Notes on the Open Economy

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April 2004

1 Introduction

In this note we study the two-country model of Table 11.4 in more detail. The model is restated here for convenience.

Table 11.4. A two-country extended Mundell-Fleming model

$$y = -\epsilon_{YR} r^* + \epsilon_{YQ} q + \epsilon_{YG} [g + \eta g^*], \tag{T3.1}$$

$$y^* = -\epsilon_{YR} r^* - \epsilon_{YQ} q + \epsilon_{YG} [g^* + \eta g], \tag{T3.2}$$

$$m - p = \epsilon_{MY} y - \epsilon_{MR} r^*, \tag{T3.3}$$

$$m^* - p^* = \epsilon_{MY} y^* - \epsilon_{MR} r^*, \tag{T3.4}$$

$$y = -\omega_N \epsilon_{NW} [w - p], \tag{T3.5}$$

$$y^* = -\omega_N \epsilon_{NW} [w^* - p^*], \tag{T3.6}$$

$$w = w_0 + \lambda p_C, \tag{T3.7}$$

$$w^* = w_0^* + \lambda^* p_C^*, \tag{T3.8}$$

$$p_C = \omega_0 + p + (1 - \alpha) q, \tag{T3.9}$$

$$p_C^* = \omega_0 + p^* - (1 - \alpha) q, \tag{T3.10}$$

Notes: All variables except the interest rate are in logarithms and starred variables refer to the foreign country. Endogenous variables are the outputs (y, y^*), the real exchange rate (q), the rate of interest (r^*), price levels (p, p^*), nominal wages (w, w^*), and consumer price indexes (p_C, p_C^*). Exogenous are government spending (g, g^*), the money stocks (m, m^*), and the wage targets (w_0, w_0^*). Note that $\omega_0 \equiv \log \Omega_0$.

We compute the effects of fiscal and monetary policy under the various regimes.

2 Nominal wage rigidity in both countries

In this case we have $\lambda = \lambda^* = 0$ so that the model reduces to:

$$y = -\epsilon_{YR} r^* + \epsilon_{YQ} q + \epsilon_{YG} \left[g + \eta g^* \right], \tag{1}$$

$$y^* = -\epsilon_{YR} r^* - \epsilon_{YQ} q + \epsilon_{YG} \left[g^* + \eta g \right], \tag{2}$$

$$m - w_0 = \delta_N y - \epsilon_{MR} r^*, \tag{3}$$

$$m^* - w_0^* = \delta_N y^* - \epsilon_{MR} r^*, \tag{4}$$

$$y = \omega_N \epsilon_{NW} \left[p - w_0 \right], \tag{5}$$

$$y^* = \omega_N \epsilon_{NW} \left[p^* - w_0^* \right], \tag{6}$$

where we have already used (5)-(6) to simplify (3)-(4) and where the composite parameter δ_N is defined as follows:

$$\delta_N \equiv \frac{1 + \epsilon_{MY} \omega_N \epsilon_{NW}}{\omega_N \epsilon_{NW}}. \tag{7}$$

Writing the system (1)-(4) in one matrix equation gives:

$$\Delta_N \begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Gamma_N \begin{bmatrix} g \\ g^* \\ m \\ m^* \end{bmatrix}, \tag{8}$$

where Δ_N and Γ_N are defined as follows:

$$\Delta_N \equiv \begin{bmatrix} 1 & 0 & \epsilon_{YR} & -\epsilon_{YQ} \\ 0 & 1 & \epsilon_{YR} & \epsilon_{YQ} \\ \delta_N & 0 & -\epsilon_{MR} & 0 \\ 0 & \delta_N & -\epsilon_{MR} & 0 \end{bmatrix}, \tag{9}$$

and:

$$\Gamma_N \equiv \begin{bmatrix} \epsilon_{YG} & \eta\epsilon_{YG} & 0 & 0 \\ \eta\epsilon_{YG} & \epsilon_{YG} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

The determinant and inverse of Δ_N are easily computed:[1]

$$|\Delta_N| = 2\delta_N \epsilon_{YQ} \left(\epsilon_{MR} + \delta_N \epsilon_{YR} \right) > 0, \tag{11}$$

