FISCAL POLICY MULTIPLIERS: THE ROLE OF MONOPOLISTIC COMPETITION, SCALE ECONOMIES AND INTERTEMPORAL SUBSTITUTION IN LABOUR SUPPLY: MATHEMATICAL APPENDIX

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CONTENTS

A.1. The Three-Stage Solution Method

The optimisation problem faced by the representative consumer can be solved in three stages. In step 1 the path of full consumption $X(\tau)$ is solved. In step 2 full consumption is allocated between its components $C(\tau)$ and $1\text{-}L(\tau)$. Finally, in step 3, $C(\tau)$ is allocated over the different varieties of the differentiated product, $C_i(\tau)$.

Stage 1.

Define the ideal cost-of-living index as $P_U(\tau)$:

$$P_U(\tau)\,U(\tau) \;=\; X(\tau), \tag{A.1a}$$

where $U(\tau)\equiv U[C(\tau),1\text{-}L(\tau)]$. In the first stage the following optimisation problem is solved for $\tau\in[t,\infty)$.

$$\begin{aligned}&\underset{\{U(\tau)\}_t^\infty}{\text{Max}} \int_t^\infty \log U(\tau)\exp\left[\alpha\,(t-\tau)\right]\mathrm{d}\tau\\ &\text{s.t.}\quad \frac{\mathrm{d}A(\tau)}{\mathrm{d}\tau} \;=\; r(\tau)A(\tau) \;+\; \left[1-t_L(\tau)\right]W(\tau) \;-\; T(\tau) \;-\; P_U(\tau)\,U(\tau).\end{aligned} \tag{A.1b}$$

This leads to the following first-order conditions:

$$U(\tau)^{-1} \;=\; \lambda_A(\tau)\,P_U(\tau), \qquad \tau\in[t,\infty), \tag{A.1c}$$

$$\frac{\mathrm{d}\lambda_A(\tau)}{\mathrm{d}\tau} \;=\; [\alpha-r(\tau)]\,\lambda_A(\tau), \qquad \tau\in[t,\infty), \tag{A.1d}$$

where $\lambda_A(\tau)$ is the co-state variable of the flow budget restriction. The integrated (life-time) budget restriction (with a NPG condition imposed) is:

$$\begin{aligned}A(t) \;+\; H(t) \;&=\; \int_t^\infty P_U(\tau)\,U(\tau)\exp\left[-\int_t^\tau r(\mu)\,\mathrm{d}\mu\right]\mathrm{d}\tau\\ &=\; \int_t^\infty \lambda_A(\tau)^{-1}\exp\left[-\int_t^\tau r(\mu)\,\mathrm{d}\mu\right]\mathrm{d}\tau,\end{aligned} \tag{A.1e}$$

where $H(t)$ is defined as:

$$H(t) \;\equiv\; \int_t^\infty\left[\left[1-t_L(\tau)\right]W(\tau) \;-\; T(\tau)\right]\exp\left[-\int_t^\tau r(\mu)\,\mathrm{d}\mu\right]\mathrm{d}\tau. \tag{A.1f}$$

The path of $\lambda_A(\tau)$ is described by (A.1d) which can be solved to yield the following:

$$\lambda_A(\tau) = \exp\left[-\int_t^{\tau}[r(\mu)-\alpha]\,d\mu\right]\lambda_A(t), \qquad \tau \geq t. \tag{A.1g}$$

Using (A.1g) in (A.1e) yields the following:

$$A(t) + H(t) = \left[1/\lambda_A(t)\right]\int_t^{\infty}\left[\exp\left[\int_t^{\tau}[\alpha - r(\mu)]\,d\mu\right]\right]^{-1}\exp\left[-\int_t^{\tau}r(\mu)\,d\mu\right]d\tau$$
$$= \left[\alpha\lambda_A(t)\right]^{-1}. \tag{A.1h}$$

But $P_U(t)U(t) \equiv X(t) = 1/\lambda_A(t)$, so that (A.1h) can be written for period t as follows.

$$X(t) = \alpha\left[A(t) + H(t)\right]. \tag{A.1i}$$

Note that equations (A.1a) and (A.1c-d) can be combined to obtain (T1.2) in Table 1 of the paper.

Stage 2

Full consumption $X(t)$ is now allocated over consumption of the composite differentiated good ($C(t)$) and leisure (1-$L(t)$).

$$\underset{\{C(t),1-L(t)\}}{\text{Max}} \quad U(t) = \left[\varepsilon_C^{\frac{1}{\sigma_{CM}}} C(t)^{\frac{\sigma_{CM}-1}{\sigma_{CM}}} + (1-\varepsilon_C)^{\frac{1}{\sigma_{CM}}}\left[1-L(t)\right]^{\frac{\sigma_{CM}-1}{\sigma_{CM}}}\right]^{\frac{\sigma_{CM}}{\sigma_{CM}-1}}$$
$$\text{s.t.} \quad C(t) + \left[1-t_L(t)\right]W(t)\left[1-L(t)\right] = X(t). \tag{A.2a}$$

This implies the following expression:

$$C(t) = \left(\frac{\varepsilon_C}{1-\varepsilon_C}\right)\left[(1-t_L(t))\,W(t)\right]^{\sigma_{CM}}\left[1-L(t)\right]. \tag{A.2b}$$

Substituting (A.2b) into (1c) yields the expressions for $L(t)$ and $C(t)$ in terms of full consumption $X(t)$.

$$L(t) = 1 - \frac{(1-\varepsilon_C)\left[(1-t_L(t))W(t)\right]^{-\sigma_{CM}}}{\left[\varepsilon_C + (1-\varepsilon_C)\left[(1-t_L(t))W(t)\right]^{1-\sigma_{CM}}\right]}X(t), \tag{A.2c}$$

$$C(t) \;=\; \frac{\varepsilon_C}{\left[\varepsilon_C \;+\; (1-\varepsilon_C)\left[(1-t_L(t))W(t)\right]^{1-\sigma_{CM}}\right]}X(t). \tag{A.2d}$$

The expression for the true price index is obtained by substituting (A.2c-d) into the instantaneous utility function (1f) and noting (A.1a):

$$P_U(t) \;=\; \left[\varepsilon_C \;+\; (1-\varepsilon_C)\left[W(t)(1-t_L(t))\right]^{1-\sigma_{CM}}\right]^{\frac{1}{1-\sigma_{CM}}}. \tag{A.2e}$$

Equations (A.2c-d) are reported in Table 1 of the paper in (T1.7)-(T1.8).

Stage 3

The agent now chooses $C_i(t)$ such that the following static maximisation program is solved.

$$\underset{\{C_i(t)\}}{\text{Max}} \;\; N(t)^{\alpha_C}\left[N(t)^{-1}\sum_{i=1}^{N(t)} C_i(t)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}} \;\; \text{s.t.} \quad \sum_{i=1}^{N(t)} P_i(t)C_i(t) \;=\; P(t)C(t). \tag{A.3a}$$

Straightforward manipulation yields the demand functions for the differentiated commodities by the agent:

$$C_i(t) \;=\; N(t)^{-(\sigma_C+\alpha_C)+\alpha_C\sigma_C}\left(\frac{P_i(t)}{P(t)}\right)^{-\sigma_C} C(t), \qquad i=1,...,N(t), \tag{A.3b}$$

where $P(t)$ is defined as:

$$P(t) \;\equiv\; N(t)^{-\alpha_C}\left[N(t)^{-\sigma_C}\sum_{i=1}^{N(t)} P_i(t)^{1-\sigma_C}\right]^{\frac{1}{1-\sigma_C}}. \tag{A.3c}$$

Equations (A.3b-c) are reported in the paper in (1g) and (1e), respectively.

A.2. The Optimisation Problem for a Representative Firm

The representative firm i aims to maximise (3b) subject to (3c-d) and (3a). The current-value Lagrangian is defined as follows.

$$\mathcal{L}(\tau) = P_i(\tau)(1+s_P)Y_i^D(\tau) - W^N(\tau)L_i(\tau) - P_I(\tau)Q_i(\tau) \\ + \lambda_K(\tau)\left[Q_i(\tau) - \delta_i(\tau)K_i(\tau)\right] + \lambda_Y(\tau)\left[F(L_i(\tau),K_i(\tau)) - f - Y_i^D(\tau)\right], \tag{A.4a}$$

where the price-elastic demand facing firm i is defined as $Y_i^D(\tau) \equiv C_i(\tau)+I_i(\tau)+G_i(\tau)$, and where s_P is an *ad valorem* product subsidy to be used below in section A.3 to show how the first-best optimum can be decentralised in some cases.

The control variables are $P_i(\tau)$, $L_i(\tau)$, and $Q_i(\tau)$, the state variable is $K_i(\tau)$, the co-state variable is $\lambda_K(\tau)$, and $\lambda_Y(\tau)$ is the Lagrange multiplier for the demand restriction. The first-order necessary conditions are:

$$\frac{\partial \mathcal{L}(\tau)}{\partial Q_i(\tau)} = 0: \quad -P_I(\tau) + \lambda_K(\tau) = 0, \tag{A.4b}$$

$$\frac{\partial \mathcal{L}(\tau)}{\partial L_i(\tau)} = 0: \quad -W^N(\tau) + \lambda_Y(\tau)\frac{\partial F(\tau)}{\partial L_i(\tau)} = 0, \tag{A.4c}$$

$$\frac{\partial \mathcal{L}(\tau)}{\partial P_i(\tau)} = 0: \quad (1+s_P)Y_i^D(\tau) + \left[P_i(\tau)(1+s_P) - \lambda_Y(\tau)\right]\left(\frac{\partial Y_i^D(\tau)}{\partial P_i(\tau)}\right) = 0, \tag{A.4d}$$

$$-\frac{\partial \mathcal{L}(\tau)}{\partial K_i(\tau)} = \dot{\lambda}_K(\tau) - R_j(\tau)\lambda_K(\tau): \quad \dot{\lambda}_K(\tau) - \left[R_j(\tau)+\delta_j(\tau)\right]\lambda_K(\tau) = -\lambda_Y(\tau)\frac{\partial F(\tau)}{\partial K_i(\tau)}, \tag{A.4e}$$

$$\dot{K}_i(\tau) = \frac{\partial \mathcal{L}(\tau)}{\partial \lambda_K(\tau)}: \quad \dot{K}_i(\tau) = Q_i(\tau) - \delta_j(\tau)K_i(\tau). \tag{A4.f}$$

