Foundations of Modern Macroeconomics

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Solutions for problems to Chapter 17

Question 1

Part (a)

The easiest way to derive the expression for the rental rate on capital is to model explicitly the investment process. If an investor purchases K_t units of capital in period t then he will obtain future rental payments, R^K_{t+1}, plus the undepreciated part of the capital stock, $(1-\delta)\,K_t$. Hence, the profit from investing is:

$$\Pi^I_t \equiv -K_t + \frac{R^K_{t+1}K_t + (1-\delta)K_t}{1+r_{t+1}}, \tag{A1}$$

where future revenues are discounted with the real interest rate, r_{t+1}. The investment profit is maximized by choice of K_t and the first-order condition (for an interior solution, with $K_t > 0$) is:

$$\frac{d\Pi^I_t}{dK_t} = -1 + \frac{R^K_{t+1} + (1-\delta)}{1+r_{t+1}} = 0, \tag{A2}$$

from which we derive that $r_{t+1} = R^K_{t+1} - \delta$.

Part (b)

The household's consolidated budget restriction is obtained by combining (2)-(3):

$$W_t = C^Y_t + \frac{C^O_{t+1}}{1+r_{t+1}}. \tag{A3}$$

The young household chooses C^Y_t and C^O_{t+1} (and, implicitly, S_t) in order to maximize Λ^Y_t (given in (1)). The Lagrangian expression for this optimization problem is:

$$\mathcal{L} \equiv \log C^Y_t + \left(\frac{1}{1+\rho}\right)\log C^O_{t+1} + \lambda\left[W_t - C^Y_t - \frac{C^O_{t+1}}{1+r_{t+1}}\right],$$

where λ is the Lagrange multiplier. The first-order conditions are the constraint and $\partial L/\partial C_t^Y = \partial L/\partial C_{t+1}^O = 0$:

$$\frac{1}{C_t^Y} = \lambda, \tag{A4}$$

$$\left(\frac{1}{1+\rho}\right)\frac{1}{C_{t+1}^O} = \frac{\lambda}{1+r_{t+1}}. \tag{A5}$$

By combining (A4)-(A5) we find the consumption Euler equation:

$$\frac{C_{t+1}^O}{C_t^Y} = \frac{1+r_{t+1}}{1+\rho}. \tag{A6}$$

According to (A6), if r_{t+1} exceeds (falls short of) ρ, then future consumption will be higher (lower) than current consumption. By substituting (A6) into the lifetime budget constraint (A3) we can find the solutions for C_t^Y and C_{t+1}^O. For C_t^Y we find:

$$\begin{aligned} C_t^Y + \frac{C_{t+1}^O}{1+r_{t+1}} &= W_t \quad \Leftrightarrow \\ C_t^Y + \frac{C_t^Y}{1+\rho} &= W_t \quad \Leftrightarrow \\ C_t^Y &= \left(\frac{1+\rho}{2+\rho}\right) W_t. \end{aligned} \tag{A7}$$

By using (A7) in, respectively, (A6) and (2) we find the expressions for C_{t+1}^O and S_t:

$$C_{t+1}^O = \left(\frac{1+r_{t+1}}{1+\rho}\right) C_t^Y = \left(\frac{1+r_{t+1}}{2+\rho}\right) W_t, \tag{A8}$$

$$S_t = W_t - C_t^Y = \frac{W_t}{2+\rho}. \tag{A9}$$

We observe that saving is proportional to wage income of the young.

The representative firm hires capital and labour from the households. After-tax profit is defined as:

$$\Pi_t \equiv (1-\tau)\,F\,(K_t, L_t) - W_t L_t - R_t^K K_t, \tag{A10}$$

where τ is an output tax used in part (e) of this question. Profit maximization yields the usual first-order conditions:

$$\frac{\partial \Pi_t}{\partial L_t} = 0: \qquad (1-\tau)F_L(K_t, L_t) = W_t, \tag{A11}$$

$$\frac{\partial \Pi_t}{\partial K_t} = 0: \qquad (1-\tau)F_K(K_t, L_t) = R_t^K. \tag{A12}$$

Since technology features CRTS, excess profits are zero ($\Pi_t = 0$).

