Non-Traded Goods, Ageing, and Pensions in Open Economies

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1. Overview of this presentation

- Output 2005-6 Netspar activities
  - highly related
  - loosely related
- Plans for 2006-7 Netspar activities
  - in progress
  - concrete
  - vague

2. Output 2005-6

2.1. Highly related to Netspar objectives

- Bettendorf, L.J.H. & B.J. Heijdra (forthcoming 2006). Population ageing and pension reform in a small open economy with non-traded goods. *Journal of Economic Dynamics and Control*.
  - realizes **main** goal of my 2005-6 Netspar project on non-traded goods
  - basic ideas presented at Netspar Pension Day of 12 May 2005
  - key features of model:
    - two sectors: traded / non-traded
    - exogenous labour supply
    - OLG structure: separate birth rate / death rate
    - endogenous real exchange rate, P_N
    - PAYG pension system

- age-dependent productivity (exogenous)
- investment goods non-traded: smooth capital dynamics
- fixed interest rate on foreign bonds

- key properties / outcomes:
  - no long-run effects of (a) demand shocks, (b) demographic parameters, and (c) pension parameters on k_i, y_i ($i = N, T$), W^j ($j = N, K$), and P_N
  - in empirically relevant case ($k_T > k_N$) there is no transitional dynamics in the real exchange rate following a shock
  - dominant supply side: factor price equalization, Stolper-Samuelson, Rybczynski

- desired extensions:
  - move beyond two-by-two case
  - introduce more detailed demographic process $\Longrightarrow$ Heijdra-Romp (2005)
  - introduce more age-dependency in household decisions $\Longrightarrow$ future plans

- Heijdra, B.J. & W.E. Romp (2005). A life-cycle overlapping-generations model of the small open economy. *SOM Research Report*, Nr. 05C04, University of Groningen, February 2005. Link: `www.heijdra.org/Ward1M.pdf`
  - Basic ambition: introduce realistic demographic structure in the Blanchard-Yaari model: Gompertz-Makeham mortality rate, $m(a) = \mu_0 + \mu_1 e^{\mu_2 a}$ [a = household's age]
  - paper presented at Netspar Lunch Seminar of 8 December 2005 [until beamer died stochastically: ”inability to withstand destruction”]
  - key features of model:
    - small open economy
    - single good
    - exogenous labour supply
    - perfect annuity markets
  - some of the key properties to be shown today

2.2. Loosely related to Netspar objectives

- Increasing order of looseness
- Heijdra, B.J. & J.E. Ligthart (forthcoming June 2006). The macroeconomic dynamics of demographic shocks. *Macroeconomic Dynamics* 10. Link to paper: `www.heijdra.org/DemoSep.pdf`
  - how do stepwise changes in birth- and/or mortality rates affect the macroeconomy?
  - key features of the model:
    - closed economy BY model
    - endogenous labour supply
    - three types of shocks: pure baby bust ($d\eta < 0$), birth-death drop ($d\eta = d\beta$), constant generational turnover ($\eta d\beta = -(\alpha + \beta)\, d\eta$)
    - analytical results

- useful for Netspar objectives in the teaching program: how do demographic shocks work out in stylized macro models?

- Heijdra, B.J. & J.E. Ligthart (forthcoming 2006). Fiscal policy, monopolistic competition, and finite lives. *Journal of Economic Dynamics and Control*. Link to paper: `www.heijdra.org/FPMCFL05.pdf`
  - how do basic fiscal policy shocks affect the macro-economy?
  - key features of the model:
    - closed economy BY model
    - monopolistic competition in the goods market
    - endogenous labour supply
    - permanent/temporary spending shocks; different modes of financing (taxes, deficits)
    - analytical results

- useful for Netspar objectives in the teaching program: how do debt-nonneutrality and imperfect competition work out in stylized macro models?

- Heijdra , B.J., J.-P. Kooiman, and J.E. Ligthart (2006). Environmental quality, the macroeconomy, and intergenerational distribution. *Resource and Energy Economics* 28, 74-104.
    - a very interesting paper

3. Presentation of Some Results 2005-6

- "overlapping-papers" presentation:
  - part of work is finished and with journal referees
  - part of work is in progress and forms part of future output
- outline:
  - Juicy bits from Heijdra-Romp (2005)
  - Ageing, human capital formation, and economic growth [Heijdra-Romp (2006a)]

3.1. Overview of Heijdra-Romp (2005)

- motivation
- model (brief)
- main properties visualized
- main lessons

3.2. Motivation

- Gompertz-Makeham Law of Mortality:

  "It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration or an increased inability to withstand destruction." (Benjamin Gompertz, 1825)

- mortality process has two components:
  - a part that is the same for all people
  - an age-dependent part

- modern statement of the law is:

$$m(\alpha) = \mu_0 + \mu_1 e^{\mu_2 \alpha}$$

where α = the household's age

- Focus of the paper
  - realistic demography in a small open economies
  - factor prices exogenous (and typically constant)
  - aggregation not necessary
  - model can be solved analytically: complementary to large-scale CGE models
  - demographic realism matters!
  - maintained assumption: actuarially fair annuities

3.3. Model: Key Assumptions

- small open economy facing constant world interest rate
- labour only factor of production (capital could be added easily)
- savings instruments:
  - foreign assets
  - government debt
  - perfect substitutes: same rate of return
- life-time uncertainty; actuarially fair life insurance
- no aggregate uncertainty
- rational agents blessed with perfect foresight

3.4. Model: Key Equations

- expected remaining lifetime utility at time t of agent born at time v ($t \geq v$)

$$\Lambda(v,t) \equiv \int_t^{\infty} \underbrace{\ln \bar{c}\,(v,\tau)}_{(a)} \underbrace{e^{M(t-v)-M(\tau-v)}}_{(b)} \underbrace{e^{\theta(t-\tau)}}_{(c)} d\tau \qquad (2.6)$$

(a) felicity: unitary intertemporal substitution elasticity

(b) lifetime uncertainty: Probability that household of age $t-v$ reaches age $\tau - v$. Process not memoryless, i.e. $M\,(t-v) - M\,(\tau - v) \neq M\,(t-\tau)$.

(c) pure discounting ($\theta > 0$): impatience

- mortality factor and mortality rate:

$$M(\tau - v) \equiv \int_0^{\tau - v} m(s)\, ds \tag{2.4}$$

  - $m(s)$ is instantaneous mortality rate, i.e. hazard rate of hazard rate of the stochastic distribution of the date of death:

$$m(s) \equiv \frac{\phi(s)}{1 - \Phi(s)}$$

  - $\phi(s)$ = density function
  - $\Phi(s)$ = distribution (or cumulative density) function
  - in **this** paper: $m(s)$ depends only on household age [stationary demography]

- budget identity:

$$\dot{\bar{a}}(v,\tau) = [r + m(\tau - v)]\,\bar{a}(v,\tau) + \bar{w}(\tau) - \bar{z}(\tau) - \bar{c}(v,\tau) \qquad (2.7)$$

  - $\bar{a}(v,\tau)$ = financial assets
  - r = world interest rate [patient country, $r > \theta$]
  - $r + m(\tau - v)$ = annuity rate of interest
  - $\bar{w}(\tau)$ = wage rate
  - $\bar{z}(\tau)$ = lump-sum tax
  - $\bar{c}(v,\tau)$ = consumption

- optimal choices of household with age $u \equiv t - v$:

$$\frac{\dot{\bar{c}}(v,\tau)}{\bar{c}(v,\tau)} = r - \theta > 0 \tag{2.9}$$

$$\bar{c}(v,t) = \frac{1}{\Delta(u,\theta)}\left[\bar{a}(v,t) + \bar{h}(v,t)\right] \tag{2.10}$$

$$\bar{h}(v,t) \equiv e^{ru+M(u)} \int_u^\infty \left[\bar{w}(s+v) - \bar{z}(s+v)\right] e^{-[rs+M(s)]} ds \tag{2.11}$$

$$\Delta(u,\lambda) \equiv e^{\lambda u+M(u)} \int_u^\infty e^{-[\lambda s+M(s)]} ds, \quad (u \geq 0,\ \lambda > 0) \tag{2.12}$$

  - $\bar{h}(v,t)$ = human wealth (market value of time endowment, using annuity rate of interest for discounting)
  - $\Delta(u,\lambda)$ = demographic factor (plays central role, e.g. $1/\Delta(u,\theta)$ is propensity to consume out of total wealth)

Lemma 1 *Let* $\Delta(u, \lambda)$ *be defined as in (2.12) and assume that the mortality rate is non-decreasing, i.e.* $m'(s) \geq 0$ *for all* $s \geq 0$*. Then the following properties can be established for* $\Delta(u, \lambda)$*:*

(i) decreasing in λ*,* $\partial\Delta(u, \lambda)/\partial\lambda < 0$*;*

(ii) non-increasing in household age, $\partial\Delta(u, \lambda)/\partial u \leq 0$*;*

(iii) upper bound, $\Delta(u, \lambda) \leq 1/[\lambda + m(u)]$*;*

(iv) $\Delta(u, \lambda) > 0$ *for* $u < \infty$*;*

(v) $\lim_{\lambda\to\infty} \Delta(u, \lambda) = 0$*;*

(vi) for $m'(s) > 0$ *and* $m''(s) \geq 0$ *it follows that* $\lim_{u\to\infty} \Delta(u, \lambda) = 0$*.*

3.5. Demographics: Theory

- birth process:

$$L(v,v) = bL(v) \tag{2.13}$$

  - $L(v,v)$ = newborn cohort at time v
  - b = birth rate [constant]
  - $L(v)$ = total population at time v

- size of cohort over time:

$$L(v,\tau) = L(v,v)\, e^{-M(\tau - v)} \tag{2.14}$$

- aggregate mortality rate, $\bar{m}$:

$$\bar{m}L(t) = \int_{-\infty}^{t} m(t-v)\, L(v,t)\, dv \tag{2.15}$$

- relative cohort weights [needed for aggregation]:

$$l(v,t) \equiv \frac{L(v,t)}{L(t)} = be^{-[n(t-v)+M(t-v)]} \tag{2.16}$$

  - $n \equiv b - \bar{m}$ = aggregate population growth rate

- for given birth rate and mortality process, (2.15)-(2.16) imply implicit solution for n:

$$\frac{1}{b} = \Delta(0,n) \tag{3.2}$$

3.6. Demographics: Estimates

- use actual demographic data for the United Stated
- projections on expected survival rates for people born in 2001
- four parametric models are estimated with nonlinear least squares:
  - constant mortality rate [Blanchard]
  - linear-in-age mortality rate
  - piece-wise linear mortality rate
  - Gompertz-Makeham
- Estimation results in **Table 1**.
- Visualisation of fit in **Figure 1**.

