}$$

The investment sub-system In this case one element of the Jacobian matrix of the investment sub-system is different: $\delta_{21}^D = \phi\delta_{21}^N$, where $\phi < 1$.[7] The use of the labour tax dampens the sensitivity of Tobin's q with respect to capital, which means that in this case a given increase in capital leads to a smaller decline in the marginal productivity of capital. The shock to the investment sub-system is:

$$\gamma_I^D(t) = \delta_{21}^N \theta_I \left[\left(\frac{\eta(1+\sigma_L)}{\epsilon} \right) \tilde{K}_G(t) - \left(\frac{\sigma_L \omega_G}{\epsilon(1-t_L)} \right) \tilde{I}_G \right], \tag{3.11}$$

where $\theta_I > 1$.[8] Notice that, in contrast to what was the case with lump-sum financing, the initial shock is negative, while the sign of the long-run shock is ambiguous.

The long-run effect on gross wages and Tobin's q is equal to that in the case of lump-sum financing ($\tilde{W}^D(\infty) = \tilde{W}^N(\infty)$, $\tilde{q}^D(\infty) = \tilde{q}^N(\infty) = 0$) whereas the long-run effect on private capital and employment (and thus on production) now crucially depends on the long-run change in the tax on labour income:

$$\tilde{K}^D(\infty) = \tilde{K}^N(\infty) - \sigma_L \tilde{t}_L^D(\infty) \qquad \tilde{L}^D(\infty) = \tilde{L}^N(\infty) - \sigma_L \tilde{t}_L^D(\infty). \tag{3.12}$$

Thus we see that a distortionary tax-financed increase in government investment will have a less expansionary effect in the long run than a lump-sum financed investment increase if (and only if) the tax rate rises in the long run. A similar conclusion can be found in Fisher and Turnovsky (1998, p. 408). They assume that it will normally be the case that higher public investment requires a higher tax rate. We can, however, derive a simple equation for the new steady-state tax rate on labour:

$$\tilde{t}_L^D(\infty) = \left(\frac{\omega_G - \eta(1+\sigma_L)t_L}{\epsilon[1-t_L(1+\sigma_L)]} \right) \tilde{I}_G. \tag{3.13}$$

[7] The composite parameter ϕ is defined as follows:

$$\phi \equiv \frac{[1+\sigma_L(1-\epsilon)][1-t_L(1+\sigma_L)]}{(1-t_L)(1+\sigma_L)-\epsilon\sigma_L}, \qquad 0 < \phi < 1.$$

[8] The composite parameter θ_I is defined as follows:

$$\theta_I \equiv \frac{(1-t_L)\,[1+\sigma_L(1-\epsilon)]}{(1-t_L)(1+\sigma_L)-\epsilon\sigma_L} \equiv \frac{(1-t_L)\phi}{1-t_L(1+\sigma_L)} > 1.$$

It is clear from this equation that the tax rate may increase or decrease in the long run, depending on the initial share of public investment (ω_G), the productivity of public capital (as measured by η), the elasticity of labour supply (σ_L), and the initial tax rate (t_L). If the share of public capital is relatively low, i.e. $\omega_G < \eta(1+\sigma_L)t_L$, the positive effect of public investment on the tax base is so strong that a lower tax rate suffices to finance the increase in public investment.

The impact effect on Tobin's q is equal to:

$$\tilde{q}^D(0) = \delta_{21}^N \theta_I \left(\frac{1+\sigma_L}{\epsilon}\right) \left[\frac{\sigma_G \eta}{r_I^D (r_I^D + \sigma_G)} - \frac{\sigma_L \omega_G}{r_I^D (1+\sigma_L)(1-t_L)}\right] \tilde{I}_G. \tag{3.14}$$

When we compare this expression to equation (3.5) we see that $\tilde{q}^D(0) > \tilde{q}^N(0)$ for low values of ω_G. So, if public capital is very scarce, the initial impulse to private investment that results from an increase in public investment is stronger in case the additional public investment is financed by labour taxation. If public capital is more abundant, however, the increase in private investment at impact is lower in case the tax on labour is used to finance the increase in public capital than when lump-sum finance is used ($\tilde{q}^D(0) < \tilde{q}^N(0)$). Note that Tobin's q falls at impact, i.e. the extremely old experience a capital loss if $\frac{\sigma_G \eta}{r_I^D + \sigma_G} < \frac{\sigma_L \omega_G}{(1+\sigma_L)(1-t_L)}$. In that case, the increase in public investment crowds out private investment so that the private capital stock declines.

The saving sub-system The impact and long-run effects on full income are:

$$\tilde{Y}_F^D(0) = \frac{-(1-t_L)\omega_G}{(1+\sigma_L)(1-t_L) - \epsilon \sigma_L} \tilde{I}_G < 0, \qquad \tilde{Y}_F^D(\infty) = \frac{(1-t_L)(\eta - \omega_G)}{1 - t_L(1+\sigma_L)} \tilde{I}_G. \tag{3.15}$$

The long-run effect on full income, and thus on human wealth, may be positive or negative, depending on η and ω_G. Notice that it follows from (2.30) and (3.15) that the absolute value of the effect on steady-state human wealth is larger the higher the initial tax rate ($\partial \left|\tilde{H}^D(\infty)\right| / \partial t_L > 0$). The comparison of (3.7) and (3.15) reveals that the effect on steady-state human wealth in case a labour tax is used is larger than in case of lump-sum taxation (i.e. $\tilde{H}^D(\infty) > \tilde{H}^N(\infty)$) iff $\eta t_L(1+\sigma_L) > \omega_G$, that is, iff the tax rate on labour falls.

