Ageing and Growth in the Small Open Economy*

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Abstract

We study the effects of demographic and fiscal shocks on the growth performance of an industrialized small open economy. We construct an overlapping-generations model which incorporates a realistic description of the mortality process. Agents engage in educational activities at the start of life and thus create human capital to be used later on in life for production purposes. Depending on the strength of the intergenerational externality in the training function, the model gives rise to exogenous or endogenous growth. Demographic shocks and fiscal stimuli give rise to slowly-converging and often non-monotonic transition paths.

JEL codes: E10, D91, O40, F41, J11.

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1 Introduction

The western world is ageing rapidly. Since the postwar period, the ageing process can be attributed both to increased longevity and reduced fertility (Lee, 2003). For example, in the Netherlands, life expectancy at birth rose from 71.5 years in 1950 to 78.5 years in 2000, whilst the annual (crude) birth rate fell from 2.3% to 1.3% of the population. Because infant mortality stayed relatively constant during that period (at 0.8% of the population), the increase in longevity must be attributed to reduced adult mortality (Vaupel, 1997). A similar demographic pattern can be observed for most OECD countries.

The objective of this paper is to investigate the effects on the economic growth performance of a small open economy of substantial demographic shocks of the type and magnitude mentioned above. It must be stressed from the outset that we restrict attention to the study of advanced industrial economies having access to well-functioning markets including the world capital market. Our study is thus intended as a contribution to the field of open-economy macroeconomics.[1] We formulate a simple analytical growth model in which finitely-lived agents accumulate both physical and human capital. Our analysis makes use of modeling insights from two main bodies of literature. First, in order to allow for demographic shocks, we employ the generalized Blanchard-Yaari overlapping-generations model reported in our earlier paper (Heijdra and Romp, 2005). In this model disconnected generations are born at each instant and individual agents face a positive and age-dependent probability of death at each moment in time. By making the mortality rate age-dependent, the model can be used to investigate changes in adult mortality.[2]

The second building block of our analysis concerns the engine of growth. Following Lu-

[1] The recent growth and development literature takes a much longer-run perspective and attempts to model the "$\cdots$ long transition process, from thousands of years of Malthusian stagnation through the demographic transition to modern growth" Galor and Weil (2000, p. 806). Clearly, in this literature, both fertility and mortality rates are endogenous; see the recent survey by Galor (2005). In this paper, we follow the macroeconomic literature by assuming that the birth rate and the mortality process are exogenous.

[2] Other papers including an age-dependent mortality process include Boucekkine *et al.* (2002), Faruqee (2003), and d'Albis (2007). Boucekkine *et al.* (2002) is discussed throughout this paper. Faruqee's (2003) analysis is flawed because he confuses the cumulative density function with the mortality rate. d'Albis (2007) characterizes the steady state in a closed economy setting. Both Faruqee (2003) and d'Albis (2007) only look at steady-state effects.

cas (1988), we assume that the purposeful accumulation of human capital forms the core mechanism leading to economic growth. More specifically, like Bils and Klenow (2000), Kalemli-Ozcan *et al.* (2000), de la Croix and Licandro (1999), and Boucekkine *et al.* (2002), we assume that individual agents accumulate human capital by engaging in full-time educational activities at the start of life. The start-up education period is chosen optimally by each individual and labour market entry is assumed to be irreversible. Depending on the parameter setting, the human capital production function (or *training function*) may include an intergenerational external effect of the "shoulders of giants" variety, as first proposed in an overlapping generations context by Azariadis and Drazen (1990). With an operative externality, an individual's training function depends positively on the economy-wide stock of human capital per worker in that individual's birth period.

In our model, the strength of the intergenerational spillover is regulated by a single non-negative parameter, ϕ. Unfortunately, there is no consensus regarding the appropriate magnitude of ϕ. For example, Kalemli-Ozcan *et al.* (2000) abstract from the intergenerational spillover altogether and thus set $\phi = 0$. In contrast, Bils and Klenow (2000) set $0 < \phi < 1$, and thus assume that the externality is operative but subject to diminishing returns. Finally, de la Croix and Licandro (1999), Boucekkine *et al.* (2002), Echevarría (2004), and Echevarría and Iza (2006) consider the knife-edge case with $\phi = 1$. Our model synthesizes the existing literature by allowing the spillover parameter to take on any value between zero and unity ($0 \leq \phi \leq 1$).

Our paper is structured as follows. In Section 2 we present the model and demonstrate its main properties. A unique solution for the optimal schooling period is derived which depends on the fiscal parameters and on the mortality process. The mortality process, in combination with the birth rate, also determines a unique path for the population growth rate. For a given initial level of per capita human capital, the model implies a unique time path for all macroeconomic variables. Depending on the strength of the intergenerational external effect, the model either displays exogenous growth ($0 \leq \phi < 1$) and ultimate convergence to constant per capita variables, or endogenous growth ($\phi = 1$) and convergence to a constant growth rate. Unlike Boucekkine *et al.* (2002), who use a linear felicity function, our model fully determines unique transition paths for all variables of interest, both at the level of individuals and in the aggregate.

In Section 3 we investigate the effects of once-off demographic changes on the population growth rate, both at impact, during transition, and in the long run. Using a general description of the mortality process, we find that a reduction in the birth rate reduces the steady-state population growth rate, whilst an increase in longevity (due to reduced adult mortality) increases this rate because average mortality falls. We also estimate the Gompertz-Makeham (G-M) mortality process, employing data for the Dutch cohort born in 1920, and use it to illustrate the rather complicated (cyclical) adjustment path resulting from once-off demographic changes. Especially for the cohort-specific mortality shock, convergence toward the new steady state is extremely slow. Indeed, due to the vintage nature of the population, more than 150 years pass until the new demographic steady state is reached. The G-M mortality process outperforms the one specified by Boucekkine *et al.* (2002, p. 344) because it fits the demographic data much better, and because it avoids the problematic prediction of a finite maximum age; a phenomenon for which no evidence exists in the modern medical or biological literature.[3]

In Section 4 we study the determinants of the optimal schooling decision in detail. An increase in the educational subsidy or the labour income tax leads to an increase in the length of the educational period. Similarly, a reduction in *adult* mortality also prompts agents to increase the schooling period. Such a shock lengthens the post-school period and increases the pecuniary benefits of schooling. In contrast, a reduction in *child* mortality has no effect on the optimal schooling period. Such a shock increases the probability of surviving the schooling period, but has no effect on the length of the working period. Finally, a baby bust also leaves the optimal schooling period unchanged because it has no effect on the individual's optimization problem. Unlike Boucekkine *et al.* (2002), who use a specific functional form for the mortality process, we reach our conclusions using the general specification for the mortality process.

Section 5 deals with the exogenous growth model, which, on the basis of the empirical evidence, we consider to be the most relevant one. Indeed, using the recent empirical study

[3] As Kirkwood puts it, "...the idea of a fixed limit to human longevity was always a little questionable but it is only now, as understanding of the ageing process improves, that the reason has become apparent. There is no mechanism that measures man's span of time and then activates a destructive process. In fact, quite the reverse is true and nearly every system in the body does its best to preserve life" (2001, p. 576). See also Kirkwood and Austad (2000) and Friedenberg (2002) on the non-existence of a fixed limit to life.

by de la Fuente and Doménech (2006), we argue that a plausible value for the intergenerational externality parameter, ϕ, lies between 0.27 and 0.40, i.e. nowhere in the vicinity of the knife-edge case considered by Boucekkine *et al.* (2002) and others. The factual evidence points firmly in the direction of positive but strongly diminishing returns to the intergenerational external effect.

In Section 5 we also study the (impact, transitional, and long-run) effects of fiscal and demographic changes on per capita human capital and the other macroeconomic variables. A positive fiscal impulse leads to an increase in the per capita stock of human capital but leaves the steady-state growth rate of the macro-variables in level terms unchanged (and equal to the steady-state population growth rate). Furthermore, whilst a reduction in the birth rate and an increase in longevity (due to reduced adult mortality) both increase the steady-state per capita human capital stock, the growth effects on level variables are opposite in sign. Again, for all shocks considered, the transitional adjustment is rather slow and often non-monotonic. For fiscal shocks, the effect on pre-shock students and workers differ because only the former can adjust their optimal schooling period.

In Section 6 we present some concluding thoughts and give some suggestions for future research. The paper also contains two appendices. Appendix A contains some key mathematical derivations. In Appendix B we briefly discuss the endogenous growth version of the model. Though this knife-edge case has been studied extensively in the theoretical literature, it is based on an unrealistically strong intergenerational external effect in human capital creation for which very little empirical backing exists (see above).

2 The model

2.1 People

2.1.1 Individual plans

At time t, an individual born at time v ($v \leq t$) has the following (remaining) lifetime utility function:

$$\Lambda(v,t) \equiv e^{M(t-v)} \int_t^\infty U\left[\bar{c}(v,\tau)\right] e^{-[\theta(\tau-t)+M(\tau-v)]} d\tau, \tag{1}$$

where $U[\cdot]$ is the felicity function, $\bar{c}(v,\tau)$ is consumption (bars denote individual variables), θ is the constant pure rate of time preference ($\theta > 0$), and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at time τ.[4] The cumulative mortality rate, $M(\tau - v)$, is defined as:

$$M(\tau - v) \equiv \int_0^{\tau - v} m(\alpha)\, d\alpha, \tag{2}$$

where $m(\alpha)$ is the instantaneous mortality rate of an agent of age α. As was pointed out by Yaari (1965), future felicity is discounted not only because of pure time preference (as $\theta > 0$) but also because of lifetime uncertainty (as $M(\tau - v) > 0$ for $\tau > v$). The felicity function is iso-elastic:

$$U[\bar{c}(v,\tau)] = \begin{cases} \dfrac{\bar{c}(v,\tau)^{1-1/\sigma} - 1}{1 - 1/\sigma} & \text{for } \sigma \neq 1 \\ \ln \bar{c}(v,\tau) & \text{for } \sigma = 1 \end{cases}, \tag{3}$$

where σ is the constant intertemporal substitution elasticity ($\sigma \geq 0$).

The budget identity is given by:

$$\dot{\bar{a}}(v,\tau) = [r + m(\tau - v)]\, \bar{a}(v,\tau) + \bar{w}(v,\tau) - \bar{g}(v,\tau) - \bar{c}(v,\tau), \tag{4}$$

where $\bar{a}(v,\tau)$ is real financial wealth, r is the constant world interest rate, $\bar{w}(v,\tau)$ is wage income, and $\bar{g}(v,\tau)$ is total tax payments (see below). As usual, a dot above a variable denotes that variable's time rate of change, e.g. $\dot{\bar{a}}(v,\tau) \equiv d\bar{a}(v,\tau)/d\tau$. Following Yaari (1965) and Blanchard (1985), we postulate the existence of a perfectly competitive life insurance sector which offers actuarially fair annuity contracts to the agents. Since someone's age is directly observable, the annuity rate of interest faced by an individual of age $\tau - v$ is equal to the sum of the world interest rate and the instantaneous mortality rate of that person. In order to avoid having to deal with a taxonomy of different cases, we restrict attention to the case of a nation populated by patient agents, i.e. $r \geq \theta$. Financial wealth can be held in the form of claims on domestic capital, $\bar{v}(v,\tau)$, domestic government bonds, $\bar{d}(v,\tau)$, or net foreign assets, $\bar{f}(v,\tau)$.

$$\bar{a}(v,\tau) \equiv \bar{v}(v,\tau) + \bar{d}(v,\tau) + \bar{f}(v,\tau). \tag{5}$$

[4] The appearance of the term $e^{M(t-v)}$ in (1) (and also in (9)-(10) below) is a consequence of the fact that the distribution of expected remaining lifetimes is not memoryless in general. Blanchard (1985) uses the memoryless exponential distribution for which $M(\alpha) = \mu_0 \alpha$ (where μ_0 is a constant) and thus $M(t - v) - M(\tau - v) = -M(\tau - t)$. Equation (1) can then be written in a more familiar format as $\Lambda(v,t) \equiv \int_t^{\infty} U[\bar{c}(v,\tau)]\, e^{-(\theta+\mu_0)(\tau - t)} d\tau$.

These assets are perfect substitutes in the agents' investment portfolios and thus attract the same rate of return.

The agent engages in full time schooling during the early stages of life and works full time thereafter.[5] The training function is given by:[6]

$$\bar{h}(v,\tau) = \begin{cases} 0 & \text{for } v \leq \tau \leq v + s(v) \\ A_H h(v)^{\phi} s(v) & \text{for } \tau > v + s(v) \end{cases}, \qquad 0 \leq \phi \leq 1, \tag{6}$$

where $\bar{h}(v,\tau)$ is the human capital of the agent, A_H is an exogenous productivity index, $h(v)$ is economy-wide human capital at time v (expressed in *per capita* terms; see below), ϕ is a parameter regulating the strength of the intergenerational external effect in knowledge creation ("standing on the shoulders of previous generations"), and $s(v)$ is the length of the schooling period chosen by an agent born at time v. Special cases of (6) were used by de la Croix and Licandro (1999, p. 257) and Boucekkine *et al.* (2002, p. 347), who set $\phi = 1$, and by Kalemli-Ozcan *et al.* (2000, pp. 5, 10), who set $\phi = 0$.

Available human capital is rented out to competitive producers so that wage income, $\bar{w}(v,\tau)$, can be written as:

$$\bar{w}(v,\tau) = w(\tau)\,\bar{h}(v,\tau), \tag{7}$$

where $w(\tau)$ is the market-determined rental rate of human capital (see below).

The tax system takes the following form. First, all through life, the agent pays a lumpsum tax. Second, during the educational phase, the agent receives a study grant from the government. Third, during working life, the agent faces a labour income tax on wage earnings.

