TAXING ENERGY TO IMPROVE THE ENVIRONMENT: MATHEMATICAL APPENDIX AND FURTHER RESULTS

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In the text of the paper we refer on several occasions to this appendix. The following results are proved in this appendix (plus many other things). In brackets the page number in the text where the particular result is first mentioned. Note that the proofs do not appear in this appendix in the same order they are referred to in the text.

A.1.1. Individual households

The Hamiltonian associated with the optimisation problem faced by the representative consumer of vintage v can be written as:

$$\mathrm{H}(v,\tau) \equiv \log\left[C(v,\tau) - \frac{L(v,\tau)^{1+1/\sigma_L}}{1+1/\sigma_L}\right] \tag{A1}$$

$$+ \lambda(v,\tau)\Big[(r+\beta)A(v,\tau) + W(\tau)\big[1-t_L(\tau)\big]L(v,\tau) + G(\tau) - C(v,\tau)\Big],$$

where $\lambda(v,\tau)$ is the co-state variable of the flow budget identity. This leads to the following first-order conditions:

$$\frac{1}{X(v,\tau)} = \lambda(v,\tau), \tag{A2}$$

$$\frac{L(v,\tau)^{1/\sigma_L}}{X(v,\tau)} = \lambda(v,\tau)\, W(\tau)\big[1-t_L(\tau)\big], \tag{A3}$$

$$\frac{\mathrm{d}\lambda(v,\tau)}{\mathrm{d}\tau} = (\alpha - r)\,\lambda(v,\tau), \tag{A4}$$

where $X(v,\tau)$ is ‘full consumption' defined in (2.2). By eliminating $\lambda(v,\tau)$ from (A2)-(A4), we obtain:

$$L(v,\tau)^{1/\sigma_L} = W(\tau)\big[1-t_L(\tau)\big], \tag{A5}$$

$$X(v,\tau) = C(v,\tau) - \left(\frac{\sigma_L}{1+\sigma_L}\right)\big[W(\tau)\,(1-t_L(\tau))\big]^{1+\sigma_L}, \tag{A6}$$

$$\frac{\dot{X}(v,\tau)}{X(v,\tau)} = r - \alpha, \tag{A7}$$

where we have also used (A5) in order to simplify (A6). Equation (A5) shows that labour supply only depends on the current after-tax wage rate as stated in the text.

The budget identity of the representative household can be rewritten by using (A5) and the definition of $X(v,\tau)$:

$$\dot{A}(v,\tau) - (r+\beta)A(v,\tau) = W(\tau)\big[1-t_L(\tau)\big]L(v,\tau) + G(\tau) - C(v,\tau) = Y_F(\tau) - X(v,\tau), \tag{A8}$$

where $Y_F(v,\tau)$=$Y_F(\tau)$ is defined as:

$$Y_F(v,\tau) \equiv \left[W(\tau)\left[1 - t_L(\tau)\right] L(v,\tau) - \frac{L(v,\tau)^{1+1/\sigma_L}}{1+1/\sigma_L} \right] + G(\tau)$$
$$= \left(\frac{1}{1+\sigma_L} \right) \left[W(\tau)(1 - t_L(\tau)) \right]^{1+\sigma_L} + G(\tau) \equiv Y_F(\tau). \tag{A9}$$

Equation (A9) coincides with equation (T1.12) in Table 1. The term in square brackets on the right-hand side on the first line represents the ‘surplus' from working, i.e. the after-tax wage income minus the instantaneous utility cost of supplying labour. In view of (A5) this expression can be expressed in terms of the after-tax wage only (done on the second line).

By integrating (A8) subject to the household’s NPG condition, the life-time budget restriction is obtained:

$$A(v,t) + H(t) = \int_t^\infty X(v,\tau) e^{(r+\beta)(t-\tau)} d\tau, \tag{A10}$$

where $H(t)$ is human wealth:

$$H(t) \equiv \int_t^\infty Y_F(\tau) e^{(r+\beta)(t-\tau)} d\tau. \tag{A11}$$

Since full income is age-independent, the same holds for human wealth. By differentiating (A11) with respect to time, equation (T1.2) in Table 1 is obtained. The path of $X(v,\tau)$ is described by (A7), which implies:

$$X(v,\tau) = X(v,t) e^{(r-\alpha)(\tau-t)}, \quad \tau \geq t. \tag{A12}$$

By using (A12) in (A10), the expression for $X(v,t)$ is obtained:

$$X(v,t) = (\alpha+\beta)\left[A(v,t) + H(t)\right]. \tag{A13}$$

Hence, full consumption is a constant proportion of total wealth.

A.1.2. Aggregate households

The aggregate variables can be calculated as the weighted integral of the values for the different generations. For example, aggregate financial wealth, $A(t)$, and full consumption, $X(t)$, are:

$$A(t) \equiv \int_{-\infty}^t A(v,t) \beta e^{\beta(v-t)} dv, \quad X(t) \equiv \int_{-\infty}^t X(v,t) \beta e^{\beta(v-t)} dv. \tag{A14}$$

The definition of aggregate full consumption implies the following expression for the time rate of change in aggregate full consumption:

$$\dot{X}(t) = \beta\left[X(t,t) - X(t)\right] + \int_{-\infty}^{t} \beta e^{\beta(v-t)} \dot{X}(v,t)\,\mathrm{d}v. \tag{A15}$$

The individual Euler equation (A7) shows that all generations face the same intertemporal trade-off, i.e. $\dot{X}(v,t)=(r-\alpha)X(v,t)$. Furthermore, newly-born agents have no financial assets ($A(t,t)=0$), and (A13) shows that for them full consumption is proportional to human wealth, i.e. $X(t,t)= (\alpha+\beta)H(t)$. Finally, the aggregate version of (A13) is $X(t)=(\alpha+\beta)[A(t)+H(t)]$. By using all three results in (A15), the aggregate modified Euler equation is obtained:

$$\begin{aligned}\dot{X}(t) &= \beta(\alpha+\beta)H(t) - \beta(\alpha+\beta)\left[A(t)+H(t)\right] + (r-\alpha)\int_{-\infty}^{t} \lambda e^{\beta(v-t)} X(v,t)\,\mathrm{d}v \\ &= (r-\alpha)X(t) - \beta(\alpha+\beta)A(t).\end{aligned} \tag{A16}$$

By using the aggregate version of (A13) (and its time derivative) and (T1.2) in (A16), equation (T1.1) in Table 1 is obtained. Finally, aggregating (A5) and (A6) with (A9) substituted in, gives (T1.8) and (T1.13).

A.2. The optimisation problem for a representative firm

The Hamiltonian for the problem facing the representative firm is:

$$\begin{aligned} G(\tau) \;=\; & F(L(\tau),N(\tau),K(\tau)) \;-\; W(\tau)L(\tau) \;-\; \left[1+t_N(\tau)\right]P_N(\tau)N(\tau) \;-\; I(\tau) \\ & +\; q(\tau)K(\tau)\left[\phi\left(\frac{I(\tau)}{K(\tau)}\right) \;-\; \delta\right]. \end{aligned} \tag{A17}$$

The first-order conditions are:

$$F_L(L(\tau),N(\tau),K(\tau)) \;=\; W(\tau), \tag{A18}$$

$$F_N(L(\tau),N(\tau),K(\tau)) \;=\; \left[1+t_N(\tau)\right]P_N(\tau), \tag{A19}$$

$$-1 \;+\; q(\tau)\phi'\left(\frac{I(\tau)}{K(\tau)}\right) \;=\; 0, \tag{A20}$$

$$\dot{q}(\tau) \;=\; \left[r+\delta-\phi\left(\frac{I(\tau)}{K(\tau)}\right)\right]q(\tau) \;-\; F_K(L(\tau),N(\tau),K(\tau)) \;+\; \frac{I(\tau)}{K(\tau)}. \tag{A21}$$

These expressions coincide with, respectively, equations (T1.6), (T1.7), (T1.9), and (T1.4) in Table 1 in the text. The proof of $V(t)=q(t)K(t)$ proceeds along the lines suggested by Hayashi (1982). We first write:

$$\frac{d}{d\tau}\left[q(\tau)K(\tau)e^{r(t-\tau)}\right] \;=\; \left[\dot{q}(\tau)K(\tau) \;+\; q(\tau)\dot{K}(\tau) \;-\; r\,q(\tau)K(\tau)\right]e^{r(t-\tau)}. \tag{A22}$$

By using (T1.3) and (A21) the term in square brackets on the right-hand side of (A22) can be written as follows:

$$\begin{aligned} [.] \;&=\; I(\tau) \;-\; K(\tau)F_K(.) \\ &=\; I(\tau) \;-\; \left[Y(\tau) \;-\; F_L(.)L(\tau) \;-\; F_N(.)N(\tau)\right] \\ &=\; I(\tau) \;-\; \left[Y(\tau) \;-\; W(\tau)L(\tau) \;-\; \left[1+t_N(\tau)\right]P_N(\tau)\right], \end{aligned} \tag{A23}$$

where we have used the linear homogeneity of the production function in the second step and equations (A18)-(A19) in the third step. By substituting (A23) in (A22) and integrating both sides from t to infinity, subject to a NPG condition, we obtain:

$$q(t)K(t) \;=\; \int_t^{\infty}\left[Y(\tau) \;-\; W(\tau)L(\tau) \;-\; \left[1+t_N(\tau)\right]P_N(\tau)N(\tau) \;-\; I(\tau)\right]e^{r(t-\tau)}d\tau \;=\; V(t). \tag{A24}$$

Hence, $V(t)=q(t)K(t)$. □

A.3. Solving the model

In this section we log-linearize the model around an initial steady state in which both debt and net foreign assets are zero (i.e. B=F=0). Readers unfamiliar with the technique of analyzing log-linearized perfect foresight models should consult Judd (1982, 1985, 1987a-b) and Bovenberg (1993, 1994). In this appendix we work with a more general specification for the production technology than in the paper. Particularly, instead of using the Cobb-Douglas production function (T1.10), we adopt a nested CES function:

$$Y(t) \;=\; F(L(t),K(t),N(t)) \;\equiv\; \left[\varepsilon_L L(t)^{(\sigma_{LZ}-1)/\sigma_{LZ}} \;+\; (1-\varepsilon_L)\, Z(t)^{(\sigma_{LZ}-1)/\sigma_{LZ}}\right]^{\sigma_{LZ}/(\sigma_{LZ}-1)}$$

$$Z(t) \;\equiv\; \left[\left(\frac{\varepsilon_K}{1-\varepsilon_L}\right) K(t)^{(\sigma_{KN}-1)/\sigma_{KN}} \;+\; \left(\frac{\varepsilon_N}{1-\varepsilon_L}\right) N(t)^{(\sigma_{KN}-1)/\sigma_{KN}}\right]^{\sigma_{KN}/(\sigma_{KN}-1)}, \qquad \text{(A25)}$$

where $0<\varepsilon_L,\varepsilon_K,\varepsilon_N<1$ are efficiency parameters (summing to unity) and σ_{LZ} and σ_{KN} are substitution elasticities. The case discussed in the text is obtained by setting $\sigma_{LZ}=\sigma_{KN}=1$.

The log-linearized model (using (A25) rather than (T1.10)) is given in Table A.1 of this appendix. In order to solve the model, it is useful to first condense the static part of the model as much as possible. By using (TA.6)-(TA.8) and (TA.10), output ($\tilde{Y}(t)$), employment ($\tilde{L}(t)$), energy use ($\tilde{N}(t)$), and the wage ($\tilde{W}(t)$) can be written in terms of the capital stock ($\tilde{K}(t)$) and the various tax rates ($\tilde{t}_N(t)$ and $\tilde{t}_L(t)$):

$$\tilde{Y}(t) \;=\; \frac{(\sigma_L+\sigma_{LZ})(1-\omega_L)\left[\omega_K\tilde{K}(t) \;-\; \sigma_{KN}\omega_N\tilde{t}_N(t)\right] \;-\; \sigma_L\,\omega_L\left[\sigma_{KN}\omega_N+\sigma_{LZ}\omega_K\right]\tilde{t}_L(t)}{\sigma_{KN}\omega_L\,\omega_N \;+\; \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}, \qquad \text{(A26)}$$

$$\tilde{L}(t) \;=\; \frac{\sigma_L(1-\omega_L)\left[\omega_K\tilde{K}(t) \;-\; \sigma_{KN}\omega_N\tilde{t}_N(t)\right] \;-\; \sigma_L\left[\sigma_{KN}\omega_L\,\omega_N+\sigma_{LZ}\omega_K\right]\tilde{t}_L(t)}{\sigma_{KN}\omega_L\,\omega_N \;+\; \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}, \qquad \text{(A27)}$$

$$\tilde{N}(t) \;=\; \frac{\left[\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L\right]\omega_K\tilde{K}(t) \;-\; \sigma_{KN}(1-\omega_L)\left[\sigma_L\,\omega_L\,\tilde{t}_L(t) \;+\; \left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\tilde{t}_N(t)\right]}{\sigma_{KN}\omega_L\,\omega_N \;+\; \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}, \qquad \text{(A28)}$$

$$\tilde{W}(t) \;=\; \frac{(1-\omega_L)\left[\omega_K\tilde{K}(t) \;+\; \sigma_L\,\omega_K\tilde{t}_L(t) \;-\; \sigma_{KN}\omega_N\tilde{t}_N(t)\right]}{\sigma_{KN}\omega_L\,\omega_N \;+\; \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}. \qquad \text{(A29)}$$

By imposing the parameter values of the Cobb-Douglas case ($\sigma_{LZ}=\sigma_{KN}=1$ so that $\omega_i=\varepsilon_i$), we obtain the expressions in (2.9)-(2.12) in the text. The impact changes in output, employment, etcetera, follow from the fact that $\tilde{K}(0)=0$.