Intergenerational welfare We can conclude that generations born in the new steady state are better off with labour taxation than with lump-sum financing iff the tax rate falls in the long run. This is not necessarily true for newly-born generations, however, as full income falls more at impact than in case of lump-sum financing if the tax rate is relatively high (i.e. $\tilde{Y}_F^D(0) < \tilde{Y}_F^N(0)$ iff $t_L > 1-\epsilon$). If, however, the initial tax rate is relatively low ($t_L < 1-\epsilon$) and still the tax rate falls in the long run, then human wealth at impact rises more under distortionary taxation than under lump-sum taxation. Consequently, in that case newly-born generations are better off with labour taxation. This will hold if the share of public investment is relatively low. As noted above, this also implies that the capital gain under labour taxation is higher than under non-distortionary taxation and consequently, the extremely old generations will gain more from the increase in public investment in this case. So, if public capital is very scarce, all generations will be better off when the increase in public investment is financed by distortionary taxation. Proposition 2 summarizes the effects of the increase in public investment financed by a variable tax on labour.

Proposition 2 *(i) Tobin's q, financial assets, and private investment fall at impact iff* $\frac{\sigma_G \eta}{r_I^D + \sigma_G} < \frac{\sigma_L \omega_G}{(1+\sigma_L)(1-t_L)}$. *They rise more than with lump-sum finance, however, if the share of public investment is low. (ii) Full income falls at impact, but rises in the long run iff* $\eta > \omega_G$. *(iii) The tax rate falls in the long-run iff* $\omega_G < \eta(1+\sigma_L)t_L$. *In that case, the positive effect on stock of private capital, production and human wealth in the steady state is larger than with lump-sum financing. (iv) Human wealth at impact rises more than under lump-sum financing if* $t_L < 1 - \epsilon$. *(v) All generations will be better off when the increase in public investment is financed by distortionary taxation if* $\omega_G < \eta(1+\sigma_L)t_L$ *and* $t_L < 1 - \epsilon$.

3.3 A time-invariant tax on labour income

In this sub-section we analyse the effects of an increase in public investment financed by a once-and-for-all increase in the labour tax. That is, the government sets the tax on labour income on such a level that the discounted value of the primary surpluses resulting from the combined public investment-labour tax policy equals zero:

$$\int_0^\infty \left[\epsilon(1-t_L)\tilde{t}_L^S + \epsilon t_L \tilde{Y}^S(t) - \omega_G \tilde{I}_G\right] e^{-rt} dt = 0. \tag{3.16}$$

The investment sub-system In this case, the Jacobian matrix of the investment sub-system is identical to the one in case of lump-sum financing. The shock to the system is different, however:

$$\gamma_I^S(t) = \gamma_I^N(t) - \delta_{21}^N \sigma_L \tilde{t}_L^S, \tag{3.17}$$

where $\tilde{t}_L^S$ is the once-and-for-all change in the tax on labour income which is equal to:

$$\tilde{t}_L^S = \left(\frac{1}{\Omega}\right)\left[\omega_G - t_L(1+\sigma_L)\left(\frac{\eta\sigma_G}{r+\sigma_G}\right)\psi_N\right]\tilde{I}_G, \tag{3.18}$$

where $0 < \Omega < \epsilon$ (and we recall that $0 < \psi_N < 1$).[9] We have argued above that ψ_N can be interpreted as the effect of the change in government investment on wages. The coefficient Ω stands for the effect of a change in the tax rate on tax revenues.

Note that it follows from (3.18) that the once-and-for-all change in the tax rate may be positive or negative depending on the scarcity of public capital. If the share of public capital is relatively low and the tax rate is relatively high, i.e. $\omega_G < t_L(1+\sigma_L)\left(\frac{\eta\sigma_G}{r+\sigma_G}\right)\psi_N$, the positive effect of public investment on wages, and thus on the tax base is so strong that a lower tax rate suffices to finance the increase in public investment. When we compare equations (3.13) and (3.18) we see that the sign of the difference between the once-and-for-all change in the tax rate and the long-run change in the tax rate in case of a variable tax is inconclusive. On the one hand, the fact that $\psi_N < 1$ tends to increase $\tilde{t}_L^S$ above $\tilde{t}_L^D(\infty)$. This is due to the fact that, as wages rise gradually, the long-run increase in wages (which is relevant for the long-run tax rate in case of policy D) exceeds the average increase in wages over the whole adjustment period (that determines the once-and-for-all change in the tax rate in policy S) On the other hand, the fact that $\Omega > \epsilon[1-t_L(1+\sigma_L)]$ tends to lower $\tilde{t}_L^S$ to a level below $\tilde{t}_L^D(\infty)$. The reason for this is that the decrease in the tax base due to lower labour supply on average over the whole adjustment period in case of policy S is higher than the long-run decrease of the tax base in case of a variable tax rate. Note, however, that if the once-and-for-all change in the tax rate is positive, then the tax rate also rises in the long run in case of a policy with a variable lump-sum tax, i.e. $\tilde{t}_L^S > 0 \Rightarrow \tilde{t}_L^D(\infty) > 0$.