Equation (A.4b) implies that $\lambda_K(\tau)=P_I(\tau)$. Equation (A.4d) can be used to solve for $\lambda_Y(\tau)$ in terms of the mark-up, $\mu_i(\tau)$, and the price chosen by the firm: $\lambda_Y(\tau)=P_i(\tau)(1+s_P)/\mu_i(\tau)$. Hence, $\lambda_Y(\tau)$ has the interpretation of marginal cost. Substituting these expressions for $\lambda_Y(\tau)$ and $\lambda_K(\tau)$ into (A.4c) and (A.4e) yields the marginal productivity conditions:

$$\frac{\partial Y_i(\tau)}{\partial L_i(\tau)} = \left(\frac{\varepsilon_i(\tau)}{\varepsilon_i(\tau)-1}\right)\left(\frac{W^N(\tau)}{(1+s_P)P_i(\tau)}\right), \tag{A.4g}$$

$$\frac{\partial Y_i(\tau)}{\partial K_i(\tau)} = \left(\frac{\varepsilon_i(\tau)}{\varepsilon_i(\tau)-1}\right)\left(\frac{P_I(\tau)}{(1+s_P)P_i(\tau)}\right)\left(R(\tau)+\delta-\frac{\dot{P}_I(\tau)}{P_I(\tau)}\right). \tag{A.4h}$$

In the absence of a product subsidy these equations coincide with (3e-f) in the text.

Profit of firm i is defined as total revenue minus payments to the production factors labour and capital:

$$\Pi_i(\tau) \equiv P_i(\tau)(1+s_P)Y_i(\tau) - W^N(\tau)L_i(\tau) - P_I(\tau)\left(R(\tau)+\delta-\frac{\dot{P}_I(\tau)}{P_I(\tau)}\right)K_i(\tau). \tag{A.4i}$$

Under free exit and entry of firms, profits of all active firms go to zero, $\Pi_i(\tau)=0$. The gross production function is homogeneous of degree λ:

$$\frac{\partial F(\tau)}{\partial L_i(\tau)}L_i(\tau) + \frac{\partial F(\tau)}{\partial K_i(\tau)}K_i(\tau) = \lambda F(L_i(\tau),K_i(\tau)) = \lambda\left[Y_i(\tau)+f\right]. \tag{A.4j}$$

By substituting the marginal productivity conditions (A.4g-h) into (A.4i) and using (A.4j), we can obtain the following expression for profit of an active firm:

$$\Pi_i(\tau) = \left(\frac{P_i(\tau)(1+s_P)}{\mu_i(\tau)}\right)\left[\mu_i(\tau)Y_i(\tau) - \lambda\left[Y_i(\tau)+f\right]\right]. \tag{A.4k}$$

Since the term in round brackets on the right-hand side is positive, the zero profit condition is:

$$\mu_i(\tau)Y_i(\tau) = \lambda\left[Y_i(\tau)+f\right] \quad \Leftrightarrow \quad \mu_i(\tau) = \lambda\eta_i(\tau), \tag{A.4l}$$

where $\eta_i(\tau)\equiv(f+Y_i(\tau))/Y_i(\tau)$ measures (local) internal scale economies due to the existence of fixed costs (Rotemberg and Woodford 1995, pp. 251-3).

A.3. Proof of Proposition 1

The current value Hamiltonian for the first-best optimum is:

$$\wp(\tau) = \varepsilon_C \log C(\tau) + (1-\varepsilon_C)\log\left[1 - L(\tau)\right]$$

$$+ \lambda_K(\tau)\left[Y(\tau) - C(\tau) - G(\tau) - \delta K(\tau)\right] - \lambda_Y(\tau)\left[Y(\tau) + N(\tau)^{\alpha_C} f - N(\tau)^{\alpha_C - \lambda} F(L(\tau), K(\tau))\right]. \quad \text{(A.5a)}$$

Government consumption cannot be negative, *i.e.* $G(\tau) \geq 0$. Then the first-order conditions are:

$$\left(\frac{\partial\wp(\tau)}{\partial N(\tau)} = 0:\right) \quad -\hat{\lambda}_Y(\tau)\hat{N}(\tau)^{-1}\left[\alpha_C f \hat{N}(\tau)^{\alpha_C} - (\alpha_C - \lambda)\hat{N}(\tau)^{\alpha_C - \lambda} F\left[\hat{L}(\tau), \hat{K}(\tau)\right]\right] = 0, \quad \text{(A.5b)}$$

$$\left(\frac{\partial\wp(\tau)}{\partial G(\tau)} \leq 0:\right) \quad -\hat{\lambda}_K(\tau) \leq 0, \quad \hat{G}(\tau) \geq 0, \quad -\hat{\lambda}_K(\tau)\hat{G}(\tau) = 0, \quad \text{(A.5c)}$$

$$\left(\frac{\partial\wp(\tau)}{\partial Y(\tau)} = 0:\right) \quad \hat{\lambda}_K(\tau) - \hat{\lambda}_Y(\tau) = 0, \quad \text{(A.5d)}$$

$$\left(\frac{\partial\wp(\tau)}{\partial C(\tau)} = 0:\right) \quad \frac{\varepsilon_C}{\hat{C}(\tau)} - \hat{\lambda}_K(\tau) = 0, \quad \text{(A.5e)}$$

$$\left(\frac{\partial\wp(\tau)}{\partial L(\tau)} = 0:\right) \quad -\frac{1-\varepsilon_C}{1-\hat{L}(\tau)} + \hat{\lambda}_Y(\tau)\hat{N}(\tau)^{\alpha_C - \lambda} F_L\left[\hat{L}(\tau), \hat{K}(\tau)\right] = 0, \quad \text{(A.5f)}$$

$$\left(\dot{\lambda}_K(\tau) - \alpha\lambda_K(\tau) = -\frac{\partial\wp(\tau)}{\partial K(\tau)}\right) \quad \dot{\hat{\lambda}}_K(\tau) = (\alpha + \delta)\hat{\lambda}_K(\tau) - \hat{\lambda}_Y(\tau)\hat{N}(\tau)^{\alpha_C - \lambda} F_K\left[\hat{L}(\tau), \hat{K}(\tau)\right], \quad \text{(A.5g)}$$

$$\left(\frac{\partial\wp(\tau)}{\partial\lambda_Y(\tau)} = 0:\right) \quad \hat{Y}(\tau) = \hat{N}(\tau)^{\alpha_C - \lambda} F\left[\hat{L}(\tau), \hat{K}(\tau)\right] - f\hat{N}(\tau)^{\alpha_C}, \quad \text{(A.5h)}$$

$$\left(\dot{K}(\tau) = \frac{\partial\wp(\tau)}{\partial\lambda_K(\tau)}:\right) \quad \dot{\hat{K}}(\tau) = \hat{Y}(\tau) - \hat{C}(\tau) - \hat{G}(\tau) - \delta\hat{K}(\tau). \quad \text{(A.5i)}$$

By using (A.5d-e), condition (A.5g) can be re-expressed in terms of the following time profile for consumption:

$$\frac{\dot{\hat{C}}(\tau)}{\hat{C}(\tau)} = \hat{N}(\tau)^{\alpha_C-\lambda}F_K\left[\hat{L}(\tau),\hat{K}(\tau)\right] - (\alpha+\delta) \equiv \hat{r}(\tau)-\alpha, \tag{A.5j}$$

where $\hat{r}(\tau)$ is the socially optimal real interest rate. Furthermore, by using (A.5d-e) condition (A.5f) can be re-written as:

$$\frac{(1-\varepsilon_C)\hat{C}(\tau)}{\varepsilon_C\left[1-\hat{L}(\tau)\right]} = \hat{N}(\tau)^{\alpha_C-\lambda}F_L\left[\hat{L}(\tau),\hat{K}(\tau)\right] \equiv \hat{W}(\tau), \tag{A.5k}$$

where $\hat{W}(\tau)$ is the socially optimal real wage rate. Equation (A.5k) shows that the marginal rate of substitution between consumption and leisure should be equated to this optimal wage. Finally, solving (A.5b) yields:

$$\hat{N}(\tau) = \left(\frac{\alpha_C-\lambda}{\alpha_C f}\right)^{\frac{1}{\lambda}}F\left[\hat{L}(\tau),\hat{K}(\tau)\right]^{\frac{1}{\lambda}}. \tag{A.5l}$$

The socially optimal plan is characterized by equation (A.5c) (noting that $\lambda_K(\tau)=\lambda_Y(\tau)>0$ by conditions (A.5d-e)) and equations (A.5h-l). Equation (A.5c) says that $\hat{G}(\tau)=0$.