A.3.1. The investment system

By using the output expression (A26) in (TA.4) and (TA.9) in (TA.3), the investment system in (q,K)-space is obtained:

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{r\omega_I}{\sigma_A \omega_A} \\ \dfrac{r\omega_L \omega_K (1-\omega_L)}{\omega_A\left[\sigma_{KN}\omega_L\omega_N + \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]} & r \end{bmatrix} \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \end{bmatrix} \tag{A30}$$

$$+ \begin{bmatrix} 0 \\ \dfrac{r\omega_K\left[\left[\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L\right]\omega_N\tilde{t}_N(t) \; + \; \sigma_L\omega_L(1-\omega_L)\tilde{t}_L(t)\right]}{\omega_A\left[\sigma_{KN}\omega_L\omega_N + \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]} \end{bmatrix},$$

where the Jacobian matrix on the right-hand side is denoted by Δ_I with typical element δ^I_{ij}. The determinant of Δ_I is unambiguously negative so that saddle-point stability of the investment system is guaranteed:

$$|\Delta_I| \;=\; -\frac{r^2\omega_K\omega_I\omega_L(1-\omega_L)}{\sigma_A\omega_A^2\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]} \;<\; 0. \tag{A31}$$

The characteristic roots of Δ_I thus alternate in sign. Designating the positive (unstable) root by r_I and the negative (stable) root by $-h_I$, the following expressions can be derived:

$$r_I \;=\; \frac{r}{2}\left[1 \;+\; \left[1 \;+\; \frac{4\omega_I\omega_K\omega_L(1-\omega_L)}{\sigma_A\omega_A^2\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]}\right]^{1/2}\right] > \; r, \tag{A32}$$

$$h_I \;=\; \frac{r}{2}\left[-1 \;+\; \left[1 \;+\; \frac{4\omega_I\omega_K\omega_L(1-\omega_L)}{\sigma_A\omega_A^2\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]}\right]^{1/2}\right] > \; 0, \tag{A33}$$

In the Appendix of the paper these inequalities are proved by directly looking at the characteristic polynomial (see also Figure A.1). This completes the proof of Result 1. □

A.3.1.1. Long-run, impact, and transition results with time-invariant tax shocks

In section 3 of the paper we consider the case of an unanticipated permanent increase in the energy tax (keeping t_L constant). In the discussion surrounding Result 7 we need to know the effects of a time-invariant (unanticipated-permanent) shock in the labour income tax rate. Since the model is linearized, we can work out the effects of both types of shock in one go. In the remainder of section A.3 we assume that the tax revenue is rebated to the households in a lump-sum fashion. Hence, the shocks facing the general investment system are:

$$\tilde{t}_N(t) \;=\; \tilde{t}_N, \qquad \tilde{t}_L(t) \;=\; \tilde{t}_L, \qquad t\in[0,\infty). \tag{A34}$$

The *long-run* effects on private capital and Tobin's q of an increase in the capital tax are obtained by substituting (A34) into (A30) and imposing the steady state:

$$\tilde{K}(\infty) \;=\; -\sigma_L\tilde{t}_L \;-\; \left(\frac{\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L}{\omega_L(1-\omega_L)}\right)\omega_N\tilde{t}_N, \qquad \tilde{q}(\infty) \;=\; 0. \tag{A35}$$

Empirical studies suggest that capital and energy are complementary inputs, i.e. that $\sigma_{LZ}>\sigma_{KN}\omega_L$ in terms of our formulation. This ensures that the term in round brackets on the right-hand side of (A35) is positive, just as in the Cobb-Douglas case discussed in the text. By using (A34) and the first expression of (A35) in (A26)-(A29) we obtain the long-run results for output, employment, energy use, and the wage rate:

$$\begin{aligned}
\tilde{Y}(\infty) \;&=\; -\sigma_L\tilde{t}_L \;-\; (\sigma_L+\sigma_{LZ})(\omega_N/\omega_L)\tilde{t}_N, \qquad \tilde{L}(\infty) \;=\; -\sigma_L\left[\tilde{t}_L+(\omega_N/\omega_L)\tilde{t}_N\right],\\
\tilde{N}(\infty) \;&=\; -\sigma_L\tilde{t}_L \;-\; \left[\frac{\omega_N\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]+\sigma_{KN}\omega_L\omega_K}{\omega_L(1-\omega_L)}\right]\tilde{t}_N, \qquad \tilde{W}(\infty) \;=\; -(\omega_N/\omega_L)\tilde{t}_N.
\end{aligned} \tag{A36}$$

The *impact* results are obtained as follows. By taking the Laplace transform of (A30) we obtain the following expression:

$$A_I(s)\begin{bmatrix}\mathfrak{L}\{\tilde{K},s\}\\ \mathfrak{L}\{\tilde{q},s\}\end{bmatrix} = \begin{bmatrix}0\\ \tilde{q}(0) \;+\; \mathfrak{L}\{\gamma_Q,s\}\end{bmatrix}, \tag{A37}$$

where we have used the fact that the private capital stock is predetermined (so that $\tilde{K}(0)=0$), γ_Q is defined as:

$$\gamma_Q \;\equiv\; \frac{r\,\omega_K\left[\sigma_L\omega_L(1-\omega_L)\tilde{t}_L+\left[\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L\right]\omega_N\tilde{t}_N\right]}{\omega_A\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]}, \tag{A38}$$

and $A_I(s)\equiv sI-\Delta_I$, so that $|A_I(s)|\equiv(s-r_I)(s+h_I)$. By pre-multiplying (A37) by adj($A_I(s)$) and evaluating the resulting expression for $s=r_I$, we obtain the initial condition for the jump in the value of Tobin's q:

$$\text{adj}(A_I(r_I))\,A_I(r_I)\begin{bmatrix}\mathcal{L}\{\tilde{K},r_I\}\\ \mathcal{L}\{\tilde{q},r_I\}\end{bmatrix} \equiv \begin{bmatrix} r_I-r & \delta_{12}^I \\ \delta_{21}^I & r_I\end{bmatrix}\begin{bmatrix}0\\ \tilde{q}(0)\;+\;\mathcal{L}\{\gamma_Q,r_I\}\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}, \tag{A39}$$

from which we derive that:

$$\tilde{q}(0) \;=\; -\mathcal{L}\{\gamma_Q,r_I\}. \tag{A40}$$

Equation (A34) implies the following Laplace transforms:

$$\mathcal{L}\{\tilde{t}_N,s\} \;=\; \frac{\tilde{t}_N}{s}, \qquad \mathcal{L}\{\tilde{t}_L,s\} \;=\; \frac{\tilde{t}_L}{s}. \tag{A41}$$

By using (A41) in (A40), we obtain the final expression for $\tilde{q}(0)$:

$$\tilde{q}(0) \;\equiv\; -\frac{\gamma_Q}{r_I} \;=\; -\left(\frac{r}{r+h_I}\right)\left[\frac{\omega_K\left[\sigma_L\omega_L(1-\omega_L)\tilde{t}_L+\left[\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L\right]\omega_N\tilde{t}_N\right]}{\omega_A\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]}\right], \tag{A42}$$

where we have used (A38) and r_I=r+h_I in the final step. Again, complementarity between capital and energy ensures that the energy tax has the same effect on Tobin's q in the general model of this appendix and the Cobb-Douglas model discussed in the text.

The *transition* results for the investment system are obtained by inverting (A37) and using (A35) and (A42):

$$\tilde{K}(t) \;=\; \tilde{K}(\infty)\left[1-e^{-h_I t}\right], \qquad \tilde{q}(t) \;=\; \tilde{q}(0)\,e^{-h_I t}. \tag{A43}$$

This completes the derivation of the results in section 3.1 of the paper (where the labour income tax rate is constant, $\tilde{t}_L$=0, and σ_{LZ}=σ_{KN}=1 so that ω_i=ε_i).

A.3.2. The saving system

The saving system is given in (TA.1)-(TA.2) and can be written in single matrix expression as follows:

$$\begin{bmatrix}\dot{\tilde{H}}(t)\\ \dot{\tilde{A}}(t)\end{bmatrix} = \begin{bmatrix} r+\beta & 0 \\ -(\alpha+\beta) & r-\alpha-\beta\end{bmatrix}\begin{bmatrix}\tilde{H}(t)\\ \tilde{A}(t)\end{bmatrix} - \begin{bmatrix} r\tilde{Y}_F(t) \\ -r\tilde{Y}_F(t)\end{bmatrix}. \tag{A44}$$

The Jacobian matrix on the right-hand side of (A44) is denoted by Δ_S with typical element δ_{ij}^S. The determinant of the Jacobian is easily computed:

$$|\Delta_S| \;=\; -(r+\beta)(\alpha+\beta-r) \;=\; -\frac{r(r-\alpha)(\omega_X-\omega_A)}{\omega_A}\;. \tag{A45}$$

By using the information on shares at the bottom of Table 2, we derive that ω_X-ω_A=ω_G+ω_L(1-t_L) /(1+σ_L)>0. This immediately shows that saddle point stability in the saving system holds and that α<r<α+β. The characteristic roots of Δ_S alternate in sign. Designating the positive (unstable) root by r_S and the negative (stable) root by -h_S, the following expressions can be derived:

$$r_S \;=\; r+\beta \;>\; 0, \qquad h_S \;=\; \alpha+\beta-r \;>\; 0. \tag{A46}$$

In the Appendix to the paper these roots are related directly to the characteristic polynomial.

The saving system thus contains one predetermined variable (financial assets) and one non-predetermined 'jumping' variable (human wealth). In the absence of bond policy, the jump in the value of financial assets, which occurs at time t=0, is due solely to the change in the value of Tobin's q and equals:

$$\tilde{A}(0) \;=\; \omega_A\tilde{q}(0) \;=\; -\frac{\omega_A\gamma_Q}{r_I}. \tag{A47}$$

where we have used (TA.14) and the fact that $\tilde{K}$(0)=$\tilde{F}$(0)=0 and $\tilde{B}$(t)=0 for all t≥0.

The forcing variable of the saving system, $\tilde{Y}_F(t)$, is time-varying and depends on the path for wages, which is determined in the investment system, and the path for lump-sum transfers, which is affected by the policy maker. In the absence of bond policy, lump-sum transfers are equal to the revenue of the energy tax as the log-linearized government budget restriction reduces to $\tilde{T}(t)$=$\tilde{G}(t)$ (see (TA.15)). By using (A26), (A29), the first expression in (A43), (TA.11), and (TA.12), we obtain the path of full income:

$$\tilde{Y}_F(t) \;=\; \tilde{Y}_F(0)e^{-h_I t} \;+\; \left[1-e^{-h_I t}\right]\tilde{Y}_F(\infty), \tag{A48}$$

with:

$$\tilde{Y}_F(0) \;=\; \left[\frac{(1-\omega_L)\omega_K-\left[\sigma_{LZ}\omega_K+\sigma_{KN}\omega_L\omega_N\right]t_L-\sigma_{KN}\omega_N(1-\omega_L)\theta_N}{\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}\right]\sigma_L\omega_L\tilde{t}_L$$
$$+\left[\frac{\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)-\sigma_{KN}\omega_L\right]-\sigma_L\sigma_{KN}\omega_L(1-\omega_L)t_L-\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right](1-\omega_L)\sigma_{KN}\theta_N}{\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}\right]\omega_N\tilde{t}_N, \tag{A49}$$

$$\tilde{Y}_F(\infty) \;=\; -\sigma_L\left[\omega_L t_L+\omega_N\theta_N\right]\tilde{t}_L \;-\; \left[\sigma_L t_L+\left(\frac{\sigma_{KN}\omega_L\omega_K+\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\omega_N}{\omega_L(1-\omega_L)}\right)\theta_N\right]\omega_N\tilde{t}_N. \tag{A50}$$

The expressions in (3.6)-(3.7) in the text are obtained by setting $\tilde{t}_L$=0 and imposing the Cobb-

Douglas parameter restrictions in (A49)-(A50).

