Intergenerational welfare effects of a tariff under monopolistic competition: Mathematical appendix

Leon J.H. Bettendorf* Ben J. Heijdra♯

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A.1 Introduction

In the text of the paper we refer on several occasions to this appendix. The following results are proved in this appendix (plus many other things). Note that the proofs do not appear in this appendix in the same order they are referred to in the text.

- Result 1. The jump in $\tilde{X}(0)$
- Result 2. Welfare effects without bond policy
- Result 3. Proof of Proposition 1
- Result 4. Egalitarian bond policy
- Result 5. Proof of Proposition 2

A.2 The two-stage solution method

The optimisation problem faced by the representative consumer can be solved in two stages. In stage 1 the dynamic problem is solved. This yields a path of full consumption, $X(v,\tau)$. In stage 2 the static allocation problem is solved. First, full expenditure is allocated between its components $C(v,\tau)$, $Z(v,\tau)$ and $L(v,\tau)$. Then $C(v,\tau)$ is allocated over the different varieties of the differentiated product, $C_i(v,\tau)$.

A.2.1 Stage 1

Define the ideal cost-of-living index in period τ as $P_U(\tau)$:

$$P_U(\tau)U(v,\tau) = X(v,\tau), \tag{A.1}$$

*OCFEB, Erasmus University, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. Phone: +31-10-408-1808. Fax: +31-10-408-9173. E-mail: bettendorf@few.eur.nl.

♯Department of Economics, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-363-7303, Fax: +31-50-363-7207, E-mail: b.j.heijdra@eco.rug.nl.

where $U(v,\tau)$ and $X(v,\tau)$ are defined in the text (see (2) and (7), respectively). In the first stage the following optimisation problem is solved.

$$\begin{aligned} &\underset{\{U(v,\tau)\}}{\text{Max}} \int_t^\infty [\log U(v,\tau)]\, \mathrm{e}^{(\alpha+\beta)(t-\tau)} d\tau \\ &\text{subject to: } \frac{dA(v,\tau)}{d\tau} = (r+\beta)\, A(v,\tau) + W(\tau) - T(\tau) - P_U(\tau)U(v,\tau). \end{aligned}$$

This leads to the following first-order conditions:

$$\frac{1}{U(v,\tau)} = \lambda(v,\tau)P_U(\tau), \tag{A.2}$$

$$\frac{d\lambda(v,\tau)}{d\tau} = (\alpha - r)\lambda(v,\tau), \tag{A.3}$$

where $\lambda(v,\tau)$ is the co-state variable of the flow budget identity. By combining (A.2)-(A.3) we obtain (8) in the text. The integrated (life-time) budget restriction (with a NPG condition imposed) is:

$$A(v,t) + H(t) = \int_t^\infty P_U(\tau)U(v,\tau)\mathrm{e}^{(r+\beta)(t-\tau)} d\tau = \int_t^\infty \lambda(v,\tau)^{-1}\mathrm{e}^{(r+\beta)(t-\tau)} d\tau, \tag{A.4}$$

where $H(t)$ is:

$$H(t) = \int_t^\infty [W(\tau) - T(\tau)]\, \mathrm{e}^{(r+\beta)(t-\tau)} d\tau. \tag{A.5}$$

From (A.3) we observe that $\lambda(v,\tau) = \lambda(v,t)e^{(r-\alpha)(t-\tau)}$. Using this in (A.4) yields:

$$(\alpha+\beta)\,[A(v,t) + H(t)] = 1/\lambda(v,t) = X(v,t).$$

Full expenditure is a constant proportion of total wealth.

A.2.2 Stage 2-a

Full expenditure $X(v,t)$ is now allocated over $C(v,t)$, $Z(v,t)$, and $1 - L(v,t)$.

$$\begin{aligned} \underset{\{C(v,t),L(v,t),Z(v,t)\}}{\text{Max}} U(v,t) &= \left[C(v,t)^\delta Z(v,t)^{1-\delta}\right]^\gamma [1 - L(v,t)]^{1-\gamma} \\ \text{subject to } X(v,t) &= P(t)C(v,t) + (1+t_M)Z(v,t) + W(t)\,[1 - L(v,t)]\,. \end{aligned}$$

Straightforward calculations yield the expressions given in (9a-c) in the text. By substituting these expressions into the sub-utility function $U(v,t)$ (defined in (2)) and noting (A.1), we recover the expression for $P_U(t)$:

$$P_U(t) \equiv \left(\frac{P(t)}{\gamma\delta}\right)^{\gamma\delta} \left(\frac{1+t_M}{\gamma(1-\delta)}\right)^{\gamma(1-\delta)} \left(\frac{W(t)}{1-\gamma}\right)^{1-\gamma}. \tag{A.6}$$

A.2.3 Stage 2-b

The agent now chooses $C_i(v,t)$ such that the following static maximisation program is solved:

$$\underset{\{C_i(v,t)\}}{\text{Max}} C(v,t) \equiv \left[\sum_{i=1}^{N(t)} C_i(v,t)^{1/\eta}\right]^\eta \text{ subject to } \sum_{i=1}^{N(t)} P_i(t)C_i(v,t) = P(t)C(v,t).$$

Straightforward manipulation yields the demand functions for the domestically produced varieties of the differentiated commodity by the agent of vintage v:

$$\frac{C_i(v,t)}{C(v,t)} = \left(\frac{P_i(t)}{P(t)}\right)^{-\eta/(\eta-1)}. \tag{A.7}$$

By substituting (A.7) into (3) the expression for $P(t)$ in (4) is obtained.

A.3 The optimisation problem for a representative firm

The representative firm i aims to maximise (14) subject to the demand restriction and the production function (13). Profit is defined as follows.

$$\Pi_i(t) = (1+s_P)P_i(t)Y_i(t) - W(t)[kY_i(t)+f],$$

where $Y_i(t) \equiv C_i(t) + E_i(t)$ is the price-elastic demand facing firm i. The first-order necessary condition is:

$$\frac{\partial \Pi_i(t)}{\partial P_i(t)} = 0: \quad P_i(t) = \frac{\mu_i(t)W(t)k}{1+s_P},$$

where the mark-up is $\mu_i(t) \equiv \epsilon_i(t)/[\epsilon_i(t)-1]$ and $\epsilon_i(t)$ is the (absolute value of the) price elasticity of demand:

$$\epsilon_i(t) \equiv -\frac{P_i(t)}{Y_i(t)}\frac{\partial Y_i(t)}{\partial P_i(t)} = \frac{\eta}{\eta-1}, \tag{A.8}$$

where the last equality is obtained by using (10) and (12). In view of (A.8) the markup is constant and equal across firms: $\mu_i = \eta$ for all active firms i.

A.4 Aggregation

Equation (T1.8) is obtained as follows. In the symmetric equilibrium all active firms charge the same price and produce the same amount, i.e. $Y_i = \bar{Y}$ and $P_i = \bar{P}$ so that $Y = N\bar{P}\bar{Y}$. Equation (4) implies $P = N^{1-\eta}\bar{P}$ so that:

$$Y/P = N^{\eta}\bar{Y}. \tag{A.9}$$

Furthermore, (13) and the zero profit condition imply that $k\bar{Y} + f = \bar{L} = \eta k\bar{Y}$. Aggregation over all active firms yields $N\bar{Y} = L/(\eta k)$, where $L \equiv N\bar{L}$ is aggregate labour supply. Solving for the number of firms yields:

$$N = L/(\eta k\bar{Y}) \tag{A.10}$$

By combining (A.9)-(A.10) we obtain $Y/P = \Omega_0 L^{\eta}$ and $N = [(\eta-1)/(\eta f)]L$, where $\Omega_0 \equiv k^{-1}\eta^{-\eta}((\eta-1)/f)^{\eta-1} > 0$. Preference for diversity thus causes aggregate real output (expressed in terms of the home good) and the equilibrium number of firms to exhibit, respectively, increasing and constant returns to labour.

A.5 Construction of the phase diagram

The construction of the phase diagram (Figure 1 in the text) proceeds as follows. First we use equations (T1.3), (T1.7) and (T1.8) to summarize labour market equilibrium (LME):

$$f(L) \equiv z_0(1-L)L^{\eta-1} = x, \tag{A.11}$$

where $x \equiv X/P$ is full expenditure expressed in terms of the domestic price index and z_0 is defined as:

$$z_0 \equiv \frac{(1+s_P)\Omega_0}{1-\gamma}. \tag{A.12}$$

It follows from (A.11) that $f(L)$ has the following first- and second-order derivatives:

$$f'(L) = z_0 L^{\eta-1}\left[(\eta-1)\left(\frac{1-L}{L}\right)-1\right], \quad f''(L) = (\eta-1)z_0\left[\eta(1-L)-2\right]L^{\eta-3}. \tag{A.13}$$

We are interested in the properties of $f(L)$ in the *economically meaningful* interval $0 \leq L \leq 1$. Clearly, $f(1) = 0$ but $f(0)$ depends on the magnitude of $\eta - 1$:

$$f(0)\left\{\begin{array}{c} = z_0 \\ = 0 \end{array}\right. \quad \Leftrightarrow \quad \eta\left\{\begin{array}{c} = \\ > \end{array}\right\} 1. \tag{A.14}$$

Finally, for $\eta = 1$, $f(L)$ is downward sloping but for $\eta > 1$ we have:

$$f'(L) \gtreqless 0 \;\Leftrightarrow\; L \lesseqgtr L_{\text{MIN}} \equiv \frac{\eta-1}{\eta}. \tag{A.15}$$

In summary, the function $f(L)$ is as drawn in the top panel in Figure A.1. (In that figure we assume that $1 < \eta < 2$ so that $f''(L) < 0$ in the feasible region). If $\eta = 1$, then $f(L)$ is the dashed line from z_0 to point d. If the diversity effect is operative ($\eta > 1$) then $f(L)$ is the solid line from the origin (point e) through a, c, and b to point d. For $L = L_{\text{MIN}}$, x reaches its maximum value (x_{MAX}) consistent with labour market equilibrium. Now consider what happens if $x = x_0$. Both points a and b are consistent with labour market equilibrium. In the bottom panel of Figure A.1 labour demand (equations (T1.3) and (T1.8) combined) and labour supply (equation (T1.7)) are depicted by, respectively, L^D and $L^S(x_0)$ (supply depends on x_0 through the wealth effect). Labour demand slopes upward because of the diversity effect. It would appear that there are two suitable equilibria but it is shown below that the labour market equilibrium in point a is unstable. Hence with an operative diversity effect the feasible range of labour market equilibria is $L \in (L_{\text{MIN}}, 1]$ and thus $x \in [0, x_{\text{MAX}})$. In terms of Figure A.1 this amounts to selecting only the employment solution on the downward sloping segment of the $f(L)$ function. In terms of parameters used in the linearized model the feasible region for L can be rewritten as:

$$0 \leq (\eta-1)\omega_{LL} < 1, \tag{A.16}$$

where $\omega_{LL} \equiv (1-L)/L$.

