Foundations of Modern Macroeconomics

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Solutions for problems to Chapter 10

Question 1

In this question we study the classic paper by Poole (1970). This paper shows how optimal economic policy was typically determined in the literature predating the rational expectations revolution of the early 1970s.

Part (a)

Equation (1) is the IS curve. Demand for goods is affected negatively by the interest rate (through investment) and there are stochastic demand shocks, represented by U_t. Equation (2) is the LM curve. The demand for real money balances (in logarithms) depends positively on output (transactions demand) and negatively on the nominal interest rate (opportunity cost of holding money). We abstract from (expected) inflation, so the nominal interest rate features in the IS curve.

Equation (3) is the *objective function* of the policy maker. The policy maker wishes to steer actual output, y_t, as closely as possible to some exogenously given *target* output level, y^*. Both positive and negative deviations of actual from target output are not appreciated by the policy maker. The policy maker must either use the interest rate or the money supply as its *instrument*.

Part (b)

In the deterministic case, both stochastic shocks are identically equal to zero ($U_t = V_t = 0$ for all t). It is easy to show that in that case both instruments serve equally well to stabilize output. In fact, they are completely identical. The equivalence result is shown in Figure 1. Assume that the economy is initially in point E_0 where output falls short of its target level ($y_t < y^*$).

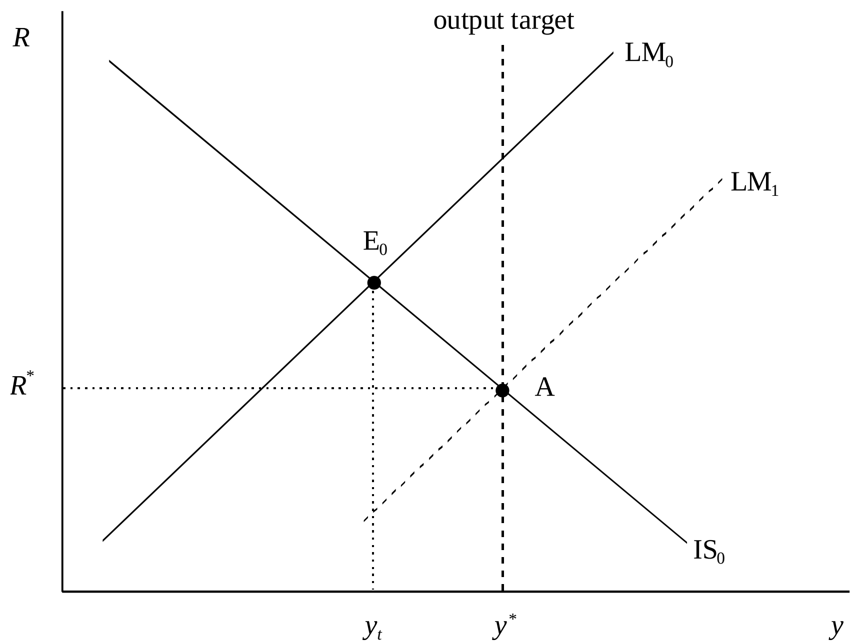

Figure 1: Money supply and interest rate instruments

Under the *interest rate instrument* the policy maker set $R = R^*$, where R^* is the interest rate for which the IS curve gives an output level equal to y^*–see point A in Figure 1. The LM curve is not very informative under the interest rate instrument because the money supply is endogenous, i.e. if the policy maker sets $R = R^*$ then the money supply will adjust such that IS_0 and the dashed LM curve, LM_1, intersect at point A. R^* can be computed by substituting $y = y^*$ (and $U_t = 0$) into the IS curve and solving for R^*:

$$y^* = \alpha_0 - \alpha_1 R^* \qquad \Leftrightarrow \qquad R^* = \frac{\alpha_0 - y^*}{\alpha_1}. \tag{A1}$$

The money supply that will result is computed by substituting $y_t = y^*$ and $R_t = R^*$ (and, of course, $p_t = 0$ and $V_t = 0$) in the LM curve (2). We obtain:

$$\begin{aligned} m_t &= \beta_0 + \beta_1 \left[\alpha_0 - \alpha_1 R^*\right] - \beta_2 R^* \\ &= \beta_0 + \alpha_0 \beta_1 - \left(\alpha_1 \beta_1 + \beta_2\right) R^*. \end{aligned} \tag{A2}$$

Under the *money supply instrument*, the policy maker sets the money supply, m^*, such that the IS-LM equilibrium occurs at an output level of y^*. This is obviously also at point A. The endogenous variables under this policy instrument are output and the interest rate. The correct money supply is thus:

$$m^* = \beta_0 + \alpha_0 \beta_1 - \left(\alpha_1 \beta_1 + \beta_2\right) R^* \tag{A3}$$

$$= \beta_0 + \alpha_0 \beta_1 - \left(\alpha_1 \beta_1 + \beta_2\right) \left(\frac{\alpha_0 - y^*}{\alpha_1}\right). \tag{A4}$$

The two instruments are equivalent and both hit the output target exactly, i.e. $y_t = y^*$ for all t and $\Omega = 0$. The instrument equivalency result breaks down in a stochastic setting, as parts (c)-(e) of this question illustrate.

