Foundations of Modern Macroeconomics

Ben J. Heijdra
University of Groningen

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Solutions for problems to Chapter 6

Question 1

Part (a)

The consolidated budget constraint of the government is given in equation (6.61) in the book:

$$[\Xi_1 \equiv] \;\; (1+r)B_0 + G_1^C + \frac{G_2^C}{1+r} + \frac{(r-r_G)G_1^I}{1+r} = t_1 Y_1 + \frac{t_2 Y_2}{1+r}, \tag{A1}$$

where Ξ_1 is the present value of the net liabilities of the government. We immediately see the golden rule of public finance: to the extent that public investment projects earn a rate of return equal to the market rate of return (so that $r_G = r$) they do not represent a net liability of the government. The government should borrow the funds to finance these investments. The advantage is that the government has no need to use distorting taxes in order to raise the revenues. The disadvantage are: (i) it is sensitive to political abuse (politicians will try to label government consumption items as if they are investment items); (ii) it is not easy to estimate the rate of return on public investment projects (politicians will have an incentive to overstate it). Private sector firms that continually invest in low-yielding projects will eventually go out of business. The government does not have that disciplining device.

Part (b)

By a temporary increase in public consumption we mean the situation in which Ξ_1 is unchanged, i.e. G_1^C rises but G_2^C falls, such that the net liabilities of the government are unchanged. It is OK to leave the tax rates unchanged and to finance the temporary increase in government consumption with debt. This can be illustrated with the aid of Figure 1. In that figure the upward sloping line (labelled $t_1 = t_2$) is the tax smoothing line, whereas the

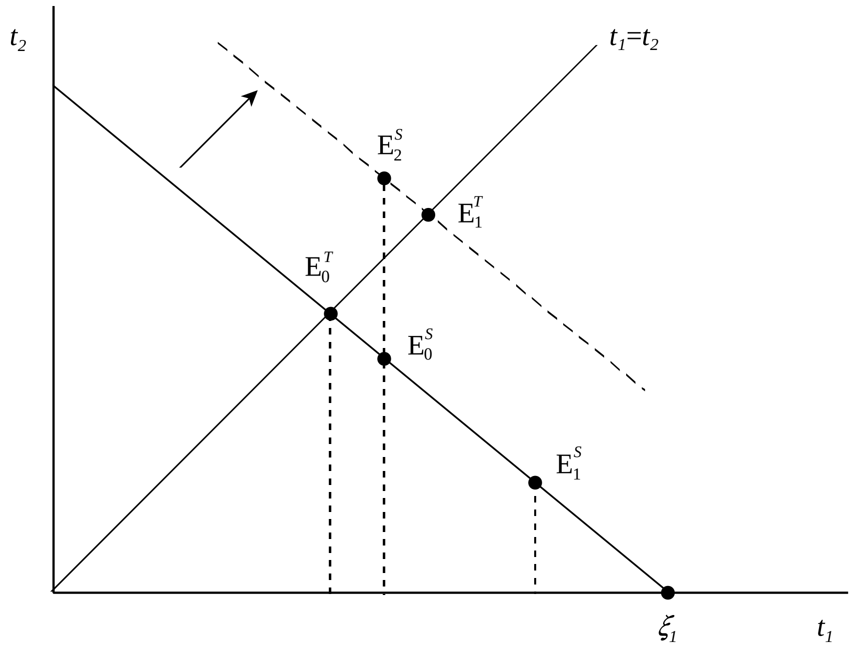

Figure 1: Temporary and permanent spending shocks

solid downward sloping line is the consolidated budget constraint of the government (equation (A1)) rewritten in terms of output shares:

$$\xi_1 = t_1 + \left(\frac{1+\gamma}{1+r}\right) t_2, \tag{A2}$$

where $\xi_1 \equiv \Xi_1/Y_1$, $\gamma \equiv Y_2/Y_1 - 1$ is the growth rate in the economy, and ξ_1 is given by:

$$\xi_1 \equiv g_1^C + \left(\frac{1+\gamma}{1+r}\right) g_2^C + \left(\frac{r-r_G}{1+r}\right) g_1^I + (1+r)b_0, \tag{A3}$$

where $g_t^C \equiv G_t^C/Y_t$, $g_1^I \equiv G_1^I/Y_1$, and $b_0 \equiv B_0/Y_1$. The deficit in period 1 can also be written in terms of output shares:

$$d_1 \equiv rb_0 + g_1^C + g_1^I - t_1, \tag{A4}$$

where $d_1 \equiv D_1/Y_1$.