To derive the link between saving by the young and the future capital stock, we need to do some bookkeeping. The economy-wide resource constraint is:

$$Y_t + (1-\delta)K_t = K_{t+1} + C_t, \tag{A13}$$

where C_t is total consumption (by young and old). The young consume $C_t^Y = W_t - S(W_t)$ (where $S(\cdot)$ is given in (A9) above) and the old consume $C_t^O = (r_t + \delta) K_t + (1-\delta)K_t$. Since we abstract from population growth, total consumption is:

$$\begin{aligned} C_t &\equiv C_t^Y + C_t^O \\ &= W_t - S(W_t) + (r_t + \delta) K_t + (1-\delta)K_t. \end{aligned} \tag{A14}$$

With a zero output tax ($\tau = 0$) we have that $F(K_t, L_t) = W_t + (r_t + \delta) K_t$ so that (A14) simplifies to:

$$Y_t + (1-\delta)K_t = C_t + S(W_t). \tag{A15}$$

Finally, by equating (A13) and (A15) we find the desired expression linking $S(W_t)$ and K_{t+1}:

$$K_{t+1} = S(W_t). \tag{A16}$$

Part (c)

According to (A11), the wage rate equals the marginal product of capital (recall that $\tau = 0$ here). By using (5) we find:

$$\begin{aligned} F_L(K_t, L_t) &= A\left[\alpha K_t^{(\sigma-1)/\sigma} + (1-\alpha)L_t^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)-1}(1-\alpha)L_t^{-1/\sigma} \\ &= (1-\alpha)A\left[\alpha K_t^{(\sigma-1)/\sigma} + 1 - \alpha\right]^{1/(\sigma-1)}, \end{aligned} \tag{A17}$$

where we have substituted $L_t = 1$ in the second line. By using (A17) and noting that $W_t = F_L(K_t, 1)$ we find:

$$\frac{W_t}{K_t} = (1-\alpha)A\,\frac{\left[\alpha K_t^{(\sigma-1)/\sigma} + 1 - \alpha\right]^{1/(\sigma-1)}}{K_t}. \tag{A18}$$

We use (A18) to derive the various limits. For the lower limit we find:

$$\lim_{K_t \to 0} \frac{W_t}{K_t} = (1-\alpha)A\,\frac{\lim_{K_t \to 0}\left[\alpha K_t^{(\sigma-1)/\sigma} + 1 - \alpha\right]^{1/(\sigma-1)}}{\lim_{K_t \to 0} K_t} = \infty. \tag{A19}$$

The upper limit is most easily computed by first rewriting (A18) somewhat:

$$\frac{W_t}{K_t} = (1-\alpha)A\,\frac{\left[\alpha + (1-\alpha)K_t^{(1-\sigma)/\sigma}\right]^{1/(\sigma-1)}K_t^{1/\sigma}}{K_t}. \tag{A20}$$

By letting $K_t \to \infty$ in (A20) we find that:

$$\lim_{K_t \to \infty} \frac{W_t}{K_t} = (1-\alpha)A\alpha^{1/(\sigma-1)} \lim_{K_t \to \infty} K_t^{(1-\sigma)/\sigma} = 0, \tag{A21}$$

