Foundations of Modern Macroeconomics

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Solutions for problems to Chapter 9

Question 1

Part (a)

The *Beveridge curve* is the combination of vacancies (V) and unemployment (U) for which the flow from employment to unemployment exactly matches the reverse flow from unemployment to employment. Put differently, it plots *equilibrium* (steady-state) unemployment as a function of the number of vacancies. In the model discussed in Chapter 9, the Beveridge curve is given by:

$$U = \frac{s}{s + f(\theta)}, \tag{A1}$$

where U is the unemployment rate, s is the (exogenous) job destruction rate, f is the job finding rate of the workers, and $\theta \equiv V/U$ is the labour market tightness variable. We typically draw the Beveridge curve in (V, U) space–see for example panel (b) of Figures 9.1-9.5. The Beveridge curve is downward sloping: for a given unemployment rate, a reduction in V leads to a fall in the instantaneous probability of finding a job (i.e. f falls). For points below the Beveridge curve the unemployment rate is thus less than the rate required for flow equilibrium in the labour market ($U < s/(s+f)$). To restore flow equilibrium (and return to the Beveridge curve) the unemployment rate must increase.

The Beveridge curve is shifted if job destruction changes or if (ceteris paribus θ) the job finding rate changes. The latter could take place if the matching process becomes more productive, e.g. because of better information transmission in the labour market (see below).

Part (b)

The *matching function* is a theoretical construct not unlike the production function. It makes the bold assumption that the number of matches (encounters between job seekers and firms with a vacancy) is a stable constant-returns-to-scale function of the number of job seekers and the number of vacancies. In the text we explain in detail how the different (instantaneous) probabilities can be distilled from the matching function–see equations (9.2) and (9.5).

Assume that the matching function is Cobb-Douglas:

$$XN = A_M \left(UN\right)^{\eta} \left(VN\right)^{1-\eta}, \tag{A2}$$

with $0 < \eta < 1$ and $A_M > 0$. The parameter A_M can be interpreted as the general productivity index in the matching process. The progression of internet technology (e.g. search engines, computerized labour market data bases, etcetera) increase A_M.

Using equation (9.2) we find that the worker finding rate of firms is:

$$q \equiv \frac{XN}{VN} = \frac{A_M \left(UN\right)^{\eta} \left(VN\right)^{1-\eta}}{VN} = A_M \left(\frac{U}{V}\right)^{\eta} = A_M \theta^{-\eta}. \tag{A3}$$

An increase in matching productivity increases the worker finding rate of firms. Using equation (9.5) we find that the job finding rate of workers is:

$$f \equiv \frac{XN}{UN} = \frac{A_M \left(UN\right)^{\eta} \left(VN\right)^{1-\eta}}{UN} = A_M \left(\frac{V}{U}\right)^{1-\eta} = A_M \theta^{1-\eta}. \tag{A4}$$

Again, an increase in matching productivity reduces the severity of the labour market friction experienced by the worker because it increases the job finding rate. (Observe that (A3) and (A4) satisfy the relationship $f = \theta q$.)

Question 2

Part (a)

The model is explained in detail in Section 9.1 in the book–see especially the detailed explanation below equations (9.25)-(9.28).

Part (b)

If the interest rate rises then several things happen. First, it follows from equation (1) that firms scale down production, i.e. they reduce the stock of capital per worker. The reduction in K leads to a reduction in the marginal product of labour (appearing in (2) and (3)) because the two production factors are cooperative. Second, it follows from equation (2) that the value of occupied job becomes smaller because $F_L - w$ is discounted more heavily.

In terms of Figure 1, the ZP condition (defined by (2)) shifts down as does the WS curve (defined by (3)). The wage rate unambiguously declines but the effect on labour market tightness appears to be ambiguous from the diagram. It can be shown mathematically, however, that θ falls as a result of the increase in r. We follow the same approach as in the appendix to Chapter 9. First we loglinearize equations (2)-(3), holding constant z, δ, and β. (We allow