[1] We have used Maple V to compute the determinant and inverse of Δ. Of course, one can also use Cramer's Rule.

$$\Delta_N^{-1} \equiv \frac{1}{|\Delta_N|} \times \tag{12}$$
$$\begin{bmatrix} \delta_N \epsilon_{YQ}\epsilon_{MR} & \delta_N \epsilon_{YQ}\epsilon_{MR} & \epsilon_{YQ}\left[\epsilon_{MR} + 2\delta_N \epsilon_{YR}\right] & -\epsilon_{YQ}\epsilon_{MR} \\ \delta_N \epsilon_{YQ}\epsilon_{MR} & \delta_N \epsilon_{YQ}\epsilon_{MR} & -\epsilon_{YQ}\epsilon_{MR} & \epsilon_{YQ}\left[\epsilon_{MR} + 2\delta_N \epsilon_{YR}\right] \\ \delta_N^2 \epsilon_{YQ} & \delta_N^2 \epsilon_{YQ} & -\delta_N \epsilon_{YQ} & -\delta_N \epsilon_{YQ} \\ -\delta_N\left[\epsilon_{MR} + \delta_N \epsilon_{YR}\right] & \delta_N\left[\epsilon_{MR} + \delta_N \epsilon_{YR}\right] & \epsilon_{MR} + \delta_N \epsilon_{YR} & -\left[\epsilon_{MR} + \delta_N \epsilon_{YR}\right] \end{bmatrix}.$$

2.1 Fiscal policy

2.1.1 Domestic fiscal policy

Using (12) and the first column of (10) we find:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Delta_N^{-1} \begin{bmatrix} \epsilon_{YG} \\ \eta\epsilon_{YG} \\ 0 \\ 0 \end{bmatrix} g = \frac{1}{|\Delta_N|} \begin{bmatrix} \delta_N (1+\eta)\,\epsilon_{YQ}\epsilon_{MR} \\ \delta_N (1+\eta)\,\epsilon_{YQ}\epsilon_{MR} \\ \delta_N^2 (1+\eta)\,\epsilon_{YQ} \\ -\delta_N (1-\eta)\left[\epsilon_{MR} + \delta_N\epsilon_{YR}\right] \end{bmatrix} \epsilon_{YG} g \tag{13}$$

By using (11) we can simplify and obtain:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \begin{bmatrix} (1+\eta)\,\epsilon_{MR} \\ (1+\eta)\,\epsilon_{MR} \\ \delta_N (1+\eta) \\ -(1-\eta)\left[\epsilon_{MR} + \delta_N\epsilon_{YR}\right]/\epsilon_{YQ} \end{bmatrix} \frac{\epsilon_{YG} g}{2\left[\epsilon_{MR} + \delta_N\epsilon_{YR}\right]}. \tag{14}$$

2.1.2 Foreign fiscal policy

Using (12) and the second column of (10) we find:

$$\begin{aligned} \begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} &= \Delta_N^{-1} \begin{bmatrix} \eta\epsilon_{YG} \\ \epsilon_{YG} \\ 0 \\ 0 \end{bmatrix} g^* \\ &= \begin{bmatrix} (1+\eta)\,\epsilon_{MR} \\ (1+\eta)\,\epsilon_{MR} \\ \delta_N (1+\eta) \\ (1-\eta)\left[\epsilon_{MR} + \delta_N\epsilon_{YR}\right]/\epsilon_{YQ} \end{bmatrix} \frac{\epsilon_{YG} g^*}{2\left[\epsilon_{MR} + \delta_N\epsilon_{YR}\right]}. \end{aligned} \tag{15}$$

Apart from the effect on the real exchange rate (which changes sign), all effects in (14) and (15) coincide.