The upshot of the discussion so far is that equilibrium employment can be expressed as a unique function of x and z_0, i.e. $L = L(x, z_0)$ with $L_x \equiv \partial L/\partial x < 0$ and $L_{z_0} \equiv \partial L/\partial z_0 > 0$. By substituting this relation in (T1.8) and also using (T1.4)-(T1.5) we get the expression for goods market equilibrium (GME):

$$\Omega_0 L(x, z_0)^{\eta} = \gamma\delta x + E_0 P^{-\sigma_T}, \tag{A.17}$$

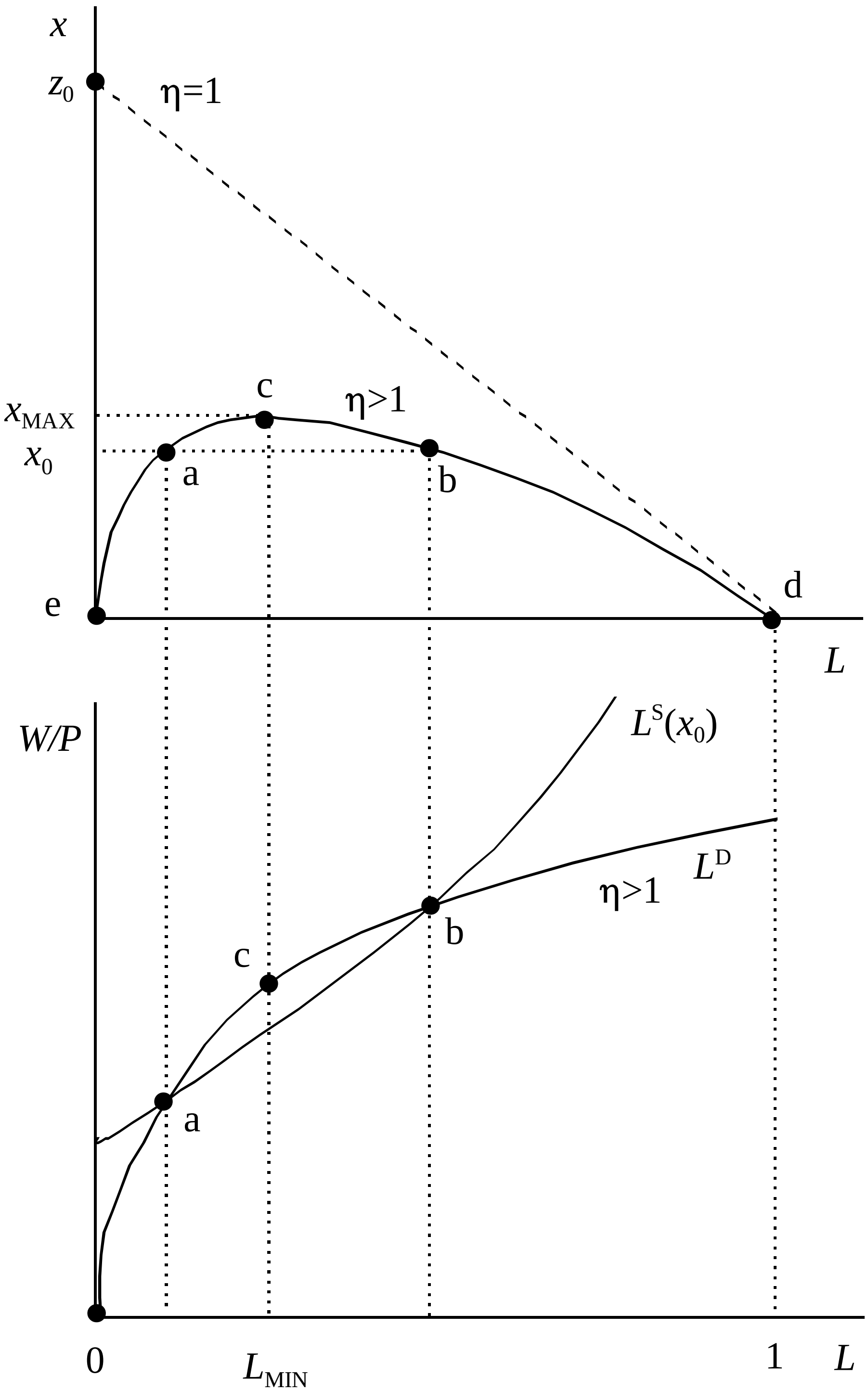

Figure A.1: Labour market equilibrium

where the left-hand side is aggregate supply of domestic goods and the right-hand side is demand for domestic goods consisting of demand by residents plus demand by foreigners (exports). An increase in x lowers supply, raises demand by residents and thus–to restore goods market equilibrium–exports must fall, i.e. P must rise. Recalling that $x \equiv X/P$, this reasoning shows that the terms of trade can be written as a unique function of X and z_0:

$$P = P(X, z_0) \tag{A.18}$$

which has the following partial derivatives:

$$P_X \equiv \frac{\partial P}{\partial X} = \frac{\Omega_0 \eta L^{\eta-1} L_x - \gamma\delta}{x\left[\Omega_0 \eta L^{\eta-1} L_x - \gamma\delta\right] - \sigma_T E_0 P^{-\sigma_T}} > 0, \tag{A.19}$$

$$P_{z_0} \equiv \frac{\partial P}{\partial z_0} = \frac{\Omega_0 P \eta L^{\eta-1} L_{z_0}}{x\left[\Omega_0 \eta L^{\eta-1} L_x - \gamma\delta\right] - \sigma_T E_0 P^{-\sigma_T}} < 0. \tag{A.20}$$

Finally, by using this expression in combination with (T1.6) and (T1.9) we obtain the following expression for the trade balance:

$$TB = TB(X, z_0, t_M) = E_0 P(X, z_0)^{1-\sigma_T} - \left(\frac{\gamma(1-\delta)}{1+t_M}\right) X, \tag{A.21}$$

with the following partial derivatives:

$$TB_X \equiv \frac{\partial TB}{\partial X} = -(\sigma_T - 1) E_0 P^{-\sigma_T} P_X - \frac{\gamma(1-\delta)}{1+t_M} < 0, \tag{A.22}$$

$$TB_{z_0} \equiv \frac{\partial TB}{\partial z_0} = -(\sigma_T - 1) E_0 P^{-\sigma_T} P_{z_0} > 0, \tag{A.23}$$

$$TB_{t_M} \equiv \frac{\partial TB}{\partial t_M} = \frac{\gamma(1-\delta)X}{(1+t_M)^2} > 0. \tag{A.24}$$

We can now describe the equilibrium dynamics. Equation (T1.1) describes the dynamics of full expenditure. The $\dot{X} = 0$ line is given by $(r - \alpha)X = \beta(\alpha + \beta)[F + B]$ which is a straight line through the origin (since $B = 0$ initially) with a positive (negative) slope for a creditor (debtor) country with $r > \alpha$ ($r < \alpha$). For points to the right (left) of the $\dot{X} = 0$ line X falls (rises) over time. This has been indicated with vertical arrows in Figure 1.

Equation (T1.2) in combination with (A.21) describes the dynamics of net foreign assets. The $\dot{F} = 0$ line is given by $rF = -TB(X, z_0, t_M)$ and slopes upwards because $TB_X < 0$. $rF_{\text{MIN}} \equiv -TB(0, z_0, t_M)$ is the lowest possible level of net foreign assets and the $\dot{F} = 0$ line has been drawn as a straight line for convenience. The dynamics follow from $\partial\dot{F}/\partial X = TB_X < 0$ i.e. for points above (below) the $\dot{F} = 0$ line, net foreign assets are falling (rising) over time. This has been indicated with horizontal arrows in Figure 1.

The model is saddle-point stable for both cases. For the creditor country ($r > \alpha$) the steady state is associated with positive net foreign assets. In contrast, for the debtor country ($r < \alpha$) steady-state net foreign assets are negative. In both cases the saddle path is upward sloping.

A.6 Model Solution

For both cases admitted by the model there is a regular saddle-point stable equilibrium. In the paper we study the comparative dynamic properties of the model by loglinearizing it around an initial steady state–see Table A.1. We use the following notational conventions. A tilde ('~')

above a variable denotes its rate of change around the initial steady-state, e.g., $\tilde{X}(t) \equiv dX(t)/X$. A variable with a tilde and a dot is the time derivative expressed in terms of the initial steady-state, for example, $\dot{\tilde{X}}(t) \equiv \dot{X}(t)/X$. The only exceptions to that convention refer to the tariff, the product subidy, the various financial assets, and the trade balance: $\tilde{t}_M \equiv dt_M/(1+t_M)$, $\tilde{s}_P \equiv ds_P/(1+s_P)$, $\tilde{B}(t) \equiv rdB(t)/Y$, $\dot{\tilde{B}}(t) \equiv r\dot{B}(t)/Y$, $\tilde{F}(t) \equiv rdF(t)/Y$, $\dot{\tilde{F}}(t) \equiv r\dot{F}(t)/Y$, and $\widetilde{TB}(t) \equiv dTB(t)/Y$.