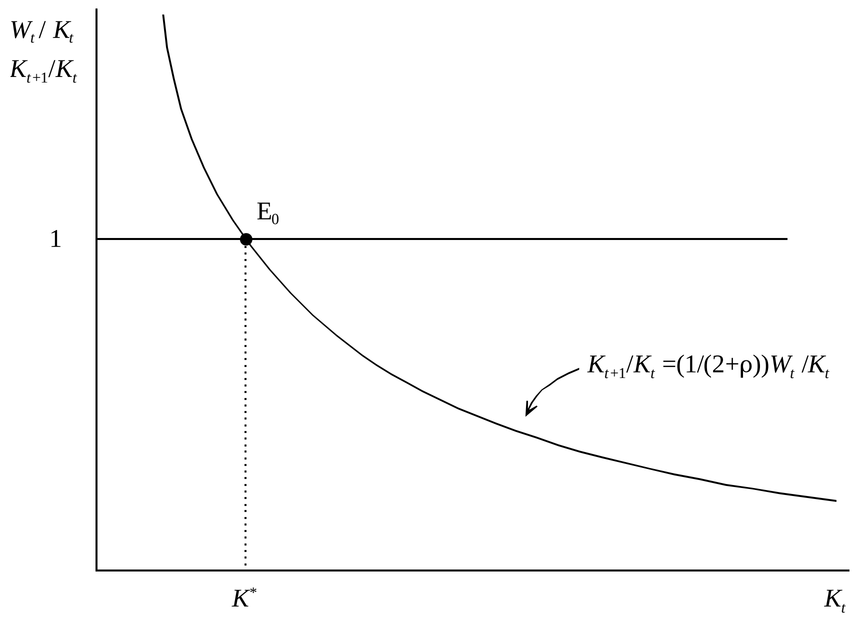

Figure 1: Hampered endogenous growth in the OLG model

where we have used the fact that $\lim_{K_t \to \infty} K_t^{(1-\sigma)/\sigma} = 0$ as $\sigma > 1$. It follows that, for $\sigma > 1$, the ratio between wages and the capital stock goes to zero as the capital stock gets large.

In Figure 1 we illustrate the dynamic properties of the model. On the vertical axis we measure W_t/K_t and K_{t+1}/K_t and on the horizontal axis K_t. The horizontal line depicts steady states, for which $K_{t+1}/K_t = 1$. The downward sloping line is the graphical representation of (A16) (with equation (A9) inserted):

$$\frac{K_{t+1}}{K_t} = \left(\frac{1}{2+\rho}\right)\frac{W_t}{K_t}. \tag{A22}$$

In view of (A19) and (A21), this line is vertical near the origin and approaches the horizontal axis as the capital stock gets large. It follows that the model possesses a unique steady state at E_0 and that growth vanishes in the long run, i.e. $\gamma_\infty^K = 0$.

Part (d)

Despite the fact that this is a "capital-fundamentalist" model, with easy substitution between capital and labour, there is no long-run growth. Intuitively this is because the savings rate is too low. Savings depends on wages of the young (see (A9)) but as K_t gets large the W_t/K_t ratio (and thus also the S_t/K_t ratio) gets smaller and smaller. This excludes the possibility of endogenous growth as there exists an upper limit on the amount of capital than can be sustained by the savings plans of the young.

Part (e)

The budget identity (2) is changed to:

$$C_t^Y + S_t = W_t + T_t^Y, \tag{A23}$$

so that (A3) becomes:

$$W_t + T_t^Y = C_t^Y + \frac{C_{t+1}^O}{1+r_{t+1}}. \tag{A24}$$

By using (A6) and (A24) we find that consumption during youth increases as a result of the transfers:

$$C_t^Y = \left(\frac{1+\rho}{2+\rho}\right)\left[W_t + T_t^Y\right]. \tag{A25}$$

By using (A25) in (A23) we find that the savings equation is given by:

$$S_t = \left(\frac{1}{2+\rho}\right)\left[W_t + T_t^Y\right]. \tag{A26}$$

Hence, saving also increases as a result of the transfers. This is obvious, as the household wants to use some of the transfers to support a higher consumption level during old age.

The government budget identity is given by:

$$\tau Y_t = T_t^Y. \tag{A27}$$

For the representative firms the expressions in (A11)-(A12) are all still valid. The resource constraint is also still as presented in (A13). Total consumption, C_t, is now:

$$\begin{aligned} C_t &\equiv C_t^Y + C_t^O \\ &= W_t + T_t^Y - S\left(W_t + T_t^Y\right) + (r_t+\delta)K_t + (1-\delta)K_t. \end{aligned} \tag{A28}$$

We find from (A10)-(A12) that:

$$(1-\tau)Y_t = W_t + (r_t+\delta)K_t. \tag{A29}$$

By using (A27) and (A29) in (A28) we find equation (A15) again. It follows that $K_{t+1} = S(W_t + \tau Y_t)$ or, by using (A26), that:

$$\frac{K_{t+1}}{K_t} = \left(\frac{1}{2+\rho}\right)\left[\frac{W_t}{K_t} + \frac{\tau Y_t}{K_t}\right]. \tag{A30}$$

Equation (A30) is the fundamental difference equation for the capital stock.