The *spending point* is defined as the point where $d_1 = 0$, and is drawn as point E_0^S in Figure 1. The *optimal taxation point* is given by point E_0^T. A temporary increase in government consumption implies that the spending point moves *along the initial budget line* from E_0^S to E_1^S. The optimal taxation point is unaffected. Since the tax rates are not changed but spending in the first period is increased, it follow from (A4) that the deficit in period 1 is increased ($dd_1/dg_1^C = 1$).

Part (c)

A permanent increase in government spending implies that ξ_1 itself increases. In terms of Figure 1, the budget line shifts out. Assuming that the spending increase takes place in the second period, the spending point moves from E_0^S to E_2^S. The optimal tax point shifts from E_0^T to E_1^T so both tax rates are increased immediately (in anticipation of the higher spending in the second period).

Part (d)

The Euler equation for the government's optimal tax plan is given by equation (6.66) in the book:

$$\frac{t_1}{t_2} = \frac{1+r}{1+\rho_G}. \tag{A5}$$

It follows that a short-sighted government has the tendency to postpone taxation, i.e. to set t_1 much lower than t_2. To figure out what happens to public debt we note that the deficit in period 1 is given by (A4) above. Since the policy maker chooses a low tax rate in the first period it runs a large deficit in that period. As a result the debt in the second period is large. Recall that $D_1 + D_2 + B_0 = 0$. By rewriting this expression in terms of income shares we get:

$$\begin{aligned} \frac{D_1}{Y_1} + \frac{D_2}{Y_2}\frac{Y_2}{Y_1} + \frac{B_0}{Y_1} &= 0 \quad \Leftrightarrow \quad d_1 + (1+\gamma)d_2 + b_0 = 0 \quad \Leftrightarrow \\ -d_2 &= \frac{d_1 + b_0}{1+\gamma}. \end{aligned} \tag{A6}$$

The surplus in the second period must be large due to the myopic nature of the policy maker.

Question 2

Part (a)

If labour supply is endogenous then the tax rates will themselves introduce distortions. Whereas income is exogenously given in the standard case discussed in the text, with endogenous labour supply Y_1 and Y_2 will depend on the tax rates in the two periods. The Ricardian experiment (a cut in t_1 and an increase in t_2) will then affect the present value of income of the households. As a result, Ricardian equivalence will not generally hold any more.

Part (b)

When households have only limited access to the capital markets Ricardian equivalence will not generally hold any more. In the text we show that a household which faces binding borrowing constraints will be unable to realize its optimal consumption point. Instead, it will choose a second-best optimal consumption plan that is restricted by income in the two periods. A tax cut in the current period moves the income-endowment point in the direction of the optimal consumption point and make the household better off. Consumption in the two periods is affected and Ricardian equivalence does not hold.

Part (c)

When new generations are born which are not altruistically linked with existing generations, then the future tax load will be carried by more shoulders. As a result, a tax cut now will make current generations better off and will prompt them to consume more. Ricardian equivalence will not hold. If, on the other hand, the new generations are altruistically linked with the present generations then Ricardian equivalence will hold again. The reason is that positive bequests will ensure that present and future generations are connected to each other.

Part (d)

When households are risk averse and future income is stochastic then they will engage in so-called *precautionary savings*. A tax cut now, matched by a tax increase later, will ensure that precautionary savings fall. The reason is that the future tax increase reduces the variance of future after-tax income. A temporary tax cut thus boosts consumption, which is inconsistent with Ricardian equivalence.