A.3.2.1. Long-run, impact, and transition results with time-invariant tax shocks

By imposing the steady state in (A44), the *long-run* effects on human and financial wealth are obtained:

$$\tilde{H}(\infty) = \frac{r\tilde{Y}_F(\infty)}{r+\beta}, \qquad \tilde{A}(\infty) = \frac{r(r-\alpha)\tilde{Y}_F(\infty)}{(r+\beta)(\alpha+\beta-r)}, \tag{A51}$$

where $\tilde{Y}_F(\infty)$ is given in (A50). These expressions are found in (3.8) in the paper.

The *impact results* are obtained as follows. By taking the Laplace transformation of (A44) we obtain:

$$\mathrm{A}_S(s)\begin{bmatrix}\mathcal{L}\{\tilde{H},s\}\\ \mathcal{L}\{\tilde{A},s\}\end{bmatrix} = \begin{bmatrix}\tilde{H}(0) - r\mathcal{L}\{\tilde{Y}_F,s\}\\ \tilde{A}(0) + r\mathcal{L}\{\tilde{Y}_F,s\}\end{bmatrix}, \tag{A52}$$

where $\tilde{A}(0)$ is given in (A47), and $\mathrm{A}_S(s)\equiv s\mathrm{I}-\Delta_S$, so that $|\mathrm{A}_S(s)|\equiv(s-r_S)(s+h_S)$. By pre-multiplying (A52) by $\mathrm{adj}(\mathrm{A}_S(s))$ and evaluating the resulting expression for $s=r_S$ we obtain the initial condition for the jump in human wealth:

$$\mathrm{adj}(\mathrm{A}_S(r_S))\,\mathrm{A}_S(r_S)\begin{bmatrix}\mathcal{L}\{\tilde{H},r_S\}\\ \mathcal{L}\{\tilde{A},r_S\}\end{bmatrix} \equiv \begin{bmatrix}\alpha+2\beta & 0\\ -(\alpha+\beta) & 0\end{bmatrix}\begin{bmatrix}\tilde{H}(0) - r\mathcal{L}\{\tilde{Y}_F,r_S\}\\ \tilde{A}(0) + r\mathcal{L}\{\tilde{Y}_F,r_S\}\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}, \tag{A53}$$

from which it follows trivially that:

$$\tilde{H}(0) = r\mathcal{L}\{\tilde{Y}_F,r_S\}. \tag{A54}$$

In view of (A48), the Laplace transform of full income can be written as a weighted average of the impact and the long-run effect:

$$\tilde{H}(0) = \left(\frac{r}{r+\beta}\right)\left[\left(\frac{r+\beta}{r+\beta+h_I}\right)\tilde{Y}_F(0) + \left(\frac{h_I}{r+\beta+h_I}\right)\tilde{Y}_F(\infty)\right]. \tag{A55}$$

This expression coincides with (3.10) in the paper.

The *transition results* are obtained as follows. First we invert the matrix $\mathrm{A}_S(s)$ in (A52) to obtain the solution in terms of Laplace transformations:

$$\begin{bmatrix}\mathcal{L}\{\tilde{H},s\}\\ \mathcal{L}\{\tilde{A},s\}\end{bmatrix} = \frac{1}{(s-r_S)(s+h_S)}\begin{bmatrix}s-r+\alpha+\beta & 0\\ -(\alpha+\beta) & s-r-\beta\end{bmatrix}\begin{bmatrix}\tilde{H}(0) - r\mathcal{L}\{\tilde{Y}_F,s\}\\ \tilde{A}(0) + r\mathcal{L}\{\tilde{Y}_F,s\}\end{bmatrix}, \tag{A56}$$

Since h_S=α+β-r (see (A46)) the first line in (A56) yields the following solution for human wealth:

$$\tilde{H}(t) = r\mathfrak{L}^{-1}\left[\frac{\mathfrak{L}\{\tilde{Y}_F, r+\beta\} - \mathfrak{L}\{\tilde{Y}_F, s\}}{s-(r+\beta)}\right], \tag{A57}$$

where we have substituted (A54). By using the path of full income (A48) in (A57), the path for human wealth can be written as:

$$\tilde{H}(t) = \tilde{H}(0)e^{-h_I t} + \left[1-e^{-h_I t}\right]\tilde{H}(\infty). \tag{A58}$$

The second line in (A56) can be used to derive the solution for financial assets. By using (A54) and noting that r_S=r+λ (see (A46)) we obtain the solution in terms of the Laplace transform:

$$(s+h_S)\mathfrak{L}\{\tilde{A},s\} = \tilde{A}(0) + r\mathfrak{L}\{\tilde{Y}_F,s\} - r(\alpha+\beta)\left[\frac{\mathfrak{L}\{\tilde{Y}_F, r+\beta\} - \mathfrak{L}\{\tilde{Y}_F, s\}}{s-(r+\beta)}\right]. \tag{A59}$$

By using the path of full income in (A59), the path for financial wealth is obtained after some simplifications:

$$\tilde{A}(t) = \tilde{A}(0)e^{-h_S t} + \mathrm{T}(h_S, h_I, t)\left[\frac{r\left[r-\alpha+h_I\right]\left[\tilde{Y}_F(0)-\tilde{Y}_F(\infty)\right]}{r+\beta+h_I}\right] + \left[1-e^{-h_S t}\right]\tilde{A}(\infty), \tag{A60}$$

where T(h_S,h_I,t) is a transition term. The properties of the transition term, mentioned in footnote 14 of the paper, are summarized in Lemma A.1.

LEMMA A.1: *Let* T(α_1,α_2,t) *be a single transition function of the form:*

$$\mathrm{T}(\alpha_1,\alpha_2,t) \begin{cases} = \dfrac{e^{-\alpha_1 t}-e^{-\alpha_2 t}}{\alpha_2 - \alpha_1} & \text{if } \alpha_1 \neq \alpha_2 \\ = t e^{-\alpha_1 t} & \text{if } \alpha_1 = \alpha_2, \end{cases}$$

with $\alpha_1>0$ *and* $\alpha_2>0$. *Then* T(α_1,α_2,t) *has the following properties: (i) (positive)* T(α_1,α_2,t)>0 $t\in(0,\infty)$, *(ii)* T(α_1,α_2,t)=0 *for* t=0 *and in the limit as* $t\to\infty$, *(iii) (single-peaked)* dT(α_1,α_2,t)/dt>0 *for* $t\in(0,\hat{t})$ *and* dT(α_1,α_2,t)/dt<0 *for* $t\in(\hat{t},\infty)$, dT(α_1,α_2,t)/dt=0 *(for* $t=\hat{t}\equiv\ln(\alpha_1/\alpha_2)/(\alpha_1-\alpha_2)$ ($\hat{t}\equiv 1/\alpha_1$ *if* $\alpha_1=\alpha_2$) *and in the limit as* $t\to\infty$), *and* dT(α_1,α_2,0)/dt=1, *(iv) (point of inflexion)* d^{2}T(α_1,α_2,t)/dt^2=0 *for* $t^*=2\hat{t}$.

PROOF: Property (i) follows by examining the two possible cases. If α_1<(>)α_2, then α_2-α_1>(<)0 and exp[-$\alpha_1 t$]>(<)exp[-$\alpha_2 t$] for all $t\in(0,\infty)$, and T(α_1,α_2,0)>0. Property (ii) follows by direct substitution. Property (iii) follows by examining dT(α_1,α_2,t)/dt:

$$\frac{\mathrm{d}\,\mathrm{T}(\alpha_1,\alpha_2,t)}{\mathrm{d}t} = \left(\frac{\alpha_1 e^{-\alpha_1 t} - \alpha_2 e^{-\alpha_2 t}}{\alpha_1 - \alpha_2}\right).$$

Property (iv) is obtained by examining $\mathrm{dT}^2(\alpha_1,\alpha_2,t)/\mathrm{d}t^2$:

$$\frac{\mathrm{d}^2\mathrm{T}(\alpha_1,\alpha_2,t)}{\mathrm{d}^2 t} = \left(\frac{\alpha_1^2 e^{-\alpha_1 t} - \alpha_2^2 e^{-\alpha_2 t}}{\alpha_2 - \alpha_1}\right).$$

Hence, $\mathrm{d}^2\mathrm{T}(\alpha_1,\alpha_2,0)/\mathrm{d}t^2=-(\alpha_1+\alpha_2)<0$, and $\lim_{t\to\infty}\mathrm{d}^2\mathrm{T}(\alpha_1,\alpha_2,t)/\mathrm{d}t^2=0$. The inflexion point is found by finding the value of $t=t^*$ where $\mathrm{d}^2\mathrm{T}(\alpha_1,\alpha_2,t)/\mathrm{d}t^2=0$. The proofs for the case for which $\alpha_1=\alpha_2$ are similar. □

This completes the derivation of the results in section 3.2 of the paper.

A.3.3. The ecological system

The path for environmental quality is computed as follows. By taking the Laplace transformation of (TA.5) we obtain:

$$(s+h_E)\mathcal{L}\{\tilde{E},s\} = -h_E\sigma_N\mathcal{L}\{\tilde{N},s\}, \tag{A61}$$

where we have used the fact that environmental quality is a predetermined variable (i.e. $\tilde{E}(0)=0$). By using (A28) and the first expression in (A43), the path of energy use under time-invariant tax shocks can be written as:

$$\tilde{N}(t) = \tilde{N}(0)e^{-h_I t} + \tilde{N}(\infty)\left[1-e^{-h_I t}\right], \tag{A62}$$

where $\tilde{N}(0)$ is

$$\tilde{N}(0) = -\frac{\sigma_{KN}(1-\omega_L)\left[\sigma_L\omega_L\tilde{t}_L + \left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\tilde{t}_N\right]}{\sigma_{KN}\omega_L\omega_N + \omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}, \tag{A63}$$

and $\tilde{N}(\infty)$ is given in (A36). By using (A62) in (A61) we obtain:

$$\mathcal{L}\{\tilde{E},s\} = -\sigma_N\left(\frac{h_E}{s+h_E}\right)\left[\frac{\tilde{N}(\infty)}{s} + \frac{\tilde{N}(0)-\tilde{N}(\infty)}{s+h_I}\right], \tag{A64}$$

which can be inverted to yield:

$$\tilde{E}(t) = -\sigma_N h_E\tilde{N}(0)\mathrm{T}(h_I,h_E,t) + \mathrm{A}(h_I,h_E,t)\tilde{E}(\infty), \quad \tilde{E}(\infty) \equiv -\sigma_N\tilde{N}(\infty), \tag{A65}$$

where $\mathrm{A}(h_I,h_E,t)$ is an S-shaped adjustment term. The properties of this term are mentioned in

Footnote 13 of the paper and are summarized in Lemma A.2.

LEMMA A.2: *Let* $A(\alpha_1,\alpha_2,t)$ *be a multiple adjustment function of the form:*

$$A(\alpha_1,\alpha_2,t) \quad \begin{aligned} &= 1 - \left(\frac{\alpha_2}{\alpha_2-\alpha_1}\right)e^{-\alpha_1 t} + \left(\frac{\alpha_1}{\alpha_2-\alpha_1}\right)e^{-\alpha_2 t} && \text{if } \alpha_1 \neq \alpha_2 \\ &= 1 - (1+\alpha_1 t)e^{-\alpha_1 t} && \text{if } \alpha_1 = \alpha_2, \end{aligned}$$

with $\alpha_1>0$ *and* $\alpha_2>0$. *Then* $A(\alpha_1,\alpha_2,t)$ *has the following properties: (i) (increasing over time)* $dA(\alpha_1,\alpha_2,t)/dt>0$ $\forall t\in(0,\infty)$, $dA(\alpha_1,\alpha_2,t)/dt=0$ (*for* t=0 *and in the limit as* $t\rightarrow\infty$), *(ii) (between 0 and 1)* $0<A(\alpha_1,\alpha_2,t)<1$ $\forall t\in(0,\infty)$ *and* $A(\alpha_1,\alpha_2,0)=1-\lim_{t\rightarrow\infty}A(\alpha_1,\alpha_2,t)=0$, *(iii) (inflexion point)* $d^2A(\alpha_1,\alpha_2,t)/dt^2=0$ *for* $t=\hat{t}\equiv\ln(\alpha_1/\alpha_2)/(\alpha_1-\alpha_2)$ ($\hat{t}\equiv1/\alpha_1$ *if* $\alpha_1=\alpha_2$).