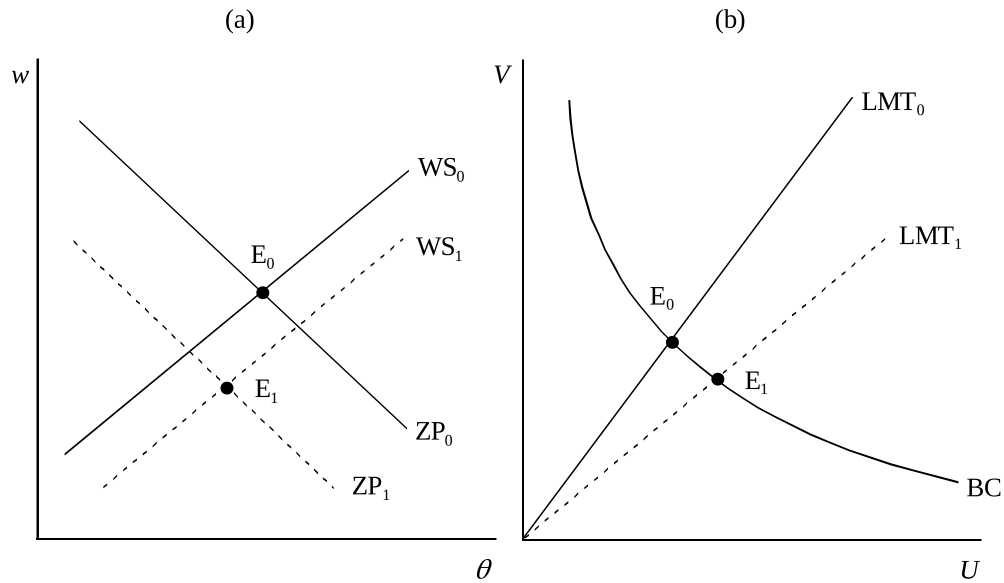

Figure 1: The effects of a higher interest rate

γ_0 to vary because we need this in part (c) of this question.) Here we show in some detail how this is done. By differentiating equation (2) we obtain:

$$\begin{aligned} \frac{\gamma_0}{q}\left[\frac{d\gamma_0}{\gamma_0}-\frac{dq}{q}\right] &= \frac{F_L-w}{r+s}\left[\frac{dF_L-dw}{F_L-w}-\left(\frac{r}{s+r}\right)\frac{dr}{r}\right] \qquad \Leftrightarrow \\ \frac{d\gamma_0}{\gamma_0}-\frac{dq}{q} &= \frac{dF_L-dw}{F_L-w}-\left(\frac{r}{s+r}\right)\frac{dr}{r}. \end{aligned} \tag{A1}$$

Differentiation of equation (3) yields:

$$\begin{aligned} dw &= \beta\left[dF_L+\theta\gamma_0\frac{d\gamma_0}{\gamma_0}+\theta\gamma_0\frac{d\theta}{\theta}\right] \\ &= \beta dF_L+\beta\theta\gamma_0\left[\frac{d\gamma_0}{\gamma_0}+\frac{d\theta}{\theta}\right]. \end{aligned} \tag{A2}$$

From the definition of $\eta(\theta)$ (in equation (9.4)) we know that:

$$\frac{dq}{q}=-\eta(\theta)\left(\frac{d\theta}{\theta}\right). \tag{A3}$$

Equations (A1)-(A3) can be used to solve $\tilde{\theta}\equiv d\theta/\theta$ and dw in terms of $\tilde{r}\equiv dr/r$, dF_L, and $\tilde{\gamma}_0\equiv d\gamma_0/\gamma_0$:

$$\begin{bmatrix} -\eta(F_L-w) & -1 \\ -\beta\gamma_0\theta & 1 \end{bmatrix}\begin{bmatrix} \tilde{\theta} \\ dw \end{bmatrix}=\begin{bmatrix} (F_L-w)\left[\tilde{\gamma}_0+\left(\frac{r}{r+s}\right)\tilde{r}\right]-dF_L \\ \beta\gamma_0\theta\tilde{\gamma}_0+\beta dF_L \end{bmatrix}. \tag{A4}$$

The matrix on the left-hand side has a negative determinant, equal to $-\left[\eta(F_L - w) + \beta\gamma_0\theta\right]$, so that it possesses a unique inverse:

$$\begin{bmatrix} \eta(w - F_L) & -1 \\ -\beta\gamma_0\theta & 1 \end{bmatrix}^{-1} = \frac{1}{\eta(F_L - w) + \beta\gamma_0\theta} \begin{bmatrix} -1 & -1 \\ -\beta\gamma_0\theta & \eta(F_L - w) \end{bmatrix}. \tag{A5}$$

Using (A5) in (A4) yields:

$$\begin{aligned} \begin{bmatrix} \tilde{\theta} \\ dw \end{bmatrix} &= \frac{1}{\eta(F_L - w) + \beta\gamma_0\theta} \begin{bmatrix} -1 & -1 \\ -\beta\gamma_0\theta & \eta(F_L - w) \end{bmatrix} \\ &\quad \times \begin{bmatrix} (F_L - w)\left[\tilde{\gamma}_0 + \left(\frac{r}{r+s}\right)\tilde{r}\right] - dF_L \\ \beta\gamma_0\theta\tilde{\gamma}_0 + \beta dF_L \end{bmatrix} \\ &= \frac{1}{\eta(F_L - w) + \beta\gamma_0\theta}\left(- \begin{bmatrix} 1 \\ \beta\gamma_0\theta \end{bmatrix} (F_L - w)\left(\frac{r}{s+r}\right)\tilde{r} \right. \\ &\quad \left. + \begin{bmatrix} 1 - \beta \\ \beta\eta(F_L - w) + \beta\gamma_0\theta \end{bmatrix} dF_L - \begin{bmatrix} (F_L - w + \beta\gamma_0\theta) \\ (1 - \eta)\beta\gamma_0\theta(F_L - w) \end{bmatrix} \tilde{\gamma}_0 \right). \end{aligned} \tag{A6}$$