In order to illustrate the possibility of endogenous growth we must first figure out what happens to Y_t/K_t as K_t gets large. We derive from (5):

$$\begin{aligned} \frac{Y_t}{K_t} &= \frac{A}{K_t}\left[\alpha K_t^{(\sigma-1)/\sigma} + (1-\alpha)L_t^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ &= A\left[K_t^{(1-\sigma)/\sigma}\left(\alpha K_t^{(\sigma-1)/\sigma} + (1-\alpha)L_t^{(\sigma-1)/\sigma}\right)\right]^{\sigma/(\sigma-1)} \\ &= A\left[\alpha + (1-\alpha)K_t^{(1-\sigma)/\sigma}\right]^{\sigma/(\sigma-1)}, \end{aligned} \tag{A31}$$

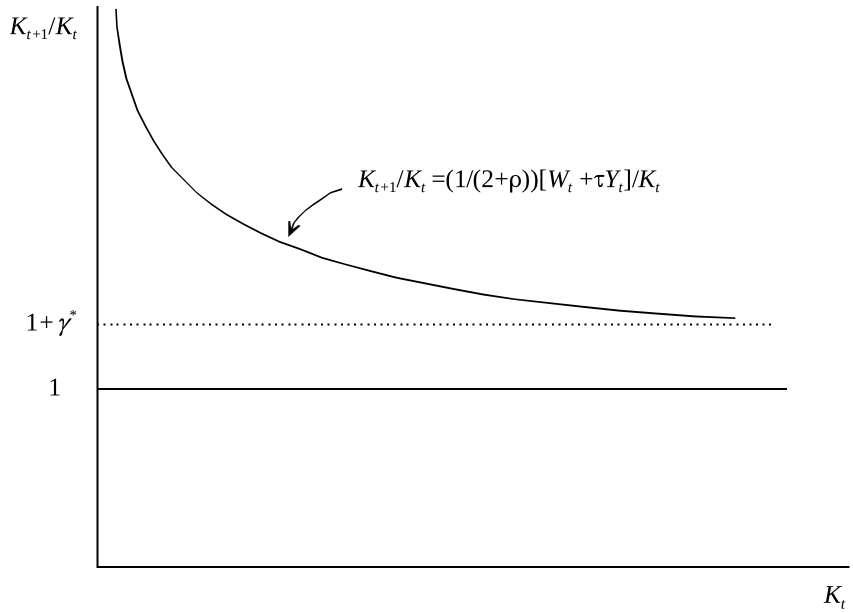

Figure 2: Endogenous growth with transfers to the young

where we have substituted $L_t = 1$ in getting to the final expression. It follows from (A31) that:

$$\lim_{K_t \to \infty} \frac{Y_t}{K_t} = A\alpha^{\sigma/(\sigma-1)} > 0. \tag{A32}$$

By taking limit on both sides of (A30) and noting (A21) and (A32) we find that:

$$\begin{aligned} \lim_{K_t \to \infty} \frac{K_{t+1}}{K_t} &= \left(\frac{1}{2+\rho}\right)\left[\lim_{K_t \to \infty} \frac{W_t}{K_t} + \tau \lim_{K_t \to \infty} \frac{Y_t}{K_t}\right] \\ &= \left(\frac{\tau}{2+\rho}\right) A\alpha^{\sigma/(\sigma-1)}. \end{aligned} \tag{A33}$$

There are now two possibilities:

- If the right-hand side of (A33) is less than unity, then there is still no endogenous growth. The economy reaches a unique steady state, just as in Figure 1.
- If the right-hand side of (A33) is larger than unity, then there is endogenous growth as K_{t+1}/K_t is bounded away from the steady-state line.

These two possibilities have been summarized mathematically in equation (6). Figure 2 illustrates the endogenous growth case. The endogenous growth rate is increasing in the output tax, τ. Intuitively, the output tax redistributes resources from the old (who dissave) to the young (who save). This enables the young to sustain the level of saving required for perpetual growth.