As before, the long-run effects on gross wages and Tobin's q are equal to those in case of lump-sum financing ($\tilde{W}^S(\infty) = \tilde{W}^N(\infty)$ and $\tilde{q}^S(\infty) = \tilde{q}^N(\infty) = 0$) while the long-run effects on private capital and employment, and thus on production, crucially depend on the change in the tax on labour income:

$$\tilde{K}^S(\infty) = \tilde{K}^N(\infty) - \sigma_L\tilde{t}_L^S \qquad \tilde{L}^S(\infty) = \tilde{L}^N(\infty) - \sigma_L\tilde{t}_L^S. \tag{3.19}$$

So the long-run effect on production may be larger or smaller than in case of lump-sum financing. The same holds for the impact effect on Tobin's q which is equal to:

$$\tilde{q}^S(0) = \tilde{q}^N(0) - \left(\frac{\sigma_L\delta_{21}^N}{r_I^N}\right)\tilde{t}_L^S. \tag{3.20}$$

[9] The composite coefficients ψ_N and Ω are defined as follows:

$$\begin{aligned}\Omega &\equiv \epsilon\left[1 - t_L(1+\sigma_L)(1-\psi_L)\right],\\ \psi_L &\equiv \frac{r\sigma_L(1-\epsilon)}{r_I^N[1+\sigma_L(1-\epsilon)]}, \qquad 0 < \psi_L < 1,\end{aligned}$$

Consequently, if public capital is relatively scarce and the tax rate is relatively high the increase in public investment leads to a once-and-for-all decrease in the tax rate and therefore boosts the economy more in the short run as well as in the long run than in case of lump-sum finance.

The saving sub-system The impact and long-run effects on full income are:

$$\tilde{Y}_F^S(0) = -\frac{(1-t_L)\epsilon}{1+\sigma_L(1-\epsilon)}\tilde{t}_L^S, \qquad \tilde{Y}_F^S(\infty) = (1-t_L)[\eta\tilde{I}_G - \epsilon\tilde{t}_L^S]. \tag{3.21}$$

So full income falls at impact iff the once-and-for-all change in the tax rate is positive, whereas the long-run effect may still be positive even though the tax rate rises if public capital has a relatively strong effect on productivity ($\eta\tilde{I}_G > \epsilon\tilde{t}_L^S$).

Intergenerational welfare It is evident from equations (3.20) and (3.21) that welfare rises for all existing and future generations if the increase in public investment provides the economy with such a strong boost that it is possible to decrease the tax rate on labour once and for all ($\tilde{t}_L^S < 0$). On the other hand, welfare will decrease for all generations if the tax rate rises very strongly. In all other cases, assessing the generational welfare effects of an increase in public investment is more complicated. When an increase of public investment is accompanied by a modest rise in the tax on labour income, the capital gain that determines the welfare effect for extremely old generations is positive, but smaller than in case of lump-sum financing. The effect on the welfare of generations born in the far future is ambiguous. On the one hand, with a modest increase in the tax on labour the negative effect of the costs of public investment on long-run full income is smaller than in case of lump-sum finance. On the other hand, the positive effect on steady-state full income due to increase in productivity is also smaller in case of a tax labour income. As a consequence, the effect on welfare of the generation born at the time of shock, that is determined by the change in human wealth human wealth at impact, is ambiguous too.

Pure efficiency effect The pure efficiency effect of an increase in public investment financed by a once-and-for-all increase in the tax on labour is clear-cut. As before, we express the pure efficiency effect as the common welfare gain that results from the impulse in combination with an egalitarian bond policy:

$$(\alpha+\beta)\left(\pi^S - \pi^N\right) = -\sigma_L t_L \epsilon \left(\frac{1-\psi_L}{\omega_X}\right)\tilde{t}_L^S, \tag{3.22}$$

where $0 < \psi_L < 1$ (see footnote 6). So, assuming that labour supply is elastic ($\sigma_L > 0$) and that the initial tax rate is positive, the difference between the common welfare gain in case of a once-and-for-all increase in the tax on labour and the common welfare gain with lump-sum financing is determined by the sign of $\tilde{t}_L^S$: the common welfare gain with labour taxation is smaller than in case a lump-sum tax is used iff the tax rate on labour rises, i.e. iff the term in square brackets on the right-hand side of (3.18) is positive. If, however, there is no tax rate on labour in the initial steady state, the efficiency effect of an increase in public investment is the same no matter whether it is financed by a lump-sum tax or by a permanent change in the tax on labour income. We analyse this special case in the next sub-section. First, we summarize the results in this section by in Proposition 3:

Proposition 3 *(i) The once-and-for-all change in the tax on labour income is positive iff* $\omega_G > t_L(1+\sigma_L)\left(\frac{\eta\sigma_G}{r+\sigma_G}\right)\psi_N$. *(ii) Iff the tax rate rises once-and-for-all then: (a) The positive effect on the stock of private capital and production in the steady state is smaller than with lump-sum financing (and may even become negative); (b) The increase in Tobin's* q, *financial assets, and private investment at impact is smaller (and may even become a decrease); (c) Full income falls at impact. (d) The common welfare gain is smaller than with lump-sum finance. Full income and human wealth rise in the long run iff* $\tilde{t}_L^S < \frac{\eta \tilde{I}_G}{\epsilon}$.