2.2 Monetary policy

2.2.1 Domestic monetary policy

Using (12) and the third column of (10) we find:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Delta_N^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} m = \frac{1}{|\Delta_N|} \begin{bmatrix} \epsilon_{YQ} \left[\epsilon_{MR} + 2\delta_N \epsilon_{YR} \right] \\ -\epsilon_{YQ}\epsilon_{MR} \\ -\delta_N \epsilon_{YQ} \\ \epsilon_{MR} + \delta_N \epsilon_{YR} \end{bmatrix} m \tag{16}$$

By using (11) we can simplify and obtain:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \begin{bmatrix} \epsilon_{MR} + 2\delta_N \epsilon_{YR} \\ -\epsilon_{MR} \\ -\delta_N \\ \left[\epsilon_{MR} + \delta_N \epsilon_{YR} \right] / \epsilon_{YQ} \end{bmatrix} \frac{m}{2\delta_N \left[\epsilon_{MR} + \delta_N \epsilon_{YR} \right]}. \tag{17}$$

2.2.2 Foreign monetary policy

Finally, using (12) and the fourth column of (10) we find:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Delta_N^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} m^* = \frac{1}{|\Delta_N|} \begin{bmatrix} -\epsilon_{YQ}\epsilon_{MR} \\ \epsilon_{YQ} \left[\epsilon_{MR} + 2\delta_N \epsilon_{YR} \right] \\ -\delta_N \epsilon_{YQ} \\ - \left[\epsilon_{MR} + \delta_N \epsilon_{YR} \right] \end{bmatrix} m^* \tag{18}$$

By using (11) we can simplify and obtain:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \begin{bmatrix} -\epsilon_{MR} \\ \epsilon_{MR} + 2\delta_N \epsilon_{YR} \\ -\delta_N \\ - \left[\epsilon_{MR} + \delta_N \epsilon_{YR} \right] / \epsilon_{YQ} \end{bmatrix} \frac{m^*}{2\delta_N \left[\epsilon_{MR} + \delta_N \epsilon_{YR} \right]}. \tag{19}$$

3 Real wage rigidity in both countries

In this case we have $\lambda = \lambda^* = 1$ so that the model reduces to:

$$y = -\epsilon_{YR} r^* + \epsilon_{YQ} q + \epsilon_{YG} \left[g + \eta g^* \right], \tag{20}$$

$$y^* = -\epsilon_{YR} r^* - \epsilon_{YQ} q + \epsilon_{YG} \left[g^* + \eta g \right], \tag{21}$$

$$y = -\omega_N \epsilon_{NW} \left[\omega_0 + w_0 + (1-\alpha) q \right], \tag{22}$$

$$y^* = -\omega_N \epsilon_{NW} \left[\omega_0 + w_0^* - (1-\alpha) q \right], \tag{23}$$

$$p = m - \epsilon_{MY} y + \epsilon_{MR} r^*, \tag{24}$$

$$p^* = m^* - \epsilon_{MY} y^* + \epsilon_{MR} r^*, \tag{25}$$

where (20)-(23) determine the real quantities (y, y^*, r^*, and q) and (24)-(25) residually determine the nominal quantities. (The model features a Classical dichotomy.) In matrix format the real system can be written as:

$$\Delta_R \begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Gamma_R \begin{bmatrix} g \\ g^* \end{bmatrix}, \tag{26}$$

where Δ_R and Γ_R are now given by:

$$\Delta_R \equiv \begin{bmatrix} 1 & 0 & \epsilon_{YR} & -\epsilon_{YQ} \\ 0 & 1 & \epsilon_{YR} & \epsilon_{YQ} \\ 1 & 0 & 0 & \delta_R \\ 0 & 1 & 0 & -\delta_R \end{bmatrix}, \tag{27}$$

and:

$$\Gamma_R \equiv \begin{bmatrix} \epsilon_{YG} & \eta\epsilon_{YG} \\ \eta\epsilon_{YG} & \epsilon_{YG} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{28}$$