Table 1: Estimated Survival Functions

	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\bar{u}}$	$\hat{\sigma}$	$\hat{n}(b)$	$1-\widehat{\Phi(100)}$
1. **Constant**	0.7026×10^{-2}	–	–	–	0.2277	0.80	**49.53**
$M(u)=\mu_0 u$	(4.92)						
2. **Linear in age**	-0.8970×10^{-2}	0.0152	–	–	0.1199	–	–
$M(u)=\mu_0 u+\mu_1^2 u^2$	(-3.83)	(12.29)					
	–	0.0104	–	–	0.1595	0.49	34.05
		(13.66)					
3. **Piece-wise linear (PWL) in age**	0.1544×10^{-2}	0.0410	–	60.85	0.0294	0.37	6.57
$M(u)=\mu_0 u+\delta(u)\mu_1^2(u-\bar{u})^2$	(6.41)	(16.12)		(43.08)			
$\delta(u)=\begin{cases}0 & \text{for } 0<u<\bar{u}\\ 1 & \text{for } u\geq\bar{u}\end{cases}$							
4. **Gompertz-Makeham (GM)**	0.5834×10^{-3}	0.3419×10^{-4}	0.0928	–	0.0018	0.37	**1.69**
$M(u)=\mu_0 u+(\mu_1/\mu_2)\left[e^{\mu_2 u}-1\right]$	(24.76)	(27.01)	(193.71)				

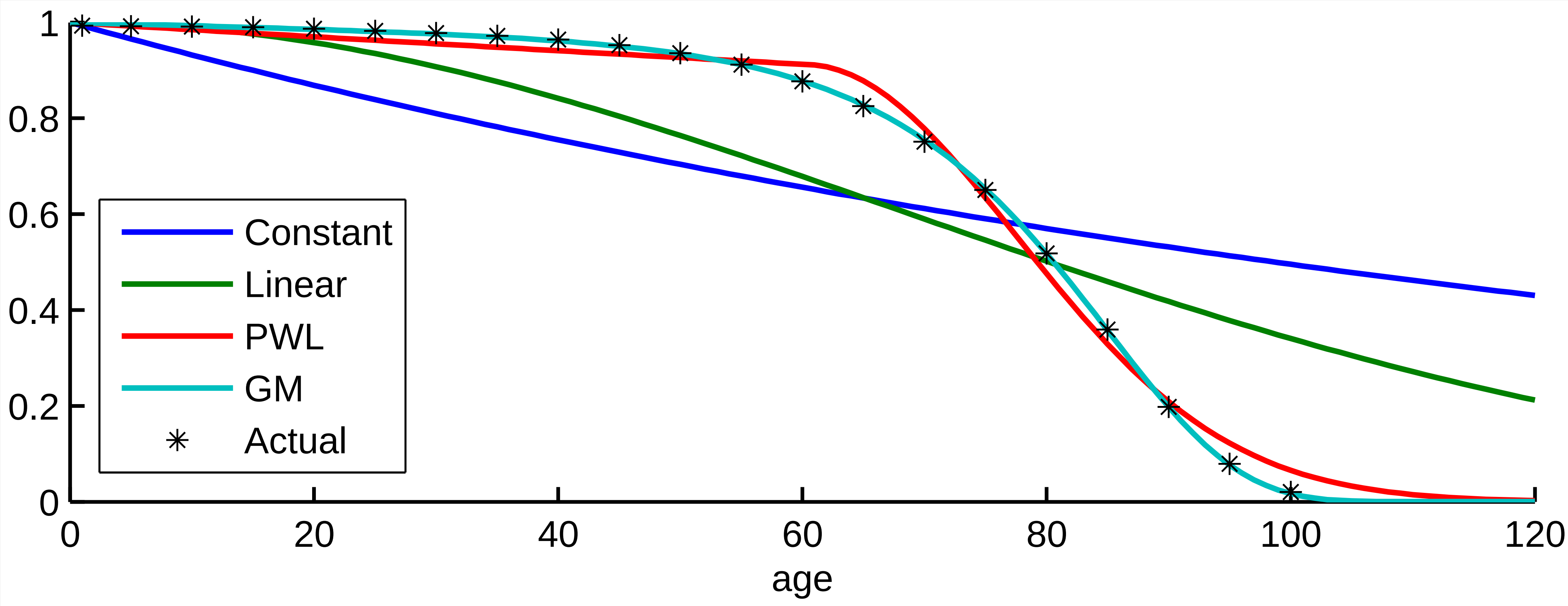


Figure 1: (a) Surviving Fraction of the Population

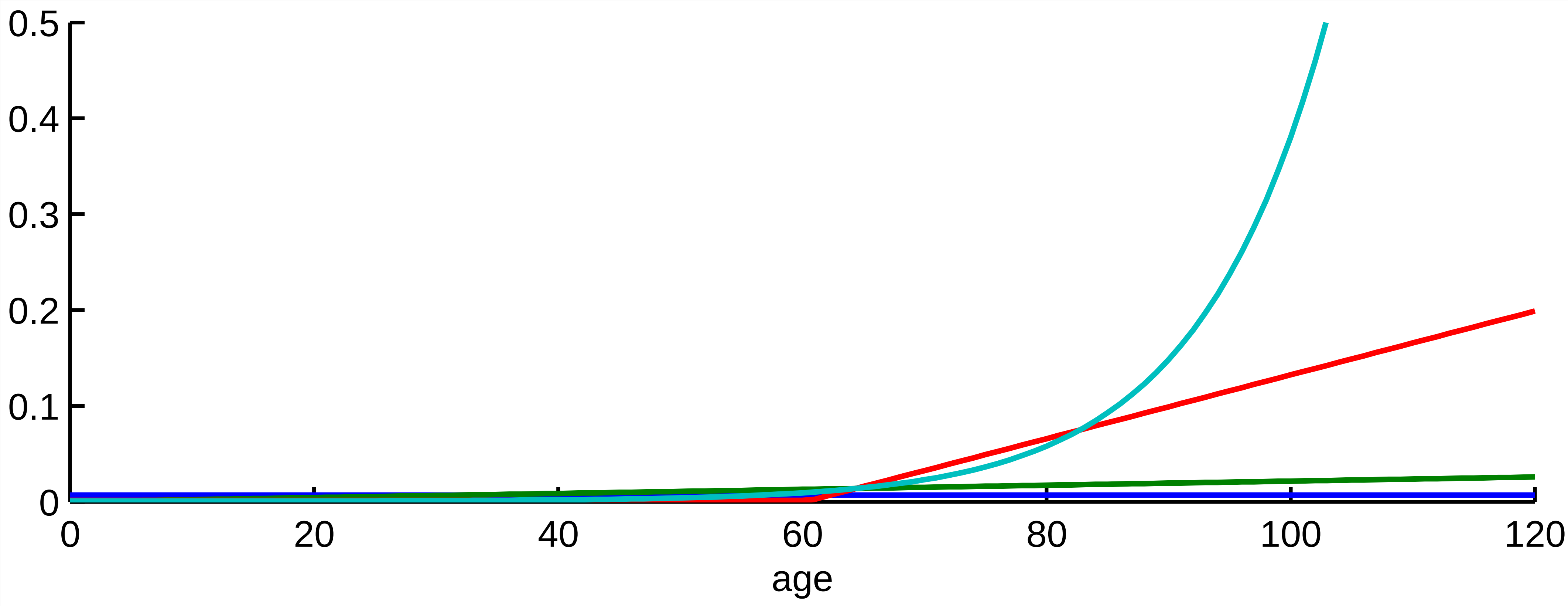


Figure 1: (b) Mortality Rate of the Population

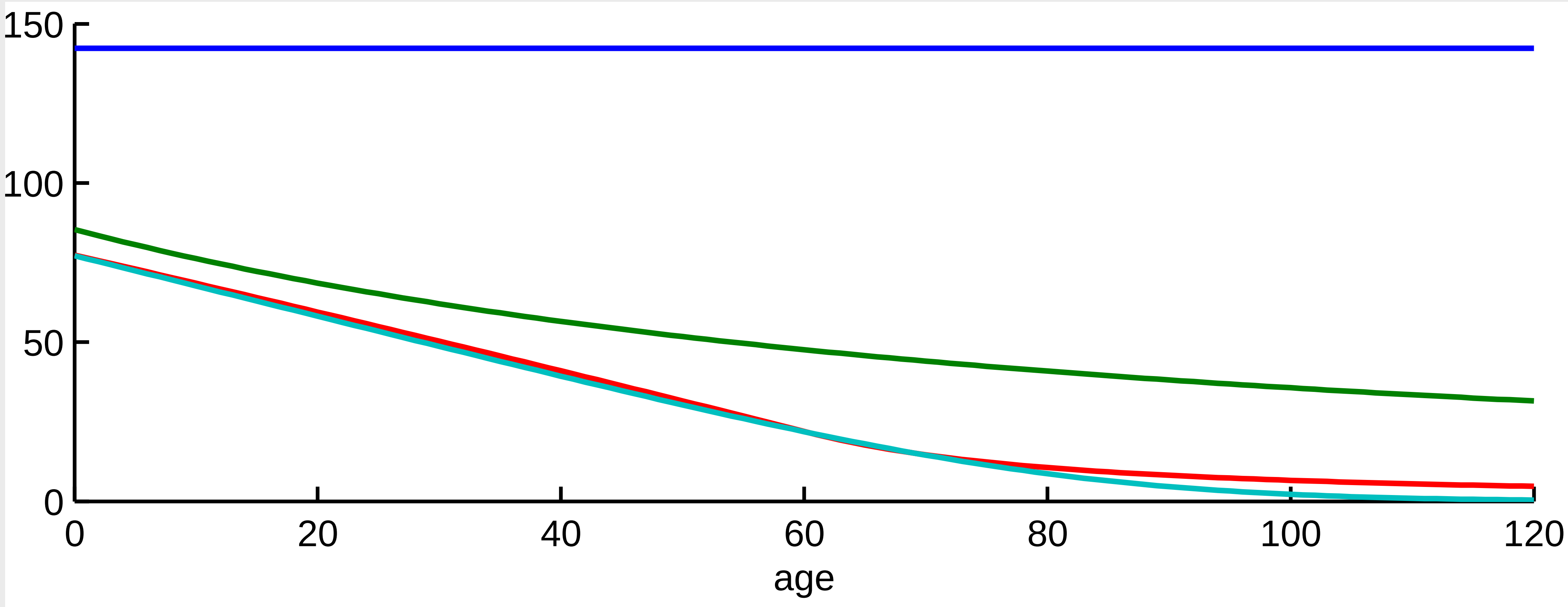


Figure 1: (c) Expected Remaining Lifetime

3.7. Steady-State Profiles

$$\frac{1}{\Delta(u,\theta)} = \left[e^{\theta u + M(u)} \int_u^{\infty} e^{-[\theta s + M(s)]} ds \right]^{-1}$$

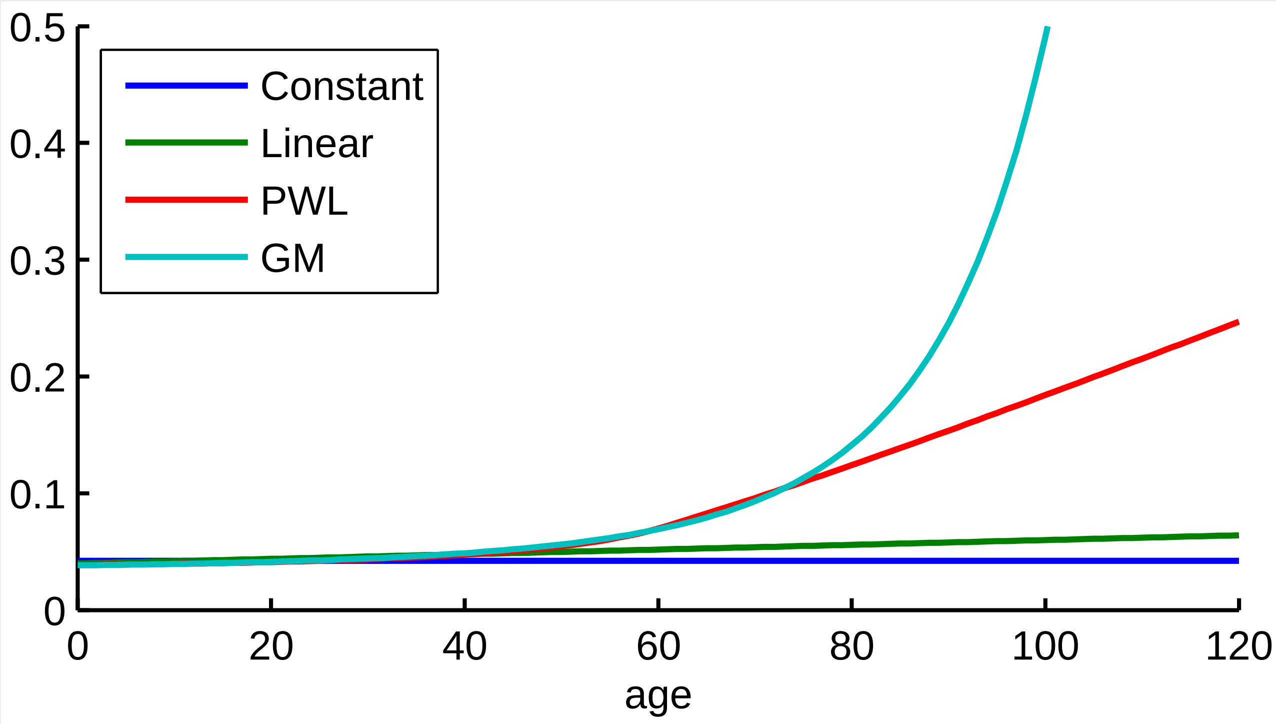


Figure 2: (a) Propensity to Consume

$$\hat{\bar{h}}\left(v,t\right)\equiv\Delta\left(u,r\right)\left[\hat{w}-\hat{z}\right]$$