3.4 No initial tax rate on labour

In this sub-section we compare the effects of the different financing modes assuming that the initial tax rate on labour is zero ($t_L = 0$). This special case has a number of important features. First, there are no first-order tax-base effects so that $\epsilon\tilde{t}_L^S = \epsilon\tilde{t}_L^D(t) = \omega_G\tilde{I}_G$. Second, the long-run effects of an increase in public investment on full income are independent of the way it is financed if the initial tax rate is zero, i.e. $\tilde{Y}_F^S(\infty) = \tilde{Y}_F^D(\infty) = \tilde{Y}_F^N(\infty)$. Third, the short-run effects on full income do depend on the financing policy, i.e. $\tilde{Y}_F^S(0) = \tilde{Y}_F^D(0) > \tilde{Y}_F^N(0)$. The reason for the full income results is that in case of labour taxation the net wage rate falls on impact, leading to a decrease in labour supply and an increase in the gross wage rate. Consequently, full income rises more in the short run if labour taxation is used. In the long run this is not true, however, as the lower labour supply then leads to a lower private capital stock so that the gross wage rate is the same irrespective of the way public investment is financed.

It follows from the analysis above that the effect on welfare of generations born in the new steady state (which is determined by steady-state human wealth) is independent of the financing mode if $t_L = 0$. The analysis also shows that the increase in human wealth at impact (which depends on the whole time path of full income) is higher when labour taxation is used. However, as noted above, the pure efficiency effect of an increase in public investment does not depend on the way this increase is financed if $t_L = 0$. Therefore, the larger increase in human wealth must come at the cost of a smaller increase of some other form of wealth. Indeed, the extra rise in human wealth at impact is mirrored by a smaller capital gain, i.e. a lower value of financial wealth, in case of labour taxation. So labour taxation shifts the welfare from capital owners to labour, i.e. from older generations to younger generations. We summarize these results in Proposition 4.

Proposition 4 *If the initial tax rate on labour income is equal to zero then: (i) The results for a time-varying tax and a once-and-for-all change in the tax rate are identical. (ii) The long-run effects on production and private capital are smaller when the tax on labour is increased, but the effect on steady-state full income is equal. (iii) The common welfare gain is independent of the financing mode. (iv) When lump-sum finance is used, full income falls more at impact, whilst Tobin's q and financial assets rise more at impact. So, using labour taxation shifts welfare from older to younger generations.*

4 Quantitative effects

In this section we present some illustrative numerical simulations of a calibrated version of the model. The following parameter values are held constant throughout the simulations:

$$\begin{array}{lllll} r = 0.05 & \delta = 0.1 & \beta = 0.02 & \epsilon = 0.7 & \eta = 0.2 \\ \sigma_L = 1 & \sigma_A = 0.5 & \sigma_G = 0.05 & \omega_I = 0.2 & \end{array} \tag{4.1}$$

The real interest rate is 5% per annum whilst the depreciation rate for the private capital stock is 10% per annum. The birth/death rate is 2% per annum, implying an expected remaining lifetime (planning horizon) of 50 years for all households. The efficiency parameters for labour and public capital are, respectively, 0.7 and 0.2. The labour supply function features a unitary wage elasticity. The elasticity of the marginal installation function for private capital is one-half, implying rather significant adjustment costs for private investment and ensuring non-degenerate private capital stock dynamics. The elasticity of the public installation function is 0.05, which means that the construction of new infrastructure proceeds only very gradually (Indeed, the half-life of a construction project is $(\ln 2)/\sigma_G = 13.86$ years.). Finally, we assume that the output share of private investment is 20%.

Once specific values for ω_G and t_L are chosen, the remaining shares and (composite) parameters can be determined by incorporating the information contained in (4.1), and by using the rate of time preference (α) as a free calibration parameter.[10] In Figures 1-3, the

[10] In particular, we find that $\omega_T \equiv T/Y = \omega_G - \epsilon t_L$, $\omega_C = 1 - \omega_I - \omega_G$, and:

$$\omega_H = \left(\frac{r}{r+\beta}\right)\left[\frac{(1-t_L)\epsilon}{1+\sigma_L} - \omega_T\right], \quad \omega_X = \omega_C - \frac{\sigma_L(1-t_L)\epsilon}{1+\sigma_L}, \quad \alpha = \frac{\omega_A\,(r-\beta) + r\omega_H}{\omega_A + \omega_H}.$$

top six panels present the impulse-response functions for the different variables, under the assumption that there is a pre-existing labour income tax ($t_L = 0.1$) and that public capital is rather scarce initially ($\omega_G = 0.01$). The bottom two panels in Figure 1-3, present the intergenerational welfare profiles for different values of ω_G and t_L respectively. The impulse consists of a step-wise increase in public investment which occurs at time $t = 0$ and is normalized to unity (i.e. $\tilde{I}_G = 1$ for $t \geq 0$).

Scenario *N* Figure 1 confirms and extends the analytical results obtained above. The public capital stock is gradually increased, private investment is non-degenerate (the adjustment speed of the investment sub-system is $h_I^N = 0.105$), and the private capital stock gradually increases over time. Tobin's q is non-monotonic and increases during the early phase of transition as public capital only gradually affects the marginal product of private capital. Output and employment increase monotonically, as do wages and full income. The lump-sum tax increases slightly at impact but falls later on during the transition because the labour income tax base expands. The fifth panel of Figure 1 shows the effect on human capital, financial wealth, and net foreign assets. Although we have de-emphasized the latter two variables, they nevertheless play an important role in the model. The key thing to note about the fifth and sixth panels is the extremely slow transition (relative to the transition speed of human wealth) of $\tilde{A}(t)$ and $\tilde{F}(t)$ (in the fifth panel) and of $\tilde{C}(t)$ and $\tilde{X}(t)$ (in the sixth panel). The intuition behind this result is as follows. The transition of human capital is characterized by the transition speeds of the investment sub-system (h_I^N) and the public capital stock (σ_G), which are both quite high. In contrast, the dynamics of financial wealth and net foreign assets not only depends on h_I^N and σ_G but also on the transition speed of the saving sub-system (h_S) which is quite low. Indeed, for the parameter values underlying Figures 1-3, we have $h_S = 0.0146$, i.e. less than 1.5% per annum.