In order to solve the loglinearized model, it is useful to first condense the static part of the model as much as possible. By using (TA.3)-(TA.9) in Table A.1, the change in real national income $(\tilde{Y}(t) - \tilde{P}(t))$, employment $(\tilde{L}(t))$, the terms of trade $(\tilde{P}(t))$, and the trade balance $(\widetilde{TB}(t))$ can be written in terms of the state variable $(\tilde{X}(t))$ and the policy variables ($\tilde{t}_M$ and $\tilde{s}_P$):

$$\tilde{Y}(t) - \tilde{P}(t) = \eta\tilde{L}(t) = -(\phi-1)\left[\left(\tilde{X}(t) - \tilde{P}(t)\right) - \tilde{s}_P\right], \tag{A.25}$$

$$\tilde{P}(t) = (1-\zeta)\tilde{X}(t) - \left(\frac{(\phi-1)\zeta}{\sigma_T\omega_X}\right)\tilde{s}_P, \tag{A.26}$$

$$\widetilde{TB}(t) = -\left[\zeta\phi + \omega_F\right]\tilde{X}(t) + (\phi-1)\xi\tilde{s}_P + (\omega_F + \omega_X)\,\tilde{t}_M, \tag{A.27}$$

where ϕ, ζ and ξ are defined as:

$$\phi \equiv \frac{1+\omega_{LL}}{1+\omega_{LL}(1-\eta)} \geq 1, \quad 0 < \zeta \equiv \frac{\sigma_T\omega_X}{\phi + \omega_X(\sigma_T - 1)} < 1, \quad 0 < \xi \equiv \left(\frac{\sigma_T - 1}{\sigma_T}\right)\zeta < 1,$$

and where we have incorporated (A.16). Note that equations (A.26) and (A.27) are the loglinearized versions of, respectively, (A.18) and (A.21). The expression for the loglinearized true price index $(\tilde{P}_U(t))$ follows in a straightforward fashion from (A.6):

$$\tilde{P}_U(t) = \gamma\delta\tilde{P}(t) + \gamma(1-\delta)\tilde{t}_M + (1-\gamma)\tilde{W}(t). \tag{A.28}$$

By using (TA.1), (TA.2), and (A.27) the dynamical system for net foreign assets and full expenditure is obtained:

$$\begin{bmatrix} \dot{\tilde{F}}(t) \\ \dot{\tilde{X}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{F}(t) \\ \tilde{X}(t) \end{bmatrix} + \begin{bmatrix} \gamma_F(t) \\ \gamma_X(t) \end{bmatrix}, \tag{A.29}$$

where the Jacobian matrix of coefficients on the right-hand side (denoted by Δ with typical element δ_{ij}), is defined as:

$$\Delta \equiv \begin{bmatrix} r & -r\left(\zeta\phi + \omega_F\right) \\ -(r-\alpha)/\omega_F & r-\alpha \end{bmatrix}. \tag{A.30}$$

The vectors of forcing terms is given by:

$$\begin{bmatrix} \gamma_F(t) \\ \gamma_X(t) \end{bmatrix} = \begin{bmatrix} r\left(\omega_X + \omega_F\right)\tilde{t}_M + r(\phi-1)\xi\tilde{s}_P \\ -\left[(r-\alpha)/\omega_F\right]\tilde{B} \end{bmatrix} \tag{A.31}$$

The determinant of Δ in (A.30) is easily computed:

$$|\Delta| = -r\zeta\phi\left(\frac{r-\alpha}{\omega_F}\right) < 0. \tag{A.32}$$

The sign is derived as follows. First, we know from section A.5 that $(r-\alpha)/\omega_F > 0$. Saddle-point stability requires $|\Delta| < 0$. It follows from equation (A.32) that $\phi > 0$ is a necessary and

Table A.1: Log-Linearized Version of the Model

$$\dot{\tilde{X}}(t) = (r-\alpha)\tilde{X}(t) - [(r-\alpha)/\omega_F]\left[\tilde{F}(t)+\tilde{B}\right] \quad \text{(TA.1)}$$

$$r^{-1}\dot{\tilde{F}}(t) = \tilde{F}(t) + \widetilde{TB}(t) \quad \text{(TA.2)}$$

$$\tilde{L}(t) = \tilde{Y}(t) - \tilde{W}(t) + \tilde{s}_P \quad \text{(TA.3)}$$

$$\tilde{Y}(t) - \tilde{P}(t) = (1-\omega_X)\tilde{C}(t) - \sigma_T\omega_X\tilde{P}(t) \quad \text{(TA.4)}$$

$$\tilde{C}(t) + \tilde{P}(t) = \tilde{X}(t) \quad \text{(TA.5)}$$

$$\tilde{Z}(t) = \tilde{X}(t) - \tilde{t}_M \quad \text{(TA.6)}$$

$$\tilde{L}(t) = \omega_{LL}\left[\tilde{W}(t) - \tilde{X}(t)\right] \quad \text{(TA.7)}$$

$$\tilde{Y}(t) - \tilde{P}(t) = \eta\tilde{L}(t) \quad \text{(TA.8)}$$

$$\widetilde{TB}(t) = -(\sigma_T - 1)\omega_X\tilde{P}(t) - (\omega_X+\omega_F)\tilde{Z}(t) \quad \text{(TA.9)}$$

Shares:
$\omega_{LL} \equiv (1-L)/L$, leisure/work ratio.
$\omega_F \equiv rF/Y$, share of asset income in national income.
$\omega_X \equiv E_0 P^{1-\sigma_T}/Y$, share of exports in national income.
Notes:
(a) We assume that $B = 0$ initially.
(b) Relationship between shares: $\omega_F + \omega_X = Z/Y > 0$.

sufficient condition for saddle point stability. It follows from the definition of ϕ that this holds iff $\omega_{LL} < (\eta-1)^{-1}$, i.e. (A.16) must hold. In that case we have $\phi \geq 1$ and $\zeta > 0$ so that $|\Delta| < 0$.

The characteristic equation of Δ, $g(\lambda) \equiv |\lambda I - \Delta|$, can be written as follows:

$$g(\lambda) = (\lambda+\lambda_1)(\lambda-\lambda_2) = \lambda^2 - (2r-\alpha)\,\lambda - r\zeta\phi\left(\frac{r-\alpha}{\omega_F}\right). \quad \text{(A.33)}$$

The roots, $-\lambda_1$ and λ_2, are the solutions to $g(\lambda) = 0$ in (A.33). It is straightforward to verify that $-\lambda_1 < 0$ and $\lambda_2 > 0$. Indeed, by using (A.33) the following expressions and inequalities for $-\lambda_1$ and λ_2 can be derived:

$$-\lambda_1 = \frac{1}{2}(2r-\alpha)\left[1-\left(1+\frac{4r\zeta\phi(r-\alpha)}{\omega_F(2r-\alpha)^2}\right)^{1/2}\right] < 0, \quad \text{(A.34)}$$

$$\lambda_2 = \frac{1}{2}(2r-\alpha)\left[1+\left(1+\frac{4r\zeta\phi(r-\alpha)}{\omega_F(2r-\alpha)^2}\right)^{1/2}\right] > 2r-\alpha > 0. \quad \text{(A.35)}$$

Obviously, in view of (A.33) (or, equivalently, (A.34)-(A.35)) we have that $\lambda_2 - \lambda_1 = 2r - \alpha$.[1]

The proof of $\lambda_2 > r$ proceeds as follows. The characteristic equation $g(\lambda)$ has two roots ($g(-\lambda_1) = g(\lambda_2) = 0$) and satisfies $g(0) = |\Delta| < 0$ (by saddle path stability) and $g'(\lambda_2) > 0$ (by (A.35)). Hence, to show that $\lambda_2 > r$ we must show that $g(r) < 0$. By using (A.33) we get:

$$g(r) = r^2 - (2r-\alpha)r - r\zeta\phi\left(\frac{r-\alpha}{\omega_F}\right) = -r\left(\frac{r-\alpha}{\omega_F}\right)(\omega_F + \zeta\phi) < 0,$$

[1] We only consider the case of a moderately impatient country, i.e. $\alpha < 2r$.

where $(\omega_F + \zeta\phi) > 0$ follows from:

$$\omega_F + \zeta\phi = \xi\phi + (1-\xi)\omega_X + \omega_F = \xi(\phi - \omega_X) + (\omega_F + \omega_X) > 0,$$

as $\phi > \omega_X$ and $\omega_F + \omega_X > 0$.

A.6.1 Long-run results

The long-run effects on the state variables of permanent shocks in the import tariff ($\tilde{t}_M$), the product subsidy ($\tilde{s}_P$), or the level of government debt ($\tilde{B}$) can be computed from the steady-state version of (A.29). After some manipulation the following expression is obtained:

$$\begin{bmatrix} \tilde{F}(\infty) \\ \tilde{X}(\infty) \end{bmatrix} \equiv \begin{bmatrix} \omega_F \\ 1 \end{bmatrix} \left(\frac{\omega_F + \omega_X}{\zeta\phi}\right) \tilde{t}_M + \begin{bmatrix} \omega_F \\ 1 \end{bmatrix} \left(\frac{(\phi-1)\xi}{\zeta\phi}\right) \tilde{s}_P - \begin{bmatrix} \zeta\phi + \omega_F \\ 1 \end{bmatrix} \frac{\tilde{B}}{\zeta\phi}. \tag{A.36}$$

In section 3 of the paper we abstract from bond policy and keep the product subsidy constant. By substituting $\tilde{s}_P = \tilde{B} = 0$ in (A.36) we get the results for section 3 of the paper. In section 4.4 we assume that egalitarian bond policy is used. Hence, the results for that section are obtained by substituting in (A.36):

$$\tilde{s}_P = 0, \;\; \tilde{B} = \left(\frac{\omega_F + \omega_X}{\omega_F + \phi\zeta}\right)\omega_F \tilde{t}_M. \tag{A.37}$$