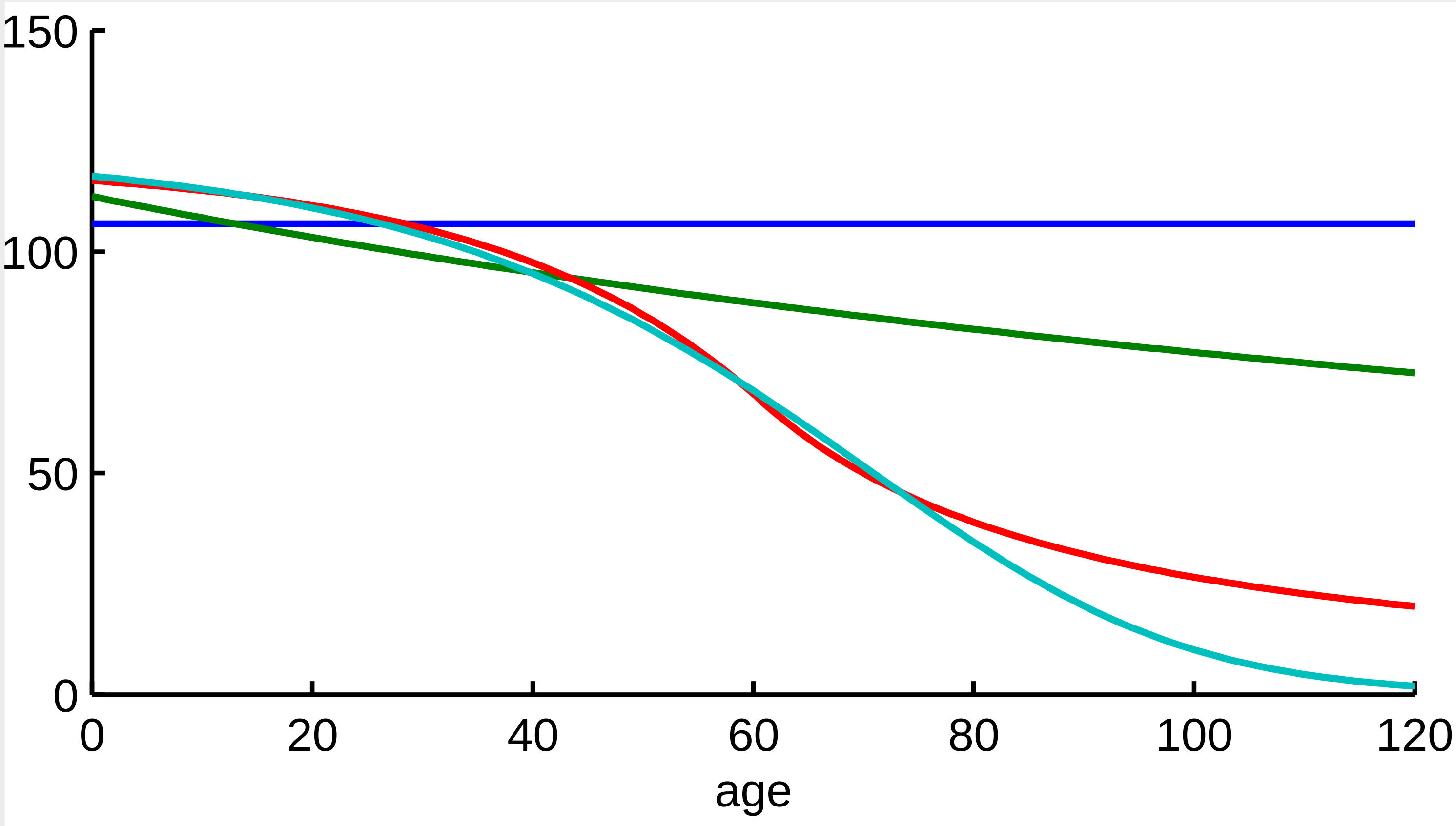


Figure 2: (b) Human Wealth

$$\hat{\bar{c}}\,(u) = \frac{\hat{\bar{h}}\,(0)}{\Delta\,(0,\theta)} e^{(r-\theta)u}$$

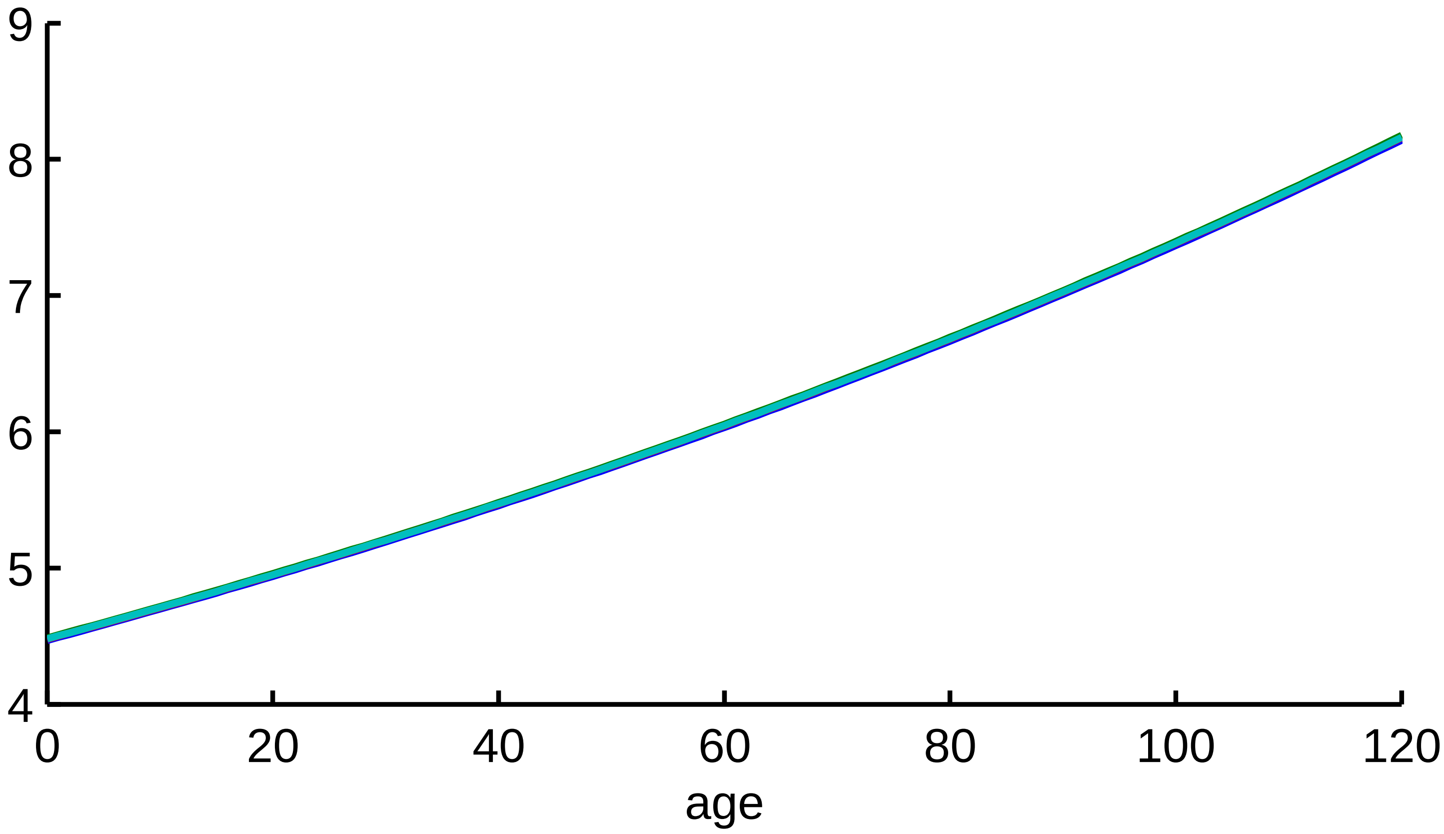


Figure 2: (c) Consumption

$$\hat{\bar{a}}\left(u\right)=\Delta\left(u,\theta\right)\hat{\bar{c}}\left(u\right)-\hat{\bar{h}}\left(u\right)$$

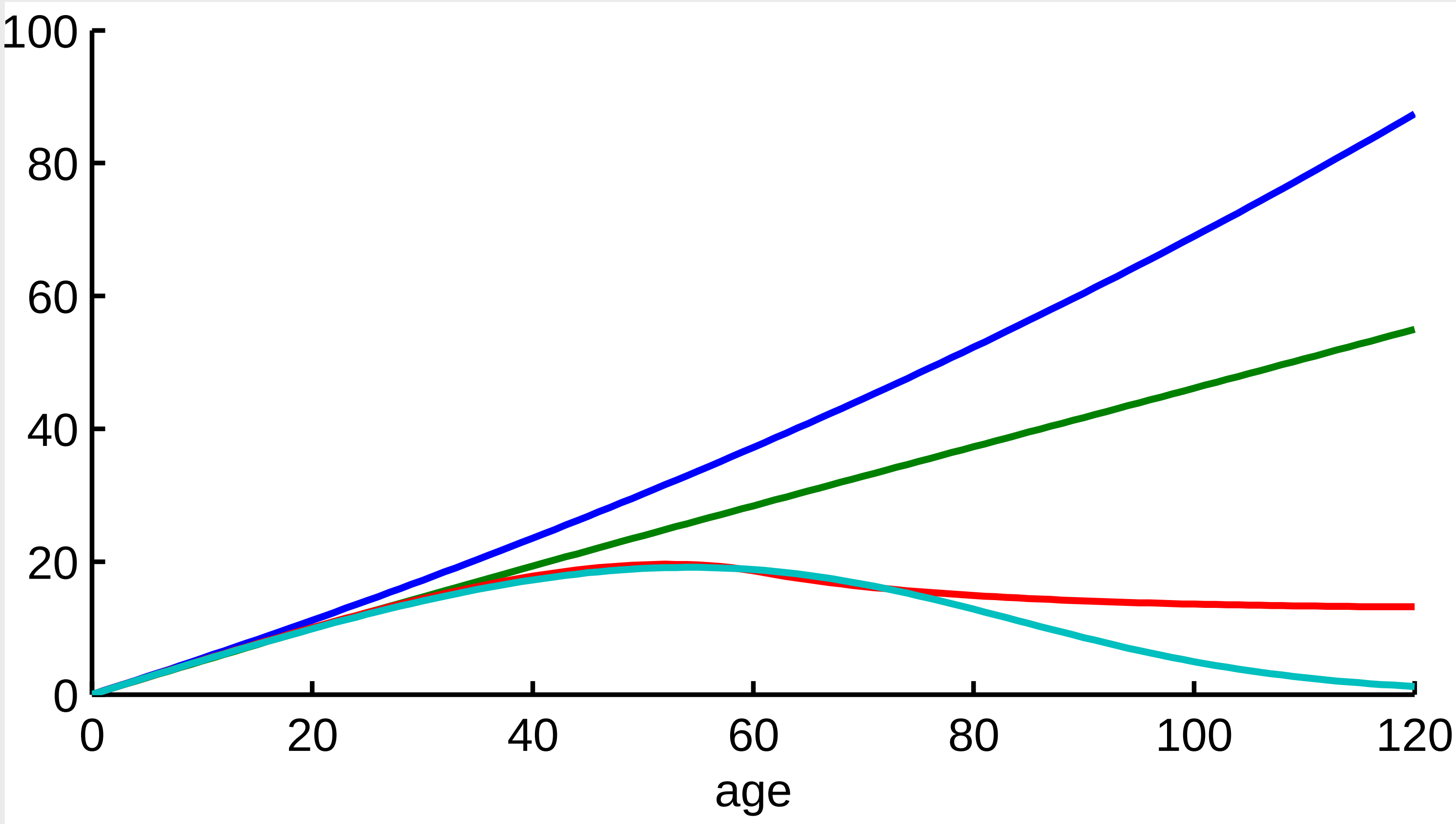


Figure 2: (d) Financial Assets

3.8. Concluding Remarks

- in the context of a small open economy [or with constant marginal product of capital] there is no need to use models based on an unrealistic description of the demographic process