The last two panels of Figure 1 show the intergenerational welfare profiles for different values of ω_G (seventh panel, where $t_L = 0.1$) and t_L (eighth panel in which $\omega_G = 0.1$). Existing generations (for $v \in [-100, 0]$) and future generations (for $t \in (0, 100)$) are plotted on the horizontal axis, whilst the marginal welfare effects ($d\Lambda^N(v, 0)$ and $d\Lambda^N(t, t)$, respectively) are plotted on the vertical axis. In the seventh panel, increasing the initial share of public capital (from its base value of $\omega_G = 0.01$) leads to a downward shift in the intergenerational welfare profile. This is to be expected because public capital gets less scarce as ω_G is increased. In Table 2 we report the pure efficiency effect, π^N, for different values of ω_G. Consider the entry for $\omega_G = 0.1$ where $\pi^N = 0.380$. Whereas most current generations would loose out in the absence of intergenerational redistribution (see the seventh panel in Figure 1), appropriately

Note that we ensure in each case that $\alpha < r < \alpha + \beta$ so that the model is well-defined and saddle-point stable.

Figure 1. An increase in public investment: Lump-sum tax

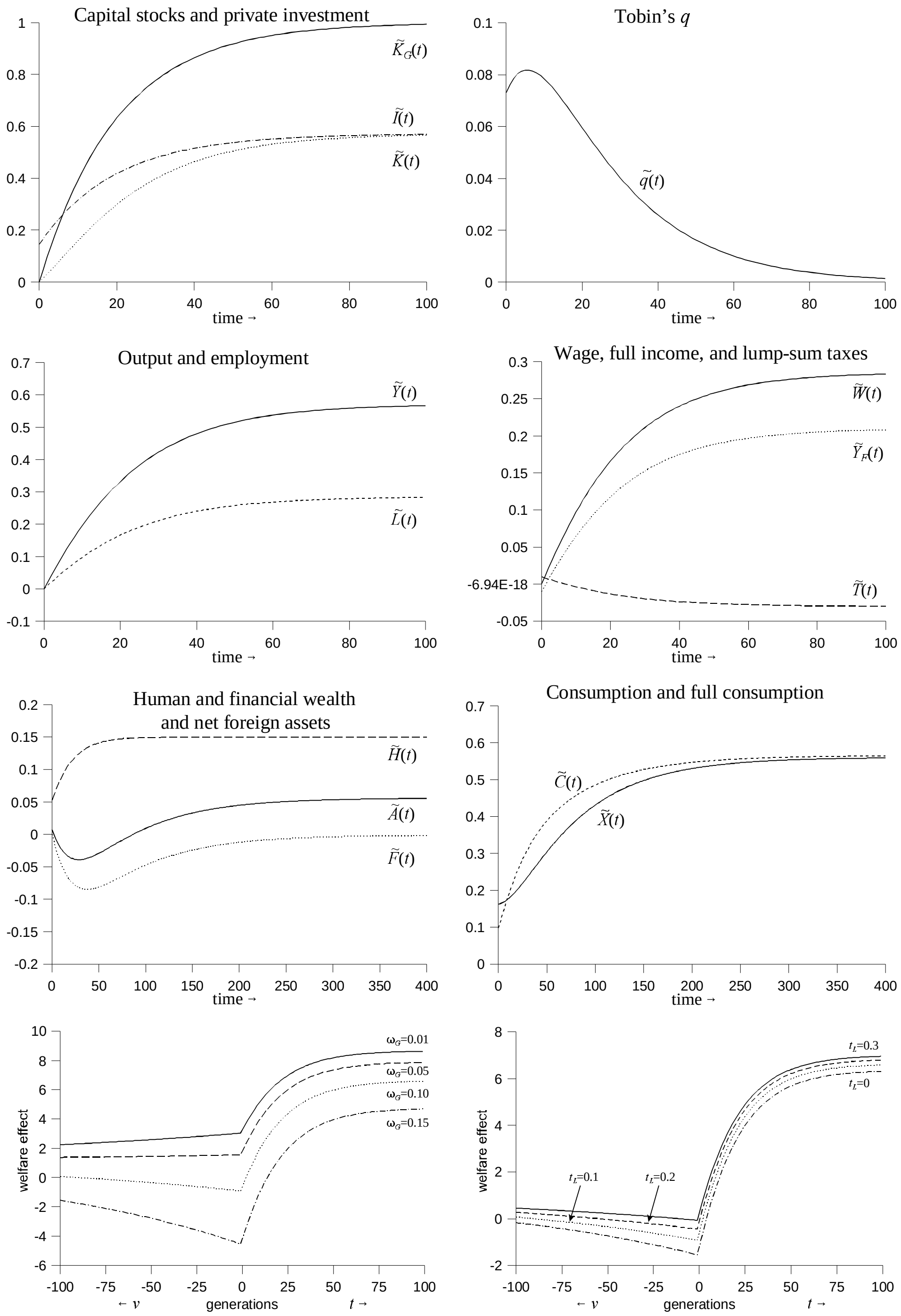


Table 2: Comparison of pure efficiency effects