PROOF: The derivative of $A(\alpha_1,\alpha_2,t)$ with respect to time is itself proportional to a single transition term with properties covered in Lemma A.1:

$$\frac{dA(\alpha_1,\alpha_2,t)}{dt} = \alpha_1\alpha_2\left(\frac{e^{-\alpha_1 t}-e^{-\alpha_2 t}}{\alpha_2-\alpha_1}\right)\geq 0,$$

for $t\in(0,\infty)$ the inequality is strict. Hence, $A(\alpha_1,\alpha_2,t)$ itself is increasing over time. Property (ii) follows from the fact that $A(\alpha_1,\alpha_2,0)=0$ and $\lim_{t\rightarrow\infty}A(\alpha_1,\alpha_2,t)=1$ plus the fact that $dA(\alpha_1,\alpha_2,0)/dt\geq0$. Property (iii) makes use of:

$$\frac{d^2A(\alpha_1,\alpha_2,t)}{d^2t} = \alpha_1\alpha_2\left(\frac{\alpha_1 e^{-\alpha_1 t}-\alpha_2 e^{-\alpha_2 t}}{\alpha_1-\alpha_2}\right).$$

Hence, $d^2A(\alpha_1,\alpha_2,0)/dt^2=\alpha_1\alpha_2>0$, and $\lim_{t\rightarrow\infty}d^2A(\alpha_1,\alpha_2,t)/dt^2=0$. The inflexion point is found by finding the value of t where $d^2A(\alpha_1,\alpha_2,t)/dt^2=0$. The solution is $\hat{t}\equiv\ln(\alpha_1/\alpha_2)/(\alpha_1-\alpha_2)$. The proofs for the case for which $\alpha_1=\alpha_2$ are similar. □

This completes the derivation of the results in section 3.3 of the paper. Section A.3 establishes Result 2. By plugging (A49)-(A50) into (A55) and setting $\tilde{t}_L$=0, an expression is obtained from which the inequality in equation (3.11) in the paper can be derived. This establishes Result 3.

A.4. Welfare analysis

The welfare implications of the different environmental policies can be derived in the manner suggested by Bovenberg (1993, 1994). The optimum utility level of generation v at time t is denoted by $\Lambda(v,t)$. It is obtained by substituting the optimum values for $X(v,\tau)$ (where τ runs from t to ∞) plus the policy-induced path for $E(\tau)$ into the utility functional (2.1): $\Lambda(v,t) \equiv \Lambda_{NE}(v,t)+\gamma_E \Lambda_E(t)$, where $\Lambda_{NE}(v,t)$ is the non-environmental component of welfare, and $\Lambda_E(t)$ is the environmental component:

$$\Lambda_{NE}(v,t) \equiv \int_t^\infty \log X(v,\tau) \exp\left[(\alpha+\beta)(t-\tau)\right] \mathrm{d}\tau, \tag{A66}$$

$$\Lambda_{E}(t) \equiv \int_t^\infty \log E(\tau) \exp\left[(\alpha+\beta)(t-\tau)\right] \mathrm{d}\tau. \tag{A67}$$

We can now analyze the two components of utility separately.

A.4.1. Non-environmental utility

The Euler equation for the household, $\dot{X}(v,\tau)=[r-\alpha]X(v,\tau)$, implies that:

$$X(v,\tau) = X(v,t) \exp\left[(r-\alpha)(\tau-t)\right], \qquad \tau \geq t. \tag{A68}$$

Substitution of this result in (A66) yields:

$$\begin{aligned} \Lambda_{NE}(v,t) &= \int_t^\infty \left[\log X(v,t) - (r-\alpha)(t-\tau)\right] \exp\left[(\alpha+\beta)(t-\tau)\right] \mathrm{d}\tau \quad \Leftrightarrow \\ &= \frac{\log X(v,t)}{\alpha+\beta} - \frac{r-\alpha}{(\alpha+\beta)^2}. \end{aligned} \tag{A69}$$

The change in utility is then equal to:

$$(\alpha+\beta)\,\mathrm{d}\Lambda_{NE}(v,t) = \frac{\mathrm{d}X(v,t)}{X(v,t)} \equiv \tilde{X}(v,t). \tag{A70}$$

We now need to distinguish between generations that are alive at the time of the shock ($v<0$) and future generations including the newly born ($v \geq 0$).

A.4.1.1. Existing generations ($v<0$)

Existing generations are born before the policy shock occurs and thus have a negative generations index, $v<0$. For an individual we have that $X(v,0)=(\alpha+\beta)[A(v,0)+H(0)]$, so that:

$$\tilde{X}(v,0) = \left[1-\alpha_H(v)\right]\frac{\mathrm{d}A(v,0)}{A(v,0)} + \alpha_H(v)\frac{\mathrm{d}H(0)}{H(0)}, \quad \text{with} \quad \alpha_H(v) \equiv \frac{H(0)}{A(v,0)+H(0)}, \tag{A71}$$

where $\tilde{X}(v,0) \equiv \mathrm{d}X(v,0)/X(v,0)$. In the steady-state we have that $X(v,0)=X(v,v)\exp[-(r-\alpha)v]$, implying:

$$(\alpha+\beta)\left[A(v,0)+H(0)\right] = (\alpha+\beta)H(0)\exp\left[-(r-\alpha)v\right] \;\Rightarrow\; \alpha_H(v) \equiv \exp\left[(r-\alpha)v\right]. \tag{A72}$$

Furthermore, we know that

$$\frac{\mathrm{d}A(0)}{A(0)} = \frac{\mathrm{d}A(v,0)}{A(v,0)}. \tag{A73}$$

Equation (A73) says that the rate of change in the value of individual assets equals the rate of change in the value of aggregate financial wealth. By using (A72)-(A73) in (A71) we obtain:

$$\tilde{X}(v,0) = (\tilde{A}(0)/\omega_A)\left[1-e^{(r-\alpha)v}\right] + (\tilde{H}(0)/\omega_H)\,e^{(r-\alpha)v}. \tag{A74}$$

where we note that by definition $\tilde{A}(0) \equiv r\mathrm{d}A(0)/Y \equiv \omega_A \mathrm{d}A(0)/A(0)$ and similarly $\tilde{H}(0) \equiv r\mathrm{d}H(0)/Y(0) \equiv \omega_H \mathrm{d}H(0)/H(0)$, where $\omega_H \equiv rH(0)/Y(0)$.

By substituting (A74) into (A70) the effect on welfare for existing generations can be written as:

$$(\alpha+\beta)\,\mathrm{d}\Lambda_{NE}(v,0) = (\tilde{A}(0)/\omega_A)\left[1-e^{(r-\alpha)v}\right] + (\tilde{H}(0)/\omega_H)\,e^{(r-\alpha)v}, \quad v<0, \tag{A75}$$

which coincides with equation (4.1) in the text.

A.4.1.2. Future generations and newborns ($v \geq 0$)

The utility change for future generations and newborns is evaluated at birth, *i.e.* we compute $\mathrm{d}\Lambda_{NE}(v,v)$ for $v=t\geq 0$. First, we know that agents are born without financial wealth, $A(v,v)=0$, so that:

$$X(v,v) = (\alpha+\beta)H(v) \quad \Rightarrow \quad \tilde{X}(v,v) = \frac{\mathrm{d}H(v)}{H(0)} = \tilde{H}(v)/\omega_H. \tag{A76}$$

The change in welfare of future generations and newborns is rewritten as:

$$(\alpha+\beta)\,\mathrm{d}\Lambda_{NE}(v,v) = \tilde{X}(v,v) = \tilde{H}(v)/\omega_H, \quad v=t\geq 0. \tag{A77}$$

This coincides with equation (4.3) in the text. Note that in subsections A.4.1.1 and A.4.1.2 $Y(0)$, $H(0)$, $A(0)$, $X(v,0)$, and $A(v,0)$ denote initial steady-state values for the respective variables, i.e. values that pertain **before** the shock occurs. In the paper itself the index '0' is dropped from steady-state values to simplify the notation. See for example the information on shares at the bottom of Table 2.

A.4.2. Environmental utility

The environmental component of total utility is given in equation (A67). By linearizing (A67) we obtain the following expression for the change in environmental utility:

$$d\Lambda_E(t) \equiv \int_t^\infty \tilde{E}(\tau) e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{A78}$$

which shows that the impact effect on environmental utility is:

$$d\Lambda_E(0) = \mathcal{L}\{\tilde{E}, \alpha+\beta\} = \frac{h_E h_I}{(\alpha+\beta+h_I)(\alpha+\beta+h_E)} \left[\frac{\tilde{E}(\infty)}{\alpha+\beta} - \frac{\sigma_N \tilde{N}(0)}{h_I} \right] > 0, \tag{A79}$$

where we have used (A65) to work out the Laplace transformation. Equation (A79) coincides with (4.4) in the paper.

In a similar fashion, the long-run effect on environmental utility can be obtained directly from (A78):

$$d\Lambda_E(\infty) \equiv \lim_{t\to\infty} \int_t^\infty \tilde{E}(\tau) e^{(\alpha+\beta)(t-\tau)} d\tau = \frac{\tilde{E}(\infty)}{\alpha+\beta}, \tag{A80}$$

which is equation (4.5) in the paper. This completes the derivation of the results in section 4.1 of the paper and thus establishes Result 4.

A.5. Proof of Proposition 4

We now turn to the proof of Proposition 4. Part (i) follows directly from (A79) since $\tilde{E}(\infty)>0$ and $\tilde{N}(0)<0$. Similarly, $d\Lambda_E(\infty)>0$ follows from (A80) and the fact that $\tilde{E}(\infty)>0$. Parts (ii) and (iii) can now be proved as follows. First we recall that the path of energy use is monotonically decreasing (see (A62) and note that $\tilde{N}(\infty)<\tilde{N}(0)<0$). By definition we have that:

$$\mathfrak{L}\{\dot{\tilde{E}},s\} \equiv s\mathfrak{L}\{\tilde{E},s\} - \tilde{E}(0) = \sigma_N\left(\frac{h_E}{s+h_E}\right)\left[-\tilde{N}(0) + \left(\frac{h_I}{s+h_I}\right)\left[\tilde{N}(0)-\tilde{N}(\infty)\right]\right], \tag{A81}$$

where we have used (A64) and $\tilde{E}(0)=0$. The term in square brackets on the right-hand side of (A81) is positive, so that environmental quality is monotonically increasing.

By using (A78) we obtain the following expression:

$$d\dot{\Lambda}_E(t) \equiv \frac{d}{dt}\left[d\Lambda_E(t)\right] = -\tilde{E}(t) + (\alpha+\beta)\,d\Lambda_E(t), \tag{A82}$$

from which it follows that $d\dot{\Lambda}_E(0)=(\alpha+\beta)d\Lambda_E(0)>0$ (as $\tilde{E}(0)=0$) and $d\dot{\Lambda}_E(\infty)=0$ (from (A80)). By using (A78) in (A82) and noting that $\tilde{E}(\tau)>\tilde{E}(t)$ (for $0\leq t<\tau<\infty$) we derive:

$$\begin{aligned} d\dot{\Lambda}_E(t) &= -\tilde{E}(t) + (\alpha+\beta)\tilde{E}(t)\int_t^\infty e^{(\alpha+\beta)(t-\tau)}d\tau + (\alpha+\beta)\int_t^\infty\left[\tilde{E}(\tau)-\tilde{E}(t)\right]e^{(\alpha+\beta)(t-\tau)}d\tau \\ &= (\alpha+\beta)\int_t^\infty\left[\tilde{E}(\tau)-\tilde{E}(t)\right]e^{(\alpha+\beta)(t-\tau)}d\tau > 0, \qquad (0\leq t<\infty). \end{aligned} \tag{A83}$$

Hence, $d\dot{\Lambda}_E(t)$ is strictly positive initially and during the transition. This proves both parts (ii) and (iii) of Proposition 4 and thus establishes Result 5.

A.6. Redistribution issues

Under an egalitarian policy, the path of lump-sum taxes must be chosen such that all current and existing generations experience the same change in welfare. The implied path of debt must satisfy the government budget restriction. In order to guarantee government solvency, we directly postulate a stable path for debt and then deduce the implied path of lump-sum taxes. The debt path is:

$$\tilde{B}(t) = b_0 + b_1 e^{-\phi_1 t} + b_2 e^{-\phi_2 t}, \tag{A84}$$

with $\phi_1,\phi_2>0$ and b_0, b_1, b_2 finite. Note that (A84) implies that $\tilde{B}(0)=b_0+b_1+b_2$ and $\tilde{B}(\infty)=b_0$.

By using (A84) in the government budget identity (TA.15) we obtain the following expression for the (log-linearized) primary deficit:

$$\tilde{D}(t) \equiv \left[\tilde{G}(t)\right]_B - \left[\tilde{T}(t)\right]_{LS} = -b_0 - \sum_{i=1}^{2} b_i (1+\phi_i/r) e^{-\phi_i t}, \tag{A85}$$

where the subscript '*LS*' refers to the lump-sum rebating scenario analyzed in section 3 of the paper, and '*B*' refers to the bond policy. Since the investment system is independent of the path of government debt, its solution is the same, and wages and tax revenue are unaffected by the bond path. Hence:

$$\left[\tilde{Y}_F(t)\right]_{LS} = \varepsilon_L (1-t_L)\left[\tilde{W}(t)-\tilde{t}_L\right]_{LS} + \left[\tilde{T}(t)\right]_{LS}, \tag{A86}$$

$$\left[\tilde{Y}_F(t)\right]_B = \varepsilon_L (1-t_L)\left[\tilde{W}(t)-\tilde{t}_L\right]_{LS} + \left[\tilde{G}(t)\right]_B = \left[\tilde{Y}_F(t)\right]_{LS} + \tilde{D}(t), \tag{A87}$$

where we have used (A85) to get from the first to the second equality in (A87).