A.6.2 Impact results

The impact results are obtained as follows. By taking the Laplace transform of (A.29) we obtain the following expression:

$$\Lambda(s) \begin{bmatrix} \mathcal{L}\{\tilde{F}, s\} \\ \mathcal{L}\{\tilde{X}, s\} \end{bmatrix} = \begin{bmatrix} \mathcal{L}\{\gamma_F, s\} \\ \tilde{X}(0) + \mathcal{L}\{\gamma_X, s\} \end{bmatrix}, \tag{A.38}$$

where we have used the fact that the stock of net foreign assets is predetermined (i.e. $\tilde{F}(0) = 0$), and where $\Lambda(s) \equiv sI - \Delta$, so that $|\Lambda(s)| \equiv (s + \lambda_1)(s - \lambda_2)$. By pre-multiplying (A.38) by $\mathrm{adj}(\Lambda(\lambda_2))$ we obtain the initial condition for the jump in full expenditure:

$$\mathrm{adj}(\Lambda(\lambda_2)\Lambda(\lambda_2) \begin{bmatrix} \mathcal{L}\{\tilde{F}, \lambda_2\} \\ \mathcal{L}\{\tilde{X}, \lambda_2\} \end{bmatrix} \equiv \begin{bmatrix} \lambda_2 - (r-\alpha) & -r(\zeta\phi + \omega_F) \\ -(r-\alpha)/\omega_F & (\lambda_2 - r) \end{bmatrix} \times \begin{bmatrix} \mathcal{L}\{\gamma_F, \lambda_2\} \\ \tilde{X}(0) + \mathcal{L}\{\gamma_X, \lambda_2\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{A.39}$$

Since the characteristic roots of Δ are distinct, $\mathrm{rank}(\mathrm{adj}(\Lambda(\lambda_i))) = 1$ and there is exactly one independent equation determining $\tilde{X}(0)$:

$$\tilde{X}(0) = -\mathcal{L}\{\gamma_X, \lambda_2\} + \left(\frac{r-\alpha}{\omega_F(\lambda_2 - r)}\right)\mathcal{L}\{\gamma_F, \lambda_2\}. \tag{A.40}$$

All results reported in the paper are based on the assumption that the shocks to the tariff, the product subsidy, and government debt are permanent and unanticipated. This implies that the Laplace transforms for these shocks take the following form:

$$\mathcal{L}\{\tilde{t}_M, s\} = \frac{\tilde{t}_M}{s}, \quad \mathcal{L}\{\tilde{s}_P, s\} = \frac{\tilde{s}_P}{s}, \quad \mathcal{L}\{\tilde{B}, s\} = \frac{\tilde{B}}{s}. \tag{A.41}$$

By using (A.31) and (A.41) in (A.40) the jump in full consumption can be computed:

$$\begin{aligned}\tilde{X}(0) &= \left(\frac{r-\alpha}{\omega_F\lambda_2}\right)\left[\left(\frac{r}{\lambda_2-r}\right)\left((\omega_F+\omega_X)\tilde{t}_M+(\phi-1)\xi\tilde{s}_P\right)+\tilde{B}\right] && \text{(A.42)}\\ &= \left(\frac{\omega_F+\omega_X}{\omega_F+\phi\zeta}\right)\tilde{t}_M+\left(\frac{(\phi-1)\xi}{\omega_F+\phi\zeta}\right)\tilde{s}_P+\left(\frac{r-\alpha}{\omega_F\lambda_2}\right)\left[\tilde{B}-\tilde{B}^*\right], && \text{(A.43)}\end{aligned}$$

where $\tilde{B}^*$ is:

$$\tilde{B}^* \equiv \left(\frac{\omega_F}{\phi\zeta+\omega_F}\right)\left[(\omega_X+\omega_F)\tilde{t}_M+\xi(\phi-1)\tilde{s}_P\right] \tag{A.44}$$

By setting $\tilde{B}=\tilde{s}_P=0$ in (A.42)-(A.44), the results for section 3 of the text are obtained. It also follows in that case that:

$$\operatorname{sgn}\left[\tilde{X}(\infty)-\tilde{X}(0)\right]=\operatorname{sgn}\left(\frac{(r-\alpha)(\omega_F+\omega_X)}{\zeta\phi(\lambda_2-r)}\right)\tilde{t}_M=\operatorname{sgn}(\omega_F), \tag{A.45}$$

where we have used the fact that $\lambda_2>r$.

A.6.3 Transition results

Since the shock administered at time $t=0$ is permanent and unanticipated, the transition path of the state variables has the following form:

$$\begin{bmatrix}\tilde{F}(t)\\ \tilde{X}(t)\end{bmatrix}=e^{-\lambda_1 t}\begin{bmatrix}0\\ \tilde{X}(0)\end{bmatrix}+\left[1-e^{-\lambda_1 t}\right]\begin{bmatrix}\tilde{F}(\infty)\\ \tilde{X}(\infty)\end{bmatrix}, \tag{A.46}$$

where λ_1 is minus the stable root of Δ which represents the transition speed in the economy. These expressions are found in the Appendix of the text.

A.7 Welfare Analysis

The welfare implications of the policy shock can be derived in the manner suggested by Judd (1982, 1998). The optimum utility level of the representative agent of vintage v at time t is denoted by $\Lambda(v,t)$:

$$\Lambda(v,t)\equiv\int_t^\infty \log[X(v,\tau)/P_U(\tau)]\mathrm{e}^{(\alpha+\beta)(t-\tau)}d\tau. \tag{A.47}$$

The Euler equation for the household, $\dot{X}(v,\tau)=(r-\alpha)X(v,\tau)$, implies that:

$$X(v,\tau)=X(v,t)\mathrm{e}^{(r-\alpha)(\tau-t)},\quad v\geq t. \tag{A.48}$$

Substitution of (A.48) in (A.47) yields:

$$\begin{aligned}\Lambda(v,t) &= -\frac{r-\alpha}{(\alpha+\beta)^2}+\int_t^\infty\left[\log X(v,t)-\log P_U(\tau)\right]\mathrm{e}^{(\alpha+\beta)(t-\tau)}d\tau\\ &= \Lambda_X(v,t)-\Lambda_D(t),\end{aligned}$$

where:

$$\Lambda_X(v,t)=-\frac{r-\alpha}{(\alpha+\beta)^2}+\frac{\log X(v,t)}{\alpha+\beta},\quad \Lambda_D(t)=\int_t^\infty \log P_U(\tau)\mathrm{e}^{(\alpha+\beta)(t-\tau)}d\tau.$$

The change in utility is calculated as $d\Lambda(v,t) = d\Lambda_X(v,t) - d\Lambda_D(t)$, with:

$$\begin{aligned} d\Lambda_X(v,t) &= \frac{1}{\alpha+\beta}\frac{dX(v,t)}{X(v,t)} = \frac{\tilde{X}(v,t)}{\alpha+\beta}, \\ d\Lambda_D(t) &= \int_t^{\infty} \tilde{P}_U(\tau)\mathrm{e}^{(\alpha+\beta)(t-\tau)}d\tau. \end{aligned}$$

The Laplace transforms of $d\Lambda_D(t)$ can be written in the following form:

$$\mathcal{L}\{d\Lambda_D, s\} = \frac{\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} - \mathcal{L}\{\tilde{P}_U, s\}}{s-(\alpha+\beta)}. \tag{A.49}$$

We now need to distinguish between generations that are alive at the time of the shock ($v < 0$) and future generations including the newly born ($v = t \geq 0$).

A.7.1 Existing generations ($v < 0$)

Existing generations are born before the policy shock occurs and have a negative generations index, $v < 0$. For an individual we have that $X(v,0) = (\alpha+\beta)[A(v,0) + H(0)]$, so that:

$$\tilde{X}(v,0) = [1-\alpha_H(v)]\,\tilde{A}(v,0) + \alpha_H(v)\tilde{H}(0), \text{ with } \alpha_H(v) \equiv \frac{H(0)}{A(v,0)+H(0)}, \tag{A.50}$$

where $\tilde{A}(v,0) \equiv dA(v,0)/A(v,0)$, $\tilde{X}(v,0) \equiv dX(v,0)/X(v,0)$, and $\tilde{H}(0) \equiv dH(0)/H(0)$. Aggregate total consumption satisfies $X(0) = (\alpha+\beta)[A(0)+H(0)]$, so that:

$$\tilde{X}(0) = [1-\omega_H]\,\tilde{A}(0) + \omega_H\tilde{H}(0), \text{ with } \omega_H \equiv \frac{H(0)}{A(0)+H(0)},$$

where $\tilde{A}(0) \equiv dA(0)/A(0)$. In the steady-state we have that $X(v,0) = X(v,v)\exp[-(r-\alpha)v]$, implying:

$$\begin{aligned} (\alpha+\beta)\,[A(v,0)+H(0)] &= (\alpha+\beta)H(0)\mathrm{e}^{-(r-\alpha)v} \\ &\Rightarrow \alpha_H(v) \equiv \mathrm{e}^{(r-\alpha)v}. \end{aligned}$$

Furthermore, we know that $\tilde{A}(v,0) = \tilde{A}(0)$ for $v < 0$, i.e. the rate of change in the value of individual assets equals the rate of change in the value of aggregate financial wealth. Combining these equations, (A.50) is written as:

$$\tilde{X}(v,0) = \tilde{A}(0) + [\alpha_H(v)/\omega_H]\left[\tilde{X}(0) - \tilde{A}(0)\right].$$

Next, it follows from (A.49) that:

$$d\Lambda_D(0) = \lim_{s\to\infty} s\mathcal{L}\{d\Lambda_D, s\} = \mathcal{L}\{\tilde{P}_U, \alpha+\beta\}.$$

The effect on welfare for existing generations ($v < 0$) can thus be written as:

$$(\alpha+\beta)d\Lambda(v,0) = \tilde{A}(0) + \left[\mathrm{e}^{(r-\alpha)v}/\omega_H\right]\left[\tilde{X}(0) - \tilde{A}(0)\right] - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\}, \tag{A.51}$$

where the change in the value of financial assets that occurs at impact can be written as:

$$\tilde{A}(0) \equiv \tilde{B}/\omega_F. \tag{A.52}$$

Note that in a debtor nation, $r < \alpha$ and $\alpha_H(v) \equiv \mathrm{e}^{(r-\alpha)v}$ goes to infinity for very old generations (i.e. as $-v$ gets large). Intuitively, agents follow a downward sloping time profile for full expenditure in that case. Extremely old generations (of which there are only very few left) have a full expenditure level close to (but greater than) zero, i.e. $X(v,0) \approx 0$ and $-F(v,0) \approx H(0)$ for these generations. Given the logarithmic felicity function, a marginal change in full expenditure results in an infinitely large change in felicity. Note also that there is a subtle problem if a policy shock were to cause a fall in human wealth at impact. If that were to happen, extremely old generations would have negative net worth, i.e. they would have to default on their foreign loans. In the scenarios studied in the paper, however, human wealth rises at impact so that default by old generations does not occur.