- using a realistic demographic process matters because.....
  - individual behaviour is different
  - impulse-response functions are different
  - transition speed is affected
  - welfare effects may be non-monotonic

4. Ageing, Human Capital Formation, and Economic Growth

4.1. Motivation

- Large empirical literature on growth effects of human capital accumulation
  - starting with Mankiw-Romer-Weil (1992 QJE), Benhabib & Spiegel (1994 JME)
  - no concensus; difficult to measure contribution of human capital to output and growth
- Small theoretical literature on effects of ageing on economic growth
  - usually quite unrealistic demography
  - exception: Boucekkine et al. (2002 JET) but they do not present macro closure (focus on human capital for knife-edge case with endogenous growth; linear felicity)

- Our aim: introduce realistic demography and develop theoretical growth implications
- Possible byproduct: implications for empirical tests

4.2. Model: Key Assumptions

- small open economy facing constant world interest rate
- two factors of production: **physical capital** (mobile) and **human capital** (immobile internationally)
- growth engine: **startup education** (possibly with intergenerational external effect) and physical capital accumulation.
- savings instruments:
  - foreign assets
  - government debt
  - **claims** on domestic capital stock
  - perfect substitutes: same rate of return

- life-time uncertainty; actuarially fair life insurance
- no aggregate uncertainty
- rational agents blessed with perfect foresight

4.3. Model: Key Equations

- demographic structure same as in Heijdra-Romp (2005) but we allow for shocks
- (expected remaining) lifetime utility at time t of agent born at time v ($t \geq v$)

$$\Lambda(v,t) \equiv e^{M(t-v)} \int_t^\infty \ln \bar{c}\,(v,\tau)\, e^{-[\theta(\tau-t)+M(\tau-v)]} d\tau \qquad (1)$$

- mortality factor and mortality rate:

$$M\,(\tau - v) \equiv \int_0^{\tau-v} m\,(\alpha)\, d\alpha \qquad (2)$$

- budget identity:

$$\dot{\bar{a}}(v,\tau) = [r + m(\tau - v)]\,\bar{a}(v,\tau) + \bar{w}(v,\tau) - \bar{g}(v,\tau) - \bar{c}(v,\tau) \qquad (3)$$

  - $\bar{a}(v,\tau)$ = financial assets
  - r = world interest rate [patient country, $r > \theta$]
  - $r + m(\tau - v)$ = annuity rate of interest
  - $\bar{w}(v,\tau)$ = wage income (see below)
  - $\bar{g}(v,\tau)$ = net tax payment (see below)
  - $\bar{c}(v,\tau)$ = consumption

- human capital creation:

$$\bar{h}(v,\tau) = \begin{cases} 0 & \text{for } v \leq \tau \leq v + s(v) \\ A_H h(v)^{\phi} s(v) & \text{for } \tau > v + s(v) \end{cases} \tag{4}$$

  - $\bar{h}(v,\tau)$ = human capital of vintage-v person at time τ (positive only after completing schooling)
  - $s(v)$ = length of schooling period (choice variable)
  - $h(v)$ = per capita human capital at time v (at birth of agent)
  - ϕ = parameter for intergenerational externality ($0 \leq \phi \leq 1$)

- wage income:

$$\bar{w}(v,\tau) = w(\tau)\,\bar{h}(v,\tau) \tag{5}$$

  - $w(\tau)$ = rental rate on human capital (time-invariant for core case, see below)

- tax/subsidy system:

$$\bar{g}\left(v,\tau\right)=\begin{cases}\left[z\left(\tau\right)-\rho\right]w\left(\tau\right)A_{H}h\left(v\right)^{\phi} & \text{for } v\leq\tau\leq v+s\left(v\right)\\ \left[z\left(\tau\right)+t_{L}s\left(v\right)\right]w\left(\tau\right)A_{H}h\left(v\right)^{\phi} & \text{for } \tau>v+s\left(v\right)\end{cases} \tag{6}$$

  - ρ = educational subsidy
  - t_L = labour income tax rate
  - z = lump-sum tax on all (poll tax; used for tax smoothing purposes)
  - all taxes/subsidies indexed smartly (to allow for ongoing growth)

- (expected remaining) lifetime budget constraint:

$$e^{M(t-v)}\int_{t}^{\infty}\bar{c}(v,\tau)e^{-[r(\tau-t)+M(\tau-v)]}d\tau=\bar{a}\left(v,t\right)+\overline{li}\left(v,t\right) \tag{7}$$

- (expected remaining) lifetime (after-tax) wage income:

$$\begin{aligned} \overline{li}\,(v,t) \equiv\; & A_H h\,(v)^{\phi}\, e^{M(t-v)} \Big[\rho \int_t^{\max\{t,v+s(v)\}} w\,(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \\ & + (1-t_L)\, s\,(v) \int_{\max\{t,v+s(v)\}}^{\infty} w\,(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \\ & - \int_t^{\infty} z\,(\tau)\, w\,(\tau)\, e^{-[r(\tau-t)+M(\tau-v)]} d\tau \Big]. \end{aligned} \qquad (8)$$

- two-stage optimization:
  - schooling choice: choose $s\,(v)$ in order to maximize $\overline{li}\,(v,t)$, eqn. (8)
  - consumption-saving choice: choose $\{\bar{c}(v,\tau)\}_t^{\infty}$ and $\{\bar{a}(v,\tau)\}_t^{\infty}$ in order to maximize lifetime utility, eqn. (1), subject to lifetime budget constraint, eqn. (7)

- optimal schooling period with constant demography and constant rental rate on human capital:

$$s^* = \frac{\rho}{1 - t_L} + \Delta(\underset{-}{s^*}, r) \tag{9}$$

  - $\Delta(u, \lambda)$ = demographic function [in case you forgot]
  - for Blanchard case: $s^* = \frac{\rho}{1-t_L} + \frac{1}{r+\mu_0}$
  - unique choice; see **Figure 1**
  - $s^* \uparrow$ if $\rho \uparrow$ or $t_L \uparrow$

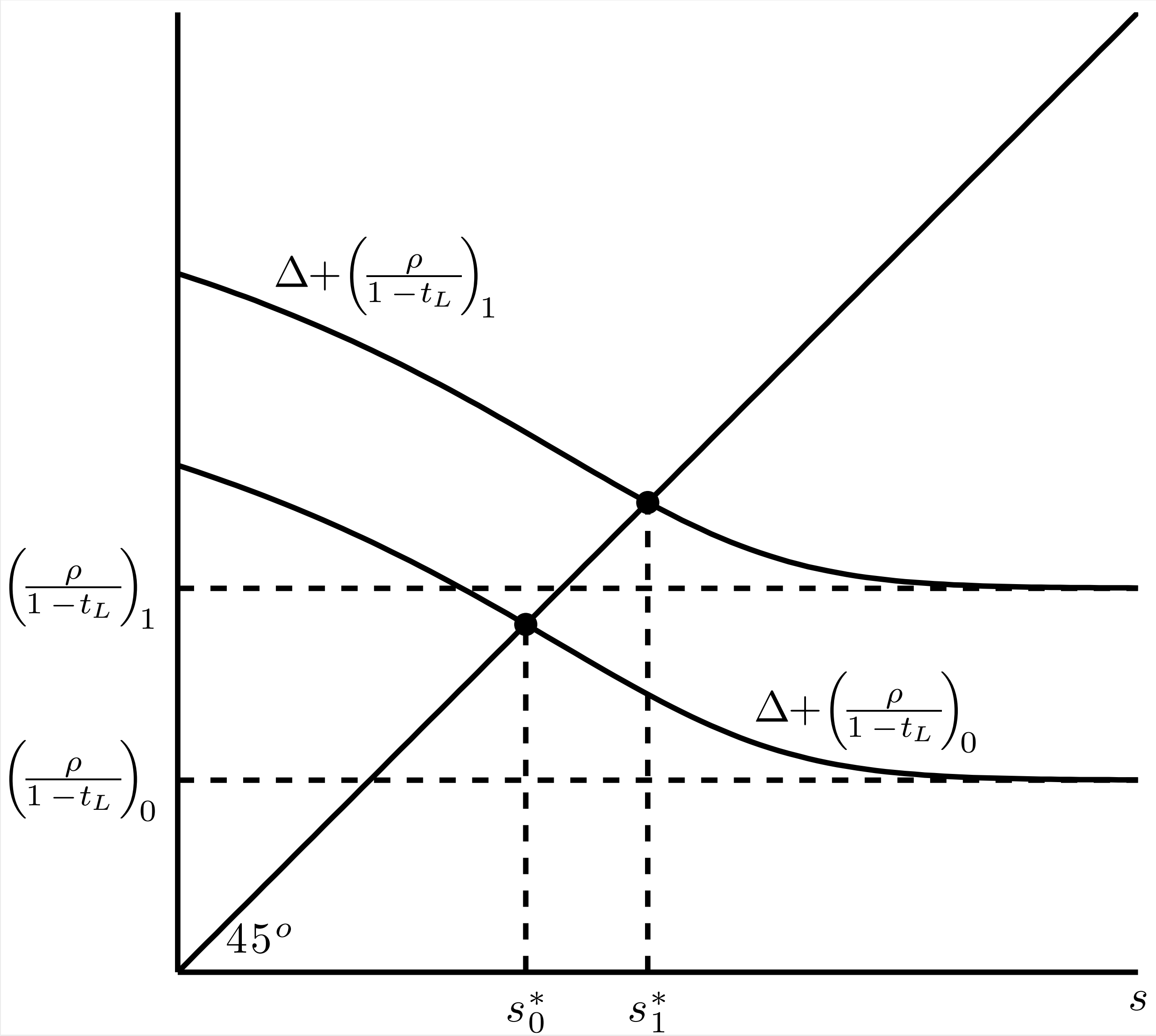


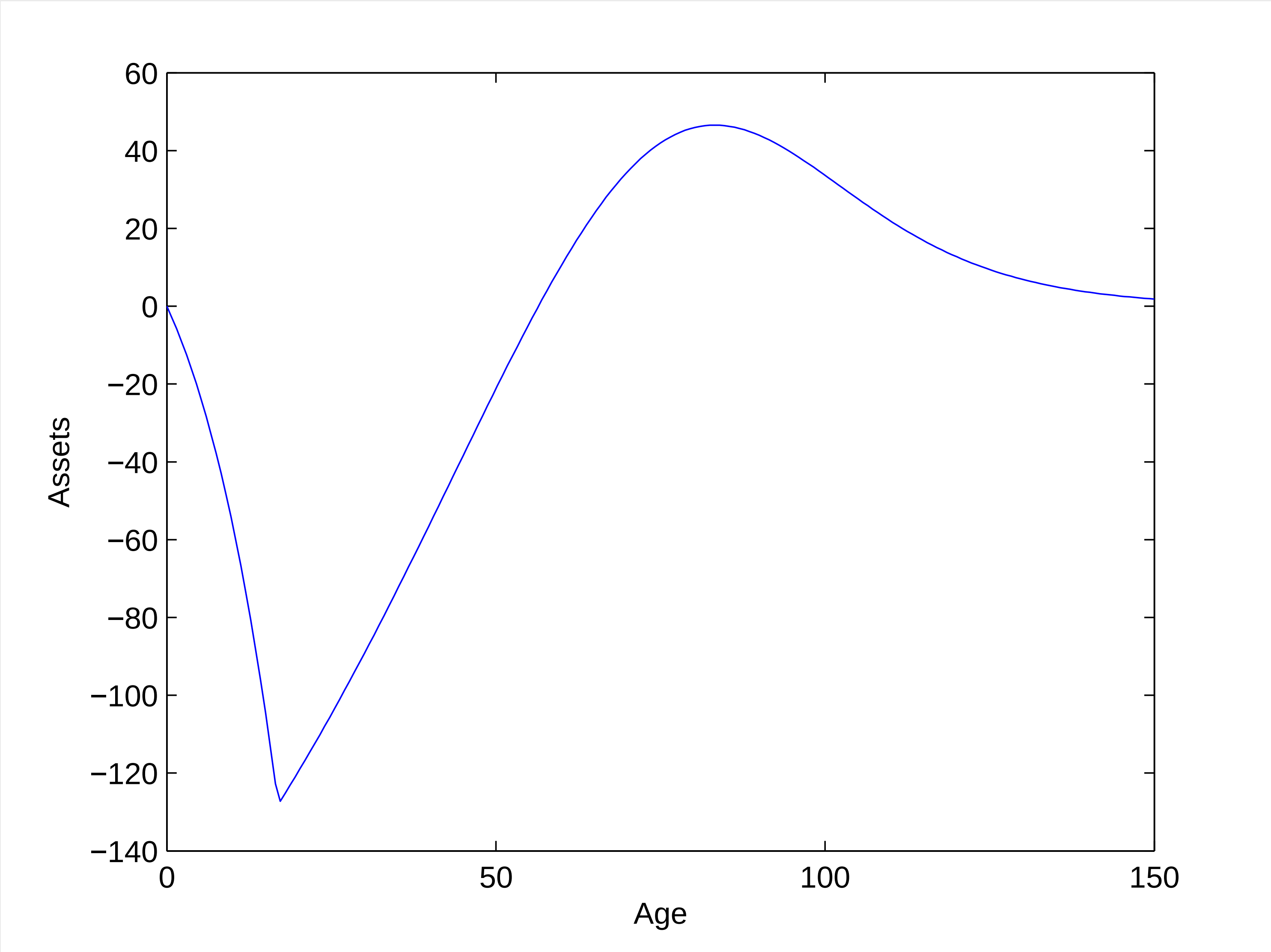
Figure 1: Optimal Schooling and Tax Change