By using (A54) and (A86)-(A87) the jump in human wealth under bond policy can be computed:

$$\left[\tilde{H}(0)\right]_B \equiv r\mathfrak{L}\{[\tilde{Y}_F]_B, r+\beta\} = \left[\tilde{H}(0)\right]_{LS} + r\mathfrak{L}\{\tilde{D}, r+\beta\}, \tag{A88}$$

where $\mathfrak{L}\{\tilde{D},r+\beta\}$ is given by:

$$\mathfrak{L}\{\tilde{D}, r+\beta\} \equiv -\frac{b_0}{r+\beta} - \sum_{i=1}^{2} \frac{b_i (1+\phi_i/r)}{r+\beta+\phi_i}. \tag{A89}$$

Similarly, by using (A57) and (A87) the path for human wealth under bond policy can be computed:

$$\left[\tilde{H}(t)\right]_B = \left[\tilde{H}(0)\right]_{LS} e^{-h_I t} + \left[1 - e^{-h_I t}\right]\left[\tilde{H}(\infty)\right]_{LS} + r\mathcal{L}^{-1}\left[\frac{\mathcal{L}\{\tilde{D}, r+\beta\} - \mathcal{L}\{\tilde{D}, s\}}{s-(r+\beta)}\right]. \tag{A90}$$

By using (A85) we obtain:

$$\mathcal{L}^{-1}\left[\frac{\mathcal{L}\{\tilde{D}, r+\beta\} - \mathcal{L}\{\tilde{D}, s\}}{s-(r+\beta)}\right] = -\frac{b_0}{r+\beta} - \sum_{i=1}^{2}\left(\frac{b_i(1+\phi_i/r)}{r+\beta+\phi_i}\right)e^{-\phi_i t}. \tag{A91}$$

Finally, by substituting (A91) into (A90) and noting (A77), we obtain the path of non-environmental welfare under bond policy:

$$\begin{aligned}\omega_H(\alpha+\beta)\,\mathrm{d}\Lambda_{NE}(t,t) &= \left[\tilde{H}(0)\right]_{LS} e^{-h_I t} + \left[1-e^{-h_I t}\right]\left[\tilde{H}(\infty)\right]_{LS} \\ &\quad - r\left[\frac{b_0}{r+\beta} + \sum_{i=1}^{2}\left(\frac{b_i(1+\phi_i/r)}{r+\beta+\phi_i}\right)e^{-\phi_i t}\right].\end{aligned} \tag{A92}$$

The path for environmental welfare does not depend on bonds and is given in (A78). With time-invariant tax shocks it can be written as:

$$\mathrm{d}\Lambda_E(t) = \frac{\tilde{E}(\infty)}{\alpha+\beta} - \left(\frac{\sigma_N h_E \tilde{N}(0) + h_I \tilde{E}(\infty)}{(h_I-h_E)(\alpha+\beta+h_E)}\right)e^{-h_E t} + \left(\frac{\sigma_N h_E \tilde{N}(0) + h_E \tilde{E}(\infty)}{(h_I-h_E)(\alpha+\beta+h_I)}\right)e^{-h_I t}. \tag{A93}$$

The effect on total welfare must equal the common effect under the egalitarian policy, which implies the following restriction:

$$\left[\mathrm{d}\Lambda(t,t) \equiv\right] \quad \mathrm{d}\Lambda_{NE}(t,t) + \gamma_E \mathrm{d}\Lambda_E(t) = \frac{\Pi}{\alpha+\beta}. \tag{A94}$$

In view of (A92) and (A93), the egalitarian policy thus requires all exponential terms in the expression for total utility to be eliminated, using the policy variables ϕ_1, ϕ_2, b_0, b_1, and b_2. An obvious requirement is that the bond policy rule contains the correct exponential terms:

$$\phi_1 = h_I, \qquad \phi_2 = h_E. \tag{A95}$$

Elimination of the exp[-$h_I t$]-term (given $\phi_1 = h_I$) by choice of b_1 yields:

$$b_1 = \left(\frac{r}{r+h_I}\right)\left[\tilde{Y}_F(0) - \tilde{Y}_F(\infty)\right]_{LS} - \gamma_E\omega_H\left(\frac{\alpha+\beta}{r+h_I}\right)\left(\frac{h_E}{h_E-h_I}\right)\left(\frac{r+\beta+h_I}{\alpha+\beta+h_I}\right)\left[\sigma_N\tilde{N}(0) + \tilde{E}(\infty)\right]. \tag{A96}$$

Similarly, elimination of the exp[-$h_E t$]-term (given $\phi_2 = h_E$) by choice of b_2 yields:

$$b_2 = \gamma_E \omega_H \left(\frac{r+\beta+h_E}{r+h_E} \right) \left(\frac{\alpha+\beta}{\alpha+\beta+h_E} \right) \left[\frac{\sigma_N h_E \tilde{N}(0) + h_I \tilde{E}(\infty)}{h_E - h_I} \right] \tag{A97}$$

The permanent term in total utility is equal to:

$$b_0 = \left[\tilde{Y}_F(\infty) \right]_{LS} + \frac{(r+\beta)\,\omega_H}{r} \left[\gamma_E \tilde{E}(\infty) - \Pi \right]. \tag{A98}$$

All existing generations (at the time of the shock) experience the same change in environmental welfare, so that the equality $(\alpha+\beta)\mathrm{d}\Lambda(v,0)=\Pi$ for $v \le 0$, reduces to $\mathrm{d}\Lambda_{NE}(0,0)=\mathrm{d}\Lambda_{NE}(-\infty,0)$, or:

$$\omega_A \left[\Pi - \gamma_E (\alpha+\beta)\,\mathrm{d}\Lambda_E(0) \right] = \left[\tilde{A}(0) \right]_B, \tag{A99}$$

where $[\tilde{A}(0)]_B$ represents the change in the value of financial assets under bond policy. In view of (TA.14) and (A84) it equals:

$$\left[\tilde{A}(0) \right]_B = \omega_A \left[\tilde{q}(0) \right]_{LS} + \tilde{B}(0) = \omega_A \left[\tilde{q}(0) \right]_{LS} + b_0 + b_1 + b_2. \tag{A100}$$

By substituting (A100) into (A99) and using (A96)-(A98) we obtain an expression which can be solved for the common welfare effect:

$$\begin{aligned} \omega_X \Pi = {} & \omega_A \left[\tilde{q}(0) \right]_{LS} + \left(\frac{r}{r+h_I} \right) \left[\tilde{Y}_F(0) \right]_{LS} + \left(\frac{h_I}{r+h_I} \right) \left[\tilde{Y}_F(\infty) \right]_{LS} \\ & + \gamma_E \omega_X \left(\frac{h_I}{r+h_I} \right) \left(\frac{h_E}{r+h_E} \right) \left[-\frac{r \sigma_N \tilde{N}(0)}{h_I} + \tilde{E}(\infty) \right]_{LS}. \end{aligned} \tag{A101}$$

This coincides with (4.6) in the paper. By substituting (A36), (A42), (A49)-(A50), (A63), and (A65) into (A101) we obtain an expression linking Π to the tax shocks:

$$\omega_X \Pi = -\left[\Gamma_L \sigma_L t_L + \Gamma_N \left(\theta_N - \theta_N^P \right) \right] \omega_N \tilde{t}_N - \left[\Sigma_L \omega_L t_L + \Sigma_N \omega_N \left(\theta_N - \theta_N^P \right) \right] \sigma_L \tilde{t}_L, \tag{A102}$$

where Γ_L, Γ_N, Σ_L, and Σ_N are positive constants:

$$\Gamma_L = \Sigma_N \equiv \frac{r \sigma_{KN} \omega_L (1-\omega_L)}{(r+h_I)\left[\sigma_{KN} \omega_L \omega_N + \omega_K \left[\sigma_{LZ} + \sigma_L (1-\omega_L) \right] \right]} + \frac{h_I}{r+h_I} > 0, \tag{A103}$$

$$\Gamma_N \equiv \frac{r\sigma_{KN}(1-\omega_L)\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]}{(r+h_I)\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]} + \frac{h_I\left[\sigma_{KN}\omega_L\omega_K+\omega_N\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]}{(r+h_I)\,\omega_L(1-\omega_L)} > 0, \qquad \text{(A104)}$$

$$\Sigma_L \equiv \frac{r\left[\sigma_{LZ}\omega_K+\sigma_{KN}\omega_L\omega_N\right]}{(r+h_I)\left[\sigma_{KN}\omega_L\omega_N+\omega_K\left[\sigma_{LZ}+\sigma_L(1-\omega_L)\right]\right]} + \frac{h_I}{r+h_I} > 0, \qquad \text{(A105)}$$

and where θ_N^P is the Pigouvian energy tax:

$$\theta_N^P \equiv \frac{\gamma_E\sigma_N\omega_X h_E}{\omega_N(r+h_E)} > 0. \qquad \text{(A106)}$$

This establishes Result 6.

A.8. Inequalities

In this section and the next we only consider the Cobb-Douglas case. It is conjectured that the results can easily be extended to the more general case, but we have refrained from doing so since little additional insight would seem to be gained in doing so. The first inequality which warrants further comment is the one relating to Tobin's q in (5.6). A direct approach yields the proof. We know that adjustment in the capital stock is monotonic, so that the induced path of the labour income tax rate is also monotonic and can be written as:

$$\tilde{t}_L(t) = \tilde{t}_L(0) e^{-h_I^* t} + \tilde{t}_L(\infty)\left[1 - e^{-h_I^* t}\right], \tag{A107}$$

with (by Assumptions 1 and 2):

$$\tilde{t}_L(0) < \tilde{t}_L(\infty) < 0. \tag{A108}$$

Equations (A107)-(A108) imply furthermore that the Laplace transform of the labour income tax rate is negative:

$$\mathcal{L}\{\tilde{t}_L, s\} < 0. \tag{A109}$$

By applying the solution approach discussed above (in section A.3.1) to the **original** investment system (A30), setting $\tilde{t}_N(t)=\tilde{t}_N$, and using $\tilde{t}_L(t)$ given in (A107) above, we obtain the following expression for the jump in Tobin's q:

$$\left[\tilde{q}(0)\right]_{TR} = -\left(\frac{r\varepsilon_K}{\omega_A\left[\varepsilon_L + (1+\sigma_L)\varepsilon_K\right]}\right)\left[(1+\sigma_L)\varepsilon_N \mathcal{L}\{\tilde{t}_N, r_I\} + \sigma_L \varepsilon_L \mathcal{L}\{\tilde{t}_L, r_I\}\right], \tag{A110}$$

where the unstable root to be used is r_I (because the original system is being used). Note that (A110) is just an alternative way to write the corresponding expression in (5.6) in the paper. By using (3.2), it is clear that (A110) can be rewritten as:

$$\left[\tilde{q}(0)\right]_{TR} = \left[\tilde{q}(0)\right]_{LS} - \left(\frac{r\varepsilon_K \sigma_L \varepsilon_L}{\omega_A\left[\varepsilon_L + (1+\sigma_L)\varepsilon_K\right]}\right)\mathcal{L}\{\tilde{t}_L, r_I\} > \left[\tilde{q}(0)\right]_{LS}, \tag{A111}$$

where (A109) is used to demonstrate the final inequality.

The impact effects on output, employment, energy use, and wages are obtained by using (A26)-(A29), noting that $\tilde{K}(0)=0$, and that $\tilde{t}_L(0)<0$ by the SRNLC. The inequalities pertaining to the two scenarios follow in a straightforward fashion. By substituting the expression for $\tilde{t}_L(0)$ (from (5.3) with $\tilde{K}(0)=0$ imposed) into (A26)-(A29), equations (5.7)-(5.10) in the paper are obtained. As is explained in the paper, no unambiguous conclusion can be obtained regarding the impact effect on full income. By using (3.6) and (5.11) we obtain the inequality mentioned in the paper. This establishes Result 7.

A.9. Inequalities welfare

We now turn to the proof of Proposition 7. Proposition 7(i) follows from (5.6) and 7(iii) from (5.5). Proposition 7(v) follows readily from the fact that $[\tilde{Y}_F(0)]_{TR}>[\tilde{Y}_F(\infty)]_{TR}$. The two remaining claims, 7(ii) and 7(iv), clearly hold for initial tax rates equal to zero. With all expressions being continuous this then implies that for all parameter settings there must be a neighbourhood of the origin $(\theta_N,t_L)=(0,0)$ where the result holds. We refer to this neighbourhood as the case with ‘sufficiently low' tax rates. We thus do not engage in a quest for specific bounds the taxes must satisfy, since that would require tedious analysis of nonlinear inequalities which does not add much insight. The zero initial taxes assumption simplifies matters substantially since it renders the long-run response of disposable full income zero (see (3.7) and (5.11)). Comparisons are consequently in terms of the initial jump for full income only. Upon substituting zero initial taxes, claim 7(ii) reduces to the statement that $[\tilde{Y}_F(0)]_{TR}$ is positive (which we showed is true). Claim 7(iv) is evidently true as $[\tilde{q}(0)]_{TR}<0$ and $[\tilde{H}(0)]_{TR}>0$.