For a given level of government consumption, $G(\tau)=G$, equations (A.5h-l) implicitly determine socially optimal paths for five macroeconomic variables, $\hat{Y}(\tau)$, $\hat{C}(\tau)$, $\hat{L}(\tau)$, $\hat{K}(\tau)$ and $\hat{N}(\tau)$ for $\tau\in[t,\infty)$, given that $K(t)=K_0$ is pre-determined. The efficiency properties of the free-entry market equilibrium can be studied by comparing the optimality conditions characterizing the socially optimal plan to the relevant conditions that emerge in the free-entry market equilibrium. The relevant expressions for the free-entry equilibrium are:

$$\dot{K}(\tau) = Y(\tau) - C(\tau) - G(\tau) - \delta K(\tau), \tag{A.5m}$$

$$\frac{\dot{C}(\tau)}{C(\tau)} = r(\tau) - \alpha, \tag{A.5n}$$

$$Y(\tau) + fN(\tau)^{\alpha_C} = N(\tau)^{\alpha_C-\lambda}F\left[L(\tau),K(\tau)\right], \tag{A.5o}$$

$$\frac{(1-\varepsilon_C)C(\tau)}{\varepsilon_C\left[1-L(\tau)\right]} = \left[1-t_L(\tau)\right]W(\tau), \tag{A.5p}$$

$$\left(\lambda \varepsilon_L \left[\frac{Y(\tau) + f N(\tau)^{\alpha_C}}{L(\tau)}\right] = \right) \quad N(\tau)^{\alpha_C - \lambda} F_L\big[L(\tau), K(\tau)\big] = \frac{\mu W(\tau)}{1 + s_P}, \tag{A.5q}$$

$$\left(\lambda (1 - \varepsilon_L) \left[\frac{Y(\tau) + f N(\tau)^{\alpha_C}}{K(\tau)}\right] = \right) \quad N(\tau)^{\alpha_C - \lambda} F_K\big[L(\tau), K(\tau)\big] = \frac{\mu\big[r(\tau) + \delta\big]}{1 + s_P}, \tag{A.5r}$$

$$\mu Y(\tau) - \lambda\big[Y(\tau) + f N(\tau)^{\alpha_C}\big] = 0. \tag{A.5s}$$

Equation (A.5m) is the capital accumulation equation which is obtained by combining (T1.1) and (T1.6) in Table 1 and imposing the simplifications of the benchmark model. Similarly, equation (A.5n) is the simplified version of the Euler equation (T1.2). Equation (A.5o) is the production function (T.9). Equation (A.5p) is obtained by combining (T1.7) and (T1.8). Equations (A.5q-r) are obtained by rewriting (A.4g-h) in terms of aggregate variables and using the free entry condition (A.4l). Finally, equation (A.5s) is obtained by writing (A.4l) in terms of aggregate variables. By solving (A.5o) and (A.5s) for the equilibrium number of firms we obtain:

$$N(\tau) = \left(\frac{\mu - \lambda}{\mu f}\right)^{\frac{1}{\lambda}} F\big[L(\tau), K(\tau)\big]^{\frac{1}{\lambda}}. \tag{A.5t}$$

Comparing the two sets of expressions for the social optimum and the market solution reveals that, provided $t_L(\tau)$=0 for all τ, the social optimum can be decentralized by setting the product subsidy equal to s_P=μ-1. In that case, as $\mu/(1+s_P)=1$, (A.5j) matches with (A.5n) and (A.5r) because $\hat{r}(\tau)=r(\tau)$, (A.5k) matches with (A.5p-q) as $\hat{W}(\tau)=W(\tau)$, (A.5i) matches with (A.5m), (A.5h) matches with (A.5o), and (A.5l) matches with (A.5t). The market characterized by Chamberlinian monopolistic competition yields the correct number of firms due to the benchmark assumption that $\alpha_C=\mu$. If this is not the case, either lump-sum payments to firms are needed to decentralise the first-best optimum and get the correct number of firms, or a second-best ‘constrained social optimum' concept must be used. See Broer and Heijdra (1996) for further details in the case of exogenous labour supply.

The assertions in Proposition 1(iii) are proved by deriving the steady-state effects of a change in the product subsidy on aggregate output, employment, the capital stock, the wage rate, and the number of firms. By doing so we can deduce comparisons for the magnitude attained by the variables in the social optimum (for which s_P=μ-1) and the market solution (for which s_P=0). This is done in section A.4.5.

A.4. Derivations of Results

In Table A.1 the log-linearized version of the complete model is given. The following notational conventions are adopted for all flow variables and the capital stock.

$$\dot{\tilde{x}}(t) \equiv \frac{\mathrm{d}\dot{x}(t)}{x(0)} = \frac{\dot{x}(t)}{x(0)}, \qquad \tilde{x}(t) \equiv \frac{\mathrm{d}x(t)}{x(0)}, \tag{A.6a}$$

where $\mathrm{d}\dot{x}(t)=\dot{x}(t)$ since we loglinearize around an initial steady-state. Hence, a variable with a tilde ('~') denotes the proportional rate of change in that variable (relative to the initial steady-state), and a variable with a tilde and a dot is the time rate of change in terms of the initial level. For the labour tax rate (t_L) and the product subsidy (s_P) the following conventions are used:

$$\tilde{t}_L(t) \equiv \frac{\mathrm{d}t_L(t)}{1-t_L(0)}, \qquad \tilde{s}_P \equiv \frac{\mathrm{d}s_P}{1+s_P(0)}. \tag{A.6b}$$

Only permanent/unanticipated shocks in the product subsidy are considered in this appendix so $\tilde{s}_P$ has no time index. Table A.2 reports the log-linearized version of the benchmark model. All results of section 3 in the text are computed with this model.

A.4.1. Local Stability and Proof of Proposition 2

The dynamical system for capital and full consumption can be derived as follows. First, equations (A2.4)-(A2.7) can be solved for $\tilde{L}(t)$, $\tilde{W}(t)$, $\tilde{r}(t)$, $\tilde{I}(t)$ and $\tilde{Y}(t)$ in terms of the state variables $\tilde{K}(t)$ and $\tilde{C}(t)$, government consumption $\tilde{G}(t)$, the labour tax rate $\tilde{t}_L(t)$, and the product subsidy $\tilde{s}_P$:

$$\mu\varepsilon_L \tilde{L}(t) = \tilde{Y}(t) - \mu(1-\varepsilon_L)\tilde{K}(t), \tag{A.6c}$$

$$\tilde{W}(t) = \left(\frac{\mu\varepsilon_L - 1}{\mu\varepsilon_L}\right)\tilde{Y}(t) + \left(\frac{\mu(1-\varepsilon_L)}{\mu\varepsilon_L}\right)\tilde{K}(t) + \tilde{s}_P, \tag{A.6d}$$

$$\tilde{Y}(t) = \mu\phi(1-\varepsilon_L)\tilde{K}(t) - (\phi-1)\left[\tilde{C}(t)+\tilde{t}_L(t)-\tilde{s}_P\right], \tag{A.6e}$$

$$\left(\frac{\alpha}{\alpha+\delta}\right)\tilde{r}(t) = \tilde{Y}(t) - \tilde{K}(t) + \tilde{s}_P, \tag{A.6f}$$

$$\omega_I \tilde{I}(t) = \tilde{Y}(t) - \omega_C \tilde{C}(t) - \omega_G \tilde{G}(t), \tag{A.6g}$$

where ϕ is a labour supply parameter that is defined as:

$$\phi \equiv \frac{1+\omega_{LL}}{1+\omega_{LL}(1-\mu\varepsilon_L)}, \qquad \phi \geq 1. \tag{A.6h}$$

By substituting (A.6g) into (A2.1) and (A.6f) into (A2.2) and using (A.6e), the general form of the dynamical system is obtained:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{C}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{K}(t) \\ \tilde{C}(t) \end{bmatrix} - \gamma(t), \tag{A.6i}$$

where the Jacobian matrix Δ has typical element δ_{ij}:

$$\Delta \equiv \begin{bmatrix} (\delta/\omega_I)\left[\mu\phi(1-\varepsilon_L)-\omega_I\right] & (\delta/\omega_I)(1-\phi-\omega_C) \\ (\alpha+\delta)\left[\mu\phi(1-\varepsilon_L)-1\right] & (\alpha+\delta)(1-\phi) \end{bmatrix}, \tag{A.6j}$$

and where $\gamma(t)$ is a vector of (potentially time-varying) forcing terms:

$$\gamma(t) \equiv \begin{bmatrix} \gamma_K(t) \\ \gamma_C(t) \end{bmatrix} \equiv \begin{bmatrix} (\delta/\omega_I)\left[\omega_G \tilde{G}(t) + (\phi-1)\left[\tilde{t}_L(t) - \tilde{s}_P\right]\right] \\ (\alpha+\delta)\left[(\phi-1)\tilde{t}_L(t) - \phi\tilde{s}_P\right] \end{bmatrix}. \tag{A.6k}$$

Local stability is investigated by examining the characteristic roots of Δ. Saddle point stability is ensured if the characteristic roots alternate in sign. Denoting the unstable (positive) root by r^* and the stable (negative) root by $-h^*$, we know that r^*-h^*=tr(Δ) and $r^*h^*=-|\Delta|$. After some manipulation, tr(Δ) can be written as:

$$\mathrm{tr}(\Delta) = \left(\frac{\delta}{\omega_I}\right)\left[(\mu-1-s_P)\phi(1-\varepsilon_L) + \omega_A\right] > 0, \tag{A.6l}$$

where have used the following relationships between the parameters and shares which are implied by the initial steady state:

$$\alpha+\delta \equiv \left(\frac{\delta}{\omega_I}\right)(1-\varepsilon_L)(1+s_P), \qquad \omega_A \equiv (1-\varepsilon_L)(1+s_P) - \omega_I. \tag{A.6m}$$

Equation (A.6l) shows that, if the product subsidy is no higher than its first-best optimum value ($s_P \leq \mu-1$), tr(Δ)>0 and at least one positive root is guaranteed (in the text s_P=0).

A necessary and sufficient condition for saddle-point stability is that the determinant of Δ be negative:

$$|\Delta| = -\left(\frac{\delta(\alpha+\delta)}{\omega_I}\right)\left[\omega_G(\phi-1)+\omega_C\phi\left(1-\mu(1-\varepsilon_L)\right)\right]. \tag{A.6n}$$

If ϕ=1 or ω_G=0, $\xi\equiv 1-\mu(1-\varepsilon_L)>0$ is a necessary and sufficient condition for saddle-point stability. If $\omega_G>0$ and $\phi>1$, then $\xi>0$ is sufficient but not necessary. To show this, assume that $\xi=0$ and $s_P=0$. In that case $\text{tr}(\Delta)\equiv(\delta/\omega_I)[\mu\phi\varepsilon_L(1-\varepsilon_L)+\omega_A]>0$ and $|\Delta|\equiv-\omega_G(\phi-1)(\alpha+\delta)(\delta/\omega_I)<0$. The formula for the stable characteristic root is:

$$-h^* \equiv \tfrac{1}{2}\text{tr}(\Delta)\left[1 - \sqrt{1 - 4|\Delta|\left(\text{tr}(\Delta)\right)^{-2}}\right] < 0.$$

If ω_G is small (but positive), the determinant is small and negative, but the trace is strictly positive. As a result, the adjustment speed h^* is positive but very low in that case.