[5] In a companion paper we study the retirement decision in the presence of a realistic pension system which includes provisions for early retirement. See Heijdra and Romp (2006).

[6] This formulation was first proposed in the context of Diamond-Samuelson style overlapping models by Azariadis and Drazen (1990, p. 510) and Tamura (1991, p. 524). Abstracting from their work experience term and using our notation, Bils and Klenow (2000, p. 1161) model the human capital production function as follows:

$$\bar{h}(v,t) = \bar{h}(v-\bar{u},t)^{\phi} e^{\zeta(s)}, \qquad \text{for } t - v > s, \tag{6'}$$

where $\bar{u}$ is interpreted as the age of the teachers (assumed to be fixed), and $\zeta(s)$ captures the productivity effect of schooling ($\zeta'(s) > 0$). Clearly, for $\zeta(s) \equiv \ln s$ the second term on the right-hand side of (6′) is equal to s. In our view, equation (6′) does not adequately capture the notion of an intergenerational externality as the link is only operative between generations v and $v - \bar{u}$, which are locked in a tango through time. In (6) the *economy-wide* stock of per capita human capital determines the initial condition facing newborns. Hence, every agent alive at time v exerts an external effect on newborns.

The tax system is thus given by:

$$\bar{g}(v,\tau) = \begin{cases} [z(\tau) - \rho]\, w(\tau)\, A_H h(v)^{\phi} & \text{for } v \le \tau \le v + s(v) \\ [z(\tau) + t_L s(v)]\, w(\tau)\, A_H h(v)^{\phi} & \text{for } \tau > v + s(v) \end{cases}, \tag{8}$$

where ρ is the *educational subsidy* rate ($\rho > 0$), t_L is the labour income tax rate ($0 \le t_L < 1$), and $z(\tau)$ represents the lumpsum part of the tax. All tax instruments are indexed to the value of marginal schooling productivity to the vintage-v individual (i.e. $A_H h(v)^{\phi}$) to ensure that the tax system continues to play a nontrivial role even in the presence of ongoing economic growth.

From the perspective of the planning date t, a young agent chooses remaining time in school ($v + s(v) - t$), and sequences for $\bar{c}(v,\tau)$ and $\bar{a}(v,\tau)$ (for $\tau \in [t,\infty)$) in order to maximize $\Lambda(v,t)$ subject to (4)-(8), a non-negativity constraint $v + s(v) \ge t$,[7] and a lifetime solvency condition. By using this solvency condition as well as equations (4)-(8), the lifetime budget constraint can be written as follows:

$$e^{M(t-v)} \int_t^{\infty} \bar{c}(v,\tau) e^{-[r(\tau-t)+M(\tau-v)]} d\tau = \bar{a}(v,t) + \overline{li}(v,t), \tag{9}$$

where we have used the fact that generations are born without financial assets (i.e. $\bar{a}(v,v) = 0$) and where $\overline{li}(v,t)$ is (remaining) lifetime after-tax wage income of the agent:

$$\begin{aligned} \overline{li}(v,t) \equiv\; & A_H h(v)^{\phi} e^{M(t-v)} \Bigg[\rho \int_t^{\max\{t,v+s(v)\}} w(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \\ & + (1-t_L)\, s(v) \int_{\max\{t,v+s(v)\}}^{\infty} w(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \\ & - \int_t^{\infty} z(\tau)\, w(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \Bigg]. \end{aligned} \tag{10}$$

According to (9), the present value of consumption expenditure (left-hand side) must equal total lifetime resources (right-hand side). In the presence of actuarially fair annuity contracts, the annuity rate of interest, $r + m(\tau - v)$, is used for discounting purposes in (9)-(10).

The following two-stage solution approach can now be used. In the first step, the agent chooses $s(v)$ in order to maximize lifetime wage income, $\overline{li}(v,t)$. Since $\bar{a}(v,t)$ is predetermined, this pushes the lifetime budget constraint out as far as possible and fixes the right-hand side of (9). In the second step, the agent chooses the optimal sequence for consumption in order to maximize $\Lambda(v,t)$ subject to (9).

[7] Older agents have already completed the educational phase ($t - v > s(v)$) and only choose paths for consumption and financial assets. Labour market entry is thus assumed to be an absorbing state.

To prepare for the discussion of the optimal solutions, we first define the *demographic discount function*, $\Delta(u,\lambda)$, in general terms as:

$$\Delta(u,\lambda) \equiv e^{\lambda u + M(u)} \int_u^\infty e^{-[\lambda\alpha + M(\alpha)]} d\alpha, \qquad \text{(for } u \geq 0\text{)}, \tag{11}$$

where $u \equiv t - v$ and $\alpha \equiv \tau - v$ denote, respectively, the agent's age in the planning period t and at some later time τ, and where λ is a parameter of the function. In our earlier paper we established a number of properties of the $\Delta(u,\lambda)$ function, which we restate for convenience in Proposition 1.

Proposition 1 Let $\Delta(u,\lambda)$ be defined as in (11) and assume that the mortality rate is non-decreasing, i.e. $m'(\alpha) \geq 0$ for all $\alpha \geq 0$. Then the following properties can be established for $\Delta(u,\lambda)$:

(i) decreasing in λ, $\dfrac{\partial\Delta(u,\lambda)}{\partial\lambda} = -e^{\lambda u + M(u)} \displaystyle\int_u^\infty [\alpha - u] e^{-[\lambda\alpha + M(\alpha)]} d\alpha < 0$;

(ii) non-increasing in the agent's age, $\dfrac{\partial\Delta(u,\lambda)}{\partial u} = (\lambda + m(u))\Delta(u,\lambda) - 1 \leq 0$;

(iii) strictly positive, $\Delta(u,\lambda) > 0$ for $u < \infty$;

(iv) $\lim_{\lambda\to\infty} \Delta(u,\lambda) = 0$;

(v) for $m'(\alpha) > 0$ and $m''(\alpha) \geq 0$, the inequality in (ii) is strict and $\lim_{u\to\infty} \Delta(u,\lambda) = 0$.

Proof: see Heijdra and Romp (2005). □

Schooling period The first-order condition for the optimal schooling period, $s^*(v)$, is obtained by using (10) and setting $d\overline{li}(v,t)/ds(v) = 0$. After some straightforward manipulations we obtain:

$$\int_{v+s^*(v)}^\infty w(\tau)\, e^{-[r(\tau-v)+M(\tau-v)]} d\tau = \left[s^*(v) - \frac{\rho}{1-t_L}\right] w(v+s^*(v))\, e^{-[rs^*(v)+M(s^*(v))]}. \tag{12}$$

As was pointed out by de la Croix and Licandro (1999, p. 258), the left-hand side of (12) is the marginal benefit of increasing the schooling period, and the right-hand side represents the marginal cost of postponing labour market entry. Clearly, both ρ and t_L reduce marginal cost. This is because, by staying in school, the agent not only receives the education subsidy but also avoids paying the labour income tax.

For the case studied in this paper, the wage rate is constant (see below), and equation (12) reduces to:

$$s^* - \frac{\rho}{1-t_L} = \Delta\left(s^*, r\right), \tag{13}$$

where $\Delta(s^*, r)$ is the demographic discount function, evaluated for $u = s^*$ and $\lambda = r$. Equation (13) determines the age at which the vintage-v individual completes his education. With a constant mortality process, the optimal schooling period is independent of the agent's date of birth. Since the left-hand side of (13) is increasing in s^* and (by Proposition 1(ii)) the right-hand side is non-increasing in s^*, it follows that the optimal schooling period is positive and unique.[8] In Section 4 below we study changes in the tax parameters and the mortality process which give rise to once-off changes in the optimal schooling period.

Consumption The first-order conditions for optimal consumption can be written as $\bar{c}(v,\tau) = e^{\sigma[r-\theta](\tau-v)}/\lambda_u$, where λ_u (> 0) is the Lagrange multiplier for the lifetime budget constraint (9). The growth rate of individual consumption is thus given by the familiar Euler equation:

$$\frac{\dot{\bar{c}}(v,\tau)}{\bar{c}(v,\tau)} = \sigma\left[r-\theta\right], \qquad \text{for } \tau \in [t, \infty). \tag{14}$$

For $r > \theta$, it follows that the agent adopts an upward sloping time profile for its consumption provided the intertemporal substitution elasticity is strictly positive ($\sigma > 0$). By using (14) in (9) the expression for the consumption *level* in the planning period is obtained:

$$\Delta\left(u, r^*\right)\bar{c}(v,t) = \bar{a}(v,t) + \overline{li}(v,t), \tag{15}$$

where $\Delta(u, r^*)$ is the demographic discount factor, evaluated for $\lambda = r^*$, and where $r^* \equiv r - \sigma[r-\theta]$ can be interpreted as the *effective* discount rate facing the agent. The marginal (and average) propensity to consume out of total wealth is equal to $1/\Delta(u, r^*)$. It follows from Proposition 1(v) that the consumption propensity rises with age. Intuitively, as one gets older the planning horizon contracts and, in the absence of bequests, one's propensity to consume out of wealth rises accordingly.[9]

[8]Indeed, for the Blanchard case with a constant death rate, μ_0, we find that $\Delta(u,\lambda) = 1/(\lambda+\mu_0)$, so that (13) simplifies to $s^* = \rho/(1-t_L) + 1/(r+\mu_0)$. Apart from the fiscal parameters, this is the expression found in de la Croix and Licandro (1999, p. 258).

[9]This mechanism is, of course, absent in the Blanchard case because the expected remaining lifetime is age-invariant in that case, and the consumption propensity equals $1/\Delta(u,r^*) = r^* + \mu_0$. Note that, with $r > \theta$, this

2.1.2 Demography

We allow for non-zero population growth by employing the analytical framework that was initially developed by Buiter (1988) and was extended to an age-dependent mortality rate by Heijdra and Romp (2005). Since we wish to study ageing shocks below, we generalize our earlier model by assuming that different cohorts may face different mortality profiles.[10] In particular, we postulate that the instantaneous mortality rate can be written as $m(\alpha, \psi(v))$, where $\psi(v)$ is a parameter that only depends on the cohort's time of birth. The corresponding cumulative mortality rate is written as $M(u, \psi(v)) \equiv \int_0^u m(\alpha, \psi(v))\, d\alpha$. Where no confusion arises, we drop the dependency of ψ on v, and the dependency of m and M on ψ.

The birth rate is exogenous but may vary over time. The size of a newborn generation at time v is proportional to the current population at that time, i.e. $L(v, v) = b(v)\, L(v)$, where $b(v)$ and $L(v)$ are, respectively the crude birth rate ($b(v) > 0$) and the population size at time v. The size of cohort v at some later time τ is given by:

$$L(v, \tau) = L(v, v)\, e^{-M(\tau - v, \psi(v))} = b(v)\, L(v)\, e^{-M(\tau - v, \psi(v))}. \tag{16}$$

By definition, the total population at time t satisfies the following expressions:

$$L(t) \equiv \int_{-\infty}^{t} L(v, t)\, dv \equiv L(v)\, e^{N(v,t)}, \tag{17}$$

where $n(\tau)$ is the instantaneous growth rate of the population at time τ, and $N(v, t) \equiv \int_v^t n(\tau)\, d\tau$ is the cumulative growth factor over the interval $t - v$. Finally, by combining (16)-(17) we obtain:

$$l(v, t) \equiv \frac{L(v, t)}{L(t)} = b(v)\, e^{-[N(v,t) + M(t - v, \psi(v))]}, \qquad t \geq v, \tag{18}$$

$$1 = \int_{-\infty}^{t} b(v)\, e^{-[N(v,t) + M(t - v, \psi(v))]} dv. \tag{19}$$

Equation (18) shows the population share of the v-cohort at some later time t. It generalizes the corresponding expression found in Heijdra and Romp (2005) to the case of a non-constant

version of the model is only valid provided $r^* + \mu_0 > 0$, i.e. σ cannot be too large, $0 \leq \sigma < (r + \mu_0) / (r - \theta)$. No such restrictions are needed for the demographic process used in the paper.

[10] In their classic paper, Lee and Carter (1992), employing US data, demonstrated a clear downward trend in the instantaneous mortality rate *at all ages* during the twentieth century, i.e. the mortality rate of an x-year old declined steadily over the period 1900-1989.

population growth rate, $n(t)$. Equation (19) implicitly determines $n(t)$ for given demographic parameters (see also Section 3.1). For an economy which has faced the same demographic environment for a long time (i.e., $b(v) = b_0$ and $M(t-v, \psi(v)) = M(t-v, \psi_0)$), the population growth rate reaches a constant steady-state value, $n(t) = \hat{n}$. Equation (19) thus reduces to $1/b_0 = \Delta(0, \hat{n})$, which is the expression reported in Heijdra and Romp (2005).

2.1.3 Per capita plans

Per capita variables are calculated as the integral of the generation-specific values multiplied by the corresponding generation weights. For example, per capita human capital is defined as:

$$h(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{h}(v,t)dv, \tag{20}$$

where $l(v,t)$ and $\bar{h}(v,t)$ are given in, respectively, (18) and (6) above. In a similar fashion, per capita consumption is given by $c(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{c}(v,t)dv$, where $\bar{c}(v,t)$ is given by (15). By differentiating $c(t)$ with respect to time and noting (14) we obtain an expression for the "Euler equation" for per capita consumption:

$$\dot{c}(t) = b\bar{c}(t,t) + \sigma[r-\theta]c(t) - n(t)c(t) - \int_{-\infty}^{t} m(t-v)l(v,t)\bar{c}(v,t)dv, \tag{21}$$

where we have used the fact that $\dot{l}(v,t)/l(v,t) \equiv -[n(t) + m(t-v)]$. Per capita consumption grows over time because new generations are born at each instant which start to consume out of human wealth (first term on the right-hand side of (21)) and because individual consumption of existing generations grows (second term). The third term on the right-hand side corrects for time-dependent population growth, whilst the fourth term corrects for (age-dependent) mortality.