The composite parameter δ_R is defined as:

$$\delta_R \equiv (1-\alpha)\, \omega_N \epsilon_{NW}. \tag{29}$$

The determinant and inverse of Δ_R are:

$$|\Delta_R| = 2\epsilon_{YR} \left(\delta_R + \epsilon_{YQ} \right), \tag{30}$$

$$\Delta_R^{-1} \equiv \frac{1}{|\Delta_R|} \begin{bmatrix} \delta_R \epsilon_{YR} & -\delta_R \epsilon_{YR} & \epsilon_{YR} \left[\delta_R + 2\epsilon_{YQ} \right] & \delta_R \epsilon_{YR} \\ -\delta_R \epsilon_{YR} & \delta_R \epsilon_{YR} & \delta_R \epsilon_{YR} & \epsilon_{YR} \left[\delta_R + 2\epsilon_{YQ} \right] \\ \delta_R + \epsilon_{YQ} & \delta_R + \epsilon_{YQ} & -\left(\delta_R + \epsilon_{YQ} \right) & -\left(\delta_R + \epsilon_{YQ} \right) \\ -\epsilon_{YR} & \epsilon_{YR} & \epsilon_{YR} & -\epsilon_{YR} \end{bmatrix}. \tag{31}$$

3.1 Fiscal policy

3.1.1 Domestic fiscal policy

Using (31) and the first column of (28) we find:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Delta_R^{-1} \begin{bmatrix} 1 \\ \eta \\ 0 \\ 0 \end{bmatrix} \epsilon_{YG} g = \begin{bmatrix} \delta_R (1-\eta) \\ -\delta_R (1-\eta) \\ (1+\eta)(\delta_R + \epsilon_{YQ}) / \epsilon_{YR} \\ -(1-\eta) \end{bmatrix} \frac{\epsilon_{YG} g}{2(\delta_R + \epsilon_{YQ})}. \tag{32}$$

3.1.2 Foreign fiscal policy

Using (31) and the second column of (28) we find:

$$\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Delta_R^{-1} \begin{bmatrix} \eta \\ 1 \\ 0 \\ 0 \end{bmatrix} \epsilon_{YG} g^* = \begin{bmatrix} -\delta_R (1-\eta) \\ \delta_R (1-\eta) \\ (1+\eta)(\delta_R + \epsilon_{YQ}) / \epsilon_{YR} \\ (1-\eta) \end{bmatrix} \frac{\epsilon_{YG} g^*}{2(\delta_R + \epsilon_{YQ})}. \tag{33}$$

3.2 Monetary policy

For obvious reasons, monetary policy does not affect any of the real variables. It is left as an exercise for the reader to determine the effects of monetary policy on price levels (and thus on the nominal exchange rate).

4 Mixed case

In the mixed case we have $\lambda = 1$ and $\lambda^* = 0$. The model can be written as:

$$\begin{aligned} y &= -\epsilon_{YR} r^* + \epsilon_{YQ} q + \epsilon_{YG} \left[g + \eta g^* \right], && (34) \\ y^* &= -\epsilon_{YR} r^* - \epsilon_{YQ} q + \epsilon_{YG} \left[g^* + \eta g \right], && (35) \\ y &= -\delta_R q - \omega_N \epsilon_{NW} \left[\omega_0 + w_0 \right], && (36) \\ m^* - w_0^* &= \delta_N y^* - \epsilon_{MR} r^*, && (37) \\ p &= m - \epsilon_{MY} y + \epsilon_{MR} r^*, && (38) \\ p^* &= \frac{y^*}{\omega_N \epsilon_{NW}} + w_0^*. && (39) \end{aligned}$$

Only domestic money is neutral. The real system, consisting of (34)-(37), can be written as follows:

$$\Delta_M \begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} = \Gamma_M \begin{bmatrix} g \\ g^* \\ m^* \end{bmatrix}, \tag{40}$$

where Δ_M and Γ_M are given by:

$$\Delta_M \equiv \begin{bmatrix} 1 & 0 & \epsilon_{YR} & -\epsilon_{YQ} \\ 0 & 1 & \epsilon_{YR} & \epsilon_{YQ} \\ 1 & 0 & 0 & \delta_R \\ 0 & \delta_N & -\epsilon_{MR} & 0 \end{bmatrix}, \tag{41}$$

and:

$$\Gamma_M \equiv \begin{bmatrix} \epsilon_{YG} & \eta\epsilon_{YG} & 0 \\ \eta\epsilon_{YG} & \epsilon_{YG} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{42}$$