A.7.2 Future generations ($v = t \geq 0$)

The utility change for future generations is evaluated at birth, i.e. we compute $d\Lambda(v,v)$ for $v = t \geq 0$. First, we know that agents are born without financial wealth, $A(v,v) = 0$, so that:

$$X(v,v) = (\alpha+\beta)H(v) \;\Rightarrow\; \tilde{X}(v,v) = \tilde{H}(v).$$

From the aggregate counterpart, $X(v) = (\alpha+\beta)[F(v) + B(v) + H(v)]$, an expression for $\tilde{H}(v)$ is obtained:

$$\omega_H \tilde{H}(v) = \tilde{X}(v) - [(1-\omega_H)/\omega_F]\left(\tilde{F}(v) + \tilde{B}(v)\right).$$

The change in welfare of future generations ($v = t \geq 0$) is rewritten as:

$$(\alpha+\beta)d\Lambda(v,v) = \tilde{X}(v,v) - (\alpha+\beta)\mathcal{L}^{-1}\left[\frac{\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} - \mathcal{L}\{\tilde{P}_U, s\}}{s-(\alpha+\beta)}\right]. \tag{A.53}$$

The Laplace transform of $\tilde{P}_U(t)$ takes the following form:

$$\mathcal{L}\{\tilde{P}_U, s\} = \frac{\tilde{P}_U(0) - \tilde{P}_U(\infty)}{s+\lambda_1} + \frac{\tilde{P}_U(\infty)}{s}. \tag{A.54}$$

This implies the following results:

$$\frac{\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} - \mathcal{L}\{\tilde{P}_U, s\}}{s-(\alpha+\beta)} = \left(\frac{1}{s}\right)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} - \left[\frac{1}{s} - \frac{1}{s+\lambda_1}\right]\left[\frac{\tilde{P}_U(0) - \tilde{P}_U(\infty)}{\alpha+\beta+\lambda_1}\right]. \tag{A.55}$$

Hence, the terms in square brackets in (A.53) can be written as weighted averages of the initial and long-run effects of the respective variables. Since the paths of the variables themselves take the same form, it follows that utility can also be written in terms of weighted impact and long-run effects. Indeed, by using the solution paths for $\tilde{X}(t)$ and $\tilde{F}(t)$, as well as (A.55) in (A.53), we obtain the following expression for $d\Lambda(t,t)$:

$$d\Lambda(t,t) = e^{-\lambda_1 t} d\Lambda(0,0) + \left[1 - e^{-\lambda_1 t}\right] d\Lambda(\infty,\infty), \tag{A.56}$$

where $d\Lambda(\infty,\infty)$ is given by:

$$\begin{aligned}(\alpha+\beta)d\Lambda(\infty,\infty) &= \frac{\tilde{X}(\infty)}{\omega_H} - \frac{(1-\omega_H)\tilde{A}(\infty)}{\omega_H} - \tilde{P}_U(\infty) \\ &= \tilde{X}(\infty) - \tilde{P}_U(\infty),\end{aligned} \tag{A.57}$$

where we have used the steady-state version of (TA.1), $\tilde{X}(\infty) = \tilde{A}(\infty)$, in the final step. This establishes Result 2.

A.8 Egalitarian bond policy

In order to prove Results 3-5, it is useful to first compute the allocation and welfare effect in the presence of bond policy and a shock in the product subsidy. By working out the most general case first, the various results to be proved can be obtained later as special cases.

The general expressions for the impact effects are given in (A.42)-(A.44). By using (A.26) in (A.43) we obtain:

$$\tilde{X}(0) - \tilde{P}(0) = \left(\frac{\omega_F + \omega_X}{\omega_F + \phi\zeta}\right)\zeta\tilde{t}_M + \left(\frac{(\phi-1)(\omega_F + \sigma_T\omega_X)}{(\omega_F + \phi\zeta)\sigma_T\omega_X}\right)\zeta\tilde{s}_P + \left(\frac{\zeta(r-\alpha)}{\omega_F\lambda_2}\right)\left[\tilde{B} - \tilde{B}^*\right]. \tag{A.58}$$

The most general long-run results, given in (A.36), can be rewritten by making use of (A.26) and (A.44):

$$\tilde{F}(\infty) = \left(\frac{\omega_F + \zeta\phi}{\zeta\phi}\right)\left[\tilde{B}^* - \tilde{B}\right], \tag{A.59}$$

$$\tilde{X}(\infty) - \tilde{P}(\infty) = \left(\frac{\omega_X + \omega_F}{\omega_F + \zeta\phi}\right)\zeta\tilde{t}_M + \left(\frac{(\phi-1)(\omega_F + \sigma_T\omega_X)}{(\omega_F + \zeta\phi)\sigma_T\omega_X}\right)\zeta\tilde{s}_P + \left[\frac{\tilde{B}^* - \tilde{B}}{\phi}\right]. \tag{A.60}$$

A.8.1 Removing transitional dynamics

It is clear from (A.58), (A.59) and (A.60) that impact and long-run results coincide for the respective variables if the bond policy takes the form $\tilde{B} = \tilde{B}^*$, i.e. $\tilde{X}(0) - \tilde{P}(0) = \tilde{X}(\infty) - \tilde{P}(\infty)$ and $\tilde{F}(0) = \tilde{F}(\infty) = 0$ in that case. Since all transition in real full expenditure is eliminated, the same also holds for the other variables. Indeed, by substituting the time-invariant value of $\tilde{X}(t)$ resulting from $\tilde{B} = \tilde{B}^*$ in equations (A.25)-(A.26) we obtain:

$$\begin{aligned} \tilde{Y}(t) - \tilde{P}(t) &= \eta\tilde{L}(t) = -\left(\frac{(\phi-1)\zeta}{\omega_F + \zeta\phi}\right) \\ &\quad \times \left[(\omega_X + \omega_F)\tilde{t}_M - \left(\frac{\sigma_T\omega_X + [1 + (\sigma_T - 1)\omega_X]\,\omega_F}{\sigma_T\omega_X}\right)\tilde{s}_P\right] \end{aligned} \tag{A.61}$$

$$\tilde{W}(t) - \tilde{P}(t) = \left(\frac{\eta - 1}{\eta}\right)\left[\tilde{Y}(t) - \tilde{P}(t)\right] + \tilde{s}_P, \tag{A.62}$$

$$\begin{aligned} \sigma_T\omega_X\tilde{P}(t) &= (\phi - \omega_X)\left[\tilde{X}(t) - \tilde{P}(t)\right] - (\phi - 1)\tilde{s}_P \\ &= \left(\frac{(\omega_X + \omega_F)\zeta}{\omega_F + \zeta\phi}\right)\left[(\phi - \omega_X)\tilde{t}_M - (\phi - 1)\tilde{s}_P\right] \end{aligned} \tag{A.63}$$

for all $t \geq 0$.

A.8.2 Eliminating all intergenerational inequity

It is straightforward to show that a bond policy of the form $\tilde{B} = \tilde{B}^*$ not only removes transitional dynamics in the macroeconomic variables but also eliminates all intergenerational inequities. The welfare effect on *existing generations* is given in (A.51). By using (A.52) and (A.43) and imposing $\tilde{B} = \tilde{B}^*$ (from (A.44)) we derive that the generation-specific term in (A.51) vanishes:

$$\tilde{X}(0) - \tilde{A}(0) = \tilde{X}(0) - \tilde{B}^*/\omega_F = 0.$$

Hence, all existing generations are affected by the shock in the same manner, i.e. $d\Lambda(v,0) = d\Lambda(0,0)$ for all $v < 0$.

For *future generations* including newborns ($v = t \geq 0$) the welfare effect is given in (A.56). It is straightforward to show that $d\Lambda(t,t) = d\Lambda(0,0)$ for all $t \geq 0$. In view of (A.56), all that needs to be done is to show that $d\Lambda(0,0) = d\Lambda(\infty,\infty)$:

$$\begin{aligned}(\alpha+\beta)d\Lambda(0,0) &= \tilde{X}(0,0) - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} \\ &= \tilde{X}(0) - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} = \tilde{X}(\infty) - \tilde{P}_U(\infty) \\ &= (\alpha+\beta)d\Lambda(\infty,\infty),\end{aligned}$$

where we have used $\tilde{X}(0,0) = \tilde{X}(0)$ in the first step and $\tilde{P}_U(0) = \tilde{P}_U(\infty)$ as well as equation (A.54) in the second step.