By setting $\tilde{\gamma}_0 = 0$ we obtain the result for $\tilde{\theta}$ from the first line of (A6):

$$\tilde{\theta} = -\frac{F_L - w}{\eta(F_L - w) + \beta\gamma_0\theta}\left(\frac{r}{s+r}\right)\tilde{r} + \frac{1 - \beta}{\eta(F_L - w) + \beta\gamma_0\theta} dF_L. \tag{A7}$$

As a result of the increase in the interest rate, the labour market tightness variable falls for two reasons. First, the firm's surplus per occupied job is discounted more heavily, thus reducing the value of occupied jobs and decreasing the supply of vacancies. This *rent discounting effect* is represented by the first term on the right-hand side of (A7). Second, the interest rate increase prompts the firm to scale down by hiring less capital. This leads to a reduction in the marginal product of capital. This *labour productivity effect* is represented by the second term on the right-hand side of (A7).

In terms of Figure 1, panel (a), the equilibrium shifts from E_0 to E_1, and both w and θ fall. In panel (b) the labour market tightness line rotates in a clockwise fashion, from LMT_0 to LMT_1, unemployment increases and vacancies fall.

Part (c)

A reduction in the firms' search costs affects both the zero profit condition (2) and the wage setting equation (3). In terms of Figure 2, the ZP curve shifts up if γ_0 falls. Intuitively, for a given value of θ, expected vacancy costs $(\gamma_0/q(\theta))$ fall so to restore zero-profit equilibrium the discounted value of rents earned on labour $(F_L - w)/(r + s))$ must fall also, i.e. the wage must rise.

The WS curve shifts down. Intuitively, in the Nash bargaining outcome, the workers capture a part of the foregone search costs $(\theta\gamma_0)$ in the form of higher wages–see equation (9.24) in the book. If these costs decline the wage falls also.

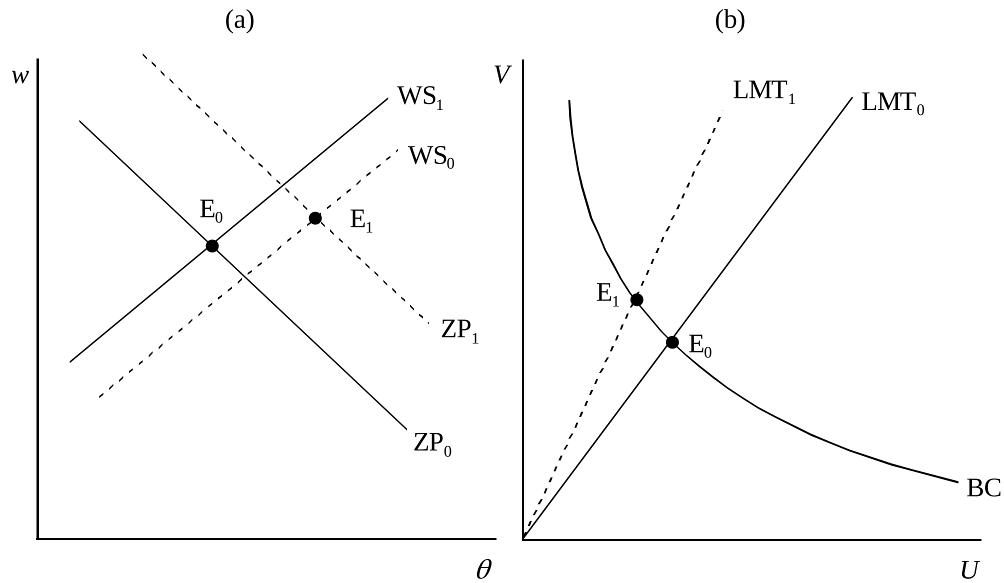

Figure 2: The effects of a reduction in firm search costs

We observe from (A6) that a decrease in γ_0 leads to an increase in both θ and w. Hence, in terms of Figure 2, panel (a), the equilibrium shifts from E_0 to E_1. In panel (b), the labour market tightness condition rotates counterclockwise from LMT_0 to LMT_1. Unemployment falls and vacancies increase.