Proposition 8, relating to the welfare consequences of the two scenarios considered, is half-way proven, like the other summarizing proposition. In fact, only claim 8(iii) warrants any comment. Using the assumption of zero initial taxes, the comparison reduces to a true inequality in terms of $[\tilde{Y}_F(0)]_{TR}$. All other claims are either proven or self-evident. This establishes Result 8.

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A.4.2. Environmental utility (old)

The environmental component of total utility is given in equation (A66). By linearizing (A66) we obtain the following expression for the change in environmental utility:

$$\mathfrak{L}\{d\Lambda_E, s\} = \frac{\mathfrak{L}\{\tilde{E}, \alpha+\beta\} - \mathfrak{L}\{\tilde{E}, s\}}{s-(\alpha+\beta)}. \tag{A112}$$

The impact effect on environmental utility can be obtained directly from (A77) by applying the initial value theorem:

$$d\Lambda_E(0) \equiv \lim_{s\to\infty} s\mathfrak{L}\{d\Lambda_E, s\} = \mathfrak{L}\{\tilde{E}, \alpha+\beta\} = \frac{h_E h_I}{(\alpha+\beta+h_I)(\alpha+\beta+h_E)}\left[\frac{\tilde{E}(\infty)}{\alpha+\beta} - \frac{\sigma_N \tilde{N}(0)}{h_I}\right], \tag{A113}$$

where we have used (A64) to work out the Laplace transformation. Equation (A78) coincides with (4.4) in the paper.

In a similar fashion, the long-run effect on environmental utility can be obtained directly from (A77) obtained by applying the final value theorem:

$$d\Lambda_E(\infty) \equiv \lim_{s\to 0} s\mathfrak{L}\{d\Lambda_E, s\} = \frac{\tilde{E}(\infty)}{\alpha+\beta}, \tag{A114}$$

which is equation (4.5) in the paper. This completes the derivation of the results in section 4.1 of the paper and thus establishes Result 4.

A.5. Old proof of Proposition 4(ii)-(iii)

In a similar fashion we write:

$$\mathfrak{L}\{\mathrm{d}\dot{\Lambda}_E,s\} \equiv s\mathfrak{L}\{\mathrm{d}\Lambda_E,s\} - \mathrm{d}\Lambda_E(0) = \frac{(\alpha+\beta)\mathfrak{L}\{\tilde{E},\alpha+\beta\} - s\mathfrak{L}\{\tilde{E},s\}}{s-(\alpha+\beta)}$$

$$= \frac{\mathfrak{L}\{\dot{\tilde{E}},\alpha+\beta\} - \mathfrak{L}\{\dot{\tilde{E}},s\}}{s-(\alpha+\beta)}, \tag{A115}$$

where we have used (A78) in going from the second to the third expression and (A80) in going from the third to the fourth. By using (A80) in (A81) we find:

$$\mathfrak{L}\{\mathrm{d}\dot{\Lambda}_E,s\} \equiv \sigma_N\left(\frac{h_E}{(\alpha+\beta+h_E)(s+h_E)}\right)\left[-\tilde{N}(0) + \left(\frac{h_I\left[s+\alpha+\beta+h_E+h_I\right]}{(\alpha+\beta+h_I)(s+h_I)}\right)\left[\tilde{N}(0)-\tilde{N}(\infty)\right]\right] > 0. \tag{A116}$$

Inverting this transform gives only terms that are positive in the time domain. An alternative proof is more general and holds for all increasing functions. We first state and prove the following Lemma.

LEMMA A.3: *Let* $\dot{x}(t)\geq 0\ \forall t\geq 0$ *and* $\dot{x}(t)>0\ \exists t\geq 0$. *Then the following result holds:*

$$\frac{\mathfrak{L}\{\dot{x},\alpha_1\} - \mathfrak{L}\{\dot{x},s\}}{s-\alpha_1} = \frac{\alpha_1\mathfrak{L}\{x,\alpha_1\} - s\mathfrak{L}\{x,s\}}{s-\alpha_1} = \alpha_1\mathfrak{L}\left[\int_t^\infty\left[x(\tau)-x(t)\right]e^{\alpha_1(t-\tau)}\mathrm{d}\tau\right] > 0,$$

where $\alpha_1>0$.

PROOF: The first equality follows from the initial value theorem and need not be proved here. To prove the second step, we write the integral on the right-hand side as:

$$\mathfrak{L}\left[\int_t^\infty\left[x(\tau)-x(t)\right]e^{\alpha_1(t-\tau)}\mathrm{d}\tau\right] = \int_0^\infty\left[\int_t^\infty x(\tau)e^{\alpha_1(t-\tau)}\mathrm{d}\tau\right]e^{-st}\mathrm{d}t - \int_0^\infty\left[\int_t^\infty x(t)e^{\alpha_1(t-\tau)}\mathrm{d}\tau\right]e^{-st}\mathrm{d}t$$

$$= \mathfrak{L}\{G,s\} - (1/\alpha_1)\mathfrak{L}\{x,s\}, \tag{$\star$}$$

where $G(t)$ is defined as:

$$G(t) \equiv \int_t^\infty x(\tau)e^{\alpha_1(t-\tau)}\mathrm{d}\tau.$$

We first note that $\mathfrak{L}\{G,s\}$ is:

$$\mathcal{L}\{G,s\} = \frac{\mathcal{L}\{x,\alpha_1\} - \mathcal{L}\{x,s\}}{s-\alpha_1}.$$

(Proof: $G(0)=\mathcal{L}\{x,\alpha_1\}$ and $\dot{G}(t)=-x(t)+\alpha_1 G(t)$. Hence, $(s-\alpha_1)\mathcal{L}\{\dot{G},s\}=G(0)-\mathcal{L}\{x,s\}=\mathcal{L}\{x,\alpha_1\}-\mathcal{L}\{x,s\}$.□) By using this result in (★) and simplifying we obtain:

$$\mathcal{L}\left[\int_t^\infty \left[x(\tau)-x(t)\right] e^{\alpha_1(t-\tau)} \mathrm{d}\tau\right] = \frac{\alpha_1 \mathcal{L}\{x,\alpha_1\} - s\mathcal{L}\{x,s\}}{\alpha_1(s-\alpha_1)},$$

which completes the proof of Lemma A.3. □

Old stuff

It is straightforward to derive that:

$$s \overset{>}{\underset{<}{=}} \alpha_1 \quad \Leftrightarrow \quad \mathcal{L}\{x,\alpha_1\} \overset{>}{\underset{<}{=}} \mathcal{L}\{x,s\},$$

which proves that numerator and denominator in (*) have the same sign. It remains to show what happens if $s=\alpha_1$. By l'Hôpital's Rule we find:

$$\lim_{(s-\alpha_1)\to 0} \frac{\mathcal{L}\{x,\alpha_1\} - \mathcal{L}\{x,s\}}{s-\alpha_1}$$

$$= \lim_{(s-\alpha_1)\to 0} \frac{\int_0^\infty x(t)e^{-st}\left[1-e^{(s-\alpha_1)t}\right]\mathrm{d}t}{s-\alpha_1}$$

$$= \lim_{(s-\alpha_1)\to 0} \frac{-\int_0^\infty x(t)e^{-st}\left[(s-\alpha_1)e^{(s-\alpha_1)t}\right]\mathrm{d}t}{-1} = 0.$$

□

Since the path of environmental quality is monotonically increasing, it follows from Lemma A.3 and (A81) that $\mathcal{L}\{\mathrm{d}\dot{\Lambda}_E,s\}\geq 0$. Equation (A77) shows that $\mathrm{d}\Lambda(t)$ satisfies the following differential equation:

$$\mathrm{d}\dot{\Lambda}_E(t) = -\tilde{E}(t) + (\alpha+\beta)\,\mathrm{d}\Lambda_E(t), \tag{A82}$$

from which we derive that $\mathrm{d}\dot{\Lambda}_E(0)=(\alpha+\beta)\mathrm{d}\Lambda_E(0)>0$ (as $\tilde{E}(0)=0$). Hence, $\mathrm{d}\dot{\Lambda}_E(t)$ is strictly positive initially and $\mathfrak{L}\{d\dot{\Lambda}_E, s\}>0$. This proves both parts (ii) and (iii) of Proposition 4 and thus establishes Result 5.

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5. Tax reform and the double dividend hypothesis

In this section we consider an alternative policy scenario, namely a ‘green' tax reform that consists of an unanticipated and permanent simultaneous increase in the energy tax rate and decrease of the labour income tax rate. In order to allow for comparison with the previously discussed scenario we require the tax reform to be budget-neutral. No deficits are allowed and we abstract from bond policy again. We also rule out lump-sum redistribution which implies we cannot have a tax reform with constant tax changes, i.e. at least one of the tax rate changes must vary over time. The natural choice in our context is to administer a time-invariant shock to the energy tax and to allow a time-varying labour income tax rate to balance the budget.

5.1. Time-varying labour income taxation

The path of the labour income tax rate that satisfies the conditions posed is found by setting $\tilde{B}(t)=\tilde{G}(t)=\tilde{T}(t)=0$ (so that (T2.15) holds trivially) and solving (T2.11) for $\tilde{t}_L(t)$:

$$\varepsilon_L(1-t_L)\tilde{t}_L(t) \;=\; -\left[\varepsilon_N(1-\theta_N)\tilde{t}_N \;+\; \omega_G\tilde{Y}(t)\right], \tag{A5}$$

where $\omega_G \equiv \varepsilon_N\theta_N + \varepsilon_L t_L \geq 0$ is the initial share of tax revenue in national income. The two terms on the right-hand side of (5.1) represent, respectively, the rate effect and the base effect. In view of the simple Cobb-Douglas production structure, the tax bases for the labour and energy tax are both proportional to aggregate output. Of course, if initial taxes are zero ($t_L=\theta_N=\omega_G=0$), there is no base effect and only the rate effect survives.

In the general case (with $\sigma_L>0$ and $\omega_G>0$), the base effect is induced by labour and energy adjustment in the impact period, and by labour, energy, and capital reallocation during transition and in the long run. By setting $\tilde{t}_N(t)=\tilde{t}_N$ in (2.9) and using (5.1), the quasi-reduced forms for output and the labour tax rate under tax reform are found:

$$\tilde{Y}(t) \;=\; \frac{(1+\sigma_L)(1-t_L)\varepsilon_K\tilde{K}(t) \;-\; \left[1-(1+\sigma_L)t_L+\sigma_L\theta_N\right]\varepsilon_N\tilde{t}_N}{(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G}, \tag{A6}$$

$$-\varepsilon_L\tilde{t}_L(t) \;=\; \frac{\omega_G(1+\sigma_L)\varepsilon_K\tilde{K}(t) \;+\; \left[\varepsilon_L+(1+\sigma_L)\varepsilon_K-(1+\sigma_L)\varepsilon_L t_L-\left[1+\sigma_L(1-\varepsilon_L)\right]\theta_N\right]\varepsilon_N\tilde{t}_N}{(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G}. \tag{A7}$$

Before moving on to the macroeconomic and distributional consequences of the tax reform policy we must reconsider stability of the investment system since the dynamics are affected by the policy under consideration. As it turns out the investment system is no longer stable for all levels of pre-existing taxation. Intuitively, stability requires the marginal product of capital (F_K) to rise as the capital stock falls. By using $\tilde{F}_K(t) \equiv \tilde{Y}(t) - \tilde{K}(t)$ and equation (5.2), we obtain the following expression for the marginal product of capital:

$$\tilde{F}_K(t) \equiv \tilde{Y}(t) - \tilde{K}(t) = -\psi(\theta_N, t_L)\tilde{K}(t) - \chi(\theta_N, t_L)\varepsilon_N \tilde{t}_N,$$

where $\psi(\theta_N, t_L)$ and $\chi(\theta_N, t_L)$ are defined as follows:

$$\psi(\theta_N, t_L) \equiv \frac{\varepsilon_L(1-t_L) - \sigma_L \omega_G}{(1-t_L)\left[\varepsilon_L + (1+\sigma_L)\varepsilon_K\right] - \sigma_L \omega_G}, \quad \chi(\theta_N, t_L) \equiv \frac{1-(1+\sigma_L)t_L + \sigma_L \theta_N}{(1-t_L)\left[\varepsilon_L + (1+\sigma_L)\varepsilon_L\right] - \sigma_L \omega_G}. \qquad \text{(A6)}$$

Stability thus requires $\psi(\theta_N, t_L) > 0$, which is trivially satisfied if labour supply is exogenous ($\sigma_L = 0$) or initial tax rates are zero ($\theta_N = t_L = \omega_G = 0$). In the general case, however, stability depends on the magnitude of pre-existing tax rates, θ_N and t_L, as can be illustrated with the aid of Figure 2.