Part (c)

If only the IS curve is subject to stochastic shocks, then the money supply and interest rate instruments are no longer equivalent. Under the interest rate instrument, the policy maker will set R^* such that the IS curve is *expected* to yield an output level of y^*, i.e. R^* will be set according to the value given in (A1). The actual output level is, of course, stochastic, because demand for goods is subject to stochastic shocks that the policy maker (and the public) cannot forecast. Hence, actual output is obtained by substituting R^* (from (A1)) into equation (1):

$$\begin{aligned} y_t &= \alpha_0 - \alpha_1 R^* + U_t \\ &= \alpha_0 - \alpha_1 \left(\frac{\alpha_0 - y^*}{\alpha_1} \right) + U_t \\ &= y^* + U_t. \end{aligned} \tag{A5}$$

Given the interest rate instrument, actual output fluctuates randomly around its target level y^*. The asymptotic variance of output will thus be:

$$\sigma_y^2 \big|_{R=R^*} = \sigma_U^2. \tag{A6}$$

Under the money supply instrument, the policy maker sets the money supply such that the expected IS-LM intersection occurs at an output level equal to y^*, i.e. it sets $m = m^*$, where m^* is given in (A3) or (A4). Since the IS curve shifts stochastically, the actual IS-LM intersection will be somewhere along the given LM curve based on $m = m^*$. To find actual output level, we first solve equations (1)-(2) for output:

$$\begin{aligned} y_t &= \alpha_0 - \frac{\alpha_1}{\beta_2} \left[\beta_0 + \beta_1 y_t + V_t - m_t + p_t \right] + U_t \qquad \Rightarrow \\ y_t &= \frac{\alpha_0 \beta_2 - \alpha_1 \beta_0 + \alpha_1 m^* + \beta_2 U_t}{\beta_2 + \alpha_1 \beta_1}, \end{aligned} \tag{A7}$$

where we have substituted $p_t = V_t = 0$ and $m_t = m^*$ for all t in going from the first to the second line. By substituting the expression for m^* (given in (A4)) into (A7), we obtain an even simpler expression:

$$y_t = y^* + \left(\frac{\beta_2}{\beta_2 + \alpha_1 \beta_1} \right) U_t. \tag{A8}$$

Actual output fluctuates randomly around its target level, just like for the interest rate instrument. The crucial difference between the two instruments is, however, that the effect of the IS shocks is dampened somewhat under the money supply instrument–the coefficient in front of U_t in (A8) is between 0 and 1. The asymptotic variance of output is:

$$\sigma_y^2\big|_{m=m^*} = \left(\frac{\beta_2}{\beta_2+\alpha_1\beta_1}\right)^2 \sigma_U^2. \tag{A9}$$

Since the term in round brackets is between 0 and 1, it follows that:

$$\sigma_y^2\big|_{m=m^*} < \sigma_y^2\big|_{R=R^*}. \tag{A10}$$

It follows that a rational policy maker (one who wants to reduce output fluctuations) chooses the money supply instrument if the LM curve does not fluctuate.

In terms of Figure 2, this result can be explained as follows. Let IS_0 be the expected position of the IS curve, i.e. equation (1) for $EU_t = 0$, and let IS_1 and IS_2 be the IS curves for a given positive and negative IS shock respectively. The deterministic equilibrium is at point E_0, where $y = y^*$ and $R = R^*$. Under the interest rate instrument, the policy maker maintains $R = R^*$ so that the economy fluctuates between points A and B and output fluctuates between y_t^A and y_t^B. Under the money supply instrument, the money supply is set such that IS_0 and LM_0 intersect at point E_0. As a result, the economy fluctuates between points C and D, and output fluctuates between y_t^C and y_t^D. Output fluctuations are smaller under the money supply rule because interest rate movements act as automatic stabilizers. With a positive IS shock, the interest rate rises under the money supply rule so that investment and thus output demand is dampened somewhat.