We have now demonstrated that with the suitable bond policy, $\tilde{B} = \tilde{B}^*$, all generations experience the same change in welfare, i.e. $d\Lambda(t,t) = d\Lambda(v,0) = d\Lambda$. To evaluate the common welfare effect on all generations, it suffices to evaluate $d\Lambda(\infty,\infty)$. First we note that (A.57) be rewritten as follows:

$$(\alpha+\beta)d\Lambda(\infty,\infty) = \left[\tilde{X}(\infty) - \tilde{P}(\infty)\right] - \left[\tilde{P}_U(\infty) - \tilde{P}(\infty)\right]. \quad \text{(A.64)}$$

Next we deduce from (A.28) that:

$$\tilde{P}_U(\infty) - \tilde{P}(\infty) = \gamma(1-\delta)\left[\tilde{t}_M - \tilde{P}(\infty)\right] + (1-\gamma)\left[\tilde{W}(\infty) - \tilde{P}(\infty)\right]. \quad \text{(A.65)}$$

By using (A.60), (A.62), (A.63), and (A.65) in (A.64), and imposing $\tilde{B} = \tilde{B}^*$, we obtain the required expression for $d\Lambda$:

$$(\alpha+\beta)d\Lambda \equiv \Phi_P(t_M, s_P)\tilde{s}_P + \Phi_M(t_M, s_P)\tilde{t}_M, \quad \text{(A.66)}$$

where $\Phi_P(.)$ and $\Phi_M(.)$ are defined as follows:

$$\Phi_P(t_M, s_P) \equiv \frac{\Psi\zeta(\phi-1)(\omega_F + \sigma_T\omega_X)}{(\omega_F + \zeta\phi)\sigma_T\omega_X} + \gamma\left(\frac{\sigma_T\omega_X + (1-\delta)(1-\omega_X)}{\sigma_T\omega_X}\right) - \Psi, \quad \text{(A.67)}$$

$$\Phi_M(t_M, s_P) \equiv \frac{\Psi\zeta(\omega_F + \omega_X)}{(\omega_F + \zeta\phi)} - \gamma(1-\delta), \quad \text{(A.68)}$$

where Ψ is defined as follows:

$$\begin{aligned}\Psi &\equiv 1 + \gamma(1-\delta)\left(\frac{\phi - \omega_X}{\sigma_T\omega_X}\right) + (1-\gamma)(\phi-1)\left(\frac{\eta-1}{\eta}\right) \\ &= 1 + \gamma(1-\delta)\left(\frac{1-\zeta}{\zeta}\right) + (1-\gamma)(\phi-1)\left(\frac{\eta-1}{\eta}\right) > 1. \quad \text{(A.69)}\end{aligned}$$

By combining (A.67)-(A.68) we can derive a useful expression relating $\Phi_M(.)$ and $\Phi_P(.)$:

$$\begin{aligned}\Phi_M(t_M, s_P) + \gamma(1-\delta) &= \left(\frac{\sigma_T\omega_X(1-\delta)}{\sigma_T\omega_X + (1-\delta)(1-\omega_X) + \delta\omega_X\left[(\sigma_T - 1)(1+t_M) - \sigma_T\right]}\right) \\ &\quad \times \left[\gamma\left(\frac{\sigma_T\omega_X + (1-\delta)(1-\omega_X)}{\sigma_T\omega_X}\right) - \Phi_P(t_M, s_P)\right]. \quad \text{(A.70)}\end{aligned}$$

This establishes Result 4, the computation of the egalitarian bond policy.

A.8.3 Optimal tariff and product subsidy

In order to establish Result 5 we now prove that that $\Phi_M(t_M, s_P) = \Phi_P(t_M, s_P) = 0$ for $t_M = 1/(\sigma_T - 1)$ and $s_P = \eta - 1$, i.e. in the first-best egalitarian optimum, the policy maker uses t_M and s_P in order to set $d\Lambda = 0$ in equation (A.66). First, we set $\Phi_M(t_M, s_P) = 0$ in (A.68) and solve for $\Psi\zeta$ which we then substitute into (A.67):

$$\begin{aligned}\Phi_P &= \frac{\gamma(1-\delta)(\phi-1)(\omega_F+\sigma_T\omega_X)}{\sigma_T\omega_X(\omega_X+\omega_F)} - \gamma(1-\delta)\left(\frac{\phi-1}{\sigma_T\omega_X}\right) - (1-\gamma)\left[1+(\phi-1)\left(\frac{\eta-1}{\eta}\right)\right] \\ &= \frac{\gamma(1-\delta)(\phi-1)(\sigma_T-1)-(1-\gamma)\sigma_T(\omega_X+\omega_F)\left[1+(\phi-1)\left(\frac{\eta-1}{\eta}\right)\right]}{\sigma_T(\omega_X+\omega_F)}. \qquad \text{(A.71)}\end{aligned}$$

The expression in (A.71) can be further simplified by noting that ω_F, ω_X, and ω_{LL} can all be written in terms of the primitive parameters η, ϕ, δ, and the pre-existing tariff and product subsidy, t_M and s_P:

$$1-\omega_X \equiv \frac{\omega_{LL}(1+s_P)\gamma\delta}{1-\gamma}, \; \omega_{LL} \equiv \frac{\phi-1}{1+\phi(\eta-1)}, \; \omega_X+\omega_F \equiv \frac{(1-\delta)(1-\omega_X)}{(1+t_M)\delta}. \qquad \text{(A.72)}$$

After some manipulations we obtain:

$$\begin{aligned}\Phi_P &= \left(\frac{\gamma(1-\delta)(\phi-1)}{\sigma_T(\omega_X+\omega_F)}\right)\left[(\sigma_T-1)-\sigma_T\left(\frac{1+s_P}{1+t_M}\right)\left(\frac{1+(\phi-1)\left(\frac{\eta-1}{\eta}\right)}{1+\phi(\eta-1)}\right)\right] \\ &= \left(\frac{\gamma(1-\delta)(\phi-1)}{(1+t_M)(\omega_X+\omega_F)}\right)\left[\left(\frac{\sigma_T-1}{\sigma_T}\right)(1+t_M)-\left(\frac{1+s_P}{\eta}\right)\right] \\ &= \left(\frac{\gamma(1-\delta)(\phi-1)}{\eta\sigma_T(1+t_M)(\omega_X+\omega_F)}\right)\left[\sigma_T(\eta-1-s_P)+\eta\left[(\sigma_T-1)t_M-1\right]\right], \qquad \text{(A.73)}\end{aligned}$$

which shows that $\Phi_P = 0$ for $t_M = 1/(\sigma_T - 1)$ and $s_P = \eta - 1$.

Next, we use $\Phi_P(t_M, s_P) = 0$ in (A.67) and solve for $\Psi\zeta$ which we then substitute into (A.68):

$$\begin{aligned}\Phi_M &= \frac{\sigma_T\omega_X(\omega_X+\omega_F)\left[\gamma(1-\delta)\left(\frac{\phi-1}{\sigma_T\omega_X}\right)+(1-\gamma)\left(1+(\phi-1)\left(\frac{\eta-1}{\eta}\right)\right)\right]}{(\phi-1)(\omega_F+\sigma_T\omega_X)} - \gamma(1-\delta) \\ &= \frac{-\omega_X\left[\gamma(1-\delta)(\phi-1)(\sigma_T-1)-(1-\gamma)\sigma_T(\omega_X+\omega_F)\left(1+(\phi-1)\left(\frac{\eta-1}{\eta}\right)\right)\right]}{(\phi-1)(\omega_F+\sigma_T\omega_X)} \\ &= \frac{-\gamma(1-\delta)\sigma_T\omega_X}{(1+t_M)(\omega_F+\sigma_T\omega_X)}\left[\left(\frac{\sigma_T-1}{\sigma_T}\right)(1+t_M)-\left(\frac{1+s_P}{\eta}\right)\right] \\ &= \left(\frac{-\gamma(1-\delta)\omega_X}{\eta(\omega_F+\sigma_T\omega_X)(1+t_M)}\right)\left[\sigma_T(\eta-1-s_P)+\eta\left[(\sigma_T-1)t_M-1\right]\right], \qquad \text{(A.74)}\end{aligned}$$

which shows that $\Phi_M = 0$ for $t_M = 1/(\sigma_T - 1)$ and $s_P = \eta - 1$. It follows from (A.73) and (A.74) jointly that $t_M^F = 1/(\sigma_T - 1)$ and $s_P^F = \eta - 1$. This establishes Result 5.

A.8.4 Knife-edge case

The results for the knife-edge case, discussed in section 4.1 of the paper, are obtained by setting $\omega_F = 0$ in the relevant expressions appearing in the previous subsections. We first note that for $\omega_F = 0$ we have:

$$\omega_X \equiv \frac{1-\delta}{1+\delta t_M}, \qquad 1-\omega_X \equiv \left(\frac{\delta(1+t_M)}{1-\delta}\right)\omega_X, \qquad \text{(A.75)}$$

so that (A.70) simplifies to:

$$\Phi_M(t_M, s_P) = \left(\frac{1-\delta}{1+\delta t_M}\right)\left[\gamma\delta\left(\frac{t_M^F - t_M}{1+t_M^F}\right) - \Phi_P(t_M, s_P)\right]. \tag{A.76}$$

Equation (A.67) can be simplified to:

$$\begin{aligned}\Phi_P(t_M, s_P) &= \left(\frac{\gamma\eta(1-\gamma)(1+\delta t_M)}{(1-\gamma)(1+t_M)+\gamma(1+s_P)(1+\delta t_M)}\right) \\ &\quad\times\left[\left(\frac{1-\delta}{1+\delta t_M}\right)\left(\frac{t_M - t_M^F}{1+t_M^F}\right) - \left(\frac{s_P - s_P^F}{1+s_P^F}\right)\right].\end{aligned} \tag{A.77}$$

By combining (A.76) and (A.77) we obtain the expression for $\Phi_M(t_M, s_P)$:

$$\begin{aligned}\Phi_M(t_M, s_P) &= -\left(\frac{\gamma(1-\delta)}{1+\delta t_M}\right)\left[\delta + \frac{(1-\gamma)(1-\delta)\eta}{(1-\gamma)(1+t_M)+\gamma(1+s_P)(1+\delta t_M)}\right]\left(\frac{t_M - t_M^F}{1+t_M^F}\right) \\ &\quad+\left(\frac{\gamma(1-\delta)}{1+\delta t_M}\right)\left(\frac{(1-\gamma)(1+\delta t_M)}{(1-\gamma)(1+t_M)+\gamma(1+s_P)(1+\delta t_M)}\right)\left(s_P - s_P^F\right).\end{aligned}$$

Apart from a trivial redefinition of $\Phi_M(.)$ this is the expression found in the text.