Turning to the wealth components, per capita financial wealth is defined as $a(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{a}(v,t)dv$. By differentiating this expression with respect to time we obtain the dynamic path of per capita financial assets:[11]

$$\dot{a}(t) = [r - n(t)]a(t) + w(t)h(t) - g(t) - c(t), \tag{22}$$

[11] In deriving (22) we have used equation (4) and noted the fact that agents are born without financial assets ($\bar{a}(t,t) = 0$).

where $g(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{g}(v,t)dv$ is per capita tax payments. We assume that the interest rate net of population growth is positive, i.e. $r > n(t)$. As in the standard Blanchard model, annuity payments drop out of the expression for per capita asset accumulation because they constitute transfers (via the life insurance companies) from the deceased to agents who continue to enjoy life.

2.2 Firms

Perfectly competitive firms use physical and human capital to produce a homogeneous commodity, $Y(t)$, that is traded internationally. The technology is represented by the following Cobb-Douglas production function:

$$Y(t) = K(t)^{\varepsilon} [A_Y H(t)]^{1-\varepsilon}, \qquad 0 < \varepsilon < 1, \tag{23}$$

where A_Y is a constant index of labour-augmenting technological change, $K(t) \equiv L(t)k(t)$ is the aggregate stock of physical capital, and $H(t) \equiv L(t)h(t)$ is the aggregate stock of human capital. The cash flow of the representative firm is given by:

$$\Pi(t) \equiv Y(t) - w(t)H(t) - I(t), \tag{24}$$

where $w(t)$ is the rental rate on human capital, and $I(t) \equiv \dot{K}(t) + \delta K(t)$ is gross investment, with δ representing the constant depreciation rate. The (fundamental) stock market value of the firm at time t is equal to the present value of cash flows, using the interest rate for discounting, i.e. $V(t) \equiv \int_t^{\infty} \Pi(\tau) e^{r(t-\tau)} d\tau$. The firm chooses paths for $I(\tau)$, $K(\tau)$, $H(\tau)$, and $Y(\tau)$ (for $\tau \in [t,\infty)$) to maximize $V(t)$ subject to the capital accumulation constraint, the production function (23) and the definition of cash flows (24). Since there are no adjustment costs on investment, the value of the firm equals the replacement value of the capital stock, i.e. $V(t) = K(t)$. In addition, the usual factor demand equations are obtained:

$$r + \delta = \varepsilon \left(\frac{A_Y h(t)}{k(t)} \right)^{1-\varepsilon} = \frac{\partial Y(t)}{\partial K(t)}, \tag{25}$$

$$w(t) = (1-\varepsilon) A_Y \left(\frac{A_Y h(t)}{k(t)} \right)^{-\varepsilon} = \frac{\partial Y(t)}{\partial H(t)}. \tag{26}$$

For each factor of production, the marginal product is equated to the rental rate. Since the fixed world interest rate pins down the ratio between human and physical capital, it follows from (26) that the wage rate is time-invariant, i.e. $w(\tau) = w$,[12] and that physical capital is proportional to human capital at all time:

$$k(t) = A_Y \left(\frac{\varepsilon}{r+\delta} \right)^{1/(1-\varepsilon)} h(t). \tag{27}$$

2.3 Government and foreign sector

In the absence of government consumption, the government (flow) budget identity in per capita terms is given by:

$$\dot{d}(t) = [r - n(t)]\, d(t) - g(t), \tag{28}$$

where $d(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{d}(v,t)dv$ is per capita government debt. Using the government solvency condition, $\lim_{\tau\to\infty} d(\tau)e^{r(t-\tau)+N(t,\tau)} = 0$, the intertemporal budget constraint of the government can be written as:

$$d(t) = \int_{t}^{\infty} g(\tau)\, e^{r(t-\tau)+N(t,\tau)} d\tau. \tag{29}$$

To the extent that there is outstanding debt (positive left-hand side), it must be exactly matched by the present value of current and future primary surpluses (positive right-hand side), using the net interest rate ($r - n(\tau)$) for discounting purposes.

By using the marginal productivity conditions (25)-(26) and noting the linear homogeneity of the production function (23) and the constancy of factor prices, we find that per capita output, $y(t) \equiv Y(t)/L(t)$, can be written as follows:

$$\begin{aligned} y(t) &= (r+\delta)\, k(t) + wh(t) \\ &= \left[(r+\delta)^{\varepsilon/(\varepsilon-1)} (\varepsilon A_Y)^{1/(1-\varepsilon)} + w \right] h(t). \end{aligned} \tag{30}$$

In going from the first to the second line we have made use of (27). It follows from the definition of gross investment that the dynamic evolution of the per capita stock of capital is

[12]With labour-augmenting technological change, $\gamma_A \equiv \dot{A}_Y/A_Y$, the wage rate grows exponentially at rate γ_A and equation (13) changes to:

$$s^* - \frac{\rho}{1-t_L} = \Delta(s^*, r-\gamma_A).$$

It follows from Proposition 1(i) that $\partial s^*/\partial\gamma_A > 0$, i.e. the schooling period depends positively on anticipated wage growth. See also Bils and Klenow (2000, p. 1161) on this issue.

given by:

$$\dot{k}(t) = i(t) - [\delta + n(t)]\, k(t), \tag{31}$$

where $i(t) \equiv I(t)/L(t)$ is per capita investment. Finally, the current account of the balance of payment, representing the dynamic change in the per capita stock of net foreign assets, $f(t)$, takes the following form:

$$\dot{f}(t) = [r - n(t)]\, f(t) + y(t) - c(t) - i(t), \tag{32}$$

where $f(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{f}(v,t)dv$.[13]

2.4 Model solution

The model is recursive and can be solved in three steps. First, for a given mortality process and with constant tax parameters ρ and t_L, equation (13) determines the optimal schooling period for each agent. Similarly, for a given birth rate, equation (19) can be solved for the population growth rate, $n(t)$. Next, conditional on the optimal value for s^*, the path for $n(t)$, and an initial condition, equation (20) can be solved for the equilibrium path of human capital, $h(t)$. Finally, the lumpsum tax z is used to balance the government's intertemporal budget restriction (29), after which the values for all remaining variables are fully determined.

The remainder of this paper is structured as follows. First, in Section 3 we use actual demographic data for the Netherlands to visualize what a realistic mortality profile looks like. In addition, we study the demographic effects of changes in adult mortality and the birth rate, both on impact, during transition, and in the long run. Next, in Section 4, we study how the optimal schooling decision is affected by changes in adult mortality and the

[13]The dynamic expression for per capita assets is given in equation (22), where $a(t) \equiv k(t) + d(t) + f(t)$ (recall that $V(t) = K(t)$). Clearly, total per capita assets $a(t)$ move smoothly over time but its constituting components ($k(t)$, $f(t)$, and $d(t)$) need not. Hence, even in the absence of discrete adjustments in government debt, the capital stock can jump as only $k(t) + f(t)$ moves smoothly over time in that case. A discrete change in $k(t)$ would be engineered by means of an asset swap. Throughout the paper, however, the world interest rate (r) is held constant so that (via (27)) the physical capital stock, $k(t)$, will evolve smoothly because the stock of human capital, $h(t)$, moves smoothly. As a result, the model also gives rise to well-defined current account dynamics—see also Figures 4-6 below.

fiscal parameters. Finally, in Section 5 we solve the general equilibrium growth model, compute analytical comparative static effects, and visualize the transitional dynamics using a plausibly calibrated version of the model.

3 Demographic shocks

In Heijdra and Romp (2005), we use Dutch demographic data[14] to estimate the parameters of the Gompertz-Makeham (G-M) mortality process. The instantaneous mortality rate associated with the G-M process takes the following format:

$$m(\alpha) = \mu_0 + \mu_1 e^{\mu_2 \alpha}, \tag{33}$$

where α is the agent's age, and the parameter estimates (and associated t-statistics) are $\hat{\mu}_0 = 0.2437 \times 10^{-2}$ (65.8), $\hat{\mu}_1 = 0.5520 \times 10^{-4}$ (20.5), and $\hat{\mu}_2 = 0.0964$ (138.2). The estimated survival function fits the data rather well.[15] It predicts an average mortality rate of 1.02% per annum and a proportion of centenarians equal to 0.1%. The dashed lines in Figure 1 illustrate several important features of the estimated G-M process. First, as panel (a) shows, the mortality rate is quite low and virtually constant up to about age 60, after which it rises exponentially. Second, as a result, the surviving fraction of the population declines steeply after age 60—see panel (b).

Two types of demographic shocks are considered in our analysis, namely a change in the birth rate and a change in the mortality process. In order to study the effects of changes in the mortality process, we write the instantaneous mortality rate as $m(\alpha, \psi)$, where ψ is a parameter.[16] We make the following assumptions regarding the effects of a change in ψ.

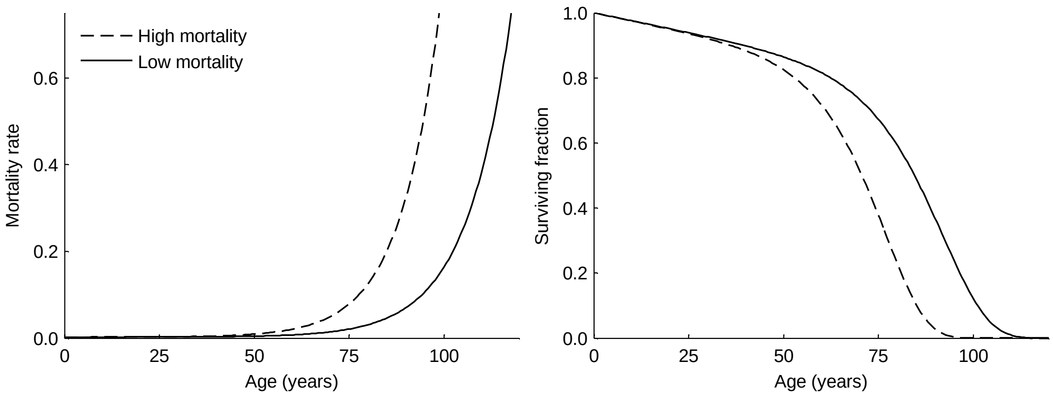
[14]We use data for the cohort born in 1920 in the Netherlands. Actual observations are available up to 2003, and projections have been used for the age range 84-105.

[15]Boucekkine *et al.* (2002) postulate the following instantaneous mortality function:

$$m(\alpha) \equiv \frac{\mu_1}{\mu_0}\left[e^{-\mu_1 \alpha} - e^{-\mu_1 \bar{\alpha}}\right]^{-1},$$

for $0 < \alpha \leq \bar{\alpha}$, where $\bar{\alpha} \equiv \ln(\mu_0)/\mu_1$ is the maximum attainable age. Using our data we find the following estimates: $\hat{\mu}_0 = 41.06$ (26.0), $\hat{\mu}_1 = 0.0429$ (86.5), $\hat{\bar{\alpha}} = 86.6$ (577.2), and $\hat{\sigma} = 0.0147$. The model incorrectly predicts that nobody from the 1920 cohort will survive until 2007, despite the fact that about 8% has managed to do so. See also Romp (2007, p. 28-32) for the estimation of five different mortality models.

[16]In the Blanchard case, which has only one parameter, μ_0 could be $-\psi$ or any decreasing function of ψ. The G-M process, stated in equation (33), depends on three parameters. Hence, the parameter *vector* is a function



(a) Mortality rate, $m(u) = \mu_0 + \mu_1 e^{\mu_2 u}$ (b) Surviving fraction, $S(u) = e^{-M(u)}$

Figure 1: Reduced Adult Mortality

Assumption 1 The mortality function has the following properties:

(i) $m(\alpha, \psi)$ is non-negative, continuous, and non-decreasing in age, $\dfrac{\partial m(\alpha, \psi)}{\partial \alpha} \geq 0$;

(ii) $m(\alpha, \psi)$ is convex in age, $\dfrac{\partial^2 m(\alpha, \psi)}{\partial \alpha^2} \geq 0$;

(iii) $m(\alpha, \psi)$ is non-increasing in ψ for all ages, $\dfrac{\partial m(\alpha, \psi)}{\partial \psi} \leq 0$;

(iv) the effect of ψ on the mortality function is non-decreasing in age, $\dfrac{\partial^2 m(\alpha, \psi)}{\partial \psi \partial \alpha} \leq 0$.

An example of a mortality shock satisfying all the requirements of Assumption 1 consists of a decrease in μ_1 or μ_2 of the G-M mortality function (see Section 5.2 for quantitative details of all shocks). In terms of Figure 1(a), the shock shifts the mortality function downward, with the reduction in mortality being increasing in age. In panel (b) the function for the surviving fraction of the population shifts to the right. The shock that we consider can thus be interpreted as a reduction in adult mortality. Of course, in view of the terminology of Assumption 1, an increase in ψ leads to an increase in the expected remaining lifetime for all ages. Note, finally, that Assumption 1 covers both the G-M process and the mortality process used by Boucekkine et al. (2002) that was mentioned in Footnote 15.

Armed with Assumption 1, the following results can be established.

of ψ, i.e. $(\mu_0, \mu_1, \mu_2) = f(\psi)$. An increase in ψ should result in such a change that the G-M mortality function decreases for all ages as ψ increases.

Proposition 2 Define $M(u,\psi)$ and $\Delta(u,\lambda,\psi)$ as:

$$M(u,\psi) \equiv \int_0^u m(\alpha,\psi)\,d\alpha, \tag{2'}$$

$$\Delta(u,\lambda,\psi) \equiv e^{\lambda u+M(u,\psi)}\int_u^\infty e^{-[\lambda\alpha+M(\alpha,\psi)]}d\alpha. \tag{11'}$$

Under Assumption 1, the following results can be established.