Part (d)

If only the LM curve is subject to stochastic shocks, then the instruments are also not equivalent. Now the IS curve is not subject to fluctuations but the LM curve fluctuates randomly. In terms of Figure 3, the IS curve is given by IS_0 and the expected position of the LM curve is LM_0. LM_1 and LM_2 are associated with, respectively, a positive and a negative money demand shock.

Under the interest rate instrument, the policy maker maintains $R = R^*$ and lets the money supply be determined endogenously. As a result, the economy stays in point E_0. (The economy does not go to points C and D because in these points the money supply is not in equilibrium.) Output is given by (A8) but, since $U_t = 0$ here, it does not fluctuate at all. There is perfect stabilization in this case and the asymptotic variance of output is zero.

Under the money supply rule, the economy fluctuates between points A and B and output fluctuates between y_t^A and y_t^B. To compute actual output under the money supply instrument, we solve equations (1)-(2) for output:

$$\begin{aligned} y_t &= \alpha_0 - \frac{\alpha_1}{\beta_2}\left[\beta_0 + \beta_1 y_t + V_t - m_t + p_t\right] + U_t \qquad \Rightarrow \\ y_t &= \frac{\alpha_0\beta_2 - \alpha_1\beta_0 + \alpha_1 m^* - \alpha_1 V_t}{\beta_2+\alpha_1\beta_1}, \end{aligned} \tag{A11}$$

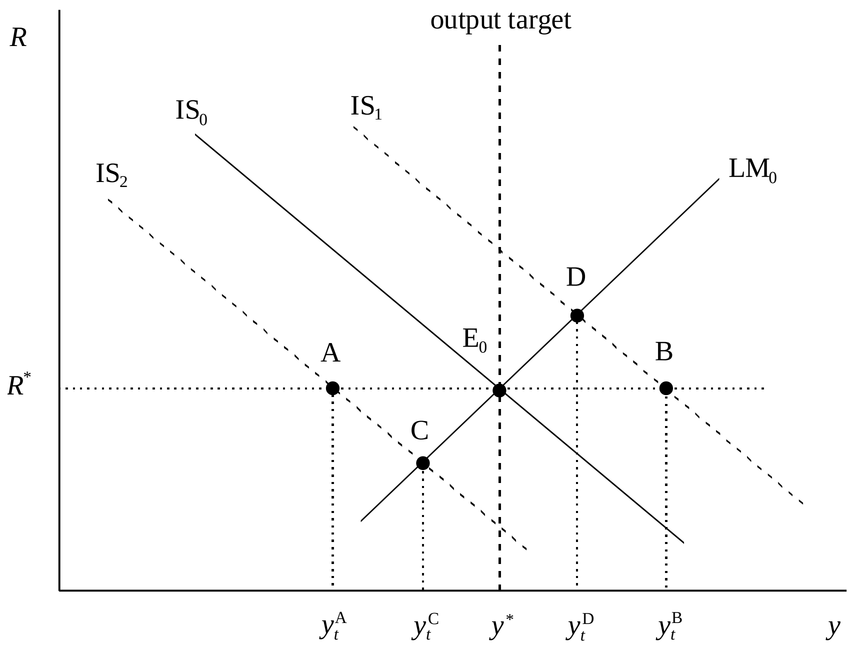

Figure 2: Instruments and output fluctuations (deterministic LM curve)

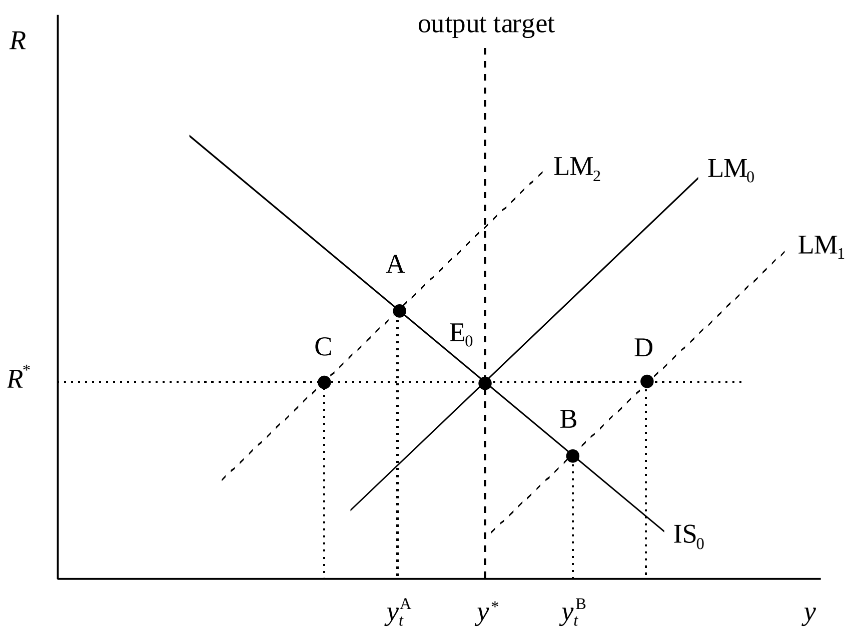

Figure 3: Instruments and output fluctuations (deterministic IS curve)