Designating the denominator and numerator of $\psi(.)$ by ψ_D and ψ_N, respectively, stability obtains if both ψ_D and ψ_N are positive (modest tax rates) or both are negative (high tax rates). In terms of initial tax rates, the two cases are represented in Figure 2 by areas 0AB and C1DE, respectively. In order to exclude the latter case, which does not appear to have much real-world relevance, we make the following assumption.

ASSUMPTION 1: *The initial tax rates θ_N and t_L satisfy:*

$$\left[\psi(\theta_N, t_L) > 0\right]: \qquad 0 \le \theta_N \le 1, \quad 0 \le t_L < 1, \quad \theta_N \le \frac{\varepsilon_L\left[1-(1+\sigma_L)t_L\right]}{\sigma_L \varepsilon_N}, \qquad \text{(NSC)}$$

i.e. the initial tax rates are relatively low and preserve macroeconomic diminishing returns to capital.

This ‘normal' stability condition (NSC) ensures that the denominator appearing in (5.2)-(5.3) is positive. It is thus also useful to sign the relationship between the change in the energy tax and the induced labour tax change, both at impact and in the long-run (see (5.3)). Indeed, if initial taxes are zero, the bracketed term in front of $\tilde{t}_N$ on the right-hand side of (5.3) is positive and the labour tax falls as a result of the increase in the energy tax. As is well known, however, this result does not extend to the general case, because the erosion of the tax base can be so severe as to prompt a positive relationship between the shock in the energy tax and the induced change in the labour tax. In order to rule out such cases, we make a number of assumptions regarding initial tax rates which will ensure that the economy operates on the upward sloping side of the Laffer curve:

ASSUMPTION 2: *The initial tax rates* θ_N *and* t_L *satisfy:*

$$\left[\tilde{t}_L(0)/\tilde{t}_N < 0\right]: \qquad \theta_N < \frac{\varepsilon_L + (1+\sigma_L)\varepsilon_K - (1+\sigma_L)\varepsilon_L t_L}{\varepsilon_L + (1+\sigma_L)(\varepsilon_K + \varepsilon_N)}, \qquad \text{(SRNLC)}$$

$$\left[\tilde{t}_L(\infty)/\tilde{t}_N < 0\right]: \qquad \theta_N < \frac{\varepsilon_L\left[1-(1+\sigma_L)t_L\right]}{\varepsilon_L + (1+\sigma_L)\varepsilon_N}, \qquad \text{(LRNLC)}$$

i.e. the initial tax rates are such as to place the economy on the upward sloping segment of the Laffer curve.

Given Assumption 1, the long-run no-Laffer-curve condition (LRNLC) is actually more restrictive than its short-run counterpart (SRNLC), i.e. LRNLC implies SRNLC. In terms of Figure 2, SC and SRNLC hold in the trapezoid 0BHG whilst SC and LRNLC obtain in the triangle OBI (which is fully contained in 0BHG). Armed with Assumptions 1 and 2, we are able to derive the modified version of Proposition 1.

PROPOSITION 6: *Suppose* $\sigma_L>0$ *and that Assumptions 1 and 2 hold. Then (i) the full model is locally saddle-point stable; (ii) the investment system has distinct characteristic roots,* $-h_I<-h_I^*<0$ *and* $r<r_I^*=r+h_I^*<r_I=r+h_I$*; (iii) the stable root satisfies* $\partial h_I^*/\partial\sigma_A<0$ *and* $h_I^*\to\infty$ *as* $\sigma_A\to0$*; and (iv) the saving system has distinct characteristic roots* $-h_S=r-(\alpha+\beta)<0$, $r_S=r+\beta>0$. PROOF: See Appendix.

The most important thing to note from Proposition 6 is that even though the dynamic properties of the economy are the same under lump-sum rebating and tax reform, the adjustment speed is lowest under the latter scenario, i.e. $h_I>h_I^*$. This has important repercussions for both the macroeconomic and distributional effects of the energy tax.

5.2. The macroeconomic effects

The capital stock and its associated shadow value exhibit similar transition patterns as before, with capital gradually decreasing to its new steady-state value and Tobin's *q* returning to its initial steady-state value upon an initial fall at impact. However, as was pointed out above, the adjustment speed is lower, $h_I^*<h_I$, and $\tilde{q}(0)$ and $\tilde{K}(\infty)$ are different as well. For the long run decrease in the capital stock we can now write:

$$\left[\tilde{K}(\infty)\right]_{LS} < \left[\tilde{K}(\infty)\right]_{TR} = -\left[\frac{1-(1+\sigma_L)t_L+\sigma_L\theta_N}{\varepsilon_L(1-t_L)-\sigma_L\omega_G}\right]\varepsilon_N\tilde{t}_N < 0, \qquad (5.5)$$

where the subscripts '*TR*' and '*LS*' refer, respectively, to the tax reform scenario and lump-sum recycling case. The decrease in the capital stock is smaller under tax reform than under lump-sum recycling. The labour supply response due to the fall in the labour tax dampens the reduction in the capital stock needed to return the marginal product of capital to its initial equilibrium value. Accordingly, for inelastic labour supply the fall in the capital stock is the same for both scenarios, $[\tilde{K}(\infty)]_{TR}=[\tilde{K}(\infty)]_{LS}=-\varepsilon_N\tilde{t}_N/\varepsilon_L$. With elastic labour supply but zero initial taxes, however, we also find $[\tilde{K}(\infty)]_{TR}=-\varepsilon_N\tilde{t}_N/\varepsilon_L$, but $[\tilde{K}(\infty)]_{LS}$ is still given by (3.1), so that the inequality in (5.5) continues to hold due to the compensating labour market effects.

The adjustment of Tobin's q at impact can be expressed as:

$$\left[\tilde{q}(0)\right]_{LS} < \left[\tilde{q}(0)\right]_{TR} = -\left(\frac{\varepsilon_K\chi(\theta_N,t_L)}{\omega_A}\right)\left(\frac{r}{r+h_I^*}\right)\varepsilon_N\tilde{t}_N < 0, \tag{5.6}$$

where $\chi(\theta_N,t_L)>0$ is given in (5.4). The shadow value of the capital stock falls at impact, though by less than under lump-sum recycling of the tax revenue. The reason is again the dampening effect of the labour market response due to the labour tax cut. This ensures that the path of the after-tax marginal product of capital under tax reform lies above the corresponding path for the lump-sum case, both at impact and during transition, i.e. $0>[\tilde{Y}(t)-\tilde{K}(t)-\tilde{t}_N]_{TR}>[\tilde{Y}(t)-\tilde{K}(t)-\tilde{t}_N]_{LS}$. Since Tobin's q represents the present value of this path, its path under tax reform also lies above the lump-sum path, i.e. $[\tilde{q}(t)]_{LS}<[\tilde{q}(t)]_{TR}<0$. In terms of Figure 1, the shift in the $\dot{q}(t)$ line is smallest under tax reform, the long-run effect is at E_2, the impact effect is at A′, and the saddle path is the dashed line through A′ and E_2.

Since capital is predetermined, the impact effect on output can be obtained from (5.2) by setting $\tilde{K}(0)=0$:

$$\left[\tilde{Y}(0)\right]_{LS} < \left[\tilde{Y}(0)\right]_{TR} = -\left[\frac{1-(1+\sigma_L)t_L+\sigma_L\theta_N}{(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G}\right]\varepsilon_N\tilde{t}_N < 0. \tag{5.7}$$

The reduction in output is dampened by the cut in the labour-income tax rate. Intuitively, the tax cut brings about a labour supply response which ensures that the gross wage is reduced further and employment picks up somewhat:

$$\left[\tilde{W}(0)\right]_{TR} = -\left[\frac{\varepsilon_L\left[1-(1+\sigma_L)t_L\right]+\sigma_L\left[\varepsilon_K-(1-\varepsilon_L)\theta_N\right]}{\varepsilon_L\left[(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G\right]}\right]\varepsilon_N\tilde{t}_N < \left[\tilde{W}(0)\right]_{LS} < 0, \tag{5.8}$$

$$\left[\tilde{L}(0)\right]_{LS} < \left[\tilde{L}(0)\right]_{TR} = \left[\frac{\sigma_L(\varepsilon_K-\theta_N)}{\varepsilon_L\left[(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G\right]}\right]\varepsilon_N\tilde{t}_N, \quad \left[\tilde{L}(0)\right]_{TR} \gtreqless 0. \tag{5.9}$$

If the pre-existing energy tax is low ($\theta_N<\varepsilon_K$) employment is boosted at impact due to the tax reform. A consequence of the dampened employment effect is that the reduction in the demand for energy is also dampened:

$$\left[\tilde{N}(0)\right]_{LS} < \left[\tilde{N}(0)\right]_{TR} = -\left[\frac{1-(1+\sigma_L)t_L+\sigma_L\varepsilon_K}{(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G}\right]\tilde{t}_N < 0. \qquad (5.10)$$

Finally, since in the tax-reform scenario tax revenue and thus transfers are held constant ($\tilde{G}(t)=\tilde{T}(t)=0$), (T2.12) and (T2.8) together imply that the path of full income is proportional to that of employment:

$$\left[\tilde{Y}_F(0)\right]_{TR} = \left[\frac{(1-t_L)(\varepsilon_K-\theta_N)}{(1-t_L)\left[\varepsilon_L+(1+\sigma_L)\varepsilon_K\right]-\sigma_L\omega_G}\right]\varepsilon_N\tilde{t}_N, \qquad (5.11)$$

where $[\tilde{Y}_F(0)]_{LS}$ is given in (3.6). No unambiguous conclusion can be drawn with respect to the relative size of the impact effect on full income under the two scenarios. After some manipulation, however, we obtain the following inequality:

$$\left[\tilde{Y}_F(0)\right]_{TR} \stackrel{<}{\underset{>}{=}} \left[\tilde{Y}_F(0)\right]_{LS} \quad\Leftrightarrow\quad \theta_N \stackrel{<}{\underset{>}{=}} \frac{\varepsilon_K-(1-\varepsilon_N)t_L}{\varepsilon_N}.$$

If both tax rates are zero initially ($\theta_N=t_L=0$), full income rises at impact due to the labour tax cut but the rise is in that case weaker than under lump-sum revenue recycling. The reason for this is that with elastic labour supply part of the labour tax cut is passed on to capital owners in the form of a lower wage level. For higher initial tax rates, however, the degree of tax shifting will be reduced and the rise in the after-tax wage rate will start to dominate the rise in transfers, so that $[\tilde{Y}_F(0)]_{TR}>[\tilde{Y}_F(0)]_{LS}$.

The long-run effects on the various macroeconomic variables can be obtained by using (5.5) and noting that $\tilde{q}(\infty)=0$ and $\tilde{K}(\infty)=\tilde{Y}(\infty)=\tilde{I}(\infty)$ in the steady state. For energy we find the following result:

$$\left[\tilde{N}(\infty)\right]_{LS} < \left[\tilde{N}(\infty)\right]_{TR} = -\left[\frac{(1-\varepsilon_K)\left[1-(1+\sigma_L)t_L\right]}{\varepsilon_L(1-t_L)-\sigma_L\omega_G}\right]\tilde{t}_N < 0. \qquad (5.12)$$

The long-run factor price frontier, $\varepsilon_L\tilde{W}(\infty)+\varepsilon_N\tilde{t}_N(\infty)=0$, ensures that the effect on the gross wage rate is the same under both scenarios. The labour tax cut thus translates into a higher after-tax wage and a smaller reduction in employment under tax reform than under lump-sum recycling:
At best there is no long-run employment effect under tax reform (if $\theta_N=0$ initially), but an employment expansion is impossible. Finally, the proportionality between employment and full

$$\left[\tilde{L}(\infty)\right]_{LS} < \left[\tilde{L}(\infty)\right]_{TR} = -\left[\frac{\sigma_L(1-\varepsilon_K)\theta_N}{\varepsilon_L\left[\varepsilon_L(1-t_L)-\sigma_L\omega_G\right]}\right]\varepsilon_N\tilde{t}_N \leq 0. \tag{5.13}$$

income under tax reform can be exploited to obtain the following result:

$$\left[\tilde{Y}_F(\infty)\right]_{LS} < \left[\tilde{Y}_F(\infty)\right]_{TR} = -\left[\frac{(1-\varepsilon_K)(1-t_L)\theta_N}{\varepsilon_L(1-t_L)-\sigma_L\omega_G}\right]\varepsilon_N\tilde{t}_N \leq 0. \tag{5.14}$$

The results in this section demonstrate that the relative changes in the macroeconomic variables under tax reform and lump-sum recycling are qualitatively similar. Essentially, all changes are less pronounced under the former policy because the labour-income tax cut acts as an automatic stabiliser.[1]

5.3. The distributional effects

Now that the macroeconomic effects of the tax reform have been determined, we can simply follow the analysis of section 4.1 in order to compute its welfare consequences. The change in financial wealth follows readily from $[\tilde{A}(0)]_{TR}=\omega_A[\tilde{q}(0)]_{TR}$. This also gives us the change in non-environmental welfare for the infinitely old:

$$\left[\mathrm{d}\Lambda_{NE}(-\infty,0)\right]_{LS} < \left[\mathrm{d}\Lambda_{NE}(-\infty,0)\right]_{TR} \equiv (\alpha+\beta)^{-1}\left[\tilde{q}(0)\right]_{TR} < 0.$$

Under both scenarios the very old are hurt, but the pain is least under the recycling policy.