Question 2

Part (a)

The government budget identity is:

$$G_t + (1+r_t)B_t = L_t T_t^Y + L_{t-1} T_t^O + B_{t+1}, \tag{A1}$$

where B_t is the total stock of government debt at the beginning of period t (which is in the hands of the old). The left-hand side of (A1) represents total spending on government consumption plus interest payment and debt redemption. The right-hand side represents total government revenue, consisting of tax revenues plus bond sales (to the young). By noting that $L_t = (1+n)L_{t-1}$, $g_t \equiv G_t/L_t$ and $b_t \equiv B_t/L_t$ we can rewrite (A1) in per capita terms as follows:

$$\begin{aligned} \frac{G_t}{L_t} + (1+r_t)\frac{B_t}{L_t} &= T_t^Y + \frac{L_{t-1}}{L_t} T_t^O + \frac{L_{t+1}}{L_t}\frac{B_{t+1}}{L_{t+1}} \quad \Leftrightarrow \\ g_t + (1+r_t)\, b_t &= T_t^Y + \frac{T_t^O}{1+n} + (1+n) b_{t+1}. \end{aligned} \tag{A2}$$

Part (b)

Instead of (17.2)-(17.3), the household faces the following budget identities:

$$C_t^Y + S_t = W_t - T_t^Y, \tag{A3}$$

$$C_{t+1}^O = (1+r_{t+1})S_t - T_{t+1}^O. \tag{A4}$$

By eliminating S_t from (A3)-(A4) we find the consolidated lifetime budget constraint:

$$\hat{W}_t \equiv W_t - T_t^Y - \frac{T_{t+1}^O}{1+r_{t+1}} = C_t^Y + \frac{C_{t+1}^O}{1+r_{t+1}}, \tag{A5}$$

where $\hat{W}_t$ thus represents after-tax human wealth of the young household.

In view of (17.5) and (1) we find that the consumption Euler equation is:

$$\frac{C_{t+1}^O}{C_t^Y} = \frac{1+r_{t+1}}{1+\rho}. \tag{A6}$$

By combining (A5) and (A6) we find the solutions for C_t^Y and C_{t+1}^O:

$$C_t^Y = \left(\frac{1+\rho}{2+\rho}\right)\hat{W}_t, \tag{A7}$$

$$C_{t+1}^O = \left(\frac{1+r_{t+1}}{2+\rho}\right)\hat{W}_t. \tag{A8}$$

Finally, by substituting (A7) into (A3) we find the saving function:

$$S_t = \left(\frac{1}{2+\rho}\right)\left[W_t - T_t^Y\right] + \left(\frac{1+\rho}{2+\rho}\right)\frac{T_{t+1}^O}{1+r_{t+1}}. \tag{A9}$$

Ceteris paribus, saving depends negatively on taxes during youth and positively on taxes during old age.

To establish the link between saving and capital formation we must do some bookkeeping. The resource constraint is:

$$Y_t + (1-\delta)K_t = K_{t+1} + C_t + G_t, \tag{A10}$$

where C_t is total consumption. Consumption by the two demographic groups is given by:

$$\begin{aligned} L_{t-1}C_t^O &= (r_t+\delta)\,K_t + (1-\delta)K_t + (1+r_t)\,B_t - L_{t-1}T_t^O, && \text{(A11)} \\ L_t C_t^Y &= L_t\left[W_t - T_t^Y - S(\cdot)\right], && \text{(A12)} \end{aligned}$$

where $S(\cdot)$ is the saving function defined in (A9) above. According to (A11), the old consume their assets (inclusive of interest payments) minus the taxes they face. By using (A11)-(A12) and noting that $Y_t = (r_t+\delta)\,K_t + W_t$ we find that total consumption can be written as:

$$\begin{aligned} C_t &\equiv L_{t-1}C_t^O + L_t C_t^Y \\ &= Y_t + (1-\delta)K_t + \left[(1+r_t)\,B_t - L_{t-1}T_t^O - L_t T_t^Y\right] - L_t S(\cdot) \\ &= Y_t + (1-\delta)K_t + B_{t+1} - G_t - L_t S(\cdot), \end{aligned} \tag{A13}$$

where we have used the government budget identity (A1) in getting to the final expression. By comparing (A10) and (A13) we find the link between household saving and capital formation:

$$\begin{aligned} K_{t+1} + C_t + G_t &= C_t + G_t - B_{t+1} + L_t S(\cdot) \quad \Leftrightarrow \\ L_t S(\cdot) &= B_{t+1} + K_{t+1}. \end{aligned} \tag{A14}$$