The inequality for the unstable characteristic root, $r^*>\omega_C(\alpha+\delta)$, can be proved as follows.[1] Define $f(s)\equiv|sI-\Delta|$. Obviously, for the stable case, $f(s)$ is a quadratic function with roots $s_1=-h^*<0$ and $s_2=r^*>0$, and $f(0)=|\Delta|<0$. All we need to show is that $f(\bar{s})<0$ for $\bar{s}\equiv\omega_C(\alpha+\delta)$. By simple substitutions we obtain (for s_P=0):

$$f(\bar{s}) = -(\alpha+\delta)^2(\phi+\omega_C-1)\left(\frac{\omega_G+\varepsilon_L\omega_C}{1-\varepsilon_L}\right)<0,$$

where we have used the first expression in (A.6m) to simplify the expression for $f(\bar{s})$.□

A.4.2. General Solution

The general solution of the model can be obtained by using the Laplace transform method developed by Judd (1982, 1985, 1987). By taking the Laplace transform of (A.6i), and using

$$\mathfrak{L}\{\dot{\tilde{C}},s\} = s\mathfrak{L}\{\tilde{C},s\} - \tilde{C}(0) \quad \text{and} \quad \mathfrak{L}\{\dot{\tilde{K}},s\} = s\mathfrak{L}\{\tilde{K},s\}, \tag{A.7a}$$

we obtain the following expression:

$$\left(sI-\Delta\right)\begin{bmatrix}\mathfrak{L}\{\tilde{K},s\}\\ \mathfrak{L}\{\tilde{C},s\}\end{bmatrix} = \begin{bmatrix}-\mathfrak{L}\{\gamma_K,s\}\\ \tilde{C}(0)-\mathfrak{L}\{\gamma_C,s\}\end{bmatrix}. \tag{A.7b}$$

Define $A(s)\equiv sI-\Delta$, so that $|A(s)|\equiv(s-r^*)(s+h^*)$. By pre-multiplying (A.7b) by $\text{adj}(A(r^*))$, we obtain the initial condition for the jump in consumption:

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[1]The form of this proof was suggested by D.P. Broer of Erasmus University.

$$\text{adj}[A(r^*)]\,A(r^*)\begin{bmatrix}\mathfrak{L}\{\tilde{K},r^*\}\\ \mathfrak{L}\{\tilde{C},r^*\}\end{bmatrix} =$$

$$\begin{bmatrix} r^*-\delta_{22} & \delta_{12} \\ \delta_{21} & r^*-\delta_{11}\end{bmatrix}\begin{bmatrix} -\mathfrak{L}\{\gamma_K,r^*\} \\ \tilde{C}(0)-\mathfrak{L}\{\gamma_C,r^*\}\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}, \tag{A.7c}$$

Since the characteristic roots of Δ are distinct, rank(adj(A(r^*)))=1 and there is exactly *one* independent equation determining the jump in full consumption, $\tilde{C}(0)$. Hence, either row of (A.7c) may be used to find $\tilde{C}(0)$:

$$-(r^*-\delta_{22})\mathfrak{L}\{\gamma_K,r^*\} + \delta_{12}\left[\tilde{C}(0)-\mathfrak{L}\{\gamma_C,r^*\}\right] = 0, \tag{A.7d}$$

$$-\delta_{21}\mathfrak{L}\{\gamma_K,r^*\} + (r^*-\delta_{11})\left[\tilde{C}(0)-\mathfrak{L}\{\gamma_C,r^*\}\right] = 0. \tag{A.7e}$$

Using either (A.7d) or (A.7e) to eliminate $\tilde{C}(0)$ from (A.7b), we obtain the general perfect foresight solution of the model in terms of Laplace transforms. Consider the first row of (A.7b) in combination with (A.7d). After some simplification it can be written as follows:

$$(s+h^*)\mathfrak{L}\{\tilde{K},s\} = -\mathfrak{L}\{\gamma_K,s\} - (r^*-\delta_{22})\left[\frac{\mathfrak{L}\{\gamma_K,s\}-\mathfrak{L}\{\gamma_K,r^*\}}{s-r^*}\right] - \delta_{12}\left[\frac{\mathfrak{L}\{\gamma_C,s\}-\mathfrak{L}\{\gamma_C,r^*\}}{s-r^*}\right], \tag{A.7f}$$

The second row of (A.7b) can be combined with (A.7e), after which we obtain:

$$(s+h^*)\mathfrak{L}\{\tilde{C},s\} = \tilde{C}(0)-\mathfrak{L}\{\gamma_C,s\} -\delta_{21}\left[\frac{\mathfrak{L}\{\gamma_K,s\}-\mathfrak{L}\{\gamma_K,r^*\}}{s-r^*}\right] - (r^*-\delta_{11})\left[\frac{\mathfrak{L}\{\gamma_C,s\}-\mathfrak{L}\{\gamma_C,r^*\}}{s-r^*}\right]. \tag{A.7g}$$

The long-run effects of the shocks $\gamma_K(\infty)$ and $\gamma_C(\infty)$ are obtained from (A.7f) and (A.7g) by applying the final-value theorem (Spiegel, 1965, p. 20).

$$\tilde{K}(\infty) \equiv \lim_{s\downarrow 0} s\mathfrak{L}\{\tilde{K},s\} = \frac{-\delta_{22}\gamma_K(\infty) + \delta_{12}\gamma_C(\infty)}{r^*h^*}, \tag{A.7h}$$

$$\tilde{C}(\infty) \equiv \lim_{s\downarrow 0} s\mathfrak{L}\{\tilde{C},s\} = \frac{\delta_{21}\gamma_K(\infty) - \delta_{11}\gamma_C(\infty)}{r^*h^*}. \tag{A.7i}$$

Equations (A.7d-i) can be used to calculate the impact, transition, and long-run results for the capital stock and full consumption once the time paths for $\gamma_K(t)$ and $\gamma_C(t)$ are specified. These paths generally depend on both the type of financing (lump-sum taxes or labour income taxes) and the type of policy experiment (permanent or temporary; anticipated or unanticipated)

A.4.3. Lump-Sum Taxes

Under lump-sum tax financing we have $\tilde{t}_L(t)$=0 and $\tilde{s}_P$=s_P=0, so that the government budget identity (A2.3) in Table A.2 can be ignored. In the case of an unanticipated permanent increase in government spending, the forcing terms are simplified to $\gamma_K(t)=\gamma_K(\infty)=(\delta\omega_G/\omega_I)\tilde{G}$ and $\gamma_C(t)=0$, *i.e.* $\tilde{G}$ is a step function with Laplace transform $\mathfrak{L}\{\tilde{G},s)=\tilde{G}/s$. In that case, it is possible to derive the following expression:

$$\frac{\mathfrak{L}\{\gamma_K,s\} - \mathfrak{L}\{\gamma_K,r^*\}}{s-r^*} = -\left(\frac{\gamma_K(\infty)}{r^*}\right)\frac{1}{s} = -\left(\frac{\delta\omega_G\tilde{G}}{\omega_I r^*}\right)\frac{1}{s}. \tag{A.7j}$$

It is also useful to recognise that:

$$\frac{1}{(s+h^*)s} = \frac{1}{h^*}\left[\frac{1}{s} - \frac{1}{s+h^*}\right] \tag{A.7k}$$

By using (A.7j-k) in (A.7f) and recognising (A.7h), we obtain the transition path for the capital stock by inverting the Laplace transforms (coinciding with the second expression in (10f) in the text):

$$\tilde{K}(t) = \mathrm{A}(h^*,t)\tilde{K}(\infty). \tag{A.7l}$$

Equation (A.7l) contains an *adjustment term*, denoted by $\mathrm{A}(h^*,t)$, about which the following useful properties can be established.

LEMMA A.1: *Let* $\mathrm{A}(\alpha_1,t)$ *be an adjustment function of the form:*

$$\mathrm{A}(\alpha_1,t) \equiv 1-e^{-\alpha_1 t},$$

with $\alpha_1>0$. *Then* $\mathrm{A}(\alpha_1,t)$ *has the following properties: (i) (positive)* $\mathrm{A}(\alpha_1,t)>0$ $t\in(0,\infty)$, *(ii)* $\mathrm{A}(\alpha_1,t)=0$ *for* $t=0$ *and* $\lim_{t\to\infty}\mathrm{A}(\alpha_1,t)=1$, *(iii) (increasing)* $\mathrm{dA}(\alpha_1,t)/\mathrm{d}t\geq 0$, *(iv) (step function as limit)* $\lim_{\alpha_1\to\infty}\mathrm{A}(\alpha_1,t)=\mathrm{u}(t)$, *where* $\mathrm{u}(t)$ *is a unit step function.*

PROOF: Properties (i) and (ii) follow by simple substitution. Property (iii) follows from the fact that $dA(\alpha_1,0)/dt=\alpha_1[1-A(\alpha_1,t)]$ plus properties (i)-(ii). Property (iv) follows by comparing the Laplace transforms of $A(\alpha_1,t)$ and $u(t)$ and showing that they converge as $\alpha_1\to\infty$. Since $\mathfrak{L}\{u(t),s\}=1/s$ and $\mathfrak{L}\{A(\alpha_1,t),s\}=1/s-1/(s+\alpha_1)$ this result follows. □

By using (A.7j-k) in (A.7g) and noting (A.7i), we find the transition path for full consumption by inverting the resulting Laplace transforms (coinciding with the first expression in (10f) in the text):

$$\tilde{C}(t) = \tilde{C}(0)\big(1-A(h^*,t)\big) + \tilde{C}(\infty)A(h^*,t), \qquad \text{(A.7m)}$$

where the jump in consumption that occurs at impact can be calculated by using either (A.7d) or (A.7e) (see equation (10a) in the text). By using equations (A.6c-g) the results for $\tilde{L}(0)$, $\tilde{W}(0)$, $\tilde{r}(0)$, $\tilde{I}(0)$ and $\tilde{Y}(0)$ are obtained by using $\tilde{C}(0)$, and the long-run results for these variables are obtained by using $\tilde{K}(\infty)$ and $\tilde{C}(\infty)$. All results have been reported in section 3.2 in the text.