		π^N	π^D	$\tilde{t}_L(0)$	$\tilde{t}_L(\infty)$	π^S	$\tilde{t}_L$
ω_G	0.01	3.237	3.272	0.017	-0.054	3.269	-0.015
	0.05	2.125	2.001	0.084	0.018	1.997	0.055
	0.10	0.380	0.005	0.169	0.107	0.001	0.143
	0.15	-1.944	-2.653	0.253	0.196	-2.657	0.230
t_L	0	0	0	0.143	0.143	0	0.143
	0.1	0.380	0.005	0.169	0.107	0.001	0.143
	0.2	0.691	0.026	0.206	0.048	0.002	0.143
	0.3	0.950	0.101	0.265	-0.071	0.004	0.143

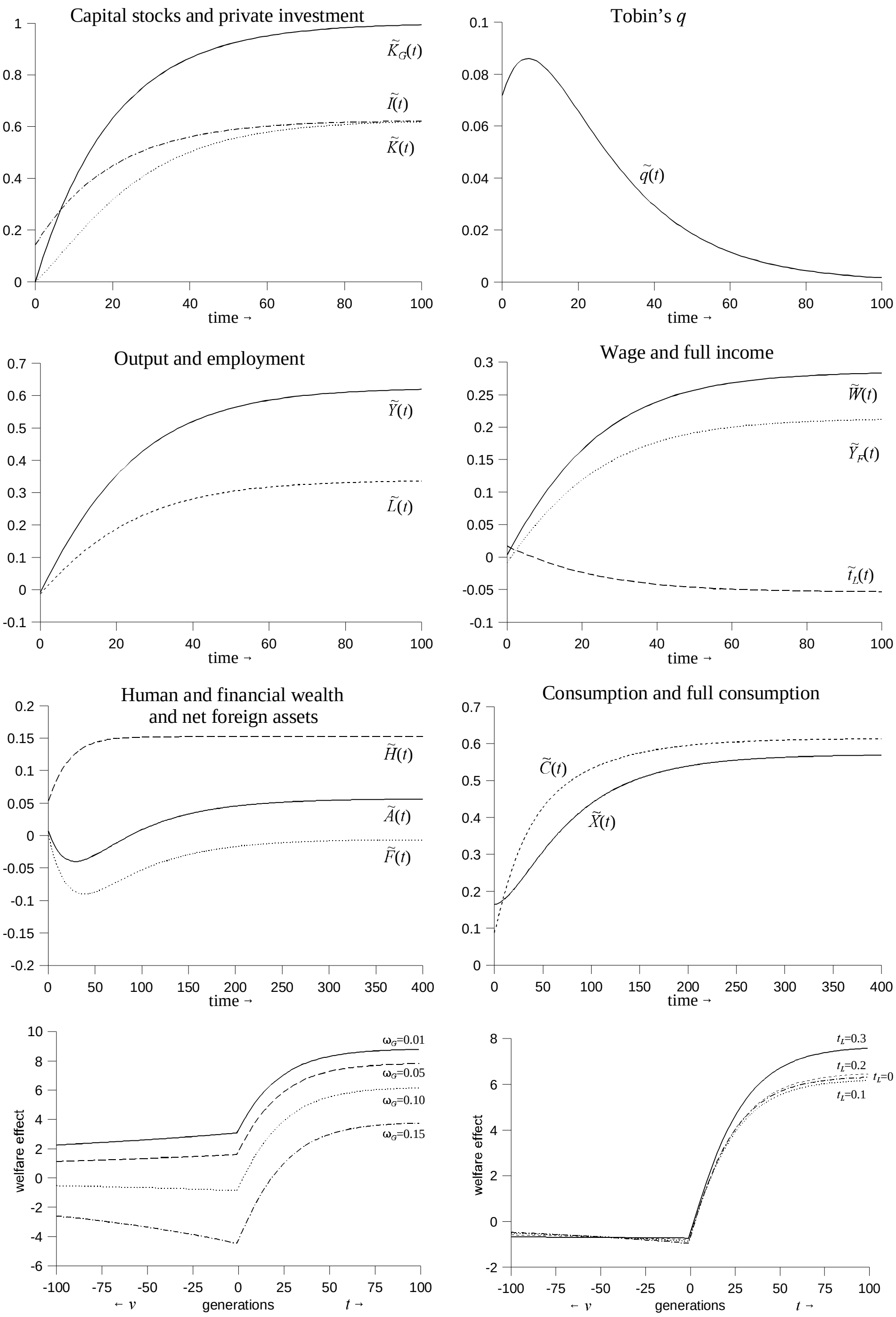
designed bond policy can make all generations strictly better off.

The results in the eighth panel of Figure 1 confirm the rather surprising result that the marginal benefit of public investment increases *for all generations* as the initial labour income tax is raised. In Table 2 we report the pure efficiency effect, π^N, associated with various values for the initial tax rate. Clearly, π^N is higher, the higher the initial labour income tax rate. Furthermore, if $t_L = 0$ initially, $\pi^N = 0$. The intuition behind this result follows from equation (3.9) above. If $t_L = 0$, the tax-wedge effect is absent and the optimal share of public investment is equal to $\omega_G^{MGR} = \frac{\eta\sigma_G}{r+\sigma_G} = 0.1$ in that case. Since the bottom half of Table 2 is based on $\omega_G = 0.1$, it follows trivially that increasing public investment does not cause first-order welfare gains.