Equation (A14) coincides with (17.58) in the text. Total saving by the young (left-hand side) equals next period's stock of assets, consisting of physical capital and government debt (right-hand side). In per capita terms, equation (A14) can be rewritten as:

$$S(\cdot) = (1+n)\left[b_{t+1} + k_{t+1}\right], \tag{A15}$$

where $k_t \equiv K_t/L_t$ is the capital labour ratio. To demonstrate the redundancy of one of b_t, T_t^Y, and T_t^O we define following so-called *effective taxes* (Ihori, 1996, p. 201):

$$\hat{T}_t^Y \equiv T_t^Y + (1+n)b_{t+1}, \qquad \hat{T}_t^O \equiv T_t^O - (1+r_t)(1+n)b_t. \tag{A16}$$

By using these definitions in (A2) we find that the government budget identity can be rewritten as:

$$\begin{aligned} g_t &= \left[T_t^Y + (1+n)b_{t+1}\right] + \frac{1}{1+n}\left[T_t^O - (1+n)(1+r_t)b_t\right] \\ &= \hat{T}_t^Y + \frac{\hat{T}_t^O}{1+n}. \end{aligned} \tag{A17}$$

Similarly, by using (A3), (A15)-(A16) we find:

$$\begin{aligned} W_t - T_t^Y - C_t^Y &= (1+n)\left[b_{t+1} + k_{t+1}\right] \quad \Leftrightarrow \\ C_t^Y &= W_t - \hat{T}_t^Y - (1+n)k_{t+1}. \end{aligned} \tag{A18}$$

Equation (A11) can be rewritten by using (A16) as:

$$\begin{aligned} C_t^O &= (1+r_t)(1+n)\left[b_t + k_t\right] - T_t^O \\ &= (1+r_t)(1+n)k_t - \hat{T}_t^O. \end{aligned} \tag{A19}$$

Since W_t and r_t only depend on k_t (see (17.15)-(17.16) in the book), the economy is fully characterized by equations (A6) and (A17)-(A19). The key thing to note is that *only the effective taxes appear in these equations*, not the separate components b_t, T_t^Y, and T_t^O. It follows that one of these three components making up the effective taxes is redundant.

Part (c)

The fundamental difference equation for this case is obtained by using (A9) and (A15) and noting that $T_t^O = T^O$ and $b_{t+1} = b_t = 0$ for all t:

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)}\left[W(k_t) - T_t^Y + \frac{1+\rho}{1+r(k_{t+1})}T^O\right], \tag{A20}$$

where for the Cobb-Douglas technology $W_t = W(k_t) \equiv \epsilon_L k_t^{1-\epsilon_L}$ and $r_t = r(k_t) \equiv (1-\epsilon_L)k_t^{-\epsilon_L}$. By differentiating (A20) with respect to k_{t+1}, k_t and T_t^Y we find:

$$\Delta dk_{t+1} = W'(k_t)dk_t - dT_t^Y, \tag{A21}$$

where Δ is defined as follows:

$$\Delta \equiv \left[(1+n)(1+\rho) + \frac{(1+\rho)\,T^O}{(1+r)^2}r'(k_{t+1})\right]. \tag{A22}$$

If $T^O \leq 0$ it follows automatically that $\Delta > 0$ (since $r'(\cdot) < 0$) but if $T^O > 0$ the sign of Δ is ambiguous. We assume that $\Delta > 0$. Since $W'(k_t) > 0$, the stability condition for the model is then:

$$0 < \frac{W'(k_t)}{\Delta} < 1 \quad \Leftrightarrow \quad 0 < \frac{dk_{t+1}}{dk_t} < 1. \tag{A23}$$