A.4.4. Labour Income Taxes

If the permanent unanticipated increase in public consumption is financed by means of the labour income tax we have $\tilde{T}(t)$=0, so that the government budget identity (A2.3) in Table A.2 reduces to:

$$\omega_G \tilde{G} = (1-t_L)\varepsilon_L \left[\tilde{t}_L(t) + \left(\frac{t_L}{1-t_L}\right)\tilde{Y}(t)\right], \qquad \text{(A.8a)}$$

where we have also used the fact that the wage bill is proportional to aggregate output (see the first expression in (A2.4)) and of course that $s_P=\tilde{s}_P=0$. By solving (A.8a) for $\tilde{t}_L(t)$ and substituting the result into equation (A.6e), the following 'quasi-reduced form' expression for $\tilde{Y}(t)$ is obtained:

$$\tilde{Y}(t) = \mu\phi\Delta_L(1-\varepsilon_L)\tilde{K}(t) - (\phi-1)\Delta_L\left[\tilde{C}(t) + \frac{\omega_G\tilde{G}}{(1-t_L)\varepsilon_L}\right], \qquad \text{(A.8b)}$$

where Δ_L is a Laffer term which is defined as:

$$\Delta_L \equiv \left[1 - (\phi-1)\left(\frac{t_L}{1-t_L}\right)\right]^{-1}. \qquad \text{(A.8c)}$$

We assume that the economy operates on the upward sloping segment of the Laffer curve, which implies that $\Delta_L>1$. By using (A.6f-g) and (A.8b) in (A2.1)-(A2.2), the system can once again be written as in (A.6i), with Δ defined as:

$$\Delta \equiv \begin{bmatrix} (\delta/\omega_I)\left[\mu\phi\Delta_L(1-\varepsilon_L)-\omega_I\right] & -(\delta/\omega_I)\left[(\phi-1)\Delta_L+\omega_C\right] \\ (\alpha+\delta)\left[\mu\phi\Delta_L(1-\varepsilon_L)-1\right] & -(\alpha+\delta)(\phi-1)\Delta_L \end{bmatrix}, \qquad \text{(A.8d)}$$

and the (time-invariant) shock term $\gamma(t)=\gamma$ as:

$$\gamma \equiv \begin{bmatrix}\gamma_K \\ \gamma_C\end{bmatrix} \equiv \begin{bmatrix} \left(\frac{\delta}{\omega_I}\right)\left[1 + \frac{(\phi-1)\Delta_L}{(1-t_L)\varepsilon_L}\right] \\ \frac{(\alpha+\delta)(\phi-1)\Delta_L}{(1-t_L)\varepsilon_L} \end{bmatrix}\omega_G\tilde{G}. \qquad \text{(A.8e)}$$

The determinant of Δ must be negative in order for saddle point stability to hold:

$$|\Delta| = -\left(\frac{(\alpha+\delta)^2}{1-\varepsilon_L}\right)\Delta_L\left[(1-\omega_I)(\phi-1)+\omega_C\left[1-\mu\phi(1-\varepsilon_L)-(\phi-1)\left(\frac{t_L}{1-t_L}\right)\right]\right] < 0. \qquad \text{(A.8f)}$$

Saddle-point stability thus ensures that the denominator of the long-run multiplier expression (11k) is positive.

The long-run results for consumption and the capital stock are obtained by using (A.8d-e) and (A.6i) in steady-state form. Once $\tilde{K}(\infty)$ and $\tilde{C}(\infty)$ are known, equations (A.6c-d) can be used to recover the expressions for $\tilde{L}(\infty)$ and $\tilde{W}(\infty)$. Obviously, we still have that $\tilde{I}(\infty)=\tilde{K}(\infty)=\tilde{Y}(\infty)$ and $\tilde{r}(\infty)=0$. The results are reported in Table A.3.

The impact result for consumption is obtained by using the shock vector (A.8e) in equation (A.7d). After some manipulation we obtain:

$$\frac{\tilde{C}(0)}{\omega_G \tilde{G}} = -\frac{\left[\varepsilon_L(1-t_L)\left[r^* + (\alpha+\delta)(\phi-1)\Delta_L\right] + (\phi-1)\Delta_L\left[r^* - (\alpha+\delta)\omega_C\right]\right]}{\varepsilon_L(1-t_L)\left[\omega_C + (\phi-1)\Delta_L\right]r^*} < 0, \tag{A.8g}$$

where we have used the fact that $r^*>\omega_C(\alpha+\beta)$ also in the presence of labour taxation (see below) in order to sign the expression. By using (A.8g) we can furthermore derive that:

$$\tilde{C}(0) + \frac{\omega_G \tilde{G}}{(1-t_L)\varepsilon_L} = \frac{\left[\omega_C - \varepsilon_L(1-t_L)\right]\left[r^* + (\alpha+\delta)(\phi-1)\right]\omega_G \tilde{G}}{\varepsilon_L(1-t_L)\left[\omega_C + (\phi-1)\Delta_L\right]r^*}. \tag{A.8h}$$

By using (A.8h) in (A.8b) and noting that $\tilde{K}(0)=0$, the expression for $\tilde{Y}(0)$ is obtained. The results for $\tilde{I}(0)$ and $\tilde{r}(0)$ follow from (A.6f-g) and the results for $\tilde{L}(0)$ and $\tilde{W}(0)$ from (A.6c-d). All results are reported in Table A.3. Since the shock is introduced instantaneously the transition paths for $\tilde{K}(t)$ and $\tilde{C}(t)$ are still of the *form* given in (A.7l) and (A.7m), respectively.

It remains to prove that $r^*>\omega_C(\alpha+\beta)$ even with labour taxation. We again define $f(s)\equiv|sI-\Delta|$, where Δ is now given in (A.8d). We need to show is that $f(\bar{s})<0$ for $\bar{s}\equiv\omega_C(\alpha+\delta)$. By simple substitutions we obtain:

$$f(\bar{s}) = -(\alpha+\delta)^2\left[\omega_C + (\phi-1)\Delta_L\right]\left(\frac{\omega_G + \varepsilon_L\omega_C}{1-\varepsilon_L}\right) < 0,$$

where we have used the first result in (A.6m) to simplify the expression for $f(\bar{s})$.□

A.4.5. Long-Run Effects of the Product Subsidy

In order to prove the assertions in Proposition 1(iii) we compute the long-run effects of a permanent increase in the product subsidy ($\tilde{s}_P>0$). By using this shock in (A.6k) (with $\tilde{G}(t)=\tilde{t}_L(t)=0$) and (A.7h-i) we obtain the long-run effects of the capital stock and consumption:

$$\tilde{K}(\infty) = \tilde{I}(\infty) = \frac{(\alpha+\delta)(\delta/\omega_I)\left[\phi - 1 + \phi\,\omega_C\right]\tilde{s}_P}{r^* h^*} > 0, \tag{A.8i}$$

$$\tilde{C}(\infty) = \frac{(\alpha+\delta)(\delta/\omega_I)\left[\phi - 1 + \phi\left[(1-\varepsilon_L)\left[\mu - 1 - s_P\right] + \omega_A\right]\right]\tilde{s}_P}{r^* h^*} > 0, \tag{A.8j}$$

where the sign of the consumption effect follows from the fact that we assume that the initial product subsidy is no higher than its first-best value ($0 \le s_P < \mu-1$). By using (A.8i-j) in (A.6g) we obtain the long-run effect on aggregate output and the number of firms:

$$\tilde{Y}(\infty) = \alpha_C \tilde{N}(\infty) = \frac{(\alpha+\delta)(\delta/\omega_I)\left[(1-\omega_G)(\phi-1)+\omega_C\mu\phi(1-\epsilon_L)\right]\tilde{s}_P}{r^*h^*} > 0, \qquad \text{(A.8k)}$$

where we have used (A1.11) plus the fact that $\tilde{\mu}(t)=0$ to conclude that $\tilde{Y}(\infty)=\alpha_C\tilde{N}(\infty)$.
By using (A.8i) and (A.8k) in (A.6c) the long-run effect on employment is obtained:

$$\tilde{L}(\infty) = \frac{(\alpha+\delta)(\delta/\omega_I)(\phi-1)\left[1-\omega_G-\mu(1-\epsilon_L)\right]\tilde{s}_P}{\mu\epsilon_L r^*h^*}. \qquad \text{(A.8l)}$$

If $\omega_G=0$, $\mu(1-\epsilon_L)<1$ is necessary for saddle-point stability and $\tilde{L}(\infty)>0$. With a positive ω_G, however, the employment effect is ambiguous because wealth and substitution effects in labour supply work in opposite directions. In terms of Figure 2 in the text, $\tilde{s}_P>0$ shifts the labour supply curve to the left because consumption (and hence wealth) rises. This is the wealth effect. $\tilde{s}_P>0$ also shifts labour demand up, both because of the direct effect and because the capital stock increases. Since labour supply is steeper than labour demand, the net effect on employment is ambiguous. If ω_G is small, however, the wealth effect is dominated by the substitution effect and employment rises.

By using (A.8i) and (A.8k) in (A.6d), the long-run effect on the wage is obtained:

$$\tilde{W}(\infty) = \frac{(\alpha+\delta)(\delta/\omega_I)\left[(\phi-1)(\mu-1+\omega_G)+\mu\phi\epsilon_L\omega_C\right]\tilde{s}_P}{\mu\epsilon_L r^*h^*} > 0. \qquad \text{(A.8m)}$$

In view of the discussion above it is clear that both the wealth and substitution effects lead to a rise in the long-run wage. There is obviously no long-run effect on the interest rate: $\tilde{r}(\infty)=0$. This completes the proof of Proposition 1(iii). □

A.5. Proofs of Extensions

All extensions are calculated with the aid of Table A.1. In all cases we retain all benchmark assumptions, except for the one whose influence is studied. Only lump-sum taxes so that the government budget restriction can be ignored as $\tilde{t}_L(t)$=0 for all t.

A.5.1. Ethier Effects

If $\alpha_I \neq \alpha_G = \alpha_C = \mu$, the relative price of new investment goods changes as a result of fiscal policy. This changes the optimal capital-labour ratio in the long run. In the long run, the key equations are:

$$\tilde{I}(\infty) = \tilde{K}(\infty), \tag{A.9a}$$

$$\tilde{K}(\infty) = \left(\frac{\alpha_I}{\mu}\right)\tilde{Y}(\infty), \tag{A.9b}$$

$$\tilde{Y}(\infty) = \omega_C \tilde{C}(\infty) + \omega_I \left[\tilde{I}(\infty) + \left(1 - \frac{\alpha_I}{\mu}\right)\tilde{Y}(\infty)\right] + \omega_G \tilde{G}, \tag{A.9c}$$

$$\tilde{Y}(\infty) = \mu\phi(1-\varepsilon_L)\tilde{K}(\infty) - (\phi-1)\tilde{C}(\infty), \tag{A.9d}$$

where we have used the fact that $\tilde{P}_I(t)$-$\tilde{P}(t)$=(1-α_I/μ)$\tilde{Y}(t)$. By solving (A.9a-d) for the long-run output effect, we obtain the expression in (12a).