- optimal choices of household with age $u \equiv t - v$:

$$\frac{\dot{\bar{c}}(v,\tau)}{\bar{c}(v,\tau)} = r - \theta > 0 \qquad (10)$$

$$\bar{c}(v,t) = \frac{1}{\Delta(u,\theta)}\left[\bar{a}(v,t) + \bar{h}(v,t)\right] \qquad (11)$$

$$\Delta(u,\lambda) \equiv e^{\lambda u + M(u)} \int_u^{\infty} e^{-[\lambda\alpha + M(\alpha)]} d\alpha, \quad (u \geq 0,\ \lambda > 0) \qquad (12)$$



Steady-State Profile of Individual Assets

4.4. Demographics

- size of cohort over time:

$$L(v,\tau) = L(v,v)\,e^{-M(\tau-v)} = bL(v)e^{-M(\tau-v)} \tag{13}$$

  - b = birth rate
  - $L(v,v)$ = size of newborn generation at time v

- total population at time t:

$$L(t) \equiv \int_{-\infty}^{t} L(v,t)\,dv \tag{14}$$

$$L(t) \equiv L(v)\,e^{N(v,t)}, \qquad N(v,t) \equiv \int_{v}^{t} n(\tau)\,d\tau \tag{15}$$

  - $n(\tau)$ = instantaneous growth rate of population at time τ

- it follows from (13)-(15) that:

$$l(v,t) \equiv \frac{L(v,t)}{L(t)} = be^{-[N(v,t)+M(t-v)]}, \qquad t \geq v \tag{16}$$

$$\frac{1}{b} = \int_{-\infty}^{t} e^{-[N(v,t)+M(t-v)]} dv \tag{17}$$

  - relative cohort size shrinks for two reasons: population growth and cohort mortality
  - eqn. (17) implicitly determines $n(t)$ for a given birth rate and demographic process
    - can be rewritten as a linear Volterra equation of the second kind with a convolution kernel (in case you wondered)

- for given birth rate and mortality process [i.e. in the long run], eqn. (17) reduces to:

$$\frac{1}{b} = \Delta(0, n) \qquad (17')$$

- stepwise changes in birth rate and/or mortality process are analyzed in the paper.

4.5. Representative Firm

- Perfect competition
- Technology:

$$Y(\tau) = A_Y K(\tau)^{\varepsilon} H(\tau)^{1-\varepsilon} \qquad 0 < \varepsilon < 1, \tag{18}$$

  - $Y(\tau)$ = output
  - A_Y = exogenous technology index
  - $K(\tau)$ = aggregate stock of physical capital
  - $H(\tau) \equiv L(\tau) h(\tau)$ = aggregate stock of human capital

- Objective function:

$$V(t) \equiv \int_t^{\infty} [Y(\tau) - w(\tau) H(\tau) - I(\tau)] e^{r(t-\tau)} d\tau \tag{19}$$

- Constraints
  - capital accumulation identity:

$$\dot{K}(\tau) = I(\tau) - \delta K(\tau) \tag{20a}$$

  - given initial capital stock:

$$K(t) = K_0 = \text{given} \tag{20b}$$

- Key features:

$$\frac{\partial Y(\tau)}{\partial K(\tau)} = r + \delta = \varepsilon A_Y \left(\frac{H(\tau)}{K(\tau)}\right)^{1-\varepsilon} \quad (22a)$$

$$\frac{\partial Y(\tau)}{\partial H(\tau)} = w(\tau) = (1-\varepsilon) A_Y \left(\frac{H(\tau)}{K(\tau)}\right)^{-\varepsilon} \quad (22b)$$

$$V(t) = K(t) \quad (22c)$$

  - r pins down H/K ratio and thus the rental rate on human capital w
  - in principle, K could jump (no adjustment costs); in practice, it moves smoothly because H does

4.6. Other Model Elements

- intertemporal budget constraint:

$$d(t) = \int_t^{\infty} g(\tau)\, e^{r(t-\tau)+N(t,\tau)} d\tau \tag{23}$$

  - $d(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{d}(v,t)dv$
  - $g(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{g}(v,t)dv$
  - pre-existing debt covered in present value terms by primary supluses, using $r - n(\tau)$ for discounting purposes

- per capita output:

$$y(t) = (r+\delta)\, k(t) + wh(t) \tag{24}$$

  - $y(t) \equiv Y(t)/L(t)$
  - $k(t) \equiv K(t)/L(t) = \int_{-\infty}^{t} l(v,t)\bar{k}(v,t)dv$

- current account of the balance of payments:

$$\dot{f}(t) = [r - n(t)]\, f(t) + y(t) - c(t) - i(t) \tag{25}$$

  - $f(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{f}(v,t)dv$
  - $c(t) \equiv \int_{-\infty}^{t} l(v,t)\bar{c}(v,t)dv$
  - $i(t) \equiv I(t)/L(t)$

4.7. Model Solution

- model is recursive
    - tax parameters (ρ and t_L) and demography determine s^*
    - for given birth rate and mortality profiles, path for population growth is determined
    - path for per capita human capital is determined
    - lump-sum poll tax (z) used to balance GBC
    - remaining variables are solved

- path for per capita human capital depends on magnitude of ϕ:
    - exogenous growth case
        - $0 \leq \phi < 1$
        - steady state **level** of per capita variables
        - long-run growth rate in levels equal to long-run population growth, n
    - endogenous growth case
        - $\phi = 1$
        - steady-state **growth rate** of per capita variables

4.8. Visualizing Demographic and Tax Shocks

- Analytical results available but focus here on visualization of transition paths
- Shocks considered:
  - demographic shock: reduction in **adult** mortality
  - tax schock: increase in ρ and/or t_L
  - combined shock
- bizarre adjustment dynamics due to vintage-structure
- link growth / human capital far from unambiguous!

4.9. Reduced Adult Mortality

- **Embodied** mortality change: from time $t = 0$, newborns face mortality function $m_1(s)$. Pre-shock agents continue to face mortality function $m_0(s)$
  - Reduced **adult** mortality, concentrate change in right-hand tail, i.e. $m_1(s) - m_0(s)$ rises with s
  - Shock increases expected remaining lifetime for post-shock agents at all ages
  - Example: Gompertz-Makeham:

$$m(\alpha) = \mu_0 + \mu_1 e^{\mu_2 \alpha}$$

    and shock is a reduction in μ_1 or μ_2
  - **Figure 2** visualizes relationship between expected lifetime at birth (horizontal axis) and steady-state population growth rate (vertical axis)

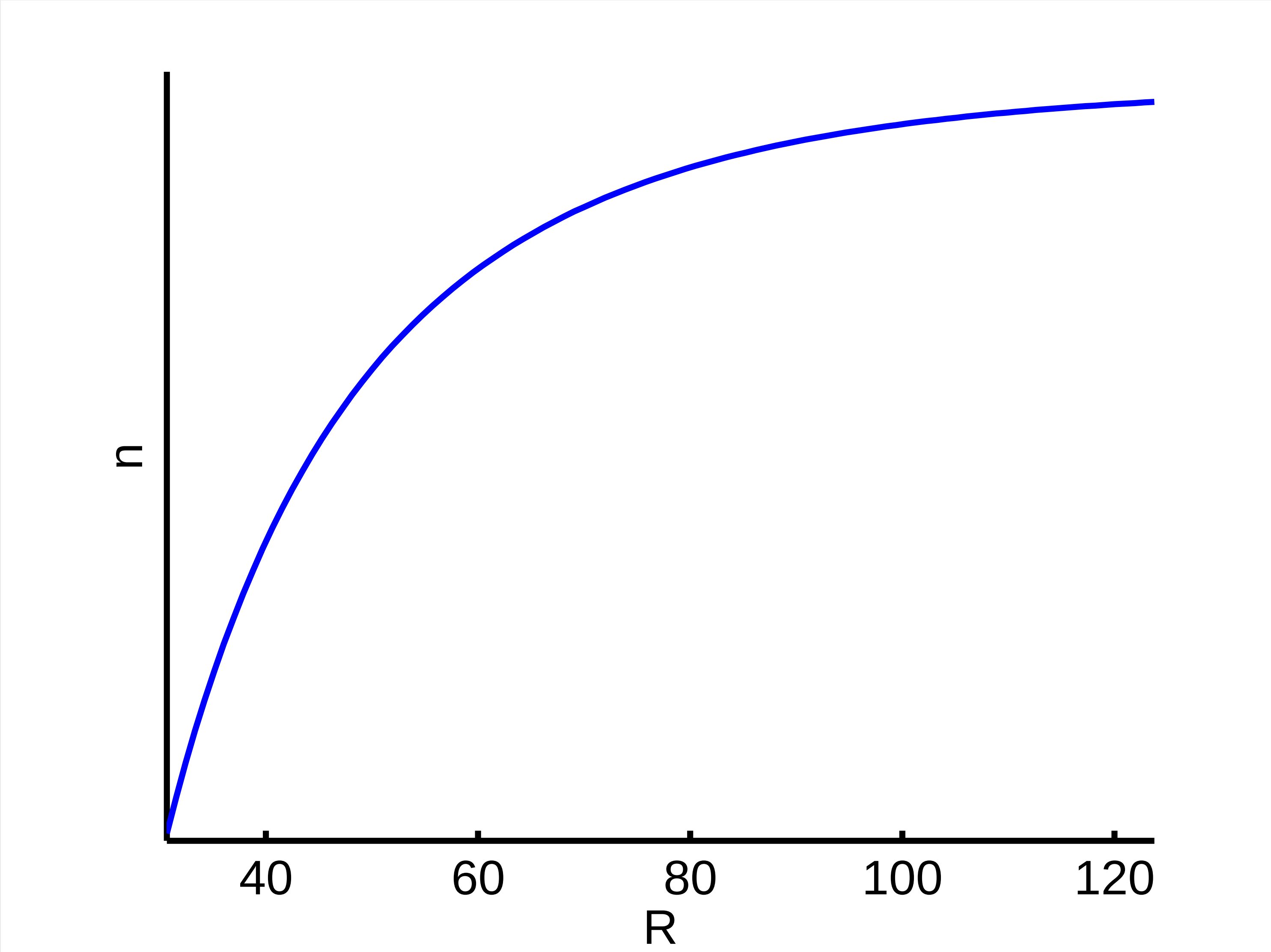


Figure 2: Expected Lifetime at Birth (R) and Steady-State Population Growth (n)

- By numerically solving eqn. (17) we obtain the time path of the instantaneous population growth rate ($n(t)$) in **Figure 3**.
  - steady-state n_1 rises (average mortality rate declines)
  - $n(t)$ overshoots its long-run equilibrium
  - transition is rather slow
- Under the assumptions regarding the mortality shock, it follows from eqn. (12) that $\Delta(u, \lambda)$ rises for all levels of u
  - in **Figure 4**, the optimal schooling choice increases from s_0^* to s_1^*
  - what matters is reduction in **adult** mortality
  - $\cdots$ a reduction in mortality rates for ages below s_0^* would have no effect on the optimal schooling choice!