The determinant and inverse of Δ_M are:

$$|\Delta_M| = \delta_R \left(\epsilon_{MR} + \delta_N \epsilon_{YR} \right) + \epsilon_{YQ} \left(\epsilon_{MR} + 2\delta_N \epsilon_{YR} \right), \tag{43}$$

$$\Delta_M^{-1} \equiv \frac{1}{|\Delta_M|} \begin{bmatrix} \delta_R \left(\epsilon_{MR} + \delta_N \epsilon_{YR} \right) & -\delta_N \delta_R \epsilon_{YR} & \epsilon_{YQ} \left(\epsilon_{MR} + 2\delta_N \epsilon_{YR} \right) & \delta_R \epsilon_{YR} \\ \epsilon_{YQ} \epsilon_{MR} & \left(\delta_R + \epsilon_{YQ} \right) \epsilon_{MR} & -\epsilon_{YQ} \epsilon_{MR} & \epsilon_{YR} \left(\delta_R + 2\epsilon_{YQ} \right) \\ \epsilon_{YQ} \delta_N & \left(\delta_R + \epsilon_{YQ} \right) \delta_N & -\delta_N \epsilon_{YQ} & -\left(\delta_R + \epsilon_{YQ} \right) \\ -\left(\epsilon_{MR} + \delta_N \epsilon_{YR} \right) & \delta_N \epsilon_{YR} & \epsilon_{MR} + \delta_N \epsilon_{YR} & -\epsilon_{YR} \end{bmatrix} \tag{44}$$

4.1 Fiscal policy

4.1.1 Domestic fiscal policy

Using (44) and the first column of (42) we find:

$$\begin{aligned}\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} &= \Delta_M^{-1} \begin{bmatrix} 1 \\ \eta \\ 0 \\ 0 \end{bmatrix} \epsilon_{YG} g \\ &= \begin{bmatrix} \delta_R \left[\epsilon_{MR} + (1-\eta)\,\delta_N \epsilon_{YR}\right] \\ \epsilon_{MR} \left[(1+\eta)\,\epsilon_{YQ} + \eta\delta_R\right] \\ \delta_N \left[(1+\eta)\,\epsilon_{YQ} + \eta\delta_R\right] \\ -\left[\epsilon_{MR} + (1-\eta)\,\delta_N \epsilon_{YR}\right] \end{bmatrix} \frac{\epsilon_{YG} g}{|\Delta_M|}. \end{aligned} \tag{45}$$

4.1.2 Foreign fiscal policy

Using (44) and the second column of (42) we find:

$$\begin{aligned}\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} &= \Delta_M^{-1} \begin{bmatrix} \eta \\ 1 \\ 0 \\ 0 \end{bmatrix} \epsilon_{YG} g^* \\ &= \begin{bmatrix} \delta_R \left[\eta\epsilon_{MR} - (1-\eta)\,\delta_N \epsilon_{YR}\right] \\ \epsilon_{MR} \left[(1+\eta)\,\epsilon_{YQ} + \delta_R\right] \\ \delta_N \left[(1+\eta)\,\epsilon_{YQ} + \delta_R\right] \\ -\epsilon_{MR} + (1-\eta)\,\delta_N \epsilon_{YR} \end{bmatrix} \frac{\epsilon_{YG} g^*}{|\Delta_M|}. \end{aligned} \tag{46}$$

4.2 Monetary policy

4.2.1 Domestic monetary policy

Domestic monetary policy does not affect any of the real variables. It is left as an exercise to the reader to see how it affects the nominal price level and the nominal exchange rate.

4.2.2 Foreign monetary policy

Using (44) and the third column of (42) we find:

$$\begin{aligned}
\begin{bmatrix} y \\ y^* \\ r^* \\ q \end{bmatrix} &= \Delta_M^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} m^* \\
&= \begin{bmatrix} \delta_R \epsilon_{YR} \\ \epsilon_{YR} \left(\delta_R + 2\epsilon_{YQ} \right) \\ -\left(\delta_R + \epsilon_{YQ} \right) \\ -\epsilon_{YR} \end{bmatrix} \frac{m^*}{|\Delta_M|}. \qquad (47)
\end{aligned}$$

5 Fiscal policy multipliers

In the text we represent the reduced-form expressions for the output levels by:

$$y = g + \zeta g^* \tag{48}$$
$$y^* = g^* + \zeta^* g \tag{49}$$

Implicitly we hold constant m, m^*, w_0, w_0^*, and ω_0 and normalize them to zero. We can then represent the various regimes in terms of ζ and ζ^*.