A.9 Intergenerational welfare effects

We now compute the intergenerational welfare effects for the case without bond policy and with a constant product subsidy (i.e. $\tilde{s}_P = \tilde{B} = 0$ in this subsection). We derive from (A.26) that $\tilde{P}(t) = (1-\zeta)\tilde{X}(t)$ so that:

$$\tilde{X}(t) - \tilde{P}(t) = \zeta\tilde{X}(t). \tag{A.78}$$

By using (TA.3), (A.25), and (A.78) we derive the following expression for real wages:

$$\tilde{W}(t) - \tilde{P}(t) = -(\phi - 1)\left(\frac{\eta - 1}{\eta}\right)\zeta\tilde{X}(t). \tag{A.79}$$

Finally, using (A.26) and (A.79) in (A.28) we get the following expression for the true price index in real terms:

$$\tilde{P}_U(t) - \tilde{P}(t) = \gamma(1-\delta)\tilde{t}_M - (\Psi - 1)\,\zeta\tilde{X}(t), \tag{A.80}$$

where Ψ is defined in (A.69) above. It follows from (A.45) and Figure 2 in the text that:

$$\text{sgn}\left[\tilde{X}(\infty) - \tilde{X}(0)\right] = \text{sgn}(r - \alpha) = \text{sgn}(\omega_F) \tag{A.81}$$

Using (A.81) in (A.80) yields the conclusion that:

$$\text{sgn}\left[\left(\tilde{P}_U(\infty) - \tilde{P}(\infty)\right) - \left(\tilde{P}_U(0) - \tilde{P}(0)\right)\right] = -\text{sgn}(\omega_F).$$

A.9.1 Existing generations

By using (A.51)-(A.52) and setting $\tilde{B} = 0$, the welfare effect on *extremely old existing generations* can be written as follows:

$$(\alpha+\beta)d\Lambda(-\infty, 0) = \begin{cases} -(\alpha+\beta)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} & \text{for } r > \alpha \\ \\ \left(\tilde{X}(0)/\omega_H\right)\lim\limits_{v\to-\infty} e^{(r-\alpha)v} - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U, \alpha+\beta\} = +\infty & \text{for } r < \alpha \end{cases}. \tag{A.82}$$

By using (A.80) and (A.26) we derive:

$$\tilde{P}_U(t) = \gamma(1-\delta)\tilde{t}_M + (1-\Psi\zeta)\,\tilde{X}(t). \tag{A.83}$$

Using (A.54) and (A.83) we derive:

$$\begin{aligned}(\alpha+\beta)\mathcal{L}\{\tilde{P}_U,\alpha+\beta\} &= \left(\frac{\alpha+\beta}{\alpha+\beta+\lambda_1}\right)\tilde{P}_U(0) + \left(\frac{\lambda_1}{\alpha+\beta+\lambda_1}\right)\tilde{P}_U(\infty) \\ &= \gamma(1-\delta)\tilde{t}_M + (1-\Psi\zeta)\,(\alpha+\beta)\mathcal{L}\{\tilde{X},\alpha+\beta\}. \end{aligned} \tag{A.84}$$

where $\mathcal{L}\{\tilde{X},\alpha+\beta\}$ satisfies:

$$\begin{aligned}(\alpha+\beta)\mathcal{L}\{\tilde{X},\alpha+\beta\} &= \left(\frac{\alpha+\beta}{\alpha+\beta+\lambda_1}\right)\tilde{X}(0) + \left(\frac{\lambda_1}{\alpha+\beta+\lambda_1}\right)\tilde{X}(\infty) \\ &= \left(\frac{\lambda_1(r+\beta+\lambda_1)}{(\lambda_2-r)(\alpha+\beta+\lambda_1)}\right)\left(\frac{\omega_F+\omega_X}{\zeta\phi}\right)\tilde{t}_M > 0. \end{aligned} \tag{A.85}$$

It follows from (A.85) (and indeed from Figure 2 in the text) that $\mathcal{L}\{\tilde{X},\alpha+\beta\} > 0$ regardless of the sign of ω_F. Hence the sign of $\mathcal{L}\{\tilde{P}_U,\alpha+\beta\}$ depends on the sign of $(1-\Psi\zeta)$. It follows from (A.69) that:

$$1-\Psi\zeta = 1-\left[\zeta+\gamma(1-\delta)\,(1-\zeta)+(1-\gamma)\zeta\,(\phi-1)\left(\frac{\eta-1}{\eta}\right)\right]. \tag{A.86}$$

The cases discussed in footnote 15 of the text are proved as follows. Both if $\gamma=1$ (exogenous labour supply) and if $\eta=1$ (no diversity effect) it follows directly from (A.86) that $1-\Psi\zeta = (1-\zeta)[1-\gamma(1-\delta)] > 0$.

The welfare effect on *newborns* at the time of the shock can be written as follows:

$$(\alpha+\beta)d\Lambda(0,0) = (1/\omega_H)\tilde{X}(0) - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U,\alpha+\beta\}. \tag{A.87}$$

By comparing (A.82) and (A.87) it follows that:

$$d\Lambda(0,0) - d\Lambda(-\infty,0) = \left(\frac{\tilde{X}(0)}{(\alpha+\beta)\omega_H}\right)\left[1-\lim_{v\to-\infty} e^{(r-\alpha)v}\right]\left\{\begin{array}{ll} >0 & \text{for } r>\alpha \\ =-\infty & \text{for } r<\alpha \end{array}\right., \tag{A.88}$$

where we have used the fact that $\tilde{X}(0) > 0$ regardless of the sign of ω_F.

A.9.2 Future generations

Equation (A.57) shows that the welfare effect on generations born in the new steady state can be written as follows:

$$\begin{aligned}(\alpha+\beta)d\Lambda(\infty,\infty) &= \tilde{X}(\infty) - \tilde{P}_U(\infty) \\ &= \left[\tilde{X}(\infty)-\tilde{P}(\infty)\right] - \left[\tilde{P}_U(\infty)-\tilde{P}(\infty)\right] \\ &= \Psi\zeta\tilde{X}(\infty) - \gamma(1-\delta)\tilde{t}_M, \end{aligned} \tag{A.89}$$

where we have used (A.78) and (A.80) in going from the second to the third line.

A.9.3 Proof of Proposition 1 (iii)

We prove part (iii) of proposition 1 by expressing $d\Lambda(-\infty,0)$ and $d\Lambda(\infty,\infty)$ in deviation from the egalitarian utility change π which would be obtained if an egalitarian bond policy were used. In particular we show that:

$$(\alpha+\beta)\left[d\Lambda(-\infty,0)-\pi\right] = \Gamma\left[\tilde{B}-\tilde{B}^*\right], \tag{A.90}$$

$$(\alpha+\beta)\left[d\Lambda(\infty,\infty)-\pi\right] = -\Psi/\phi\left[\tilde{B}-\tilde{B}^*\right], \tag{A.91}$$

where $\tilde{B}^*$ is:

$$\tilde{B}^* = \left(\frac{\omega_F+\omega_X}{\omega_F+\phi\zeta}\right)\omega_F\tilde{t}_M, \tag{A.92}$$

and we keep s_P constant ($\tilde{s}_P = 0$). We first show how (A.90)-(A.91) can be obtained from, respectively, (A.82) and (A.89). Then we complete the proof of Proposition 1(iii) by demonstrating that $\Gamma > 0$ for $r > \alpha$. (For $r < \alpha$, $d\Lambda(-\infty,0) \to \infty$ and, since $d\Lambda(\infty,\infty)$ is finite, it is obvious that $d\Lambda(\infty,\infty) < d\Lambda(-\infty,0)$).

We first note that $\tilde{X}(0)$ and $\tilde{X}(\infty)$ can be written as:

$$\tilde{X}(0) = \tilde{X}^* + \left(\frac{r-\alpha}{\omega_F\lambda_2}\right)\left[\tilde{B}-\tilde{B}^*\right], \tag{A.93}$$

$$\tilde{X}(\infty) = \tilde{X}^* - \left(\frac{1}{\zeta\phi}\right)\left[\tilde{B}-\tilde{B}^*\right], \tag{A.94}$$

where $\tilde{X}^*$ is the effect on full expenditure under the egalitarian policy:

$$\tilde{X}^* = \left(\frac{\omega_F+\omega_X}{\omega_F+\zeta\phi}\right)\tilde{t}_M. \tag{A.95}$$

In deriving (A.93) we have used (A.43) and (A.92) and for (A.94) we have used (A.60), (A.78), and (A.92). It follows that the Laplace transform for $\tilde{X}$ can be written as:

$$\begin{aligned}(\alpha+\beta)\mathcal{L}\{\tilde{X},\alpha+\beta\} &= \left(\frac{\alpha+\beta}{\alpha+\beta+\lambda_1}\right)\tilde{X}(0) + \left(\frac{\lambda_1}{\alpha+\beta+\lambda_1}\right)\tilde{X}(\infty) \\ &= \tilde{X}^* + \left[\left(\frac{\alpha+\beta}{\alpha+\beta+\lambda_1}\right)\left(\frac{r-\alpha}{\omega_F\lambda_2}\right) - \left(\frac{\lambda_1}{\alpha+\beta+\lambda_1}\right)\left(\frac{1}{\zeta\phi}\right)\right]\left[\tilde{B}-\tilde{B}^*\right] \\ &= \tilde{X}^* + \left(\frac{\lambda_1(\alpha+\beta-r)}{r\zeta\phi(\alpha+\beta+\lambda_1)}\right)\left[\tilde{B}-\tilde{B}^*\right].\end{aligned} \tag{A.96}$$

The egalitarian policy is such that all current generations enjoy the same utility change, i.e.:

$$\tilde{X}^* = \tilde{B}^*/\omega_F. \tag{A.97}$$

By using (A.84), (A.96) and (A.97) in (A.51) we get the expression for the common welfare change:

$$\begin{aligned}(\alpha+\beta)d\Lambda(-\infty,0)^* &= \tilde{B}^*/\omega_F - (\alpha+\beta)\mathcal{L}\{\tilde{P}_U,\alpha+\beta\}^* \\ &= \tilde{X}^* - \gamma(1-\delta)\tilde{t}_M - (1-\Psi\zeta)\tilde{X}^* \\ &= \Psi\zeta\tilde{X}^* - \gamma(1-\delta)\tilde{t}_M \equiv (\alpha+\beta)d\Lambda.\end{aligned} \tag{A.98}$$