where we have set $p_t = U_t = 0$ and $m_t = m^*$ for all t in going from the first to the second line. By substituting (A4) into (A11) we obtain an even simpler expression for output:

$$y_t = y^* - \left(\frac{\alpha_1}{\beta_2 + \alpha_1 \beta_1}\right) V_t. \tag{A12}$$

The asymptotic variance of output under the money supply instrument is thus:

$$\sigma_y^2\big|_{m=m^*} = \left(\frac{\alpha_1}{\beta_2 + \alpha_1 \beta_1}\right)^2 \sigma_V^2. \tag{A13}$$

Since output can be perfectly stabilized under the interest rate instrument, the rational policy maker prefers this instrument over the money supply instrument.

Part (e)

To solve the general instrument choice problem, we first solve (1)-(2) for output. After some manipulation we obtain for the interest rate instrument:

$$y_t = y^* + U_t, \tag{A14}$$

and for the money supply instrument:

$$y_t = y^* + \frac{\beta_2 U_t - \alpha_1 V_t}{\beta_2 + \alpha_1 \beta_1}. \tag{A15}$$

The asymptotic variance of output under the interest rate instrument is obtained from (A14):

$$\sigma_y^2\big|_{R=R^*} = \sigma_U^2. \tag{A16}$$

The asymptotic variance under the money supply rule is obtained from (A15):

$$\sigma_y^2\big|_{m=m^*} = \frac{\beta_2^2 \sigma_U^2 + \alpha_1^2 \sigma_V^2}{(\beta_2 + \alpha_1 \beta_1)^2}, \tag{A17}$$

where we have used the fact that U_t and V_t are independent random variables (so that $EU_t V_t = 0$).

It is easy to rewrite the quadratic loss function, given in equation (3), in terms of the asymptotic variance of output:

$$\Omega \equiv E\left[y_t - y^*\right]^2 = E\left[y_t - Ey_t\right]^2 \equiv \sigma_y^2, \tag{A18}$$

where we have used the fact that $Ey_t = y^*$ under both instruments (see equations (A14) and (A15)). Hence, the rational policy maker chooses that policy instrument for which σ_y^2 is lowest. By comparing (A16) and (A17), we find that the money supply instrument is the preferred instrument if (and only if):

$$\frac{\beta_2^2 \sigma_U^2 + \alpha_1^2 \sigma_V^2}{(\beta_2 + \alpha_1 \beta_1)^2} < \sigma_U^2 \qquad \Leftrightarrow \qquad \frac{\sigma_U^2}{\sigma_V^2} > \frac{\alpha_1}{\beta_1 (2\beta_2 + \alpha_1 \beta_1)}. \tag{A19}$$

According to (A19), the money supply instrument is optimal if the variance of the IS shocks is large relative to that of the LM shocks. The intuition behind this result is provided by the answer to part (c). Interest rate fluctuations dampen the output fluctuations in that case.

Question 2

Part (a)

Equation (1) is the Lucas supply curve. Equation (2) is the objective function of individual i in the population.

Part (b)

If we assume that all citizens have the same coefficient of relative inflation aversion, it does not matter who is the central banker. We can thus use the model that is discussed in detail in Subsection 10.1.2 of the book. If the Dutch central banker can somehow commit to follow the behaviour of some (more inflation-averse) central banker (say, the German one), then he/she can effectively mitigate the effects of dynamic inconsistency somewhat. Even if the other central banker follows discretionary policy, the resulting inflation rate will be lower than the inflation rate the domestic central banker would choose. As a result, welfare would be higher.

Part (c)

This question is studied in detail in Section 10.2 of the book. By choosing a person more conservative than him-/herself, the median voter commits to a lower discretionary inflation rate. Like in part (b) of this question, this leads to an increase in social welfare because the adverse effects of dynamic inconsistency are mitigated somewhat.

Question 3

Part (a)

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Part (b)

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Part (c)

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Part (d)

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References

Poole, W. (1970). Optimal choice of monetary policy instruments in a simple stochastic macro model. *Quarterly Journal of Economics*, 84:197–216.