If $\alpha_G \neq \alpha_I = \alpha_C = \mu$, the relative price of the public good changes as a result of fiscal policy. In the long run, the key equations are (A.9a), (A.9d) and

$$\tilde{K}(\infty) = \tilde{Y}(\infty), \tag{A.9e}$$

$$\tilde{Y}(\infty) = \omega_C \tilde{C}(\infty) + \omega_I \tilde{I}(\infty) + \omega_G \left[\tilde{G} + \left(1 - \frac{\alpha_G}{\mu}\right)\tilde{Y}(\infty)\right], \tag{A.9f}$$

where we have used the fact that $\tilde{P}_G(t)$-$\tilde{P}(t)$=(1-α_G/μ)$\tilde{Y}(t)$. By solving (A.9a), (A.9d), and (A.9e-f) for the long-run output effect, we obtain the expression in (12b).

If $\alpha_G = \alpha_I = \alpha_C \neq \mu$, the relative prices $P_G(t)/P(t)$ and $P_I(t)/P(t)$ are both constant but the aggregate scale economies are now different from μ. The key equations are (A.9a), (A.9e) and:

$$\tilde{Y}(\infty) = \omega_C \tilde{C}(\infty) + \omega_I \tilde{I}(\infty) + \omega_G \tilde{G}, \tag{A.9g}$$

$$\tilde{Y}(\infty) = \frac{\alpha_C(1-\varepsilon_L)(1+\omega_{LL})\tilde{K}(\infty) - \alpha_C\varepsilon_L\omega_{LL}\tilde{C}(\infty)}{1+\omega_{LL}(1-\alpha_C\varepsilon_L)}, \tag{A.9h}$$

By solving (A.9a), (A.9e) and (A.9g-h) for the long-run output effect, equation (12c) in the text is obtained.

A.5.2. Intratemporal Substitution Effects

If we allow for a general value for the substitution elasticity between composite consumption and labour supply (σ_{CM}), the key equations are (A.9a), (A.9e), (A.9g) and:

$$\tilde{Y}(\infty) = \frac{\mu(1-\varepsilon_L)(1+\theta_{LW}\omega_{LL})\tilde{K}(\infty) - \mu\varepsilon_L\omega_{LL}\tilde{X}(\infty)}{1+\theta_{LW}\omega_{LL}(1-\mu\varepsilon_L)}, \tag{A.10a}$$

$$\tilde{C}(\infty) = (\sigma_{CM}-\theta_{LW})\left[\frac{\mu(1-\varepsilon_L)\tilde{K}(\infty) + \omega_{LL}(1-\mu\varepsilon_L)\tilde{X}(\infty)}{1+\theta_{LW}\omega_{LL}(1-\mu\varepsilon_L)}\right] + \tilde{X}(\infty), \tag{A.10b}$$

where $\theta_{LW} \equiv [\omega_C\sigma_{CM}+\omega_L\omega_{LL}]/[\omega_C+\omega_L\omega_{LL}]$. By using (A.9e) and (A.10a-b) we can express $\tilde{C}(\infty)$ in terms of $\tilde{Y}(\infty)$ only:

$$\tilde{C}(\infty) = \left(\frac{\mu\varepsilon_L\sigma_{CM}\omega_{LL} - [1-\mu(1-\varepsilon_L)](1+\sigma_{CM}\omega_{LL})}{\mu\varepsilon_L\omega_{LL}}\right)\tilde{Y}(\infty). \tag{A.10c}$$

By using (A.9a), (A.9e), and (A.10c) in (A.9g) and simplifying, the expression in (13a) is obtained.

If we allow for a general value for the substitution elasticity between capital and labour in the gross production function (σ_{KL}), the key equations are (A.9a), (A.9g), and:

$$\tilde{Y}(\infty) = \frac{\mu(1-\omega_L)(\sigma_{KL}+\omega_{LL})\tilde{K}(\infty) - \mu\omega_L\omega_{LL}\sigma_{KL}\tilde{C}(\infty)}{\sigma_{KL}+\omega_{LL}[1-\omega_L(\mu\sigma_{KL}+1-\sigma_{KL})]}, \tag{A.10d}$$

$$\tilde{K}(\infty) = \left[\frac{\mu\sigma_{KL}+1-\sigma_{KL}}{\mu}\right]\tilde{Y}(\infty). \tag{A.10e}$$

By using (A.10d-e), we can express $\tilde{C}(\infty)$ in terms of $\tilde{Y}(\infty)$:

By using (A.9a), (A.9g), and (A.10e-f), the expression in (13b) is obtained.

$$\tilde{C}(\infty) = -\left(\frac{\mu\omega_L - (\mu-1)\left[1+\omega_{LL}+(\sigma_{KL}-1)(1-\omega_L)\right]}{\mu\omega_L\omega_{LL}}\right)\tilde{Y}(\infty). \qquad \text{(A.10f)}$$

A.5.3. Mark-up Effects under Free Entry

In the text below equation (15a) it is asserted that the markup has no first-order effect if μ equals $\alpha_C=\alpha_I=\alpha_G$ initially. It asserted in the text that the markup drops out of both the aggregate production function and the labour demand function (even for $\sigma_{KL}\neq 1$). This has been shown in the text for the aggregate productivity index in (15a). For labour demand it is shown as follows. By solving (A1.11) for $\tilde{N}(t)$ and substituting the result into the general labour demand expression (A1.4), we obtain in successive steps:

$$\lambda\eta\tilde{L}(t) = \left[1+(\lambda-1)\sigma_{KL}\right]\left[\eta\left(\tilde{Y}(t) + \left(\frac{\eta}{\eta-1}\right)\tilde{\mu}(t)\right) - \left(\frac{\eta}{\eta-1}\right)\tilde{\mu}(t)\right] - \lambda\eta\sigma_{KL}\tilde{\mu}(t)$$

$$+ \frac{\eta(1-\sigma_{KL})(\lambda-\alpha_C)}{\alpha_C}\left[\tilde{Y}(t) + \left(\frac{\eta}{\eta-1}\right)\tilde{\mu}(t)\right] - \lambda\eta\sigma_{KL}\tilde{W}(t) \quad \Leftrightarrow$$

$$\lambda\eta\tilde{L}(t) = \lambda\eta\left[\sigma_{KL} + \frac{1-\sigma_{KL}}{\alpha_C}\right]\tilde{Y}(t) - \lambda\eta\sigma_{KL}\tilde{W}(t) + \left[\frac{\eta(1-\sigma_{KL})(\lambda\eta-\alpha_C)}{\alpha_C(\eta-1)}\right]\tilde{\mu}(t) \quad \Leftrightarrow$$

$$\tilde{L}(t) = \left[\sigma_{KL} + \frac{1-\sigma_{KL}}{\alpha_C}\right]\tilde{Y}(t) - \sigma_{KL}\tilde{W}(t), \qquad \text{(A.11)}$$

where we have used the fact that $\mu=\lambda\eta$ (due to free entry) and $\mu=\alpha_C$ (by assumption) in the final step. Hence, the markup drops out of the labour demand expression even if $\sigma_{KL}\neq 1$.

The only place where $\tilde{\mu}(t)$ appears in the model is in equation (A1.10). Hence, the change in the markup is determined residually under free entry/exit if $\mu=\alpha_C=\alpha_I=\alpha_G$ initially.□

A.5.4. Mark-up Effects under Restricted Entry

Under restricted entry and with a constant mark-up, the key equations are given by (A.9a), (A.9g), and:

$$\tilde{Y}(\infty) = \eta\tilde{K}(\infty), \qquad \text{(A.12a)}$$

$$\tilde{Y}(\infty) = \eta\left[\tilde{W}(\infty) + \tilde{L}(\infty)\right], \qquad \text{(A.12b)}$$

$$\tilde{L}(\infty) \;=\; \omega_{LL}\big[\tilde{W}(\infty) \;-\; \tilde{C}(\infty)\big], \tag{A.12c}$$

$$\tilde{Y}(\infty) \;=\; \mu\varepsilon_L \tilde{L}(\infty) \;+\; (\lambda\eta - \mu\varepsilon_L)\tilde{K}(\infty), \tag{A.12d}$$

Note that the excess profit rate, π, can be written as:

$$\pi \equiv \frac{P_i Y_i}{TC_i} - 1 = \frac{\mu}{\lambda\eta} - 1, \tag{A.12e}$$

so that, if the economy is initially in a zero-profit equilibrium, $\mu=\lambda\eta$ and (A.12d) is identical to (A2.7). Equations (A.12a-b) are, however, still different from the expressions (A2.4) which hold under free entry/exit.

By combining (A.12a-d) we obtain the following expression for $\tilde{C}(\infty)$:

$$\tilde{C}(\infty) \;=\; -\left(\frac{\mu\varepsilon_L/\eta \;-\; (\lambda-1)(1+\omega_{LL})}{\mu\varepsilon_L\omega_{LL}}\right)\tilde{Y}(\infty). \tag{A.12f}$$

Using (A.9a), (A.12f), and (A.12a) in (A.9g), we obtain the expression (15c) in the text.

With a variable markup, (A.12a-b) are replaced by, respectively:

$$\tilde{Y}(\infty) \;=\; \eta\big[\tilde{K}(\infty) \;+\; \tilde{\mu}(\infty)\big], \tag{A.12g}$$

$$\tilde{Y}(\infty) \;=\; \eta\big[\tilde{L}(\infty) \;+\; \tilde{\mu}(\infty) \;+\; \tilde{W}(\infty)\big], \tag{A.12h}$$

and (A.12f) becomes:

$$\tilde{C}(\infty) \;=\; -\left(\frac{\mu\varepsilon_L/\eta - (\lambda-1)(1+\omega_{LL})}{\mu\varepsilon_L\omega_{LL}}\right)\tilde{Y}(\infty) \;+\; \frac{1}{\omega_{LL}}\left(1 \;-\; \frac{\lambda\eta(1+\omega_{LL})}{\mu\varepsilon_L}\right)\tilde{\mu}(\infty). \tag{A.12i}$$

By using (A.9a), (A.12g) and (A.12i) in (A.9g), we obtain the multiplier expression (15h) in the text.