Scenario *D* In Figure 2 we present the impulse-response functions and intergenerational welfare profiles for the distorting tax scenario. Qualitatively the effects are very similar to the ones presented in Figure 1. This is not surprising in view of the fact that both Figures are based on a very low initial share of public investment ($\omega_G = 0.01$). Two features of Figure 2 are worth stressing. First, in the eighth panel there is no longer an unambiguous pattern between the intergenerational welfare profile and the initial tax rate. Second, in the fourth panel, the distorting labour income tax falls during transition. Because public capital is very scarce initially, the labour income tax is actually reduced in the long run ($\tilde{t}_L(\infty) < 0$–see also equation (3.13) above). Because this tax is distorting the labour supply decision, this tax reduction forms an additional source of welfare gains.

In Table 2 we report the pure efficiency effect, π^D, for various values of ω_G (top panel) and t_L (bottom panel). Consider the top panel first. As was to be expected, π^D falls as

Figure 2. An increase in public investment: Time-varying labour tax



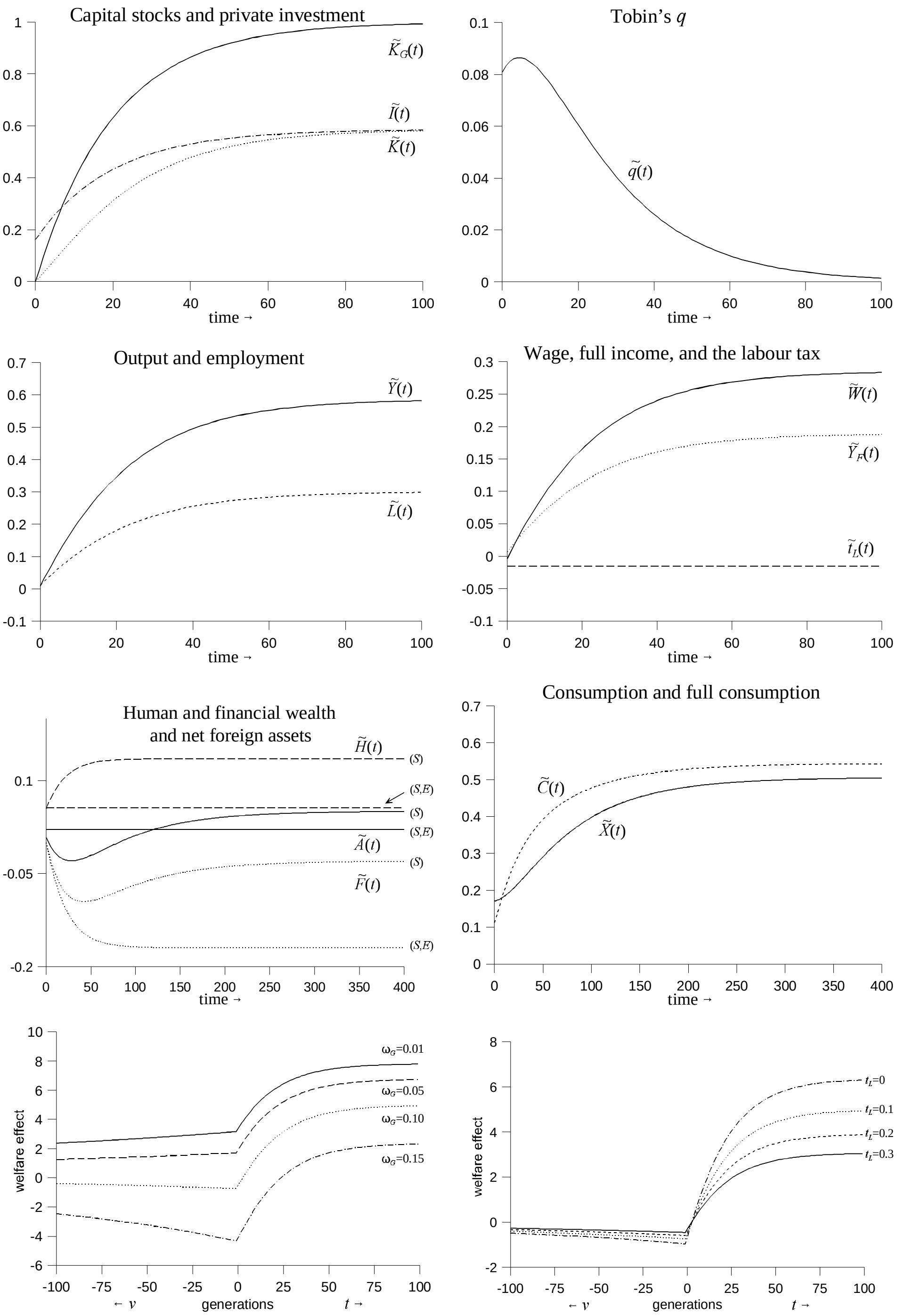
public capital gets less and less scarce (and ω_G rises). Comparing the entries for π^D and π^N for the base case with $\omega_G = 0.01$ (scarce public capital), we find that it is actually better to finance the policy shock with the distorting tax. In this case, the public investment policy under the D-scenario delivers a side-benefit in the form of a less distorting tax system in the long run (note that $\tilde{t}_L(\infty) = -0.54$ in this case). For higher values of ω_G we find the more conventional result that π^D falls short of π^N: the tax rate has to rise and the tax system becomes more distortionary.