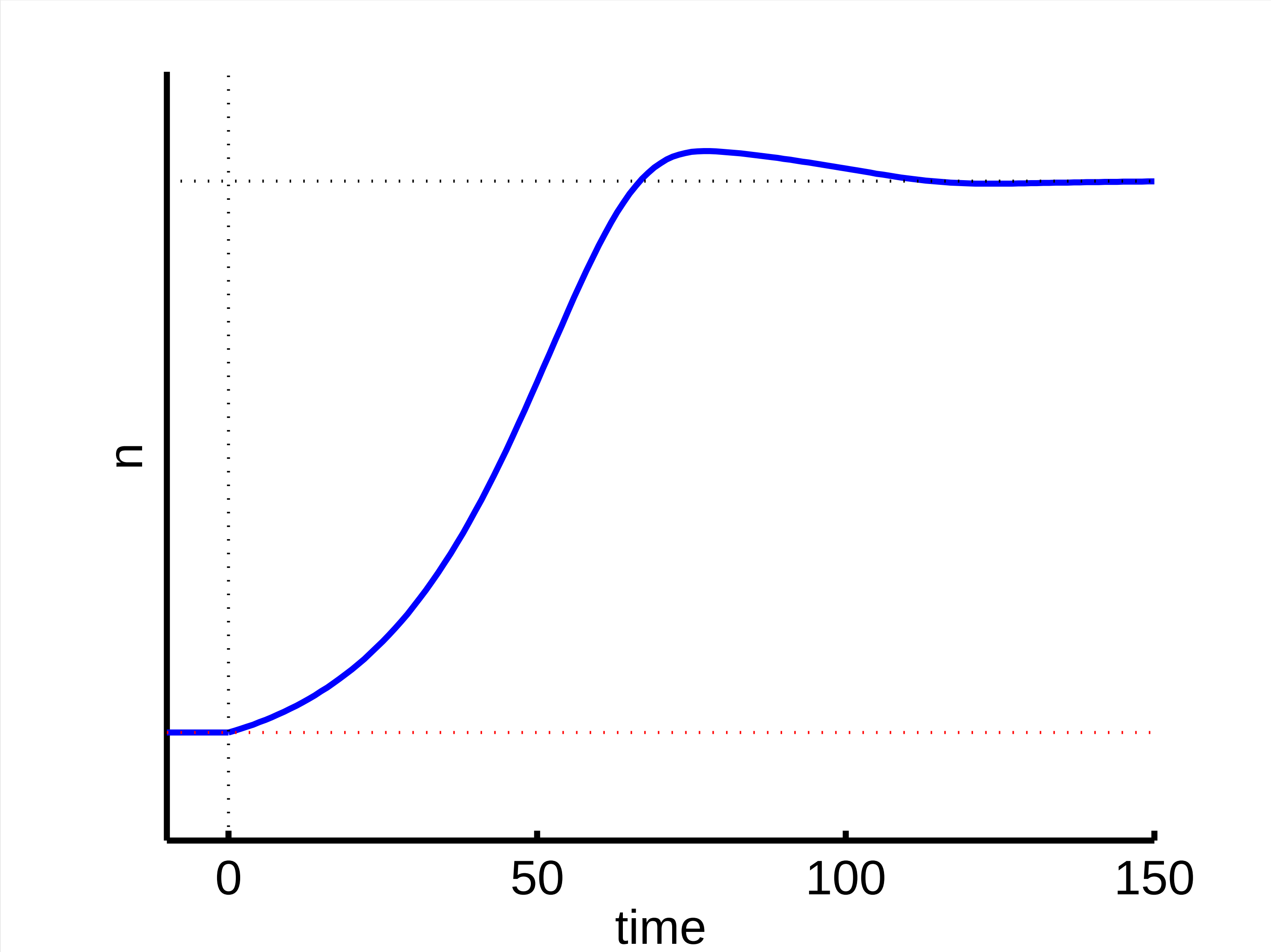


Figure 3: Mortality Change and the Population Growth Rate

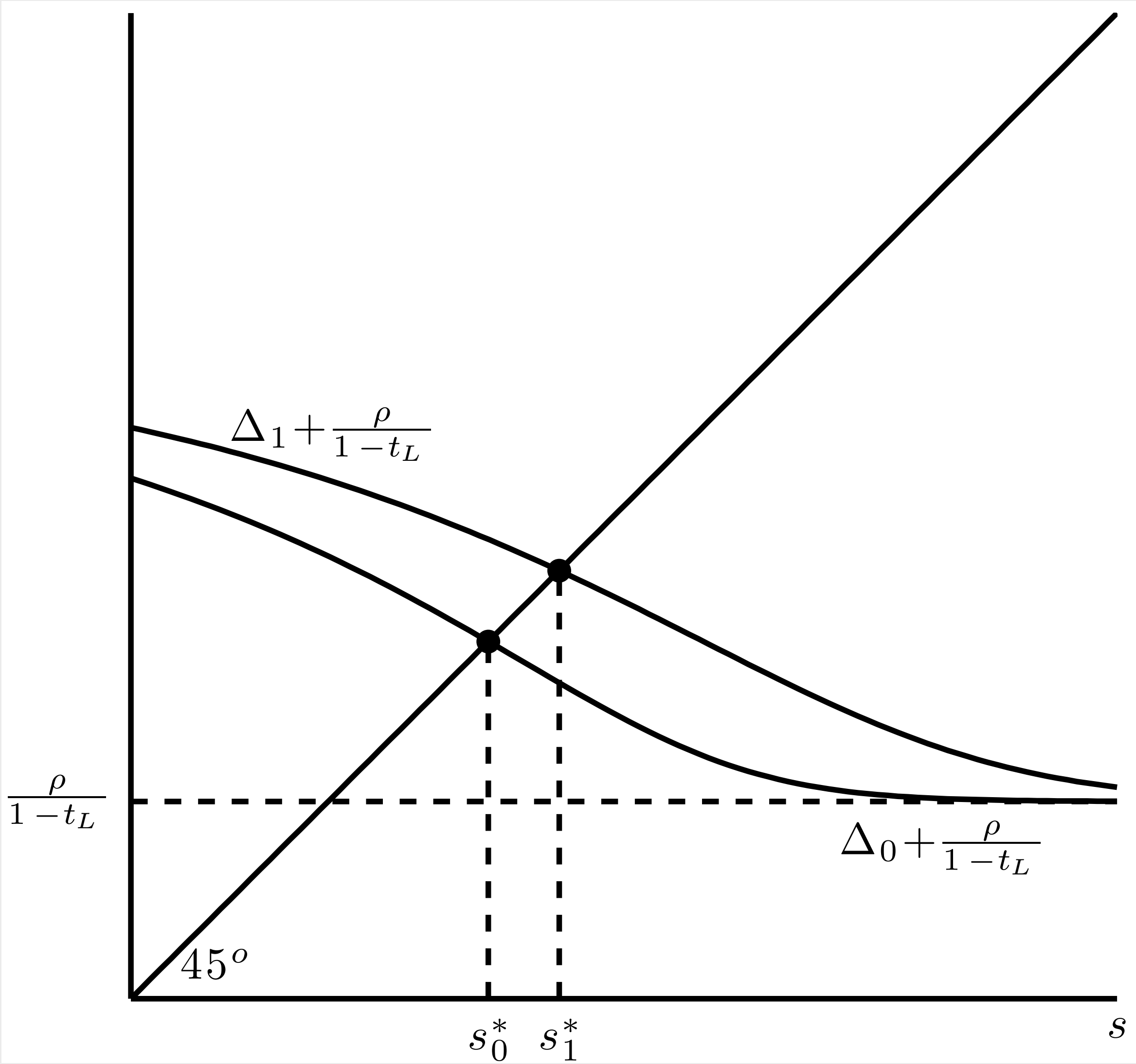


Figure 4: Optimal Schooling and Mortality Change

4.9.1. Exogenous Growth

- $0 \leq \phi < 1$

- Long-run human capital (per capita):

$$h^{1-\phi} = A_H s_1^* b \int_{s_1^*}^{\infty} e^{-n_1 u - M_1(u)} du \qquad (26)$$

  - pure schooling effect is positive for dynamically efficient case ($r > n$):

$$\frac{\partial h^{1-\phi}}{\partial s} = A_H b e^{-ns-M(s)} \left[\Delta(s, \underline{n}) - \Delta(s, \underline{r})\right] > 0 \qquad (27)$$

  - mortality shock affects both the schooling decision and the discounting factor. Both lead to increase in h

  - **Figure 5** illustrates the case with persistence parameter $\phi = 0.2$

- Transitional dynamics distinguishes three phases
  - $0 \leq t < s_0^*$: system stays in old steady state. Only pre-shock students enter the labour market (only post-shock students increase schooling period, from s_0^* to s_1^*)
  - $s_1^* - s_0^* \leq t < s_1^*$: no new labour market entrants (oldest of the post-shock students not yet finished)
  - $t \geq s_1^*$: new labour market entrants are all post-shock agents
- Things get worse before they get better in the long run

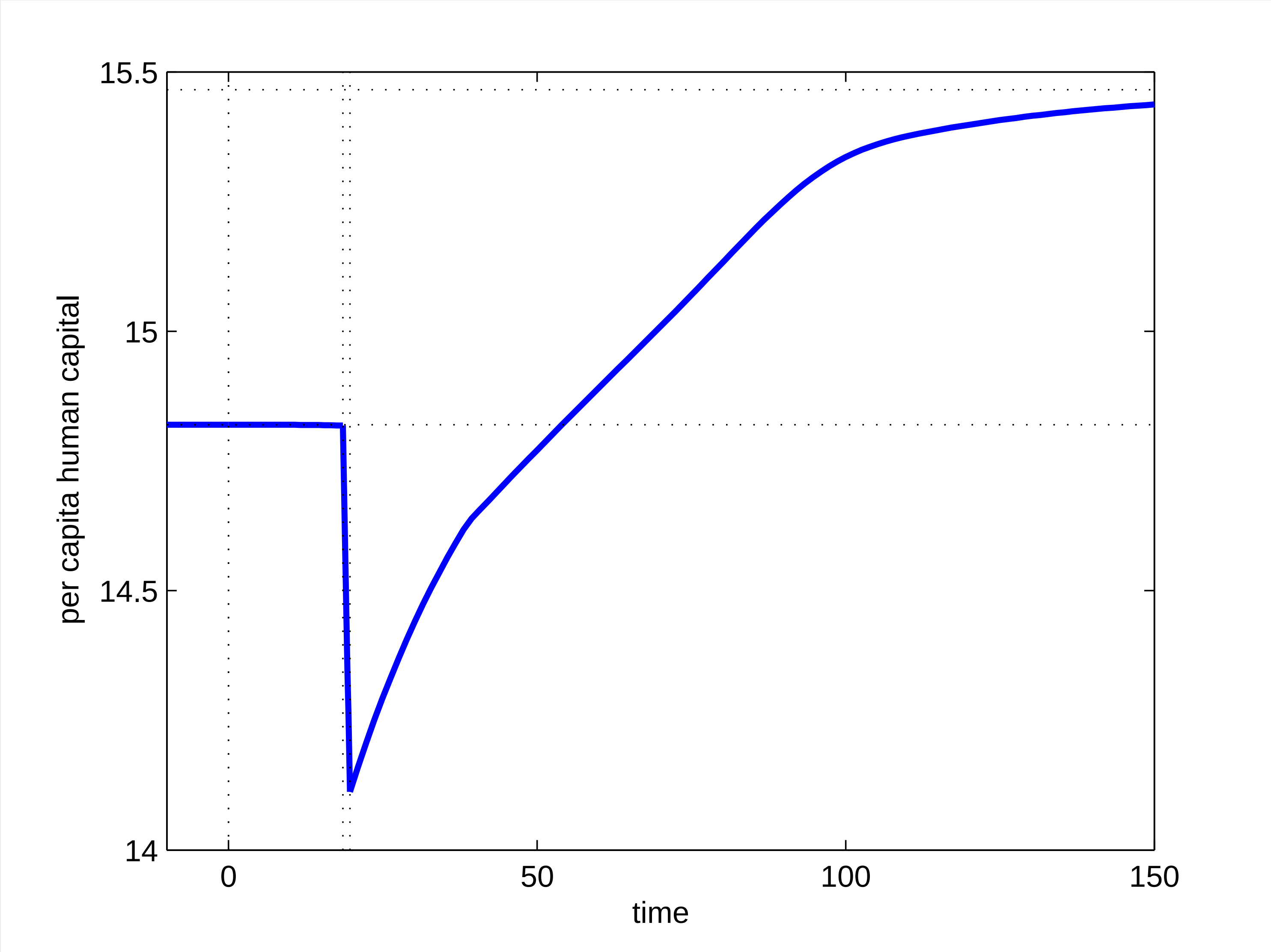


Figure 5: Per Capita Human Capital and Mortality Change ($\phi = 0.2$)