In the sequel we assume that (A23) is satisfied. By using (A2) and noting that $b_{t+1} = b_t = 0$ and $T_t^O = T^O$ we find that the tax on the young changes according to $dT_t^Y = dg$. By using this result in (A21) we find the impact and long-run multipliers:

$$\frac{\partial k_{t+1}}{\partial g} = -\frac{1}{\Delta} < 0, \qquad \frac{dk_\infty}{dg} = -\frac{1}{\Delta - W'(k_\infty)} < 0. \tag{A24}$$

At impact, the capital stock is predetermined ($dk_t = 0$) and the increase in government consumption crowds out investment ($\partial k_{t+1}/\partial g < 0$). In the long run the capital stock is crowded out even further because the future wage rate declines and the future interest rate increases (both these effects depress future saving).

Part (d)

The fundamental difference equation for this case is obtained by using (A9) and (A15) and noting that $T_t^Y = T^Y$ and $b_{t+1} = b_t = 0$ for all t:

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} \left[W(k_t) - T^Y + \frac{1+\rho}{1+r(k_{t+1})} T_{t+1}^O \right]. \tag{A25}$$

The government budget constraint simplifies for this case to:

$$T_t^O = (1+n)\,(g_t - T^Y). \tag{A26}$$

By substituting (A26) into (A25) we find:

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} \left[W(k_t) - T^Y + \frac{(1+\rho)(1+n)}{1+r(k_{t+1})} \left(g_{t+1} - T^Y\right) \right]. \tag{A27}$$

The stability condition is still given by (A23) and the impact and long-run multipliers are:

$$\frac{\partial k_{t+1}}{\partial g} = \frac{1}{(1+r)\,\Delta} > 0, \qquad \frac{dk_\infty}{dg} = \frac{(1+\rho)(1+n)}{(1+r)\,[\Delta - W'(k_\infty)]} > 0. \tag{A28}$$

At impact there is a positive effect on capital formation because the young anticipate higher future taxes and save more as a result (see also (A9) above). In the long run two things happen. First, the wage rises because there is more capital per worker. Second, the interest rate falls (due to capital formation) and the present value of taxes during old age ($T_{t+1}^O/(1+r_{t+1})$) rises. Both effects explain that the long-run effect is larger than the short-run effect on capital.

The lesson we learn from parts (c)-(d) is that the effect on the capital stock of an increase in public consumption depends very much on the financing method employed by the policy maker. If the young must finance this fiscal policy then capital formation is harmed. The opposite holds if the old must pay for the additional public consumption.

Part (e)

Since the government does not possess a full set of age-specific taxes, there is no redundant tax parameter any more. Indeed, in view of (A16) and the assumption that $T_t^O = 0$ for all t it follows that changing b must change both $\hat{T}_t^Y$ and $\hat{T}_t^O$. In this setting government debt has real effects. By using (A2) we find that the government budget identity simplifies to:

$$g + (1+r_t)b = T_t^Y + (1+n)b, \tag{A29}$$

so that the change in taxes on the young satisfies:

$$dT_t^Y = (r_t - n)\,db > 0, \tag{A30}$$

where the sign follows from the assumption that the economy is dynamically efficient (so that $r_t > n$). The fundamental difference equation for the capital stock is obtained by using (A9) and (A15):

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)} \left[W(k_t) - T_t^Y \right] - b. \tag{A31}$$

The stability condition for the model is still as in (A23) (with $T^O = 0$ imposed). The impact effect on capital of an increase in b is:

$$\begin{aligned} \frac{\partial k_{t+1}}{\partial b} &= -\frac{1}{(1+n)(2+\rho)} \frac{dT_t^Y}{db} - 1 \\ &= -\left[1 + \frac{r_t - n}{(1+n)(2+\rho)} \right] < -1, \end{aligned} \tag{A32}$$

where we have used (A30) to get from the first to the second line. The increase in debt crowds out capital formation because the tax on the young has risen. The long-run effect on the capital stock is:

$$\frac{dk_\infty}{db} = -\frac{r - n + (1+n)(1+\rho)}{(1+n)(1+\rho) - W'(k_\infty)} < 0. \tag{A33}$$

References

Ihori, T. (1996). *Public Finance in an Overlapping Generations Economy.* Macmillan Press, London.