In the bottom panel of Table 2, we observe that π^D rises as the initial tax gets larger, although the effects are rather small (compared to the lump-sum case) because required changes in the time-varying tax are rather large (and distortionary). We also find that for $t_L = 0$, the investment policy causes no first-order welfare effects. There is no tax-wedge effect in that case and all financing modes are equivalent (see Proposition 4(i) and equation (3.22) above).

Scenario *S* In Figure 3 we present the impulse-response functions and intergenerational welfare profiles for the tax smoothing case. As for the previous case, the information contained in Figure 3 is very similar (in a qualitative sense) to that in Figure 1. For that reason we focus on some of the key features. First, in the fourth panel we observe that a once-off *reduction* in the labour income tax is possible ($\tilde{t}_L^S = -0.015$ for $\omega_G = 0.01$ in Table 2). Second, in the seventh panel we observe that the intergenerational welfare profile shifts down as ω_G increases. Third, in the eighth panel we find that the effect of the pre-existing labour income tax on the intergenerational welfare profile is ambiguous. As t_L rises, the welfare profile shifts up for existing generations but it shifts down for future generations. Finally, in the fifth panel we plot the effects on human and financial wealth and on net foreign assets, both when only tax smoothing is practised (label "(S)") and when tax smoothing is accompanied by egalitarian intergenerational redistribution (label "(S,E)"). The key thing to note about the latter scenario is that bond policy eliminates all transitional dynamics from both human and financial wealth but not from the current account.

In the top panel of Table 2 we report the pure efficiency effect, π^S, for different values of ω_G. Some features are worth noting. First, just as for the previous two scenarios, π^S falls as ω_G gets larger. Second, for $\omega_G = 0.01$ public capital is very scare and π^S exceeds π^N because the reduction in the labour income tax rate ($\tilde{t}_L^S = -0.015$) furnishes additional welfare gains in the tax smoothing scenario (see also the analytical results in equation (3.22) above). Third, for all value of ω_G considered we obtain the rather surprising result that π^D exceeds π^S, i.e. tax smoothing is actually sub-optimal. The common welfare gain is larger if the labour income tax falls over time than if it features a once-off adjustment. (The same

Figure 3. An increase in public investment: Time-invariant labour tax



feature is observed in the bottom panel of Table 2.) This result suggests that the overall distortiveness of the tax system is reduced by having a relatively high tax rate during the early phases of transition (when the tax base is still low) and having a relatively low tax rate later on (when the tax base has grown).

5 Conclusion

We have studied the macroeconomic and intergenerational welfare effects of an increase in public investment in the context of a small open economy populated by overlapping-generations of finite-lived households. It was shown that these effects depend not only on the particular financing mode chosen by the policy maker, but also on the pre-existing tax on labour income. In all cases, output, before-tax wages and private capital increase monotonically over time. However, the long-run effect on private capital and output is, ceteris paribus, smaller if the tax rate on labour rises in the long run. But the policy shock may in fact boost the tax base to such an extent that it is possible to decrease the tax rate on labour once-and-for-all, especially if the initial share of public investment is low, the initial tax rate on labour is high, and labour supply is highly elastic. In that case, all generations are better off with the permanent decrease in the distortionary tax than under lump-sum financing. Of course, if it is possible to decrease the tax on labour once-and-for-all, it is also possible to decrease the long-run tax rate under a balanced-budget policy with time-varying labour tax rates. In that case it is not necessarily true, however, that all generations gain more than under non-distortionary taxation. In particular, young existing generations may be worse off.

In many cases the tax rate on labour will have to rise in order to finance the increase in public investment. As noted above, this comes at the cost of lower long-run production than under lump-sum finance. Even stronger, we have shown that the pure efficiency effect of additional public investment, as measured by the common welfare gain that can be realized when the government neutralizes all intergenerational effects, is lower than under lump-sum finance if it is necessary to raise the labour tax permanently in order to finance the additional investment. So, why would a government use distortionary taxation? One reason may be that the labour taxation not only affects efficiency, but also the intergenerational distribution of welfare. In particular, we have shown that distortionary taxation shifts welfare from older to younger generations as compared to lump-sum finance. Because, under lump-sum finance, older generations typically gain from an increase in public investment–whereas the effect on welfare of younger existing generations typically is ambiguous–this may be crucial for the political support among existing generations for a policy of raising the public capital stock.

The political support for a proposal to increase public investment not only depends on the mode of financing but also on the initial tax on labour income. As noted above, the initial tax rate is an important factor in determining whether the tax on labour can fall or has to rise under distortionary taxation. Furthermore, we have shown that the political support for a lump-sum financed increase in public capital critically depends on the initial tax rate. A higher initial labour tax rate has a positive influence on the welfare effect on young existing generations. As the welfare gain for old existing generations is independent of the initial tax on labour, this may turn the median voter from an opponent into a supporter of the proposal. Finally, it is shown in the paper that the pure efficiency effect of public investment is a positive function of the initial labour tax rate. Consequently, the optimal share of public investment is larger, the higher is the initial tax on labour income.

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