A.5.5. No Intertemporal Substitution in Labour Supply

In order to study the crucial role played by the intertemporal substitution effect in labour supply, we compute the multiplier in the absence of this effect. Instead of using equation (1f) in the text, we use the following sub-utility function:

$$U\left[C(\tau), L(\tau)\right] = C(\tau) - \left(\frac{\gamma_L}{1+\theta}\right) L(\tau)^{1+\theta}, \tag{A.13a}$$

with $\theta>0$ and $\gamma_L>0$. The units in (A13a) must of course be chosen such that $U(\tau)>0$. This can be ensured by choosing γ_L appropriately. The Hamiltonian associated with the optimisation problem faced by the representative consumer can be written as:

$$\begin{aligned} \mathrm{H}(\tau) \equiv \log\left[C(\tau) - \left(\frac{\gamma_L}{1+\theta}\right) L(\tau)^{1+\theta}\right] \\ + \lambda_A(\tau)\left[r(\tau)A(\tau) + W(\tau)L(\tau) - T(\tau) - C(\tau)\right], \end{aligned} \tag{A.13b}$$

where $\lambda_A(\tau)$ is the co-state variable of the flow budget identity. This leads to the following first-order conditions:

$$\frac{1}{U(\tau)} = \lambda_A(\tau), \tag{A.13c}$$

$$\frac{\gamma_L L(\tau)^{\theta}}{U(\tau)} = \lambda_A(\tau) W(\tau), \tag{A.13d}$$

$$\frac{\mathrm{d}\lambda_A(\tau)}{\mathrm{d}\tau} = \left[\alpha - r(\tau)\right]\lambda_A(\tau). \tag{A.13e}$$

By eliminating $\lambda_A(\tau)$ from (A.13c-e), we obtain the following expressions characterizing household behaviour:

$$\begin{aligned} W(\tau) = \gamma_L L(\tau)^{\theta}, \qquad \dot{U}(\tau) = \left[r(\tau) - \alpha\right] U(\tau), \\ U(\tau) = C(\tau) - \left(\frac{\gamma_L^{-1/\theta}}{1+\theta}\right) W(\tau)^{\frac{1+\theta}{\theta}}. \end{aligned} \tag{A.13f}$$

The first expression in (A.13f) shows that labour supply only depends on the real wage.

The expressions appearing in (A.13f) can be log-linearized as follows:

$$\tilde{W}(t) \;=\; \theta\tilde{L}(t), \quad \dot{\tilde{U}}(t) \;=\; \alpha\tilde{r}(t), \quad \omega_C\tilde{C}(t) \;=\; \omega_U\tilde{U}(t) \;+\; (\varepsilon_L/\theta)\,\tilde{W}(t), \tag{A.13g}$$

with:

$$\dot{\tilde{U}}(t) \;\equiv\; \mathrm{d}\dot{U}(t)/U \;=\; \dot{U}(t)/U, \quad \tilde{U}(t) \;\equiv\; \mathrm{d}U(t)/U, \quad \omega_U \;\equiv\; U/Y. \tag{A.13h}$$

The full model consists of the expressions in (A.13g), (A2.1), (A2.4), (A2.5), and (A2.7). Using the standard solution procedures, the following expressions can be obtained:

$$\tilde{Y}(t) \;=\; (1+\theta)\tilde{L}(t) \;=\; \left(\frac{1+\theta}{\theta}\right)\tilde{W}(t) \;=\; \left(\frac{\alpha_C(1-\varepsilon_L)(1+\theta)}{1+\theta-\alpha_C\varepsilon_L}\right)\tilde{K}(t), \tag{A.13i}$$

where we assume that the denominator is positive. By using (A.13i) in the second expression of (A2.4) and in (A2.5), respectively, the following expressions are obtained:

$$\begin{aligned} \alpha\tilde{r}(t) \;&=\; \left(\frac{(\alpha+\delta)\left[\alpha_C\varepsilon_L-(1+\theta)\left[1-\alpha_C(1-\varepsilon_L)\right]\right]}{1+\theta-\alpha_C\varepsilon_L}\right)\tilde{K}(t), \\ \omega_I\tilde{I}(t) \;&=\; \left(\frac{1+\theta-\varepsilon_L}{1+\theta}\right)\tilde{Y}(t) \;-\; \omega_U\tilde{U}(t) \;-\; \omega_G\tilde{G}. \end{aligned} \tag{A.13j}$$

Finally, by substituting the expressions in (A.13j) into (A2.1) and the second expression in (A.13g), respectively, the dynamical system for $\tilde{K}(t)$ and $\tilde{U}(t)$ is obtained:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{U}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{K}(t) \\ \tilde{U}(t) \end{bmatrix} - \begin{bmatrix} (\delta\omega_G/\omega_I)\,\tilde{G} \\ 0 \end{bmatrix}, \tag{A.13k}$$

where the Jacobian matrix Δ is:

$$\Delta \;\equiv\; \begin{bmatrix} (\delta/\omega_I)\left[\left(\dfrac{1+\theta-\varepsilon_L}{1+\theta-\alpha_C\varepsilon_L}\right)\alpha_C(1-\varepsilon_L)-\omega_I\right] & -(\delta\omega_U/\omega_I) \\ (\alpha+\delta)\left[\dfrac{\alpha_C\varepsilon_L-(1+\theta)\left[1-\alpha_C(1-\varepsilon_L)\right]}{1+\theta-\alpha_C\varepsilon_L}\right] & 0 \end{bmatrix}. \tag{A.13l}$$

Local stability is again investigated by examining the characteristic roots of Δ. Saddle point stability is ensured if the characteristic roots alternate in sign. A necessary and sufficient condition for saddle-point stability is that the determinant of Δ be negative:

$$|\Delta| = \frac{\delta\omega_X(\alpha+\delta)\left[\alpha_C\varepsilon_L - (1+\theta)\left[1-\alpha_C(1-\varepsilon_L)\right]\right]}{\omega_I\left[1+\theta-\alpha_C\varepsilon_L\right]} < 0. \tag{A.13m}$$

There must be diminishing returns to capital for saddle-point stability to hold.

By using (A.13k) in the steady state, it is clear that an increase in government consumption does not affect the long-run capital stock at all. By (A.13i-j) it follows that the output employment, the wage, and the interest rate are unaffected also. The long-run effect on sub-utility is thus unambiguously negative:

$$\frac{\mathrm{d}U(\infty)}{\mathrm{d}G} = \frac{\mathrm{d}C(\infty)}{\mathrm{d}G} = -1. \tag{A.13n}$$

Hence, regardless of the intratemporal substitution elasticity of labour supply (θ), there is one-for-one crowding out of private by public consumption. This demonstrates the crucial importance of the intertemporal substitution effect in labour supply.☐

A.5.6. Indivisible Labour

In the text it is asserted that Hansen's (1985) indivisible labour solution is obtained by setting $\omega_{LL}\to\infty$. This can be shown as follows. In the Hansen model, the felicity function of the representative household is linear in leisure, and the household solves:

$$\underset{\{C(\tau),L(\tau)\}_t^\infty}{\text{Max}} \int_t^\infty \left[\log C(\tau) + \gamma_L\left[1-L(\tau)\right]\right]\exp\left[\alpha(t-\tau)\right]\mathrm{d}\tau$$

$$\text{s.t.} \quad \frac{\mathrm{d}A(\tau)}{\mathrm{d}\tau} = r(\tau)A(\tau) + \left[1-t_L(\tau)\right]W(\tau) - T(\tau) - C(\tau). \tag{A.14a}$$

The first-order conditions for this problem are:

$$C(\tau)^{-1} = \lambda_A(\tau), \qquad \tau\in[t,\infty), \tag{A.14b}$$

$$\gamma_L = \lambda_A(\tau)W(\tau)\left[1-t_L(\tau)\right], \qquad \tau\in[t,\infty), \tag{A.14c}$$

$$\frac{\mathrm{d}\lambda_A(\tau)}{\mathrm{d}\tau} = [\alpha - r(\tau)]\lambda_A(\tau), \qquad \tau\in[t,\infty), \tag{A.14d}$$

where $\lambda_A(\tau)$ is the co-state variable of the flow budget restriction. By using (A.14b) in (A.14c) and (A.14d), the household's optimal plans reduce to:

$$\frac{\dot{C}(\tau)}{C(\tau)} = r(\tau) - \alpha, \tag{A.14e}$$

$$\gamma_L C(\tau) = W(\tau)\left[1 - t_L(\tau)\right]. \tag{A.14f}$$

Equation (A.14e) is identical to (T1.2) for the benchmark model (with $C(\tau)=\varepsilon_C X(\tau)$). Equation (A.14f) can be log-linearized:

$$\tilde{C}(t) = \tilde{W}(t) - \tilde{t}_L(t). \tag{A.14g}$$

Equation (A.14g) coincides with the expression in (T2.6) for $\omega_{LL}\rightarrow\infty$. This proves that setting $\omega_{LL}\rightarrow\infty$ yields the indivisible labour model. □