4.9.2. Endogenous Growth

- $\phi = 1$
- Steady-state growth path:

$$h(t) = \int_{\infty}^{t-s} l(t-v)\bar{h}(v,t)dv$$
$$= A_H s \int_{\infty}^{t-s} l(t-v)h(v)dv$$

  - no past demographic shocks ($l\,(v,t) = l\,(t-v)$)
  - can be written as Volterra equation (following a shock)

- human capital growth converges to a constant, λ. Growth rate determined implicitly by:

$$\frac{1}{b} = A_H s \int_s^\infty e^{-[\lambda+n]u-M(u)} du$$

  - pure schooling effect is positive for dynamically efficient case ($r > n + \lambda$):

$$\frac{\partial \lambda}{\partial s} = \omega_0 \left[\Delta(s, \lambda+n) - \Delta(s, r) \right] > 0$$

  - mortality shock affects s, n, and $M(u)$. Total effect ambiguous, though for realistic life expectancy at birth, λ falls as a result of the shock.
  - **Figure 6** illustrates this case

- Transitional dynamics distinguishes three phases
  - $0 \leq t < s_0^*$: system stays in old steady state growth path. Only pre-shock students enter the labour market (only post-shock students increase schooling period, from s_0^* to s_1^*)
  - $s_1^* - s_0^* \leq t < s_1^*$: no new labour market entrants (oldest of the post-shock students not yet finished) and growth rate collapses
  - $t \geq s_1^*$: new labour market entrants are all post-shock agents and growth rate jumps up
  - echo effect
- Growth rate increases in the short run but decreases in the very long run!

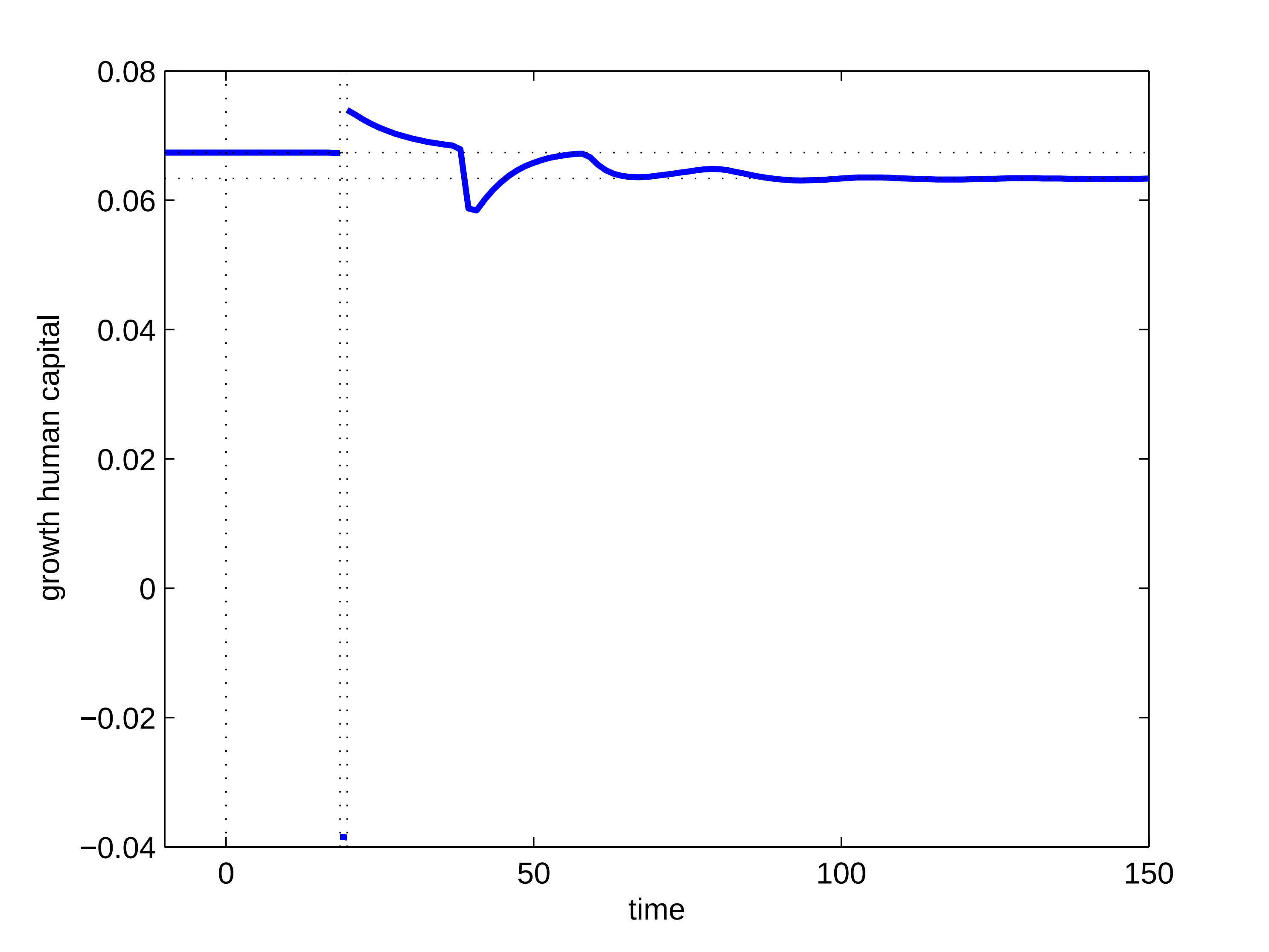


Figure 6: Per Capita Human Capital Growth and Mortality Change ($\phi = 1$)

4.10. Education-Promoting Tax Shock

- At time $t = 0$, the educational subsidy (ρ) or the labour income tax (t_L) is increased permanently.
  - Leaving school is assumed to be an **absorbing state**: pre-shock workers continue to work (facing the wage income profile corresponding to s_0^* and the new tax rate)
  - Pre-shock students and all post-shock newborns increase schooling period from s_0^* to s_1^*. See **Figure 7**.
  - Nothing happens to the population growth rate

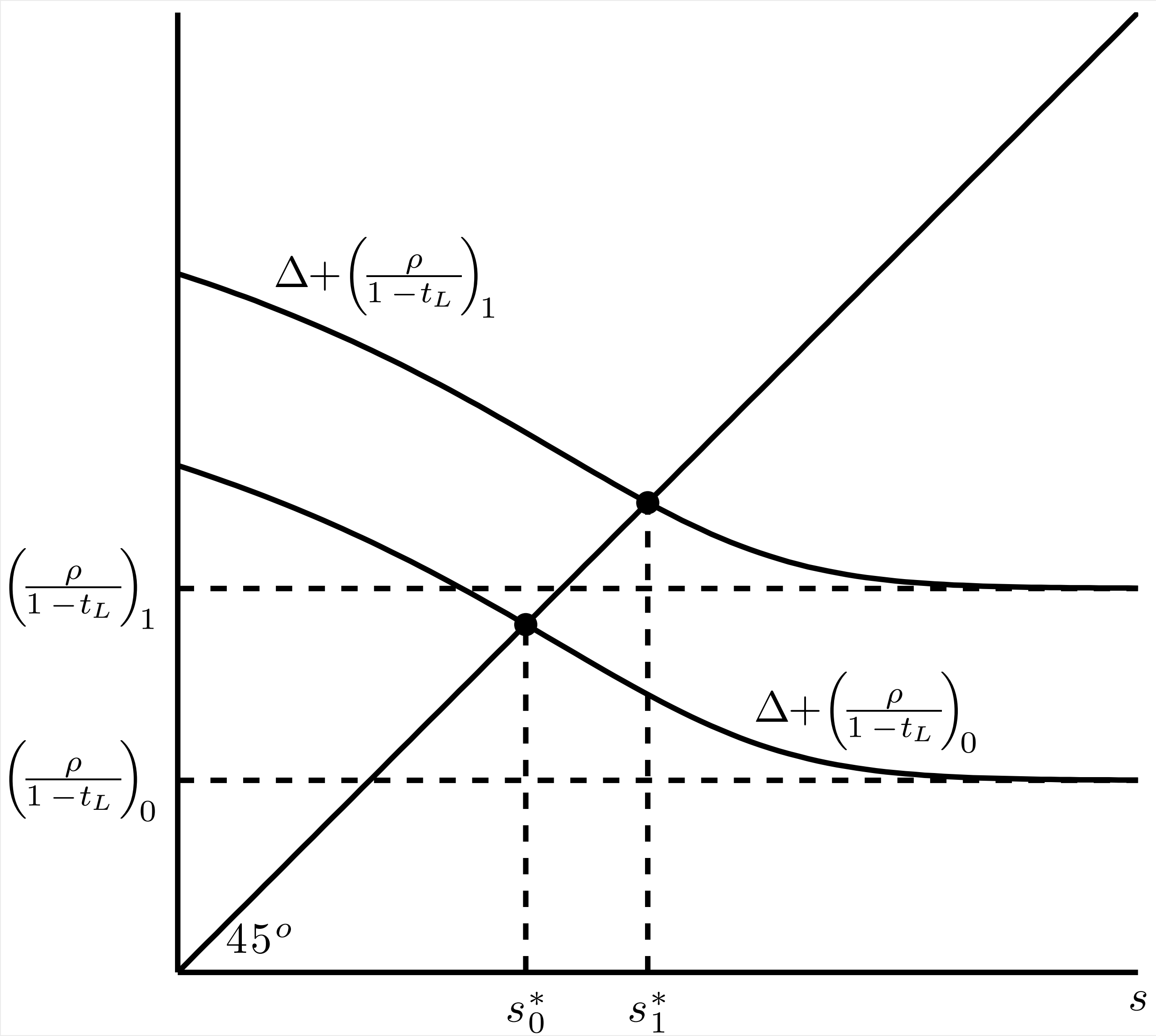


Figure 7: Optimal Schooling and Taxes

4.10.1. Exogenous Growth

- $0 \leq \phi < 1$
- Transitional dynamics distinguishes three phases
  - $0 \leq t < s_1^* - s_0^*$: no new labour market entrants (pre-shock students increase schooling period, from s_0^* to s_1^*)
  - $s_1^* - s_0^* \leq t < s_1^*$: new labour market entrants only pre-shock students
  - $t \geq s_1^*$: new labour market entrants only post-shock students (who also choose s_1^*)
- $h(t)$ only increases in the very long run. See **Figure 8**!