(i) $\dfrac{\partial M(u,\psi)}{\partial\psi} = \displaystyle\int_0^u \frac{\partial m(\alpha,\psi)}{\partial\psi}d\alpha \le 0;$

(ii) $\dfrac{\partial^2 M(u,\psi)}{\partial u\partial\psi} = \dfrac{\partial m(u,\psi)}{\partial\psi} \le 0;$

(iii) $\dfrac{\partial\Delta(u,\lambda,\psi)}{\partial\psi} = e^{\lambda u+M(u,\psi)}\displaystyle\int_u^\infty \left[\frac{\partial M(u,\psi)}{\partial\psi} - \frac{\partial M(\alpha,\psi)}{\partial\psi}\right] e^{-[\lambda\alpha+M(\alpha,\psi)]}d\alpha > 0.$

Proof: Items (i) and (ii) follow from simple differentiation and noting Assumption 1(iii). Item (iii) follows from differentiation of (11′) and (ii). □

3.1 Population growth

Demographic changes affect the growth rate of the population, both at impact, during transition, and in the long run. Armed with Propositions 1 and 2, we can easily compute the long-run effects of changes in the birth rate and the mortality process. Indeed, since equation (19) reduces in the steady state to $b\Delta(0,\hat{n},\psi) = 1$, it follows that $\hat{n}$ is an implicit function of b and ψ, the partial derivatives of which are given by:

$$\frac{\partial\hat{n}}{\partial b} = -\frac{\Delta(0,\hat{n},\psi)}{b\partial\Delta(0,\hat{n},\psi)/\partial\hat{n}} > 0, \tag{34}$$

$$\frac{\partial\hat{n}}{\partial\psi} = -\frac{\partial\Delta(0,\hat{n},\psi)/\partial\psi}{\partial\Delta(0,\hat{n},\psi)/\partial\hat{n}} > 0. \tag{35}$$

The signs in (34)-(35) follow from Propositions 1(i) and 2(iii). Not surprisingly, an increase in the birth rate and an increase in longevity both lead to an increase in the steady-state growth rate of the population.

To compute the transition path for the growth rate of the population we assume that at time $t = 0$ both the mortality process and the birth rate change in a stepwise fashion.[17] The mortality shock is assumed to be *embodied*, i.e. it only affects generations born from time

[17]More gradual transitions in the mortality rate and birth rate give rise to comparable patterns, except that the transition speed is slower. By assuming stepwise changes we thus over-estimate the speed of adjustment.

$t = 0$ onwards. In particular, the mortality process for pre-shock cohorts (with a negative generation index, $v < 0$) is described by $M(t-v, \psi_0)$ and $m(t-v, \psi_0)$, whereas post-shock cohorts (with $v \geq 0$) face the mortality process described by $M(t-v, \psi_1)$ and $m(t-v, \psi_1)$. In a similar fashion, the pre-shock and post-shock birth rates are denoted by, respectively, b_0 and b_1. The system is initially in a demographic steady state and the pre-shock population growth rate is denoted by $\hat{n}_0$ (defined implicitly by the condition $1/b_0 = \Delta(0, \hat{n}_0, \psi_0)$.

As a consequence of the demographic changes, the path for the population growth rate is implicitly determined by the following expression:

$$1 = b_0 \int_{-\infty}^{0} e^{-M(t-v,\psi_0)-N(v,t)} dv + b_1 \int_{0}^{t} e^{-M(t-v,\psi_1)-N(v,t)} dv, \tag{36}$$

where $N(v,t) \equiv \int_v^t n(\tau)\, d\tau$ (see also (17) above).[18] In Figure 2 we plot the transition path for $n(t)$ for both types of demographic shocks. Panel (a) depicts the path for a baby bust. There is an immediate downward jump at impact ($n(0) = \hat{n}_0 - b_0 + b_1$) followed by gradual cyclical adjustment. Adjustment is rather fast because the birth rate change applies to the entire (pre-shock and post-shock) population alike. Panel (b) of Figure 2 depicts the adjustment path following a decrease in adult mortality. Nothing happens at impact and the population growth rate rises only gradually to its long-run steady-state value. Transition is much slower than for the baby bust because the ageing shock is embodied, i.e. the shock only applies to post-shock generations and pre-shock generations only die off gradually during the demographic transition.

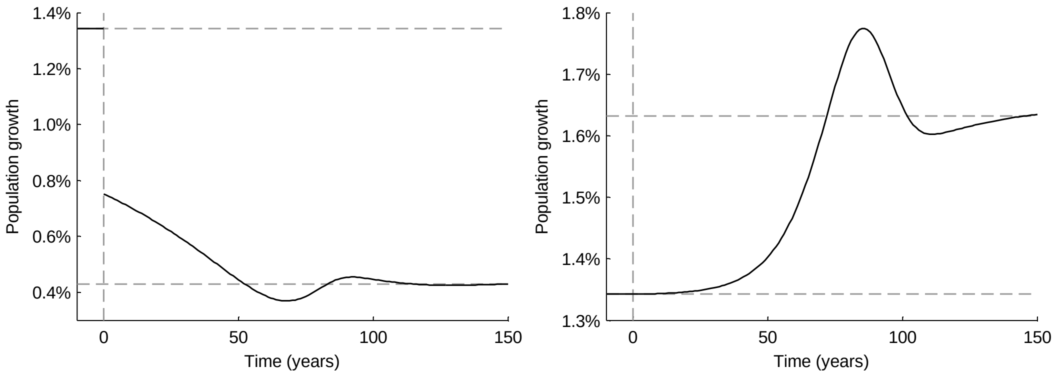
4 Determinants of schooling

In this section we study the comparative static effect on the optimal schooling period of stepwise changes in the demographic process and the fiscal parameters.

Reduced adult mortality By using equation (13), and noting the definition (11′), the comparative static effect on the optimal schooling period of a reduction in adult mortality can be computed:

$$\frac{\partial s^*}{\partial \psi} = \frac{\partial \Delta / \partial \psi}{1 - \partial \Delta / \partial s^*} > 0, \tag{37}$$

[18] Equation (36) can be rewritten in the form of a linear Volterra equation of the second kind with a convolution-type kernel for which efficient numerical solution algorithms are available. See Romp (2007).



(a) Baby bust

(b) Reduced adult mortality

Figure 2: Population Growth Rate

where the sign follows from the fact that $\partial\Delta/\partial s^* \leq 0$ (see Proposition 1(ii)) and $\partial\Delta/\partial\psi > 0$ (see Proposition 2(iii)). An increase in longevity prompts agents to increase their human capital investment at the beginning of life. In terms of Figure 3(a), the initial optimum, s_0^*, occurs at the intersection of the line labeled $\Delta_0 + \rho/\,(1 - t_L)$ and the 45° line. The mortality shock shifts the demographic discount function to the right, and increases the optimal schooling period from s_0^* to s_1^*.

What is the intuition behind our result? Bils and Klenow argue that a higher life expectancy (as captured in their model by an increase in the exogenous planning horizon) leads to an increase in the optimal schooling period "since it affords a longer working period over which to reap the wage benefits of schooling" (2000, p. 1164). Similarly, de la Croix and Licandro (1999, p. 258) and Kalemli-Ozcan *et al.* (2000, p. 11), using the Blanchard demography, show that a decrease in the death probability leads to an increase in the expected planning horizon for all agents and an increase in the optimal schooling period.

Our model clarifies that the crucial determinant of the schooling decision is *adult life expectancy*, not the expected planning horizon at birth. In our model, a decrease in child mortality increases expected remaining life time at birth but leaves the optimal schooling period unchanged. Such a shock merely increases each individual's probability of actually living long enough to finish school and enter the labour market. In terms of Figure 3(a), reduced child mortality flattens the left-hand section of the line $\Delta_0 + \rho/\,(1 - t_L)$ but the equilibrium solution stays at s_0^*.[19] [20] In contrast, a decrease in adult mortality increases the expected working period, and thus boosts the schooling period conform the mechanism identified by Bils and Klenow (2000). Of course, with the Blanchard demography one cannot distinguish between child mortality and adult mortality because the death probability is age-independent.

Fiscal stimulation By using equation (13), the comparative static effects of fiscal changes can be computed:

$$\frac{\partial s^*}{\partial \rho} = \frac{1}{(1-t_L)\,(1-\partial\Delta/\partial s^*)} > 0, \tag{38}$$

$$\frac{\partial s^*}{\partial t_L} = \frac{\rho}{(1-t_L)^2\,(1-\partial\Delta/\partial s^*)} > 0, \tag{39}$$

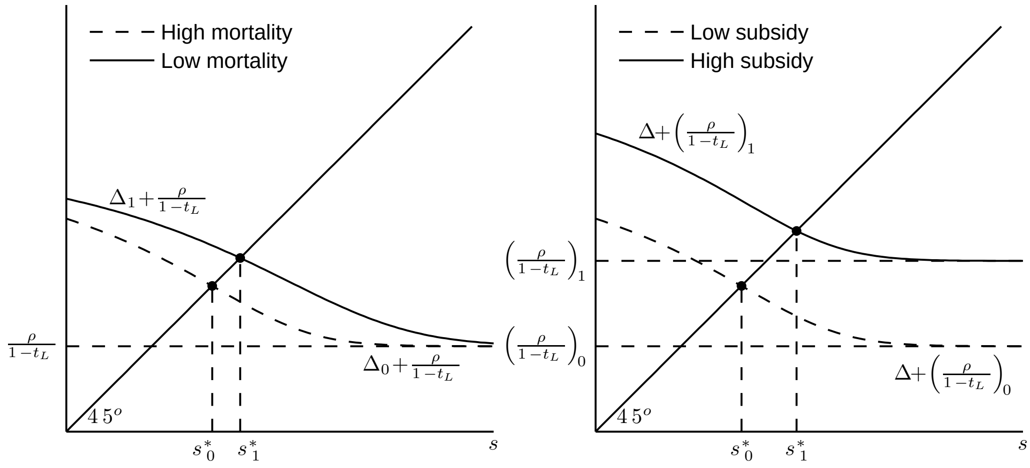
where the signs follow from the fact that $\partial\Delta/\partial s^* \leq 0$ (see Proposition 1(ii)). Not surprisingly, an increase in the educational subsidy leads to a reduction in the opportunity cost of schooling and a longer optimal schooling period. Interestingly, provided the educational subsidy is strictly positive, an increase in the marginal labour income tax also increases the optimal schooling period. Because the educational subsidy is untaxed, the *effective* subsidy affecting the schooling decision is $\rho/\,(1-t_L)$, which is increasing in t_L. In terms of Figure 3(b), an increase in either ρ or t_L shifts the optimum from s_0^* to s_1^*.

5 Exogenous growth

In Section 4 it was shown that both fiscal and demographic shocks lead to a change in the optimal schooling period. In this section we study the resulting transitional and long-run effects on human capital formation for the case in which the intergenerational knowledge transfer incorporated in the training function (6) is either absent ($\phi = 0$) or subject to diminishing returns ($0 < \phi < 1$). For such values of ϕ, the model implies a unique steady-state

[19] Boucekkine *et al.* also distinguish age-dependent mortality and argue that "an increase in life expectancy increases the optimal length of schooling" (2000, pp. 352, 370). They thus fail to notice that the mechanism producing this result runs via reduced old-age mortality, not via increased life expectancy in general.

[20] Bils and Klenow (2000, p. 1175) also report that their model implies an unrealistically high sensitivity of the optimal schooling period with respect to life expectancy that is close to unity. In contrast, in the calibrated version of our model, $ds^*/dR\,(0) = 0.06$ which comes close to the empirical estimate mentioned by Bils and Klenow (2000, p. 1175n27). In our model, $R\,(0) \equiv \Delta\,(0,0)$ represents life-expectancy at birth.



(a) Reduced adult mortality

(b) Fiscal impulse

Figure 3: Schooling Period

level of per capita human capital, i.e. the long-run growth rate in the economy is exogenous (and equal to the population growth rate). The knife-edge case, with $\phi = 1$, gives rise to endogenous growth and is studied in Appendix B.

This section proceeds as follows. First, in Section 5.1 we analytically characterize the steady-state and study its sensitivity with respect to fiscal and demographic shocks. Next, in Section 5.2 we visualise the rather complicated transitional dynamics associated with the various shocks for a plausibly parameterized model which incorporates the estimated G-M process introduced above (see the discussion below equation (33)).

5.1 Long-run effects

In the long-run equilibrium, equation (20) gives rise to the following expression for the steady-state stock of per capita human capital, $\hat{h}$:

$$\hat{h}^{1-\phi} = A_H s^* b \int_{s^*}^{\infty} e^{-[\hat{n}u+M(u,\psi)]} du. \tag{40}$$

Equation (40) clearly shows the various mechanisms affecting $\hat{h}$, namely (i) the birth rate, (ii) the optimal schooling decision of agents, s^*, which itself depends on the fiscal and mortality parameters (ρ, t_L, ψ), (iii) the population growth rate, $\hat{n}$, which depends on (b, ψ), and (iv) the cumulative mortality factor, $M(u, \psi)$, which depends on the mortality parameter ψ.