Question 3

Part (a)

According to (1), lifetime utility (V) is the sum of felicity in the current period and weighted felicity in the second period. The household discounts future felicity because it exhibits time preference. Equations (2) and (3) are budget *identities*, i.e. they hold by definition. We obtain the lifetime budget *constraint* by setting $A_2 = 0$. It makes no sense for the household to die with positive assets (i.e. $A_2 \leq 0$) and capital markets will not allow the household to die indebted (i.e. $A_2 \geq 0$). Combing the two inequalities yields $A_2 = 0$ as the solvency condition. By substituting (2) into (3) and setting $A_2 = 0$ we find the household budget constraint:

$$C_1 + \frac{C_2}{1+r} = (1+r)A_0 + \left[(1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r}\right] \equiv \Omega. \tag{A1}$$

Part (b)

The government budget identities are:

$$rB_0 + G_1 - t_1Y_1 = B_1 - B_0, \tag{A2}$$

$$rB_1 + G_2 - t_2Y_2 = B_2 - B_1 = -B_1, \tag{A3}$$

where B_i is government debt in period i. The solvency condition for the government is $B_2 = 0$. By combing (A2) and (A3) and setting $B_2 = 0$ we find the budget constraint of the government:

$$(1+r)B_0 + G_1 + \frac{G_2}{1+r} = t_1Y_1 + \frac{t_2Y_2}{1+r}. \tag{A4}$$

Since government bonds are the only financial asset in this economy, $A_i = B_i$. By using this in (A1) and (A4) we find:

$$\begin{aligned}(1+r)A_0 &= C_1 + \frac{C_2}{1+r} - \left[(1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r}\right] \\ &= t_1Y_1 + \frac{t_2Y_2}{1+r} - \left[G_1 + \frac{G_2}{1+r}\right],\end{aligned} \tag{A5}$$

or:

$$C_1 + \frac{C_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}. \tag{A6}$$

The tax parameters drop out of the rewritten household budget constraint. The path of taxes does not matter for the real equilibrium in the economy.

Part (c)

The Lagrangean associated with the optimization problem is:

$$\mathcal{L} \equiv \frac{C_1^{1-1/\sigma} - 1}{1-1/\sigma} + \beta\left(\frac{C_2^{1-1/\sigma} - 1}{1-1/\sigma}\right) + \lambda\left[\Omega - C_1 - \frac{C_2}{1+r}\right]. \tag{A7}$$

The first-order conditions are the budget constraint (A1) and:

$$\frac{\partial \mathcal{L}}{\partial C_1} = C_1^{-1/\sigma} - \lambda = 0, \tag{A8}$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \beta C_2^{-1/\sigma} - \frac{\lambda}{1+r} = 0. \tag{A9}$$

By combining (A8)-(A9), the so-called consumption Euler equation is obtained:

$$\begin{aligned}\lambda &= C_1^{-1/\sigma} = \beta(1+r)C_2^{-1/\sigma} \quad \Rightarrow \quad \left(\frac{C_2}{C_1}\right)^{1/\sigma} = \beta(1+r) \quad \Rightarrow \\ \frac{C_2}{C_1} &= [\beta(1+r)]^{\sigma}.\end{aligned} \tag{A10}$$

Note that (A10) is the same as equation (6.15) in the book if $\sigma = 1$ (recall that $\beta \equiv 1/(1+\rho)$).

Next we find the levels of C_1 and C_2 by combining (A10) and the budget constraint (A1). We obtain:

$$\begin{aligned} C_1 + \frac{[\beta(1+r)]^{\sigma} C_1}{1+r} &= \Omega \\ C_1 \left[1+\beta^{\sigma}(1+r)^{\sigma-1}\right] &= \Omega \\ C_1 &= \frac{\Omega}{[1+\beta^{\sigma}(1+r)^{\sigma-1}]}. \end{aligned} \tag{A11}$$

It follows from (A11) and (A10) that C_2 is:

$$C_2 = [\beta(1+r)]^{\sigma} C_1 = \frac{[\beta(1+r)]^{\sigma} \Omega}{[1+\beta^{\sigma}(1+r)^{\sigma-1}]}. \tag{A12}$$

If $\sigma = 1$, then (A11) and (A12) coincide with the expressions found in equation (6.16) in the book.