By using (A.98) we can write $d\Lambda(-\infty,0)$ (for the relevant case with $r>\alpha$) as:

$$\begin{aligned}(\alpha+\beta)d\Lambda(-\infty,0) &= \tilde{B}/\omega_F-(\alpha+\beta)\mathcal{L}\{\tilde{P}_U,\alpha+\beta\} \\ &= \frac{\tilde{B}-\tilde{B}^*}{\omega_F}+\frac{\tilde{B}^*}{\omega_F}-(\alpha+\beta)\mathcal{L}\{\tilde{P}_U,\alpha+\beta\}^* \\ &\quad -(1-\Psi\zeta)\left(\frac{\lambda_1(\alpha+\beta-r)}{r\zeta\phi(\alpha+\beta+\lambda_1)}\right)\left[\tilde{B}-\tilde{B}^*\right] \\ &= (\alpha+\beta)d\Lambda+\Gamma\left[\tilde{B}-\tilde{B}^*\right], \end{aligned} \tag{A.99}$$

where Γ is defined as:

$$\begin{aligned}\Gamma &\equiv \frac{1}{\omega_F}-(1-\Psi\zeta)\left(\frac{\lambda_1(\alpha+\beta-r)}{r\zeta\phi(\alpha+\beta+\lambda_1)}\right) \\ &= \frac{\Gamma_1}{r\zeta\phi(\alpha+\beta+\lambda_1)\omega_F}+\Psi\zeta\left(\frac{\lambda_1(\alpha+\beta-r)}{r\zeta\phi(\alpha+\beta+\lambda_1)}\right), \end{aligned} \tag{A.100}$$

where $\Gamma_1 \equiv r\zeta\phi(\alpha+\beta+\lambda_1)-\lambda_1(\alpha+\beta-r)\omega_F$. By using $|\Delta|=-\lambda_1\lambda_2$ from (A.32) we get:

$$\begin{aligned}\Gamma_1 &= \frac{\omega_F\lambda_1}{(r-\alpha)}\left[\lambda_2(\alpha+\beta+\lambda_1)-(r-\alpha)(\alpha+\beta-r)\right] \\ &= \frac{\omega_F\lambda_1}{(r-\alpha)}(\lambda_1+r)(\lambda_1+r+\beta)\end{aligned}$$

where we have substituted $\lambda_2=\lambda_1+2r-\alpha$ to get from the first to the second line. Since $\Gamma_1>0$ it follows that $\Gamma>0$.

We derive (A.91) as follows. By using (A.94) and (A.98) in (A.89) we get:

$$\begin{aligned}(\alpha+\beta)d\Lambda(\infty,\infty) &= \Psi\zeta\tilde{X}(\infty)-\gamma(1-\delta)\tilde{t}_M \\ &= \Psi\zeta\left(\tilde{X}^*-\left(\frac{1}{\zeta\phi}\right)\left[\tilde{B}-\tilde{B}^*\right]\right)-\gamma(1-\delta)\tilde{t}_M \\ &= (\alpha+\beta)d\Lambda-\left(\frac{\Psi}{\phi}\right)\left[\tilde{B}-\tilde{B}^*\right].\end{aligned}$$

It follows from (A.90)-(A.91) that in the absence of bond policy, $d\Lambda(-\infty,0)<d\Lambda$ and $d\Lambda(\infty,\infty)>d\Lambda$. This completes the proof of Proposition 1 and establishes Result 3.

A.10 Optimal policy when $r=\alpha$

In the knife-edge case when the world rate of interest is exactly equal to the rate of time preference ($r=\alpha$) the model features a zero characteristic root ($\lambda_1=0$ in (A.34) and $\lambda_2=r$ in (A.35)) and the dynamics is degenerate (see also Blanchard (1985, p. 230)). Essentially the model is static as households neither save nor dissave. The steady-state is associated with $F=0$ (as $B=0$ by assumption) and all generations have the same level of (human) wealth and full expenditure, i.e. $X(v,t)=X(t)=(\alpha+\beta)H(t)$. Hence despite the fact that there are overlapping generations of finitely lived agents, all agents are essentially identical. Apart from their age there is no difference between generations. This property of the zero-root model is attractive because it allows us to conduct a standard welfare analysis in which the social planner maximizes the lifetime utility of the reprentative household.

A.10.1 Social optimum

The social planner chooses C, Z, L, and P in order to maximize lifetime utility of the representative generation:

$$\Lambda \equiv \gamma\delta \log C + \gamma(1-\delta) \log Z + (1-\gamma) \log [1-L], \tag{A.101}$$

subject to the following constraints:

$$\Omega_0 L^\eta = C + E_0 P^{-\sigma_T}, \tag{A.102}$$
$$Z = E_0 P^{1-\sigma_T}, \tag{A.103}$$

where we ignore the (now superfluous) time index. Equation (A.102) is the resource constraint in the so-called *constrained social optimum* when the planner cannot influence the size of individual firms because lump-sum taxes/transfers at the firm level are deemed to be absent (see Dixit and Stiglitz (1977) and Broer and Heijdra (2000)). Intuitively, (A.102) says that production (left-hand side) equals domestic consumption plus exports (right-hand side). Equation (A.103) is the trade balance constraint, requiring imports (left-hand side) to equal export earnings (right-hand side). The constraint is static because the social planner, like the households, has no reason to use the current account to smooth consumption.

The Lagrangian of the social plan is:

$$\begin{aligned} \mathcal{H}^{SP} \equiv\ & \gamma\delta \log C + \gamma(1-\delta) \log Z + (1-\gamma) \log [1-L] + \lambda_Y \left[\Omega_0 L^\eta - C - E_0 P^{-\sigma_T}\right] \\ & + \lambda_T \left[E_0 P^{1-\sigma_T} - Z\right], \end{aligned}$$

so that the first-order conditions for an interior solution to the social optimum are the constraints and:

$$\frac{\gamma\delta}{C} = \lambda_Y \tag{A.104a}$$
$$\frac{\gamma(1-\delta)}{Z} = \lambda_T \tag{A.104b}$$
$$\frac{1-\gamma}{1-L} = \lambda_Y \eta \Omega_0 L^{\eta-1} \tag{A.104c}$$
$$\frac{\lambda_T}{\lambda_Y} = \frac{1}{P}\left(\frac{\sigma_T}{\sigma_T - 1}\right) \tag{A.104d}$$

A.10.2 Decentralized economy

In the decentralized economy, the representative household chooses C, Z, and L in order to maximize (A.101) subject to the (static) budget restriction:

$$W(1-L) + PC + (1+t_M)Z = W - T. \tag{A.105}$$

The Lagrangian of the household plan is:

$$\begin{aligned} \mathcal{H}^{H} \equiv\ & \gamma\delta \log C + \gamma(1-\delta) \log Z + (1-\gamma) \log [1-L] \\ & + \lambda_H [W - T - W(1-L) - PC - (1+t_M)Z], \end{aligned}$$

so that the first-order conditions for an internal solution to the household optimum are the constraint and:

$$\frac{\gamma\delta}{C} = \lambda_H P \tag{A.106a}$$

$$\frac{\gamma(1-\delta)}{Z} = \lambda_H(1+t_M) \tag{A.106b}$$

$$\frac{1-\gamma}{1-L} = \lambda_H W. \tag{A.106c}$$

The remaining expressions affecting the decentralized solution are:

$$W/P = \left(\frac{1+s_P}{\eta}\right)\eta\Omega_0 L^{\eta-1} \tag{A.107a}$$

$$Y/P = \Omega_0 L^{\eta} \tag{A.107b}$$

$$s_P Y = T + t_M Z \tag{A.107c}$$

$$Y/P = C + E_0 P^{-\sigma_T} \tag{A.107d}$$

$$Z = E_0 P^{1-\sigma_T}. \tag{A.107e}$$

The only expression warranting some comment is (A.107a) which is the markup pricing rule. It is derived as follows. First, by using $P = \bar{P}N^{1-\eta}$ in the pricing rule (equation (15) in the text with $P_i = \bar{P}$ imposed) we get $W/P = [(1+s_P)/(\eta k)]\, N^{\eta-1}$. We also know that $N = [(\eta-1)/(\eta f)]L$ (see below (A.10)) so that W/P can be rewritten as in (A.107a).

A.10.3 Matching first-order conditions

By using, respectively, (A.106a), (A.106c) and (A.107a) and (A.104a) and (A.104c) we obtain:

$$\frac{(1-\gamma)/(1-L)}{\gamma\delta/C} = \left(\frac{1+s_P}{\eta}\right)\eta\Omega_0 L^{\eta-1}, \tag{A.108}$$

$$\frac{(1-\gamma)/(1-L)}{\gamma\delta/C} = \eta\Omega_0 L^{\eta-1}. \tag{A.109}$$

Similarly, by combining respectively (A.106a) and (A.106b) and (A.104a), (A.104b) and (A.104d) we derive:

$$\frac{(1-\delta)/Z}{\delta/C} = \frac{1+t_M}{P}, \tag{A.110}$$

$$\frac{(1-\delta)/Z}{\delta/C} = \frac{1}{P}\left(\frac{\sigma_T}{\sigma_T-1}\right). \tag{A.111}$$

By matching, respectively, (A.108) and (A.109) and (A.110) and (A.111) we derive that the decentralized market equilibrium replicates the social optimum for the following values of the tariff and product subsidy:

$$1+s_P = \eta, \qquad 1+t_M = \left(\frac{\sigma_T}{\sigma_T-1}\right). \tag{A.112}$$

These values coincide with the optimal values in the egalitarian solution for the general model (with $r \neq \alpha$). The optimal product subsidy is set to exploit the product diversity effect whereas the optimal tariff exploits national market power.

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