Pure schooling shock In order to facilitate the interpretation of our results, we first study the effects of a change in the schooling period in isolation. By differentiating equation (40) with respect to s^* and simplifying we obtain:

$$\begin{aligned}\frac{\partial \hat{h}^{1-\phi}}{\partial s^*} &= A_H b e^{-[\hat{n}s^*+M(s^*,\psi)]}\left[\Delta(s^*,\hat{n})-s^*\right] \\ &= A_H b e^{-[\hat{n}s^*+M(s^*,\psi)]}\left[\Delta(s^*,\hat{n})-\Delta(s^*,r)-\frac{\rho}{1-t_L}\right], \end{aligned} \tag{41}$$

where we have used (13) to arrive at the second expression. In the absence of an educational subsidy ($\rho=0$), a pure schooling shock unambiguously leads to an increase in the per capita stock of human capital. Indeed, since the interest rate exceeds the steady-state growth rate of the population ($r>\hat{n}$), it follows from Proposition 1(i) that $\Delta(s^*,\hat{n})>\Delta(s^*,r)$ so that $\partial\hat{h}^{1-\phi}/\partial s^*>0$ in that case. With a non-zero educational subsidy, equation (41) shows that the effect on $\hat{h}$ of a pure schooling shock is no longer unambiguous because a sufficiently high effective educational subsidy will render the term in square brackets negative even for the case with $r>\hat{n}$. Intuitively, in such a case the economy is "over-educated", i.e. agents study for too long a period and thus have too short a career as productive workers. Because in actual economies r is much greater than $\hat{n}$ and educational subsidies are typically quite low, we make the following assumption which rules out over-education and ensures that $\partial\hat{h}^{1-\phi}/\partial s^*$ is positive.

Assumption 2 The steady-state net interest rate $r-\hat{n}$ is sufficiently positive to ensure that $\Delta(s^*,\hat{n})>\Delta(s^*,r)+\rho/\left(1-t_L\right)$.

Fiscal shock A fiscal shock, consisting of an increase in either ρ or t_L, affects the steady-state per capita human capital stock according to:

$$\frac{\partial \hat{h}^{1-\phi}}{\partial\left[\rho/\left(1-t_L\right)\right]}=\frac{\partial \hat{h}^{1-\phi}}{\partial s^*}\frac{\partial s^*}{\partial\left[\rho/\left(1-t_L\right)\right]}>0, \tag{42}$$

where the sign follows from (38)-(39) above. The fiscal shock leads to an increase in the optimal schooling period which, in view of Assumption 2, leads to an increase in $\hat{h}$.

Birth rate shock A change in the birth rate affects steady-state per capita human capital both directly and via its effect on the steady-state population growth rate. By differentiating equation (40) with respect to b and simplifying we obtain:

$$\frac{\partial \hat{h}^{1-\phi}}{\partial b} = A_H s^* \left[\int_{s^*}^{\infty} e^{-[\hat{n}u + M(u,\psi)]} du - b \frac{\partial \hat{n}}{\partial b} \int_{s^*}^{\infty} u e^{-[\hat{n}u + M(u,\psi)]} du \right] < 0, \tag{43}$$

where the sign follows from Lemma A.1 in Appendix A. Intuitively, a higher birth rate leads to an upward shift in the steady-state path of the human capital stock in *level* terms, but also induces an increase in the population growth rate. The latter effect dominates the former so that *per capita* human capital declines in the steady state.

Mortality shock The mortality change is by far the most complicated shock under consideration because it affects the schooling period, s^*, the population growth rate, $\hat{n}$, and the cumulative mortality factor, $M(u, \psi)$. By differentiating (40) with respect to ψ we obtain:

$$\frac{\partial \hat{h}^{1-\phi}}{\partial \psi} = \frac{\partial \hat{h}^{1-\phi}}{\partial s^*} \frac{\partial s^*}{\partial \psi} + A_H s^* b \frac{\partial}{\partial \psi} \int_{s^*}^{\infty} e^{-[\hat{n}u + M(u,\psi)]} du > 0, \tag{44}$$

where the sign follows from (37), (41), and Lemma A.2 in Appendix A. The first composite term on the right-hand side is straightforward: increased longevity boosts the optimal schooling period which in turn increases per capita human capital in the steady state. The second term on the right-hand side represents the joint effect of increased longevity on the integral appearing on the right-hand side of (40). An increase in ψ has two effects on the discounting factor of that integral. First, the population growth rate is increased ($\partial \hat{n} / \partial \psi > 0$) leading to heavier discounting and a lower value for the integral. Higher population growth constitutes a higher drag on human capital as the cake must be shared over ever more people. This effect leads to a decrease in per capita human capital. Second, the cumulative mortality factor is decreased for higher age levels ($\partial M(u, \psi) / \partial \psi < 0$) leading to reduced discounting and a higher integral. Educated people live longer as a result of the shock and per capita human capital increases as a result. Lemma A.2 in the Appendix shows that, under our set of assumptions regarding mortality change, the first effect is dominated by the second and, ceteris paribus the schooling period, *human-capital deepening* occurs as a result of increased longevity, i.e. the second composite term on the right-hand side of (44) is positive.

Balanced growth Up to this point attention has been restricted to steady-state per capita human capital. This focus is warranted because all remaining variables are uniquely related to $\hat{h}$. Indeed, it follows directly from, respectively, (27) and (30), that $\hat{k}$ and $\hat{y}$ are both proportional to $\hat{h}$. Furthermore, the steady-state versions of (21), (28), (31), and (32) determine unique values for $\hat{c}$, $\hat{d}$, $\widehat{\imath}$, and $\hat{f}$ as a function of $\hat{h}$, $\hat{n}$, and the parameters. Hence, in level terms the steady-state growth rate for output, consumption, investment, physical capital, human capital, financial assets, net foreign assets, and debt is equal to the steady-state population growth rate, $\hat{n}$.

5.2 Transitional dynamics

In this subsection we compute and visualise the transitional effects of fiscal and demographic shocks using a plausibly calibrated version of the model.[21] We set the world interest rate at $r = 0.055$, the pure rate of time preference at $\theta = 0.03$, the intertemporal substitution elasticity at $\sigma = 1$, the capital depreciation rate at $\delta = 0.07$, and the efficiency parameter for physical capital at $\varepsilon = 0.3$.

The human capital externality parameter is set at $\phi = 0.3$. We rationalize this choice as follows. In a recent paper, de la Fuente and Doménech (2006, p. 12) formulate an aggregate production function of the form:

$$\ln y_i(t) = \ln TFP_i(t) + \alpha_1 \ln k_i(t) + \alpha_2' \ln s_i(t), \tag{45}$$

where i is the country index, TFP_i is total factor productivity, k_i is capital per worker, and s_i measures education attainment, i.e. the average years of education of *employed* workers. Since their data on educational attainment refers to the total (rather than the employed) population, they postulate the relationship $\ln s_i(t) = \beta_1 \ln \bar{s}_i(t) - \beta_2 \ln PR_i(t)$, where $\bar{s}_i$ measures population average education attainment (i.e. average years of schooling in the adult population), and PR_i is the participation rate (i.e. the proportion of employed adults). Substituting this expression into (45) they derive the equation to be estimated:

$$\ln y_i(t) = \ln TFP_i(t) + \alpha_1 \ln k_i(t) + \alpha_2 \ln \bar{s}_i(t) + \alpha_3 \ln PR_i(t), \tag{46}$$

where $\alpha_2 \equiv \alpha_2' \beta_1$ and $\alpha_3 \equiv -\alpha_2' \beta_2$. They present panel data estimates for the parameters, using different specifications for $\ln TFP_i(t)$, and find large and highly significant values for

[21] Kalemli-Ozcan *et al.* (2000) restrict attention to the steady state. Boucekkine *et al.* (2002, pp. 363-365) only show the adjustment path in the endogenous growth rate following a drop in the birth rate. This shock leaves the optimal schooling period unchanged, so that all transitional dynamics is entirely attributable to changes in the growth rate of the population.

α_2 ranging from 0.378 to 0.958 (de la Fuente and Doménech, 2006, p. 14). They argue on the basis of meta-estimation that the lower bound for the key parameter of interest, α_2', lies in the range of 0.752 to 0.844 for the fixed-effect regressions. They conclude that "...investment in human capital is an important growth factor whose effect on productivity has been underestimated in previous studies because of poor data quality" (de la Fuente and Doménech, 2006, p. 28).

What does this say about our ϕ parameter? In the steady state our model implies the following relationship:

$$\ln \hat{y} = \alpha_0 + \varepsilon \ln \hat{k} + \frac{1-\varepsilon}{1-\phi} \ln s^*, \tag{47}$$

where $\alpha_0 \equiv (1-\varepsilon) \ln A_Y + \frac{1-\varepsilon}{1-\phi} \ln \left(bA_H \int_{s^*}^{\infty} e^{-[\hat{n}u + M(u,\psi)]} du \right)$. Ignoring the fact that in equation (47) the constant term itself depends negatively on s^*, we find that $\hat{\alpha}_1$ is an estimate for ε and $\hat{\alpha}_2'$ is an estimate for $(1-\varepsilon) / (1-\phi)$. De la Fuente and Doménech find estimates for $\hat{\alpha}_1$ in the range 0.448 to 0.491, so that the implied estimate for ϕ is given by $\hat{\phi} \equiv (\hat{\alpha}_2' + \hat{\alpha}_1 - 1) / \hat{\alpha}_2'$ which ranges from 0.266 to 0.397.[22] Our chosen value of ϕ falls within this range.

On the demographic side, we interpret the estimated G-M demography as the truth and choose the birth rate, b, such that $\hat{n} = 0.0134$ (the average population growth rate during the period 1920-1940). This yields a value of $b = 0.0237$ (which falls in between the observed birth rates for 1920 ($= 0.028$) and 1940 ($= 0.02$)). The estimated G-M model yields an expected remaining lifetime at birth of 65.5 years. We compute the implied wage rate from the factor price frontier and find $w = 1.019$. The initial lumpsum tax follows from the government solvency condition for an initial debt level of $\hat{d}_0 = -2.112$ and fiscal parameters $\rho = 4.915$ and $t_L = 0.15$. The implied value for the lumpsum tax is $z_0 = 0.2645$. Finally, for the scaling variables we use $A_H = A_Y = 1$. The initial age at which agents leave school and enter the labour market is $s_0^* = 21.82$ years. The initial steady state has the following main features: $\hat{a}_0 = 7.8$, $\widehat{li}_0 = 647.2$, $\hat{h}_0 = 36.1$, $\hat{y}_0 = 52.6$, $\hat{c}_0 = 37.2$, $\hat{i}_0 = 10.5$, $\hat{k}_0 = 126.2$, and $\hat{f}_0 = -116.2$. The output shares of consumption, investment, and net exports are, respectively, 0.71, 0.20, and 0.09.

[22] Of course, this is only a very tentative estimate for ϕ for at least two reasons. First, the data may not represent observations for the steady state. Second, the procedure ignores the fact that α_0 itself also depends on s^*. This may lead to an under-estimate for ϕ.

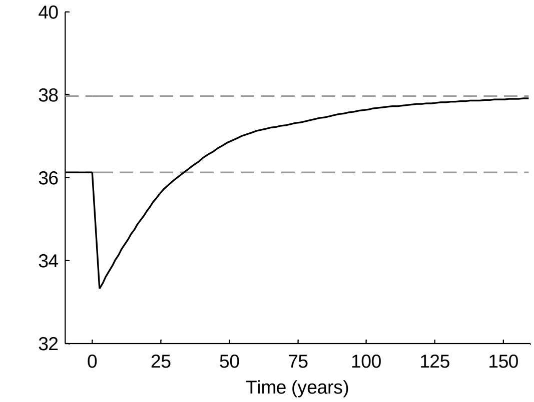
The economy is initially in a steady-state equilibrium, the stepwise shock occurs at time $t = 0$, and we refer to pre-shock ($v < 0$) and post-shock agents ($v \geq 0$). In the interest of brevity, we focus the discussion on the transition path of per capita human capital. As is seen readily from (27) and (30), the time paths for $k(t)$ and $y(t)$ are proportional to that of $h(t)$. The remaining variables of the model (such as $d(t)$, $i(t)$, $f(t)$, $li(t)$, $a(t)$, and $c(t)$) feature more complicated dynamic adjustment paths. Where no confusion can arise we drop the "per capita" adjective in the intuitive discussion of our results.

Fiscal shock In Figure 4 we illustrate the transitional dynamics associated with a lumpsum-tax financed fiscal education impulse, consisting of a 50% increase in the educational subsidy, from $\rho_0 = 4.915$ to $\rho_1 = 7.372$. There is no effect on the demography so the population growth rate is unchanged ($n(t) = \hat{n}_0$). The human capital of pre-shock workers is unaffected because labour market entry is an *absorbing state*, i.e. workers cannot go back to school by assumption. Pre-shock students, however, react to the improved fiscal incentives by extending their schooling period from $s_0^* = 21.8$ to $s_1^* = 24.5$. As a result, in the time interval $0 \leq t < s_1^* - s_0^*$ there are no new labour market entrants and human capital declines sharply as a result of the mortality process—see Figure 4(a). Labour market entry resumes for $t \geq s_1^* - s_0^*$ and the entrants have a higher level of education, so human capital starts to rise as a result. During the interval $s_1^* - s_0^* \leq t < s_1^*$ entry consists entirely of pre-shock students, whereas for $t \geq s_1^*$ only post-shock cohorts enter the labour market. Since these cohorts choose the same schooling period s_1^*, adjustment in human capital is monotonic. For $t \to \infty$, the system reaches a new steady-state which features a higher stock of human capital (see also (42) above).