Finally, by noting that $S_1 \equiv A_1 - A_0$ we find:

$$\begin{aligned} S_1 &= A_1 - A_0 = rA_0 + (1-t_1)Y_1 - C_1 \\ &= rA_0 + (1-t_1)Y_1 - \frac{\Omega}{[1+\beta^{\sigma}(1+r)^{\sigma-1}]} \\ &= \left(\frac{[\beta(1+r)]^{\sigma}[rA_0+(1-t_1)Y_1] - (1-t_2)Y_2}{(1+r)[1+\beta^{\sigma}(1+r)^{\sigma-1}]}\right), \end{aligned} \tag{A13}$$

where we have used the definition of Ω in the final step.

Part (d)

If there is a broad-based income tax, which includes interest income in the tax base, then the savings decision will be distorted. This is explained in detail in section 1.2 of Chapter 6.

Part (e)

If interest income is taxed, then the household budget constraint becomes:

$$C_1 + \frac{C_2}{1+r^*} = [1+r(1-t_1)]A_0 + \left[(1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r^*}\right] \equiv \Omega^*, \tag{A14}$$

where $r^* \equiv r(1-t_2)$. Retracing the steps performed in part (c) we find:

$$C_1 = \frac{\Omega^*}{[1+\beta^{\sigma}(1+r^*)^{\sigma-1}]}, \tag{A15}$$

$$C_2 = \frac{[\beta(1+r^*)]^{\sigma}\Omega^*}{[1+\beta^{\sigma}(1+r^*)^{\sigma-1}]}. \tag{A16}$$

The effects of taxes in the two periods are as follows. The effect of t_1 operates only via household wealth and leaves the intertemporal trade-off between C_1 and C_2 unchanged:

$$\frac{\partial \Omega^*}{\partial t_1} = -(rA_0 + Y_1) < 0. \tag{A17}$$

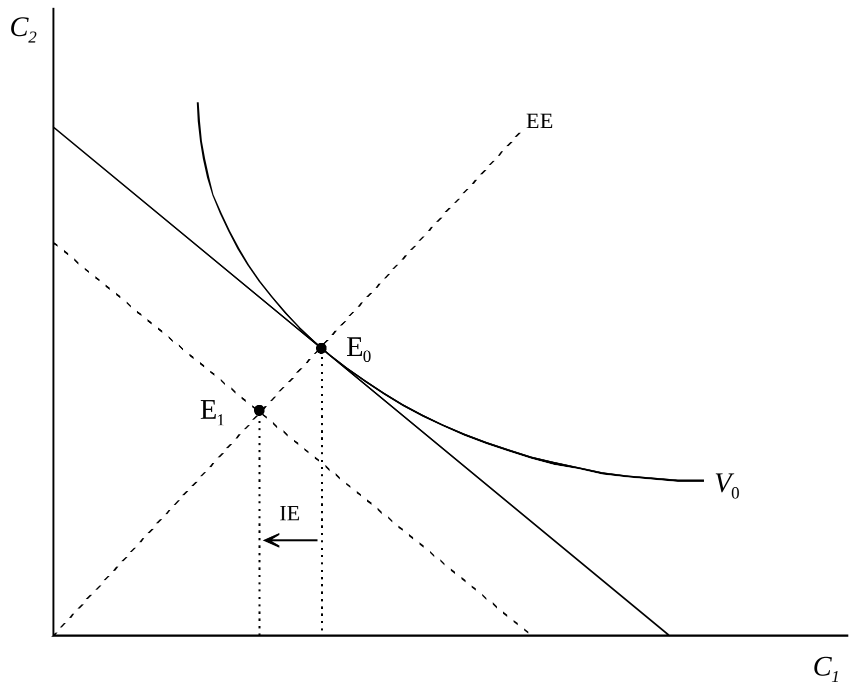

Figure 2: Increase in current tax rate

In terms of Figure 2, the lifetime budget constraint shifts inward in a parallel fashion. If the initial equilibrium is at E_0 (where lifetime utility is V_0), after the tax increase the new equilibrium will be at E_1 (where lifetime utility is lower). The straight line from the origin, labelled EE, is the Euler equation:

$$\frac{C_2}{C_1} = \beta^{\sigma} \left[1 + r(1 - t_2)\right]^{\sigma}. \tag{A18}$$

Since t_1 does not affect the intertemporal trade-off between C_1 and C_2, E_0 and E_1 lie on the same EE line. The move from E_0 to E_1 only causes an income effect (IE).