The welfare effect for the generations born in the new steady state, $[\mathrm{d}\Lambda_{NE}(\infty,\infty)]_{TR}$, can be derived in a similar manner, by using (3.8), (4.3) and (5.14):

$$\left[\mathrm{d}\Lambda_{NE}(\infty,\infty)\right]_{LS} \leq \left[\mathrm{d}\Lambda_{NE}(\infty,\infty)\right]_{TR} \equiv \frac{r\left[\tilde{Y}_F(\infty)\right]_{TR}}{(r+\beta)(\alpha+\beta)\,\omega_H} \leq 0.$$

Hence, steady-state generations are also hurt under the tax-reform scenario, though by less than under the lump-sum scenario.

For the complete picture of changes in non-environmental welfare under tax reform we need the effect on human wealth at impact. From (3.10) and (4.3) we can conclude that the generations born at the time of the tax reform benefit from it, but we cannot determine how their gain relates to their gain under the previous scenario. The problem is caused by the fact that the impact effect on human wealth is a weighted average of the impact and long-run effects on full income (see (3.10)), but that both the weights are different between the scenarios (as $h_I>h_I^*$) and the quantities to be weighted are different (as $[\tilde{Y}_F(0)]_{TR}<[\tilde{Y}_F(0)]_{LS}$ for low tax rates and

$[\tilde{Y}_F(\infty)]_{TR}>[\tilde{Y}_F(\infty)]_{LS}$). No generally valid conclusions can be drawn, though some special cases are nevertheless instructive. If capital is highly mobile ($\sigma_A \to 0$) then both h_I and h_I^* are extremely high relative to β, and the long-run effect on full income will dominate the expression (3.10), so that $[\tilde{H}(0)]_{TR}>[\tilde{H}(0)]_{LS}$. On the other hand, if the generational-turnover effect is strong (β is high), the short-run effect on full income dominates and we conclude $[\tilde{H}(0)]_{TR}<[\tilde{H}(0)]_{LS}$ for low tax rates. This last inequality also holds generally for zero initial taxes and thus, by a continuity argument, for initial taxes close to zero. We conclude that the picture from the macroeconomic variables above broadly translates to non-environmental welfare; viz. the changes are less pronounced under tax reform.

The effects on environmental welfare are quite straightforward. Indeed, by using (5.10) and (5.12) in (4.4) (imposing $\tilde{E}(\infty)=-\sigma_N\tilde{N}(\infty)$) and noting the adjustment speed is now h_I^*, we immediately conclude that the rise in environmental welfare is smaller under tax reform:

$$\left[d\Lambda_E(0)\right]_{LS} > \left[d\Lambda_E(0)\right]_{TR} = \left(\frac{h_I^*}{\alpha+\beta+h_I^*}\right)\left(\frac{h_E}{\alpha+\beta+h_E}\right)\left[-\frac{\sigma_N\tilde{N}(0)}{h_I^*} + \frac{\tilde{E}(\infty)}{\alpha+\beta}\right] > 0.$$

The smaller reduction of the capital stock and the slower adjustment speed both directly deliver a smaller increase in environmental utility at impact. In the long run, only the capital stock effect survives (see (4.5)), which ensures that $[dU_E(\infty)]_{LS}>[dU_E(\infty)]_{TR}$. We summarize our findings from this section in the following propositions. The first of these generalizes Proposition 3 to the case of tax reform.

PROPOSITION 7: *The changes in the path of non-environmental welfare indexed by tax reform satisfy (i)* $[d\Lambda_{NE}(-\infty,0)]_{TR}<0$, *(ii)* $0<[d\Lambda_{NE}(0,0)]_{TR}$ *for low initial tax rates, and (iii)* $[d\Lambda_{NE}(\infty,\infty)]_{TR}\leq 0$. *Furthermore, (iv)* $[d\Lambda_{NE}(-\infty,0)]_{TR}<[d\Lambda_{NE}(0,0)]_{TR}$ *for low initial tax rates and (v)* $[d\Lambda_{NE}(\infty,\infty)]_{TR}<[d\Lambda_{NE}(0,0)]_{TR}$ *for all feasible rates of pre-existing taxation. Finally, the path of changes in environmental welfare satisfies (vi)* $0<[d\Lambda_E(0)]_{TR}<[d\Lambda_E(\infty)]_{TR}$ *for all feasible initial tax rates.* PROOF: See text and Heijdra and Van der Horst (1998).

The inequalities stated are exactly the same as in Proposition 3, though the conditions required by some are not necessarily the same. The next proposition specifies how the welfare effects of the two policies considered in this paper, viz. lump-sum recycling and tax reform, relate:

PROPOSITION 8: *Under the assumptions made, the paths of changes in welfare resulting from tax increase and tax reform satisfy (i)* $[d\Lambda_{NE}(-\infty,0)]_{LS}<[d\Lambda_{NE}(-\infty,0)]_{TR}$, *for all feasible rates of pre-existing taxation, (ii)* $[d\Lambda_{NE}(-0,0)]_{TR}<[d\Lambda_{NE}(-\infty,0)]_{LS}$ *for low initial tax rates, and (iii)* $[d\Lambda_{NE}(\infty,\infty)]_{LS}\leq[d\Lambda_{NE}(\infty,\infty)]_{TR}$ *for all feasible rates of pre-existing taxation. The paths of changes in* *environmental welfare satisfy (iv)* $[d\Lambda_E(0)]_{TR}<[d\Lambda_E(0)]_{LS}$ *and (v)* $[d\Lambda_E(\infty)]_{TR}< [d\Lambda_E(\infty)]_{LS}$ *for all feasible initial tax rates.*

The tax reform policy is thus qualitatively the same as the recycling policy in the sense that all changes are in the same direction. Of course, for specific generations the consequences of the two policies may differ but the broad picture is more or less the same. The main differences that exist between the welfare profiles generated by the two policies are that (i) tax reform generates slower adjustments and consequently affects more generations' welfare, but (ii) at the same time the changes are less pronounced under tax reform.

5.4. Is there a double dividend?

Over the past few years a vast literature has emerged dealing with the so-called double dividend hypothesis (See Parry (1995) and Goulder (1995) for recent papers on this topic). There are actually two versions of this hypothesis. In general, the double dividend hypothesis states that using the revenues of environmental taxes to cut distortionary taxes on labour or consumption may yield two dividends. The first dividend is the improvement in environmental quality. The second dividend is defined as a rise in employment in the European literature, whereas in the American literature it is more generally defined as the extra benefits derived from the reduction in pre-existing distortions in the economy. Up to now most studies on double dividend have been conducted by means of static representative agent models.

We focus on the European definition of the double dividend. In the short run a simultaneous increase in both environmental quality and employment will occur provided the initial energy tax is low. Equation (5.9) shows that in that case employment rises at impact due to the labour tax cut, and the process of capital decumulation is set in motion which leads to a gradual improvement in environmental quality, and an immediate green dividend, i.e. $[d\Lambda_E(0)]_{TR}>0$, without immediately offsetting the employment effects of the labour tax cut. This is sufficient for establishing a double dividend as both dividends are generation-independent so that redistributional aspects play no role here.

The environmental gain is persistent and growing as we have seen, but in the long run the employment dividend will disappear. The decrease in the capital stock depresses labour demand and the concomitant erosion of the tax base necessitates a gradual increase in the labour tax rate which in turn depresses labour supply. Both effects result in lower employment levels. Indeed, as was shown in (5.13), at best there is no long-run employment effect under tax reform (if $\theta_N=0$ initially), but an employment expansion is impossible.

Table A.1: Log-linearized version of the generalized model

$$\dot{\tilde{A}}(t) = \left[r-(\alpha+\beta)\right]\tilde{A}(t) - (\alpha+\beta)\tilde{H}(t) + r\tilde{Y}_F(t) \qquad \text{(TA.1)}$$

$$\dot{\tilde{H}}(t) = (r+\beta)\tilde{H}(t) - r\tilde{Y}_F(t) \qquad \text{(TA.2)}$$

$$\dot{\tilde{K}}(t) = (r\omega_I/\omega_A)\left[\tilde{I}(t) - \tilde{K}(t)\right] \qquad \text{(TA.3)}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) + (r/\omega_A)\left[\omega_L\tilde{W}(t) + \omega_N\tilde{t}_N(t)\right] \qquad \text{(TA.4)}$$

$$\dot{\tilde{E}}(t) = -h_E\left[\tilde{E}(t) + \sigma_N\tilde{N}(t)\right] \qquad \text{(TA.5)}$$

$$\tilde{Y}(t) - \tilde{L}(t) = \sigma_{LZ}\tilde{W}(t) \qquad \text{(TA.6)}$$

$$\tilde{N}(t) = \tilde{K}(t) - (\sigma_{KN}/\omega_K)\left[\omega_L\tilde{W}(t) + (1-\omega_L)\tilde{t}_N(t)\right] \qquad \text{(TA.7)}$$

$$\tilde{L}(t) = \sigma_L\left[\tilde{W}(t) - \tilde{t}_L(t)\right] \qquad \text{(TA.8)}$$

$$\tilde{q}(t) = \sigma_A\left[\tilde{I}(t) - \tilde{K}(t)\right] \qquad \text{(TA.9)}$$

$$\tilde{Y}(t) = \omega_L\tilde{L}(t) + \omega_K\tilde{K}(t) + \omega_N\tilde{N}(t) \qquad \text{(TA.10)}$$

$$\tilde{T}(t) = \omega_L t_L\left[\tilde{W}(t)+\tilde{L}(t)\right] + \omega_L(1-t_L)\tilde{t}_L(t) + \theta_N\omega_N\tilde{N}(t) + \omega_N\tilde{t}_N(t) \qquad \text{(TA.11)}$$

$$\tilde{Y}_F(t) = \omega_L(1-t_L)\left[\tilde{W}(t) - \tilde{t}_L(t)\right] + \tilde{G}(t) \qquad \text{(TA.12)}$$

$$\omega_C \tilde{C}(t) = \omega_X \tilde{X}(t) + \sigma_L \left[\tilde{Y}_F(t) - \tilde{G}(t)\right], \quad \omega_X \tilde{X}(t) = \left((\alpha+\beta)/r\right)\left[\tilde{A}(t) + \tilde{H}(t)\right] \qquad \text{(TA.13)}$$

$$\tilde{A}(t) = \omega_A \left[\tilde{K}(t) + \tilde{q}(t)\right] + \tilde{B}(t) + \tilde{F}(t) \qquad \text{(TA.14)}$$

$$\dot{\tilde{B}}(t) = r\left[\tilde{B}(t) + \tilde{G}(t) - \tilde{T}(t)\right] \qquad \text{(TA.15)}$$

Steady-state shares:

ω_L	$F_L L/Y = WL/Y$	Share of labour in national income.
ω_N	$F_N N/Y = (1+t_N)P_N N/Y$	Share of energy in national income.
ω_K	$F_K K/Y$	Share of capital in national income.
ω_I	I/Y	Share of firm investment in national income.
ω_X	X/Y	Share of full consumption in national income.
ω_A	$rA/Y = rqK/Y$	Share of asset income in national income.
ω_C	C/Y	Share of consumption in national income.
ω_G	$G/Y = T/Y$	Share of transfers and tax revenue in national income.

Relationships between shares and parameters:

$\omega_C = \omega_A + (1-t_L)\omega_L + \omega_G$
$\omega_C = 1 - \omega_I - \omega_N(1-\theta_N)$
$\omega_C = \omega_X + \sigma_L \omega_L (1-t_L)/(1+\sigma_L)$
$\omega_A + \omega_I = \omega_K$
$\omega_G = \omega_L t_L + \omega_N \theta_N$
$\omega_L + \omega_K + \omega_N = 1$
$\theta_N \equiv t_N/(1+t_N)$

Notes: (a) We have used the normalization $B=F=0$ initially.

(b) $\sigma_A \equiv -(I/K)(\phi''/\phi') \geq 0$, represents the degree of concavity of the installation cost function. A low value for σ_A implies that physical capital is highly mobile, with the limiting case of $\sigma_A=0$ (no adjustment costs) representing perfect mobility of capital.

(c) The relative price of energy is assumed to be constant.

1. To complete the discussion of the macroeconomic effects, it is noted that the transition paths for the variables discussed in this section can all be written as a weighted average of the impact effect and the long-run effect, with respective time-varying weights $\exp[-h_I^* t]$ and $1-\exp[-h_I^* t]$.