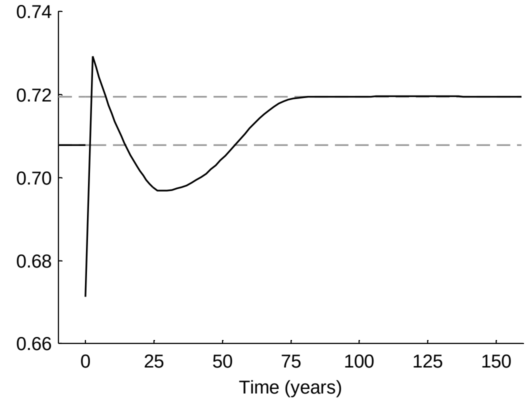
Panels (b)-(f) of Figure 4 illustrate the adjustment paths of the other macroeconomic variables. In panel (b) consumption falls at impact due to the once-off increase in the lumpsum tax needed to finance the increase in the educational subsidy. During transition, however, consumption increases non-monotonically as a result of the increase in lifetime income caused by the increase in human capital. In panel (e) the path for government debt is illustrated. Debt fluctuates during transition because the government engages in tax smoothing with respect to the lumpsum tax, z. The current account dynamics is illustrated in panel (f). At impact, the reduction in consumption and investment dominates the reduction in output, so that net exports increase and the stock of net foreign assets rises sharply. During transition, however, net foreign assets gradually fall during the first two decades of adjustment after which they rise to a permanently higher level. In a similar fashion, the path for total assets is non-monotonic due to the population heterogeneity that exists during transition. Indeed, during transition three broad cohort types coexist, namely pre-shock workers (who base their savings decisions on the pre-shock schooling choice s_0^*), pre-shock students (who switched from s_0^* to s_1^* at time $t = 0$ and changed their savings plans accordingly), and post-shock cohorts (who all choose s_1^* and, provided $\phi > 0$, face changing initial conditions because human capital changes over time).

Birth rate shock In Figure 5 we illustrate the transitional dynamics associated with a baby bust, that is the birth rate drops once and for all by 25% from $b_0 = 0.0237$ to $b_1 = 0.0178$. Nothing happens to the optimal schooling choice, but the population growth rate falls in a non-monotonic fashion from $\hat{n}_0 = 0.0134$ to $\hat{n}_1 = 0.0043$ as is illustrated in Figure 2(a). The sharp increase in human capital in Figure 5(a) is entirely attributable to the fast reduction in $n(t)$ during the early phase of transition. At time $t = s_0^*$, the population growth rate is close to its new steady state and the slope of the per capita human capital stocks flattens out. This is because the flow of labour market entrants is smaller than before as it consists entirely of post-shock newborns. In the new steady state, per capita human capital increases as a result of the baby bust (see also (43) above). For completeness sake, the paths for the remaining macroeconomic variables are also illustrated in panels (b)-(f) of Figure 5.

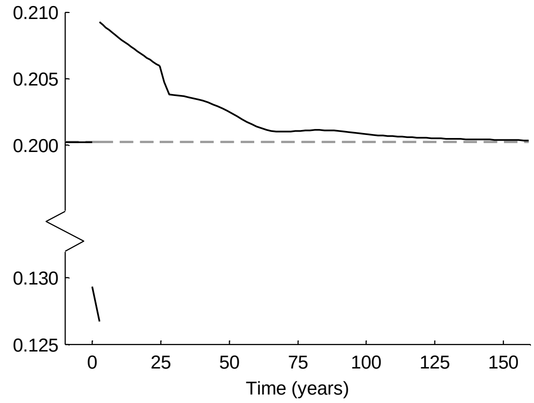
Mortality shocks In Figure 6 we illustrate the transitional dynamics associated with an adult mortality shock leading to increased longevity. The μ_1-parameter of the G-M process is reduced by 50% and the μ_2-parameter by 10% leading to an increase of the expected lifetime at birth from $R_0(0) \equiv \Delta_0(0,0) = 65.5$ to $R_1(0) \equiv \Delta_1(0,0) = 77.6$. In the face of increased longevity, post-shock cohorts choose a longer schooling period ($s_1^* = 22.5$ instead of $s_0^* = 21.8$). Furthermore, the shock perturbs the demographic steady-state and causes a rather slow non-monotonic increase in the population growth rate, from $\hat{n}_0 = 0.0134$ to $\hat{n}_1 = 0.0163$ as is illustrated in Figure 2(b). The transition in human capital passes through the following phases. During the interval $0 \leq t < s_0^*$ nothing happens to human capital because only pre-shock students (facing an unchanged mortality process) enter the labour market and the



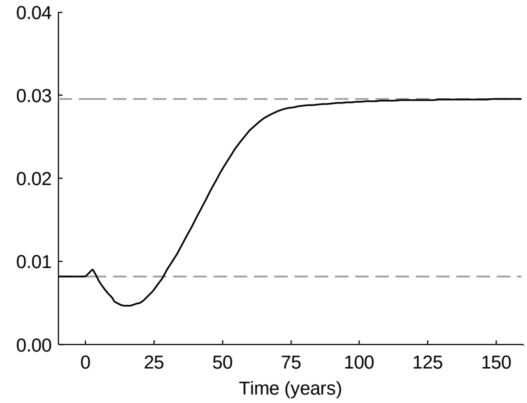
(a) Human capital ($h(t)$)



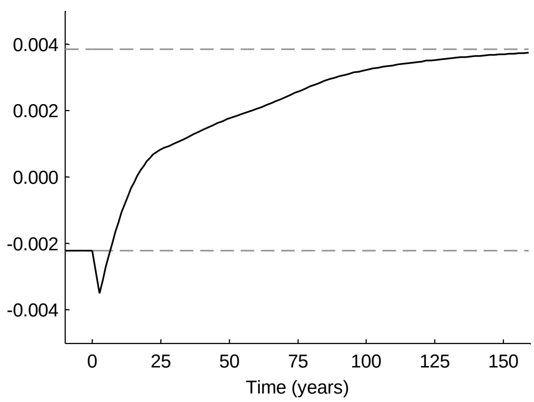
(b) Consumption ($c(t)/\hat{y}$)



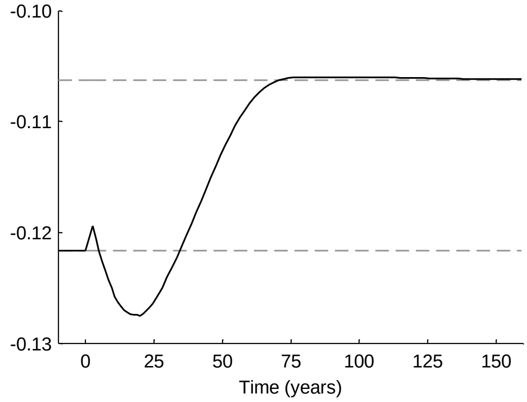
(c) Investment ($i(t)/\hat{y}$)



(d) Assets ($ra(t)/\hat{y}$)

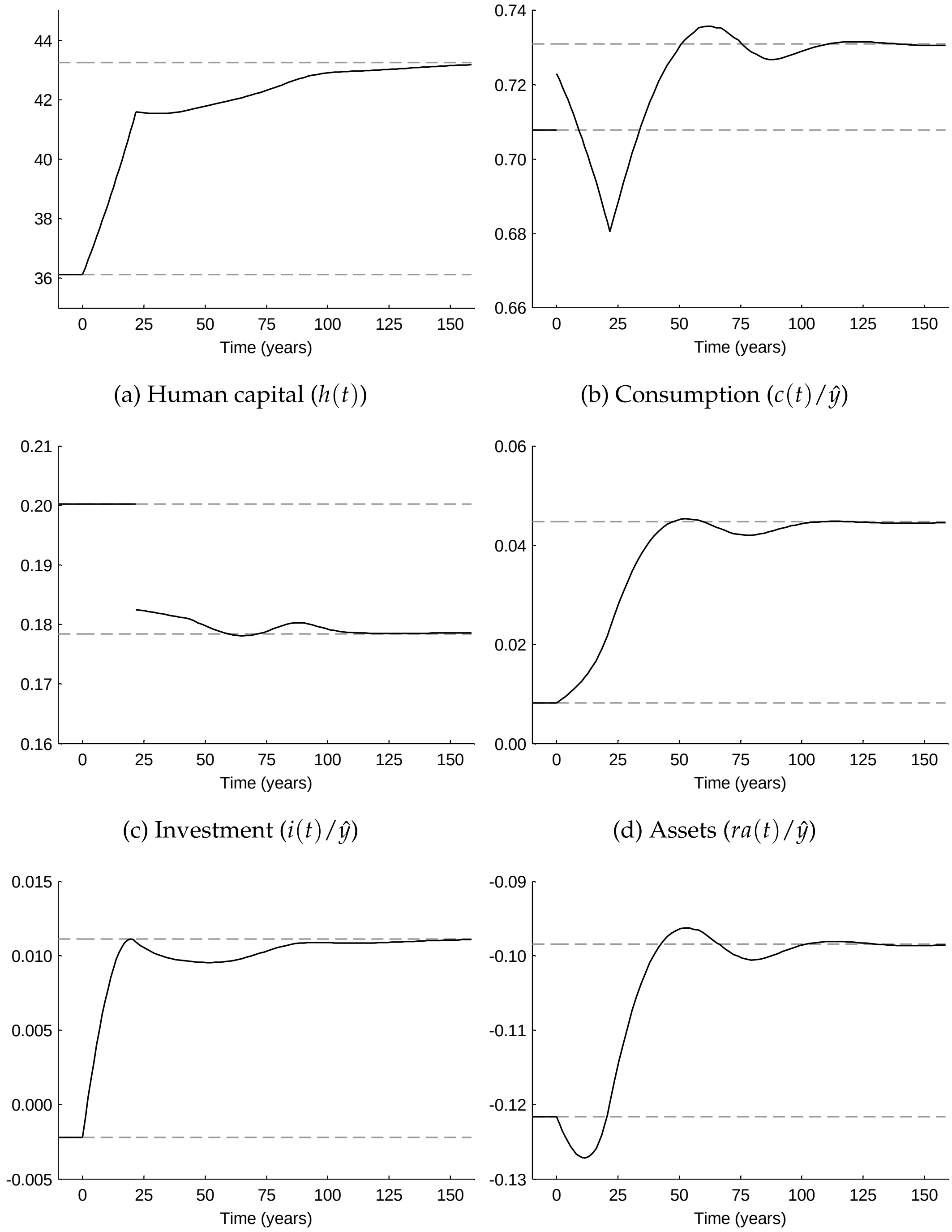


(e) Government debt ($rd(t)/\hat{y}$)



(f) Foreign assets ($rf(t)/\hat{y}$)

Figure 4: Aggregate effect of a fiscal impulse



(a) Human capital ($h(t)$)

(b) Consumption ($c(t)/\hat{y}$)

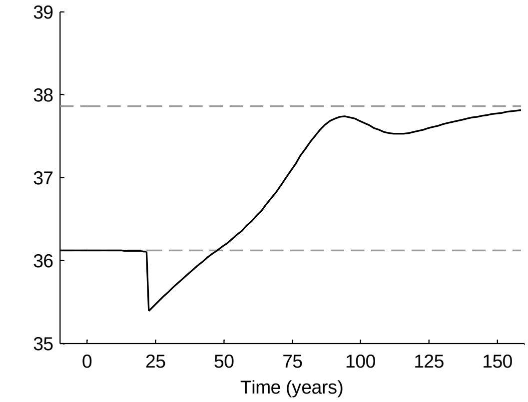
(c) Investment ($i(t)/\hat{y}$)

(d) Assets ($ra(t)/\hat{y}$)

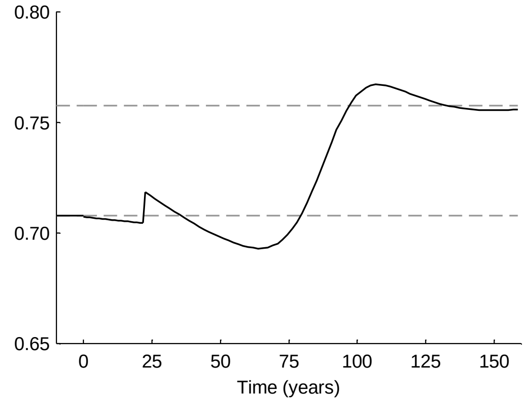
(e) Government debt ($rd(t)/\hat{y}$)

(f) Foreign assets ($rf(t)/\hat{y}$)

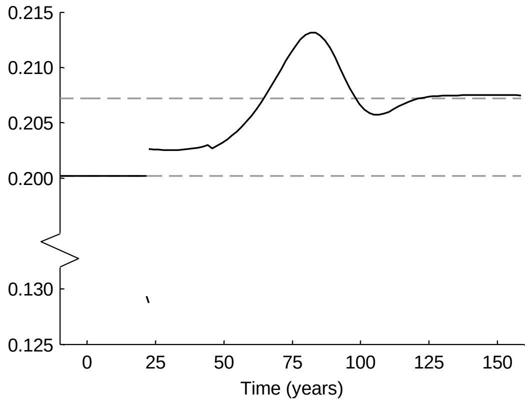
Figure 5: Aggregate effect of a baby bust



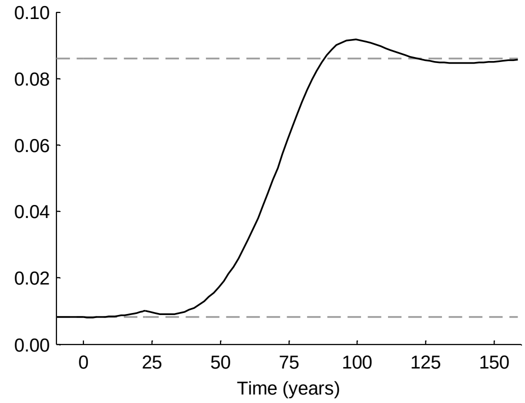
(a) Human capital ($h(t)$)



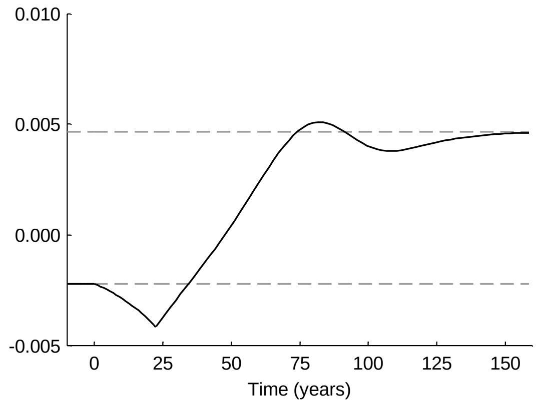
(b) Consumption ($c(t)/\hat{y}$)



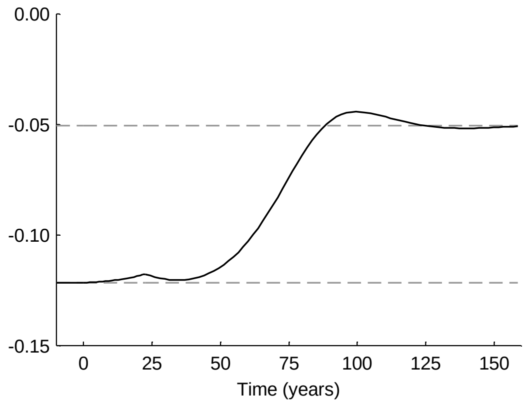
(c) Investment ($i(t)/\hat{y}$)



(d) Assets ($ra(t)/\hat{y}$)



(e) Government debt ($rd(t)/\hat{y}$)



(f) Foreign assets ($rf(t)/\hat{y}$)

Figure 6: Aggregate effect of reduced adult mortality

mortality process for pre-shock workers has not changed. For $s_0^* \leq t < s_1^*$ there are no new labour market entrants at all because post-shock students choose a schooling period s_1^*. Human capital declines sharply because (a) pre-shock cohorts die off at the rate implied by the pre-shock mortality process, and (b) the population growth rate increases. For $t \geq s_1^*$ post-shock cohorts enter the labour market. The closer the birth rate of such cohorts is to s_1^*, the worse are their initial conditions in the human capital formation process. Indeed, the cohort born at time $t = s_1^*$ faces low schooling productivity because $h(s_1^*)$ is quite low. As is clear from Figure 6(a), human capital increases in a non-monotonic fashion after $t = s_1^*$, where the bump after about 90 years is due to the corresponding maximum in the population growth rate at that time—see Figure 2(b).