5.1 Nominal wage rigidity in both countries

From (14) and (15) we find that:

$$y = [g + g^*] \frac{(1+\eta)\,\epsilon_{MR}\epsilon_{YG}}{2\left[\epsilon_{MR} + \delta_N \epsilon_{YR}\right]}, \tag{50}$$
$$y^* = [g + g^*] \frac{(1+\eta)\,\epsilon_{MR}\epsilon_{YG}}{2\left[\epsilon_{MR} + \delta_N \epsilon_{YR}\right]}. \tag{51}$$

Normalizing the common multiplier by unity, we can represent this case by $\zeta = \zeta^* = 1$.

5.2 Real wage rigidity in both countries

From (32) and (33) we find that:

$$y = [g - g^*] \frac{(1-\eta)\,\delta_R \epsilon_{YG}}{2\left(\delta_R + \epsilon_{YQ}\right)}, \tag{52}$$
$$y^* = [-g + g^*] \frac{(1-\eta)\,\delta_R \epsilon_{YG}}{2\left(\delta_R + \epsilon_{YQ}\right)}. \tag{53}$$

Normalizing the common multiplier by unity, we can represent this case by $\zeta = \zeta^* = -1$.

5.3 Mixed case

From (45) and (46) we find:

$$y = \left[\left[1 + \frac{(1-\eta)\,\delta_N \epsilon_{YR}}{\epsilon_{MR}}\right] g + \left[\eta - \frac{(1-\eta)\,\delta_N \epsilon_{YR}}{\epsilon_{MR}}\right] g^*\right] \frac{\delta_R \epsilon_{MR} \epsilon_{YG}}{|\Delta_M|}, \tag{54}$$
$$y^* = \left[\left[\frac{(1+\eta)\,\epsilon_{YQ}}{\delta_R} + \eta\right] g + \left[1 + \frac{(1+\eta)\,\epsilon_{YQ}}{\delta_R}\right] g^*\right] \frac{\delta_R \epsilon_{MR} \epsilon_{YG}}{|\Delta_M|}. \tag{55}$$

In the text we implicitly assume that foreign fiscal policy constitutes a beggar-thy-neighbour policy, i.e. we assume that $\eta \epsilon_{MR} < (1-\eta)\,\delta_N \epsilon_{YR}$ in (54). In the first step we normalize the common multiplier appearing in (54)-(55), i.e. we set:

$$\frac{\delta_R \epsilon_{MR} \epsilon_{YG}}{|\Delta_M|} = 1. \tag{56}$$

In the second step, we transform our measure of g and g^* by writing:

$$\bar{g} \equiv \frac{g}{1+\frac{(1-\eta)\delta_{N}\epsilon_{YR}}{\epsilon_{MR}}}, \tag{57}$$

$$\bar{g}^* \equiv \frac{g^*}{1+\frac{(1+\eta)\epsilon_{YQ}}{\delta_{R}}}. \tag{58}$$

Using (56)-(58) we can then write (54)-(55) in the form of (48)-(49), though with $\bar{g}$ and $\bar{g}^*$ appearing, where ζ and ζ^* are defined as follows:

$$\zeta = \frac{\eta-\frac{(1-\eta)\delta_{N}\epsilon_{YR}}{\epsilon_{MR}}}{1+\frac{(1+\eta)\epsilon_{YQ}}{\delta_{R}}} < 0, \tag{59}$$

$$0 < \zeta^* = \frac{\frac{(1+\eta)\epsilon_{YQ}}{\delta_{R}}+\eta}{1+\frac{(1-\eta)\delta_{N}\epsilon_{YR}}{\epsilon_{MR}}} < 1. \tag{60}$$

Note that in the text we ignore this transformation and work directly with (59)-(60).