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Table A.1: Log-Linearized Version of the Complete Model

$$\dot{\tilde{K}}(t) = \delta\left[\tilde{I}(t) - \tilde{K}(t)\right] \qquad \text{(A1.1)}$$

$$\dot{\tilde{X}}(t) = \alpha\tilde{r}(t) \qquad \text{(A1.2)}$$

$$\omega_G\left[\tilde{G}(t) + \tilde{P}_G(t) - \tilde{P}(t)\right] = \omega_T\tilde{T}(t) + (1-t_L)\omega_L\left[\tilde{t}_L(t) + \left(\frac{t_L}{1-t_L}\right)\left[\tilde{L}(t) + \tilde{W}(t)\right]\right]$$

$$(1+s_P)\left[\tilde{s}_P + \left(\frac{s_P}{1+s_P}\right)\tilde{Y}(t)\right] \qquad \text{(A1.3)}$$

$$\lambda\eta\tilde{L}(t) = \left[1+(\lambda-1)\sigma_{KL}\right]\left[\tilde{Y}(t) + \alpha_C(\eta-1)\tilde{N}(t)\right] - \lambda\eta\sigma_{KL}\tilde{\mu}(t) + \eta(1-\sigma_{KL})(\lambda-\alpha_C)\tilde{N}(t) - \lambda\eta\sigma_{KL}\tilde{W}(t) \qquad \text{(A1.4)}$$

$$\tilde{K}(t) - \tilde{L}(t) = \sigma_{KL}\left[\tilde{W}(t) + \tilde{P}(t) - \tilde{P}_I(t) - \left(\frac{\alpha}{\alpha+\delta}\right)\tilde{r}(t) - \left(\frac{1}{\alpha+\delta}\right)\left[\dot{\tilde{P}}(t) - \dot{\tilde{P}}_I(t)\right]\right] \qquad \text{(A1.5)}$$

$$\tilde{Y}(t) = \omega_C\tilde{C}(t) + \omega_I\left[\tilde{I}(t) + \tilde{P}_I(t) - \tilde{P}(t)\right] + \omega_G\left[\tilde{G}(t) + \tilde{P}_G(t) - \tilde{P}(t)\right] \qquad \text{(A1.6)}$$

$$\tilde{C}(t) = \omega_M(\sigma_{CM}-1)\left[\tilde{W}(t) - \tilde{t}_L(t)\right] + \tilde{X}(t) \qquad \text{(A1.7)}$$

$$\tilde{L}(t) = \omega_{LL}\left[\left[\sigma_{CM} + \omega_M(1-\sigma_{CM})\right]\left[\tilde{W}(t) - \tilde{t}_L(t)\right] - \tilde{X}(t)\right] \qquad \text{(A1.8)}$$

$$\tilde{Y}(t) = (\alpha_C - \lambda\eta)\tilde{N}(t) + \mu\omega_L\tilde{L}(t) + (\lambda\eta - \mu\omega_L)\tilde{K}(t) \qquad \text{(A1.9)}$$

$$-\frac{\mu}{(\mu-1)^2}\tilde{\mu}(t) = \omega_C(\sigma_C - \sigma_G)\left[\tilde{C}(t) - \tilde{Y}(t)\right] + \omega_I(\sigma_I - \sigma_G)\left[\tilde{I}(t) + \tilde{P}_I(t) - \tilde{P}(t) - \tilde{Y}(t)\right] \qquad \text{(A1.10)}$$

$$\tilde{\mu}(t) = \left(\frac{\eta-1}{\eta}\right)\left[\alpha_C\tilde{N}(t) - \tilde{Y}(t)\right] \qquad \text{(A1.11)}$$

$$\tilde{P}_G(t) - \tilde{P}(t) = (\alpha_C - \alpha_G)\tilde{N}(t), \qquad \tilde{P}_I(t) - \tilde{P}(t) = (\alpha_C - \alpha_I)\tilde{N}(t) \qquad \text{(A1.12)}$$

Shares and parameters:

ω_T	T/Y.	Share of lump-sum taxes in real output.
ω_L	WL/Y.	Share of before tax wage income in real output.
ω_M		Share of gross spending on leisure in full consumption, $\omega_M \equiv (1-t_L)\omega_L\omega_{LL}/[\omega_C+(1-t_L)\omega_L\omega_{LL}]$ and $0<\omega_M<1$.

η	$(f+\bar{Y})/\bar{Y}$	Scale parameter due to fixed cost. If entry/exit is free then $\mu=\lambda\eta$.
ω_G	GP_G/PY.	Share of government spending on differentiated goods in output.
ω_C	C/Y.	Share of private consumption in real output
ω_I	IP_I/PY.	Share of investment spending on differentiated goods in output, $\omega_C+\omega_I+\omega_G=1$.
ω_{LL}	$(1-L)/L$	Ratio between leisure and labour.
t_L		Proportional tax rate on labour levied on households.

$r^*h^*\equiv[(\alpha+\delta)^2/(1-\varepsilon_L)][\omega_G(\phi-1)+\phi\omega_C(1-\mu(1-\varepsilon_L))]>0$.

Note:

Under restricted entry/exit, $\tilde{N}(t)=0$, and the zero pure-profit condition (A1.11) is irrelevant. Under free entry/exit, simplifications are obtained by noting that $\mu=\lambda\eta$.

Table A.2: Log-Linearized Version of the Benchmark Model

$$\dot{\tilde{K}}(t) \; = \; \delta\left[\tilde{I}(t) - \tilde{K}(t)\right] \tag{A2.1}$$

$$\dot{\tilde{C}}(t) \; = \; \alpha\,\tilde{r}(t) \tag{A2.2}$$

$$\begin{aligned} \omega_G \tilde{G}(t) \; = \; & \omega_T \tilde{T}(t) \; + \; (1-t_L)\varepsilon_L\left[\tilde{t}_L(t) \; + \; \left(\frac{t_L}{1-t_L}\right)\left[\tilde{L}(t) \; + \; \tilde{W}(t)\right]\right] \\ & - \; (1+s_P)\left[\tilde{s}_P \; + \; \left(\frac{s_P}{1+s_P}\right)\tilde{Y}(t)\right] \end{aligned} \tag{A2.3}$$

$$\tilde{L}(t) \; = \; \tilde{Y}(t) - \tilde{W}(t) + \tilde{s}_P, \qquad \tilde{K}(t) \; = \; \tilde{Y}(t) - \left(\frac{\alpha}{\alpha+\delta}\right)\tilde{r}(t) + \tilde{s}_P \tag{A2.4}$$

$$\tilde{Y}(t) \; = \; \omega_C \tilde{C}(t) + \omega_I \tilde{I}(t) + \omega_G \tilde{G} \tag{A2.5}$$

$$\tilde{L}(t) \; = \; \omega_{LL}\left[\tilde{W}(t) - \tilde{t}_L(t) - \tilde{C}(t)\right] \tag{A2.6}$$

$$\tilde{Y}(t) \; = \; \mu\left[\varepsilon_L \tilde{L}(t) \; + \; (1-\varepsilon_L)\tilde{K}(t)\right] \tag{A2.7}$$

Definitions:

ε_L	*WL*/*Y*.	Share of before tax wage income in real output, $0<\varepsilon_L<1$.
ω_A	*rK*/*Y*.	Share of income from financial assets in real output, $\omega_A=\omega_C+\omega_T-(1-t_L)\varepsilon_L$ and $\omega_A=(1-\varepsilon_L)(1+s_P)-\omega_I$, $\omega_A>0$.
ω_G	*G*/*Y*.	Share of government spending on differentiated goods in output, $0\leq\omega_G<1$.
ω_C	*C*/*Y*.	Share of private consumption in real output, $0<\omega_C<1$.
ω_I	*I*/*Y*.	Share of investment spending on differentiated goods in output, $\omega_C+\omega_I+\omega_G=1$, $0<\omega_I<1$.
ω_{LL}	(1-*L*)/*L*	Ratio between leisure and labour.
ω_T	*T*/*Y*.	Share of lump-sum taxes in real output, $\omega_G+s_P=\omega_T+t_L\varepsilon_L$.
t_L		Proportional tax rate on labour levied on households, $t_L\geq0$.
μ	$\sigma_C/(\sigma_C-1)$	Gross markup, $\mu>1$.
s_P		Ad valorem product subsidy, $s_P\geq0$.

Table A.3: The Effects of Fiscal Policy in the Benchmark Model under Labour Income Taxation

	The shock in government spending is normalized to $\omega_G\tilde{G}=1$	
	Impact Effect (t=0)	*Long-Run Effect* ($t\rightarrow\infty$)
$\tilde{K}(t)$	0	$\frac{(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]}{r^*h^*(\omega_I/\delta)\varepsilon_L(1-t_L)}$
$\tilde{C}(t)$	$-\frac{\left[\varepsilon_L(1-t_L)\left[r^*+(\alpha+\delta)(\phi-1)\Delta_L\right]+(\phi-1)\Delta_L\left[r^*-(\alpha+\delta)\omega_C\right]\right]}{\varepsilon_L(1-t_L)\left[\omega_C+(\phi-1)\Delta_L\right]r^*}$	$-\frac{(\alpha+\delta)\Delta_L\left[(\phi-1)\omega_A+\varepsilon_L(1-t_L)\phi\left[1-\mu(1-\varepsilon_L)\right]\right]}{r^*h^*(\omega_I/\delta)\varepsilon_L(1-t_L)}$
$\tilde{Y}(t)$	$\frac{(\phi-1)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]\left[r^*+(\alpha+\delta)(\phi-1)\right]}{\varepsilon_L(1-t_L)\left[\omega_C+(\phi-1)\Delta_L\right]r^*}$	$\frac{(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]}{r^*h^*(\omega_I/\delta)\varepsilon_L(1-t_L)}$
$\tilde{I}(t)$	$\frac{(\alpha+\delta)(\phi-1)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]}{\varepsilon_L(1-t_L)\omega_I r^*}$	$\frac{(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]}{r^*h^*(\omega_I/\delta)\varepsilon_L(1-t_L)}$
$\tilde{L}(t)$	$\frac{(\phi-1)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]\left[r^*+(\alpha+\delta)(\phi-1)\right]}{\mu\varepsilon_L^2(1-t_L)\left[\omega_C+(\phi-1)\Delta_L\right]r^*}$	$\frac{(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]\left[1-\mu(1-\varepsilon_L)\right]}{r^*h^*(\omega_I/\delta)\mu\varepsilon_L^2(1-t_L)}$
$\tilde{W}(t)$	$\frac{(\mu\varepsilon_L-1)(\phi-1)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]\left[r^*+(\alpha+\delta)(\phi-1)\right]}{\mu\varepsilon_L^2(1-t_L)\left[\omega_C+(\phi-1)\Delta_L\right]r^*}$	$\frac{(\mu-1)(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]}{r^*h^*(\omega_I/\delta)\mu\varepsilon_L(1-t_L)}$
$\tilde{r}(t)$	$\frac{(\phi-1)(\alpha+\delta)\Delta_L\left[\varepsilon_L(1-t_L)-\omega_C\right]\left[r^*+(\alpha+\delta)(\phi-1)\right]}{\alpha\varepsilon_L(1-t_L)\left[\omega_C+(\phi-1)\Delta_L\right]r^*}$	0