5.3 Discussion

The main findings of this section are as follows. Provided the intergenerational externality parameter is below the knife-edge value of unity, the stock of per capita human capital settles at a constant level in the long run. Balanced growth in consumption, investment, output, employment, and human and physical capital is thus entirely due to population growth as in the celebrated Solow-Swan model. Fiscal incentives, though causing permanent level effects, only produce temporary growth effects. In contrast, demographic shocks change both levels and the population growth rate in the long run. In particular, the baby bust reduces long-run growth whilst increased longevity—due to reduced adult mortality—increases it. It is thus an empirical issue whether ageing countries, experiencing the combined demographic shock mentioned in the Introduction, will ultimately converge to a lower or a higher long-run rate of economic growth. Since convergence is extremely slow and the transition path may be non-monotonic, time series tests for the exogenous growth model will be hard to conduct given the paucity of data.

6 Concluding remarks

We have studied how fiscal incentives and demographic shocks affect the growth performance of a small open economy populated by disconnected generations of finitely-lived agents facing age-dependent mortality and constant factor prices. Among other things, the paper highlights the crucial role played by the strength of the intergenerational external effect in the training function faced by individual agents. Provided this external effect is non-zero, as the empirical evidence suggests, the vintage nature of the model gives rise to very slow and rather complicated dynamic adjustment. This feature of the model may help explain why robust empirical results linking education and growth have been so hard to come by.

Throughout our paper we compare and contrast our findings with those of Boucekkine *et al.* (2002). We have chosen their paper as a point of departure for two reasons. First, it is by far the most sophisticated treatment in our specific area of interest, i.e. demography-based macroeconomics. Second, it is the paper most closely associated with ours and thus shares a lot of common features. In one dimension, however, the analysis of Boucekkine *et al.* (2002) is more general than ours in that their model explains both the schooling decision and the retirement decision. We have decided to study these two decisions in separate papers. The current paper focuses on the education decision made early on in life, and ignores the retirement decision. Our companion paper, Heijdra and Romp (2006), ignores the education decision and focuses on the retirement decision that agents make at the onset of old-age.

There is both a practical and a fundamental reason why we think it is fruitful to study schooling and retirement in isolation. First, by zooming in on one decision at a time, simple and intuitive insights are much easier to come by. A more detailed simultaneous treatment can always be implemented in the context of a Computable General Equilibrium (CGE) model. Second, and more fundamentally, it allows us to expand the model in other, potentially more interesting, directions. In the current paper, for example, we chose to introduce a system of taxes and educational subsidies which impinges directly on the education decision.

In Heijdra and Romp (2006), we ignore schooling and instead endogenize the agent's retirement decision in the presence of a stylized public pension system which includes realistic institutional features such as an early entitlement age (EEA) and a statutory retirement age (Gruber and Wise, 1999). We find that most actual pension systems give rise to a kink in the lifetime income function which acts as an early retirement trap, i.e. agents find it optimal to retire at the EEA, rather than at the optimum retirement age that would have been chosen in the absence of a pension system. Comparative static exercises must take into account the policy-induced kink. Even inframarginal demographic shocks fail to induce agents to work beyond the EEA, and large fiscal changes are not potent enough to get individuals out of the trap. Increasing the EEA appears to be a low cost measure to counteract the adverse effects of the various demographic shocks.

Appendix A: Useful Lemmas

In order to determine the effect on steady-state human capital of a change in the birth rate in equation (43), we make use of the following Lemma.

Lemma A.1 By using (34) in (43) we obtain:

$$\frac{\partial \hat{h}^{1-\phi}}{\partial b} = \frac{A_H s^*}{b} \Psi\left(s^*\right),$$

where $\Psi\left(s\right)$ is defined as:

$$\Psi(s) \equiv \frac{\int_s^\infty e^{-[\hat{n}u+M(u,\psi)]}du}{\int_0^\infty e^{-[\hat{n}u+M(u,\psi)]}du} - \frac{\int_s^\infty ue^{-[\hat{n}u+M(u,\psi)]}du}{\int_0^\infty ue^{-[\hat{n}u+M(u,\psi)]}du},$$

with $\hat{n} > 0$ and $M(u,\psi)$ as defined in equation (2′). The following results can be established:

(i) $\Psi(s) \leq 0$ for all $s \geq 0$,

(ii) $\Psi(0) = 0$,

(iii) $\lim_{s\to\infty} \Psi(s) = 0$.

Proof. Results (ii) and (iii) follow directly from the definition of $\Psi(s)$. Differentiation with respect to s gives

$$\frac{\partial \Psi}{\partial s} = e^{-[\hat{n}s+M(s,\psi)]}\left[\frac{s}{\int_0^\infty e^{-[\hat{n}u+M(u,\psi)]}du} - \frac{1}{\int_0^\infty ue^{-[\hat{n}u+M(u,\psi)]}du}\right], \tag{A.1}$$

which is continuous in s and has only one root. The second derivative is positive in this unique stationary point, so it is a global minimum. Together with (ii) and (iii) this implies result (i). □

In order to determine the effect on steady-state human capital of a change in adult mortality in equation (44), we make use of the following Lemma.

Lemma A.2 Define $\Xi(s,\psi)$ for $s \geq 0$ as:

$$\Xi(s,\psi) = \int_s^\infty e^{-[\hat{n}u+M(u,\psi)]}du.$$

Then $\dfrac{\partial \Xi(s,\psi)}{\partial \psi} \geq 0$ for all $s > 0$, where the equality holds if and only if $\dfrac{\partial^2 m(u,\psi)}{\partial u \partial \psi} = 0$.

Proof. For the sake of readability define

$$\begin{aligned}\Xi_\psi(s,\psi) &\equiv \frac{\partial \Xi(s,\psi)}{\partial \psi} \\ &= \int_s^\infty \frac{\partial M(u,\psi)}{\partial \psi} e^{-[\hat{n}u+M(u,\psi)]} du - \frac{\partial \hat{n}}{\partial \psi} \int_s^\infty u e^{-[\hat{n}u+M(u,\psi)]} du. \end{aligned} \tag{A.2}$$

Note that $\lim_{s\to\infty} \Xi_\psi(s,\psi) = 0$ and:

$$\frac{\partial \hat{n}}{\partial \psi} = \frac{\int_0^\infty \frac{\partial M(u,\psi)}{\partial \psi} e^{-[\hat{n}u+M(u,\psi)]} du}{\int_0^\infty u e^{-[\hat{n}u+M(u,\psi)]} du}. \tag{A.3}$$

By substituting (A.3) into (A.2) we find that $\Xi_\psi(0,\psi) = 0$. The stationary points of $\Xi_\psi(s,\psi)$ with respect to s are determined by the roots of:

$$\frac{\partial \Xi_\psi(s,\psi)}{\partial s} = e^{-[\hat{n}u+M(u,\psi)]} \left[\frac{\partial M(s,\psi)}{\partial \psi} - s \frac{\partial \hat{n}}{\partial \psi} \right]. \tag{A.4}$$

From Proposition 2 we know that $\frac{\partial M(s,\psi)}{\partial \psi}$ is non-positive, non-increasing and concave in s. This implies together with $\frac{\partial \Xi_\psi(0,\psi)}{\partial s} = 0$ that $\frac{\partial \Xi_\psi(s,\psi)}{\partial s}$ has at most two roots (one at $s = 0$) or is 0 everywhere (if $\frac{\partial \Xi_\psi(0,\psi)}{\partial s} = 0$ on the interval $[0, s^*]$, $0 \leq s^* \ll \infty$, then $\lim_{s\to\infty} \Xi_\psi(s,\psi) = 0$ does not hold). If $\frac{\partial \Xi_\psi(s,\psi)}{\partial s} = 0$ for all $s \geq 0$, then $\Xi_\psi(s,\psi) = 0$ for all $s \geq 0$. This last situation only occurs if $\frac{\partial M(s,\psi)}{\partial \psi}$ is linear in s, i.e. if $\frac{\partial^2 m(u,\psi)}{\partial u \partial \psi} = 0$.

If $\frac{\partial^2 m(u,\psi)}{\partial u \partial \psi} < 0$ for some $s \geq 0$, then $\Xi_\psi(s,\psi)$ has exactly two stationary points for a given ψ, one at $s = 0$ and one at $s = s^* > 0$. Concavity of $\frac{\partial M(s,\psi)}{\partial \psi}$ implies that the stationary point at $s = s^*$ is a maximum. Since $\frac{\partial \Xi(s,\psi)}{\partial \psi}$ goes to 0 as $s \to \infty$ and is continuous, $\frac{\partial \Xi(s,\psi)}{\partial \psi}$ must be positive for all $s > 0$, otherwise there would be a minimum somewhere at $s > s^*$. This completes the proof. □

Appendix B: Endogenous growth

In the main body of the paper we have restricted attention to the case for which the intergenerational knowledge externality is relatively weak (i.e. $0 \leq \phi < 1$) and the system reaches a steady state in terms of per capita levels. In this appendix we study the knife-edge case for which the intergenerational knowledge transfer is very strong and subject to constant returns ($\phi = 1$). It has been studied extensively in the literature, despite the rather spectacular lack of empirical support for this knife-edge case. See, see among others, Azariadis and Drazen (1990), de la Croix and Licandro (1999), Boucekkine *et al.* (2002), Echevarría (2004), and Echevarría and Iza (2006).

B.1 Long-run effects

The steady-state growth path for per capita human capital can be written as follows:

$$\begin{aligned} \hat{h}(t) &= \int_{-\infty}^{t-s^*} l(t-v)\hat{\bar{h}}(t-v)dv \\ &= A_H s^* b \int_{-\infty}^{t-s^*} e^{-[\hat{n}(t-v)+M(t-v,\psi)]}\hat{h}(v)dv, \end{aligned} \tag{B.1}$$

where we have used (6) and (18) to arrive at the second expression. As Romp (2007, pp. 91-94) shows, the steady-state growth path features the following key properties: (i) there is a unique steady-state growth rate of per capita variables, and (ii) all per capita variables feature uniform convergence to their respective steady-state growth path.

Denoting the steady-state growth rate by $\hat{\gamma}$, it follows that along the balanced growth path we have $\hat{h}(v) = \hat{h}(t)\, e^{-\hat{\gamma}(t-v)}$. By using this result in (B.1) and simplifying we obtain the implicit definition for $\hat{\gamma}$:

$$1 = A_H s^* b \int_{s^*}^{\infty} e^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}du. \tag{B.2}$$

Clearly, the model implies a *scale effect* in the growth process, i.e. a productivity improvement in the human capital production function gives rise to an increase in the steady-state growth rate ($\partial\hat{\gamma}/\partial A_H > 0$). Equation (B.2) can also be used to compute the effect on the asymptotic growth rate of the fiscal and demographic shocks.

Pure schooling shock Just as in Subsection 5.1 above, the interpretation of our results is facilitated by first considering a pure schooling shock. By differentiating (B.2) with respect to $\hat{\gamma}$ and s^*, and gathering terms we find:

$$\begin{aligned} \frac{\partial\hat{\gamma}}{\partial s^*} &= \frac{e^{-[(\hat{\gamma}+\hat{n})s^*+M(s^*,\psi)]}\left[\Delta\left(s^*,\hat{\gamma}+\hat{n}\right)-s^*\right]}{s^*\int_{s^*}^{\infty} ue^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}du} \\ &= \frac{e^{-[(\hat{\gamma}+\hat{n})s^*+M(s^*,\psi)]}}{s^*\int_{s^*}^{\infty} ue^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}du}\left[\Delta\left(s^*,\hat{\gamma}+\hat{n}\right)-\Delta\left(s^*,r\right)-\frac{\rho}{1-t_L}\right] > 0, \end{aligned} \tag{B.3}$$

where we have used equation (13) to arrive at the final expression. The sign of $\partial\hat{\gamma}/\partial s^*$ is determined by the term in square brackets on the right-hand side of (B.3). By appealing to the endogenous-growth counterpart of Assumption 2 (with $\hat{n}$ replaced by $\hat{n}+\hat{\gamma}$) we find that the steady-state growth rate increases as a result of the pure schooling shock.