An increase in the tax rate in the second period has more complicated effects. First, it follows from the definition of r^* that the after-tax interest rate falls:

$$\frac{\partial r^*}{\partial t_2} = -r < 0. \tag{A19}$$

Second, it follows from the definition of Ω^* in (A14) that wealth falls:

$$\frac{\partial \Omega^*}{\partial t_2} = Y_2 \left(\frac{-(1 + r^*) + r(1 - t_2)}{(1 + r^*)^2} \right) = -\frac{Y_2}{(1 + r^*)^2} < 0. \tag{A20}$$

In terms of Figure 3, the equilibrium shifts from E_0 to E_1. We can decompose the total effect into the income effect (IE), the substitution effect (SE), and the human wealth effect (HWE). The decrease in the after-tax interest rate, given by (A19), is represented by the counter-clockwise rotation of the budget line from its initial position to the dashed line aa.

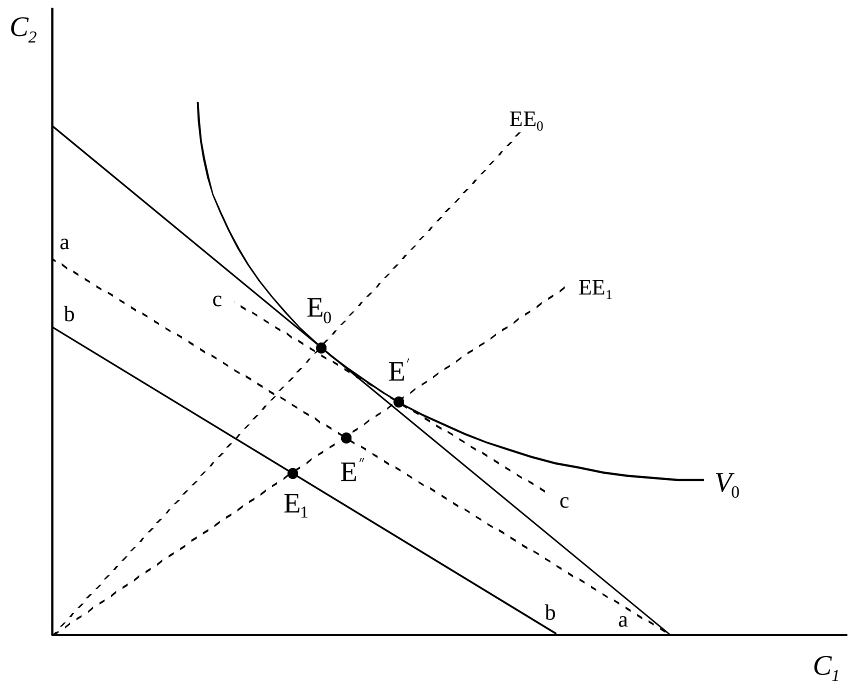

Figure 3: Increase in future tax rate

The decrease in wealth, as given in (A20), is represented by the parallel shift of the aa line to the bb line. In order to discover the pure substitution effect we draw the auxiliary line cc, which is parallel to both aa and bb, in order to find the tangency point E′ along the old indifference curve, V_0.

The total effect can now be decomposed as follows:

- The substitution effect (SE) is represented by the move from E_0 to E′. The decrease in the after-tax interest rate causes an increase in consumption in the first period and a decrease in future consumption.
- The income effect (IE) is represented by the move from E′ to E″. Since the household is poorer as a result of the tax increase, less of both goods is consumed.
- The human wealth effect (HWE) is represented by the move from E″ to E_1. Though future income is discounted less heavily (because r^* falls) the reduction in after-tax future income dominates this discounting effect (see (A20)). Because the household is poorer, consumption of both goods is decreased.