Fiscal shock An increase in the educational subsidy or the labour income tax affects the steady-state growth rate via its positive effect on the schooling period. Indeed, we deduce from (38)-(39) and (B.3) that:

$$\frac{\partial \hat{\gamma}}{\partial \left[\rho / \left(1 - t_L\right)\right]} = \frac{\partial \hat{\gamma}}{\partial s^*} \frac{\partial s^*}{\partial \left[\rho / \left(1 - t_L\right)\right]} > 0. \tag{B.4}$$

Birth rate shock The growth effects of a birth rate change are computed most readily by restating the shock in terms of the steady-state population growth rate, $\hat{n}$, and noting the monotonic relationship between $\hat{n}$ and b stated in (34) above. Indeed, by substituting the steady-state version of (19) into (B.2) we find an alternative implicit expression for $\hat{\gamma}$:

$$\int_0^\infty e^{-[\hat{n}u + M(u,\psi)]} du = A_H s^* \int_{s^*}^\infty e^{-[(\hat{\gamma}+\hat{n})u + M(u,\psi)]} du. \tag{B.5}$$

Since the birth rate shock leaves the schooling period unchanged, it follows from (B.5) that:

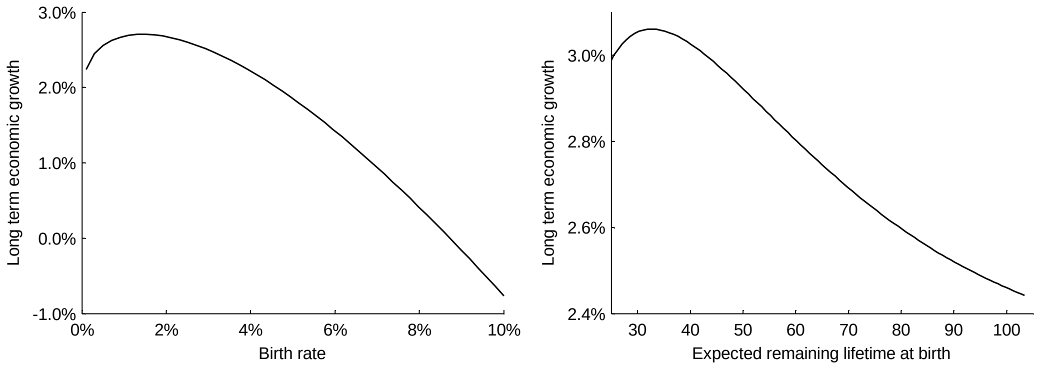
$$\frac{\partial \hat{\gamma}}{\partial b} = \frac{\partial \hat{\gamma}}{\partial \hat{n}} \frac{\partial \hat{n}}{\partial b} = \frac{\partial \hat{n}}{\partial b} \left[\frac{\int_0^\infty u e^{-[\hat{n}u + M(u,\psi)]} du}{A_H s^* \int_{s^*}^\infty u e^{-[(\hat{\gamma}+\hat{n})u + M(u,\psi)]} du} - 1 \right] \gtreqless 0. \tag{B.6}$$

Despite the fact that $\partial \hat{n} / \partial b > 0$, the growth effect of a birth rate change is ambiguously because the term in square brackets on the right-hand side of (B.6) cannot be signed a priori. Indeed, using the calibrated version of the model, we find that the relationship between $\hat{\gamma}$ and b is hump-shaped. As is illustrated in Figure B.1(a), the growth rate rises with the birth rate for low birth rates, but is decreasing for higher birth rates. For the calibrated model, the maximum growth rate is attained at a birth rate of 1.25% per annum.

Mortality shock Just as in the exogenous growth model, increased longevity constitutes by far the most complicated shock studied here. Indeed, as can be seen from equation (B.2) above, a mortality shock affects three distinct items featuring in the implicit expression for the steady-state growth rate, $\hat{\gamma}$, namely (a) the optimal schooling period, s^*, (b) the steady-state growth rate of the population, $\hat{n}$, and (c) the cumulative mortality factor, $M(u, \psi)$. By differentiating (B.2) with respect to $\hat{\gamma}$ and ψ (and recognising the dependence of s^* and $\hat{n}$ on ψ) we find after some steps:

$$\frac{\partial \hat{\gamma}}{\partial \psi} = \frac{\partial \hat{\gamma}}{\partial s^*} \frac{\partial s^*}{\partial \psi} - \frac{\partial \hat{n}}{\partial \psi} + \frac{\int_{s^*}^\infty -\frac{\partial M(u,\psi)}{\partial \psi} e^{-[(\hat{\gamma}+\hat{n})u + M(u,\psi)]} du}{\int_{s^*}^\infty u e^{-[(\hat{\gamma}+\hat{n})u + M(u,\psi)]} du} \gtreqless 0. \tag{B.7}$$

The overall growth effect of increased longevity is ambiguous. The first composite term on the right-hand side of (B.7) represents the schooling effect, which is positive (see (37) and



(a) Growth and the birth rate

(b) Growth and life expectancy at birth

Figure B.1: Growth and demography

(B.3)). The third term on the right-hand side represents the cumulative mortality effect and is also positive (given Proposition 2(i)). The ambiguity thus arises because the second term on the right-hand side exerts a negative influence on growth, i.e. increased longevity boosts the steady-state population growth rate (see (35) above) which in turn slows down growth.

In Figure B.1(b) we use the calibrated version of the model to plot the relationship between the steady-state growth rate and a measure of longevity, namely life expectancy at birth, $R(0,\psi) \equiv \Delta(0,0,\psi)$. Except for very low values of $R(0,\psi)$, there is negative relationship between long-term growth and longevity.

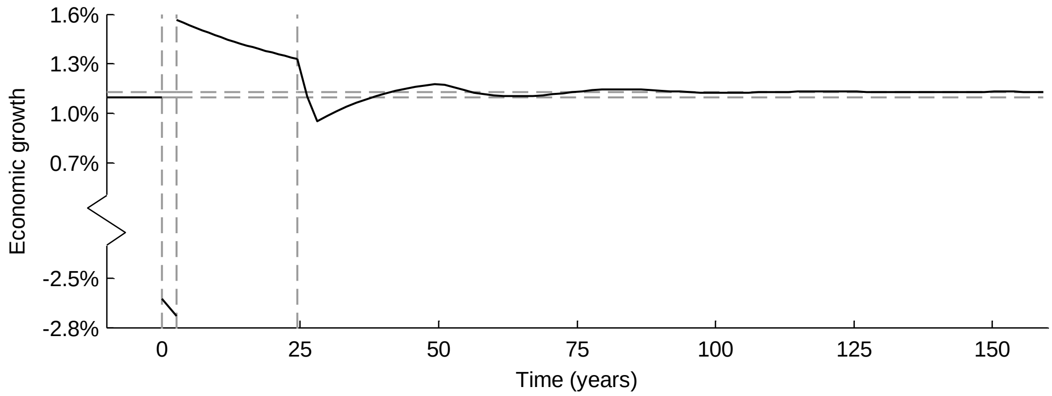
B.2 Transitional dynamics

In this subsection we visualise the transitional effects of fiscal and demographic shocks in the endogenous growth model. Unlike Boucekkine *et al.* (2002), we are able to study shocks which change the optimal schooling period. For reasons of space we ignore the adjustment paths for the remaining macroeconomic variables and restrict attention to the growth rate of per capita human wealth, $\gamma(t) \equiv \dot{h}(t)/h(t)$. Except for ϕ and A_H, we use the same calibration values as before (see Subsection 5.2). Because the model contains a scale effect, we set $A_H = 0.13$ and obtain a realistic steady-state growth rate, $\hat{\gamma}_0 = 1.096\%$. The discussion here can be quite brief because, following a shock, the transition proceeds along the same phases as in the exogenous growth model.

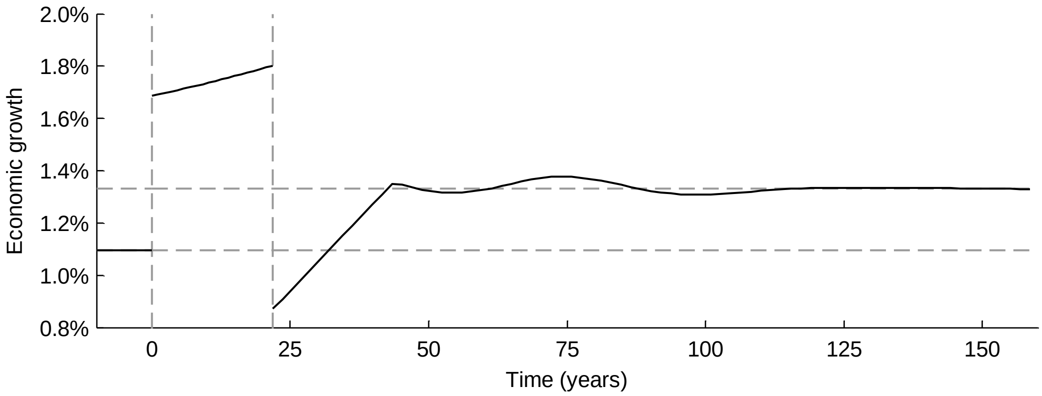
Fiscal shock Figure B.2(a) illustrates the path for $\gamma(t)$ following a 20% increase in the educational subsidy. For $0 \leq t < s_1^* - s_0^*$ there are no new labour market entrants and the growth rate collapses. Then, for $s_1^* - s_0^* \leq t < s_1^*$ pre-shock students enter the labour market and the growth rate jumps above its initial steady-state level. Finally, for $t \geq s_1^*$ the growth rate converges in a non-monotonic fashion to its long-run value, i.e. $\lim_{t\to\infty} \gamma(t) = \hat{\gamma}_1 = 1.111\%$, where $\hat{\gamma}_1$ exceeds the initial steady-state growth rate $\hat{\gamma}_0$ (see equation (B.4) above).

Birth rate shock In Figure B.2(b) the transitional effects of a baby bust are illustrated. There is no effect on the optimal schooling period but the population growth rate falls from $\hat{n}_0$ to $\hat{n}_1$—see Figure 2(a). Growth jumps sharply due to the fast reduction in $n(t)$ that occurs at impact and immediately hereafter. Intuitively, pre-shock students enter the labour market but their human capital is spread out over fewer people than before the shock so that growth in per capita terms increases sharply. About twenty-two years after the shock, $n(t) \approx \hat{n}_1$ and there is a sharp decline in growth. This is because the post-shock students start to enter the labour market. Despite the fact that they have higher human capital than existing workers, as a group they are not large enough to maintain the previous growth in per capita human capital. Thereafter, the growth rate converges in a non-monotonic fashion to its long-run level $\hat{\gamma}_1 = 1.193\%$, which is higher than the initial steady-state growth rate, i.e. $\hat{\gamma}_1 > \hat{\gamma}_0$. Given our calibration, the economy lies to the right of the peak in the curve for $\hat{\gamma}$ in Figure B.1(a) so that a baby bust increases long-run growth.

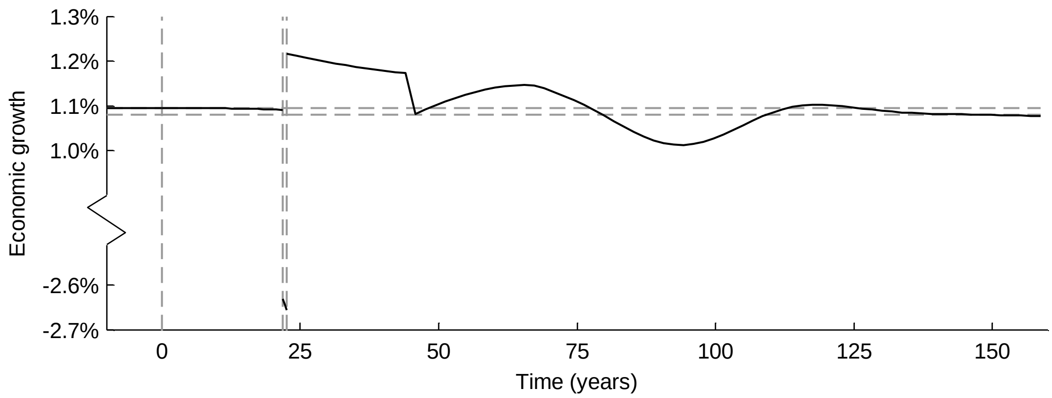
Mortality shock In Figure B.2(c) the effect on the growth rate of increased longevity of generations born after time $t = 0$ is illustrated. Just as for the exogenous growth model, nothing happens to growth for the period $0 \leq t < s_0^*$ because only pre-shock agents enter the labour market and the same type of agents die off. For $s_0^* \leq t < s_1^*$ there are no new labour market entrants and the growth rate collapses. At time $t = s_1^*$ the oldest of the post-shock cohorts enter the labour market and as a result growth is boosted again. For $t > s_1^*$, the growth rate converges non-monotonically towards the new steady-state growth rate $\hat{\gamma}_1 = 1.088\% < \hat{\gamma}_0$. In terms of Figure B.1(b), the calibration places the economy on the downward sloping segment of the $\hat{\gamma}$ curve so increased longevity reduces the long-run growth rate.



(a) Fiscal impulse



(b) Baby bust



(c) Reduced adult mortality

Figure B.2: Per capita human capital growth

B.3 Discussion

The main findings of this section are as follows. For the calibrated model, the long-run growth rate in per capita human capital increases as a result of a positive fiscal impulse or a fall in the birth rate. Increased longevity, however, reduces this long-run growth rate. The transition path in the growth rate is cyclical and rather complex for all shocks considered, and the new equilibrium is reached only very slowly.

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