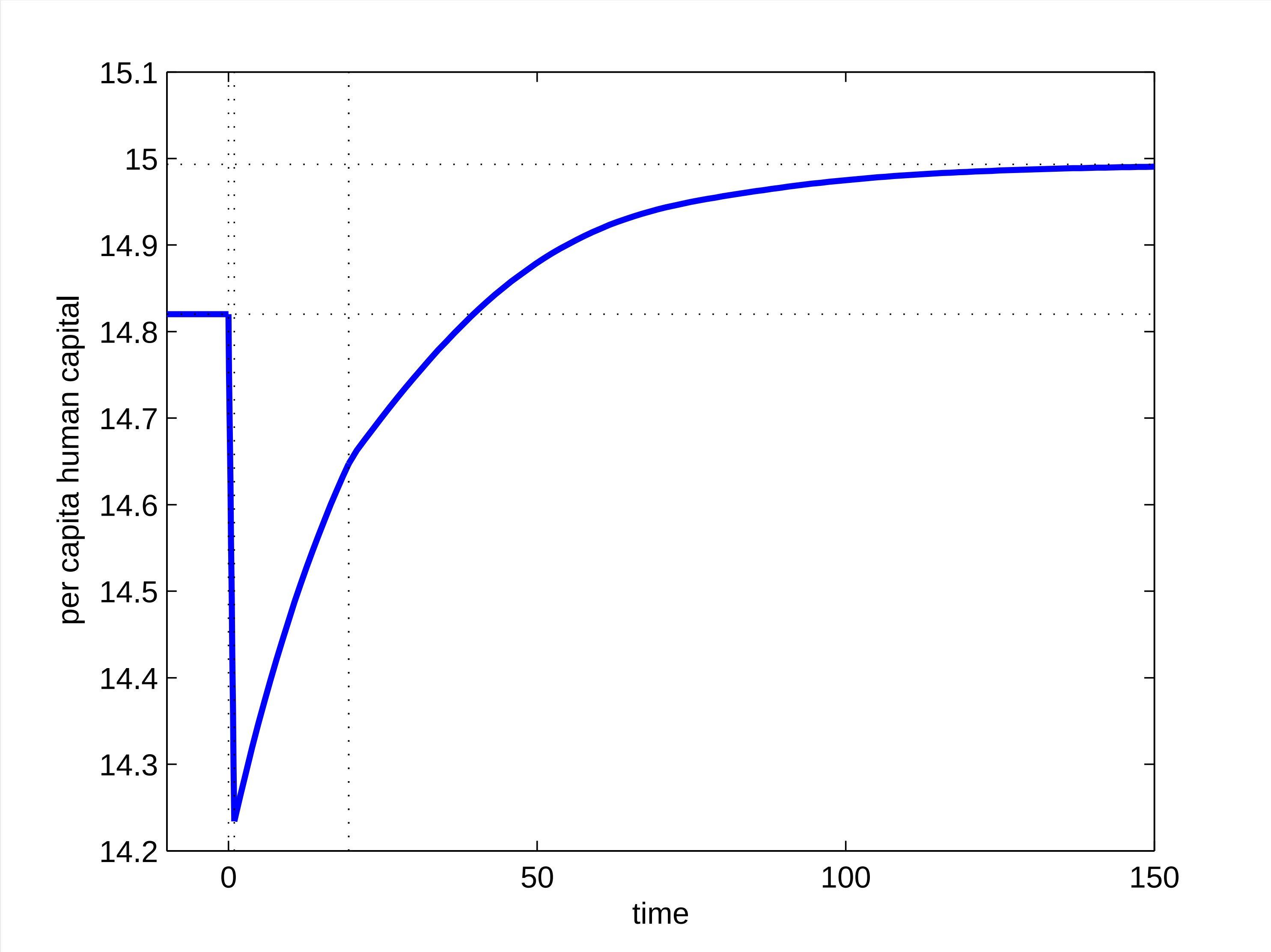


Figure 8: Per Capita Human Capital and Taxes ($\phi = 0.2$)

4.10.2. Endogenous Growth

- $\phi = 1$
- Transitional dynamics distinguishes three phases
  - $0 \leq t < s_1^* - s_0^*$: no new labour market entrants (pre-shock students increase schooling period, from s_0^* to s_1^*)
  - $s_1^* - s_0^* \leq t < s_1^*$: new labour market entrants only pre-shock students
  - $t \geq s_1^*$: new labour market entrants only post-shock students (who also choose s_1^*)
- growth in $h(t)$ increases in the short run but decreases the very long run. See **Figure 9**!

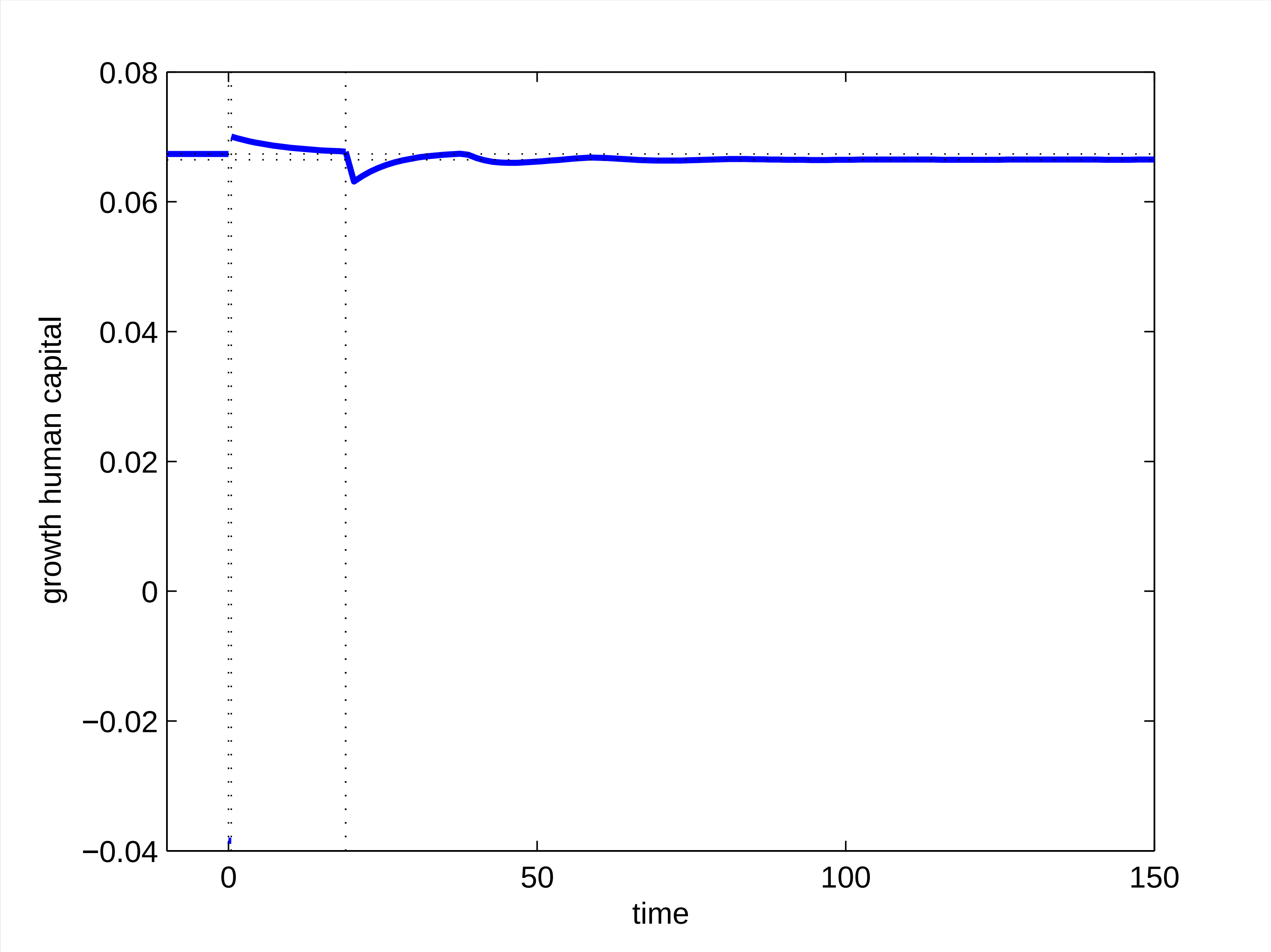


Figure 9: Per Capita Human Capital Growth and Taxes ($\phi = 1$)

4.11. Combined Shock

- Reduction in adult mortality and positive tax incentive
- Transitional dynamics distinguishes four phases
  - $0 \leq t < s_1^* - s_0^*$: no new labour market entrants (pre-shock students increase schooling period, from s_0^* to s_1^*; post-shock students choose $s_2^* > s_1^*$)
  - $s_1^* - s_0^* \leq t < s_1^*$: new labour market entrants only pre-shock students
  - $s_1^* \leq t < s_2^*$: no new labour market entrants
  - $t \geq s_2^*$: new labour market entrants only post-shock students
- **Figure 10** shows the adjustment path for the exogenous growth case ($0 \leq \phi < 1$)
- **Figure 11** shows the adjustment path for the endogenous growth case ($\phi = 1$)

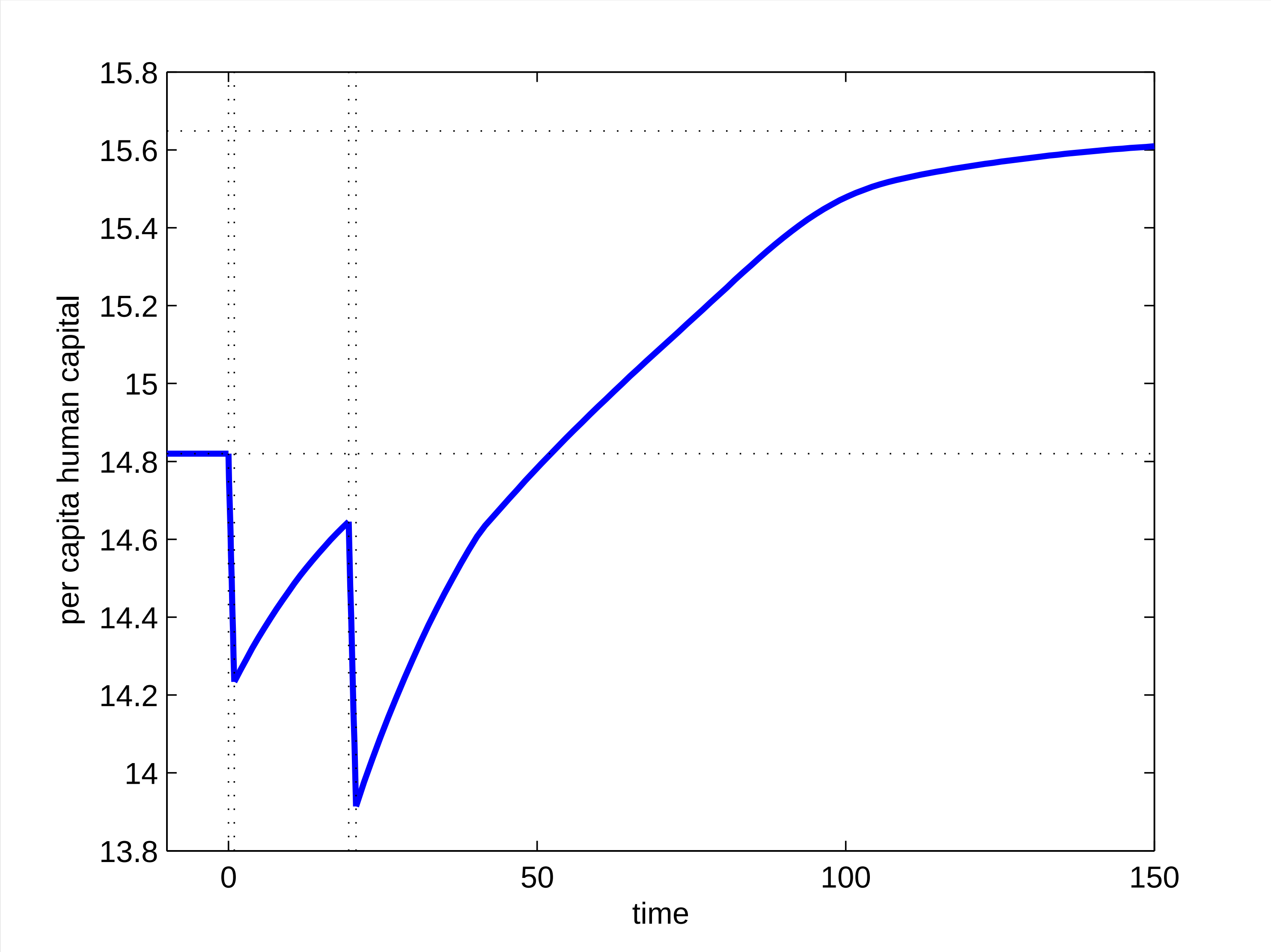


Figure 10: Per Capita Human Capital Under Combined Shock ($\phi = 0.2$)

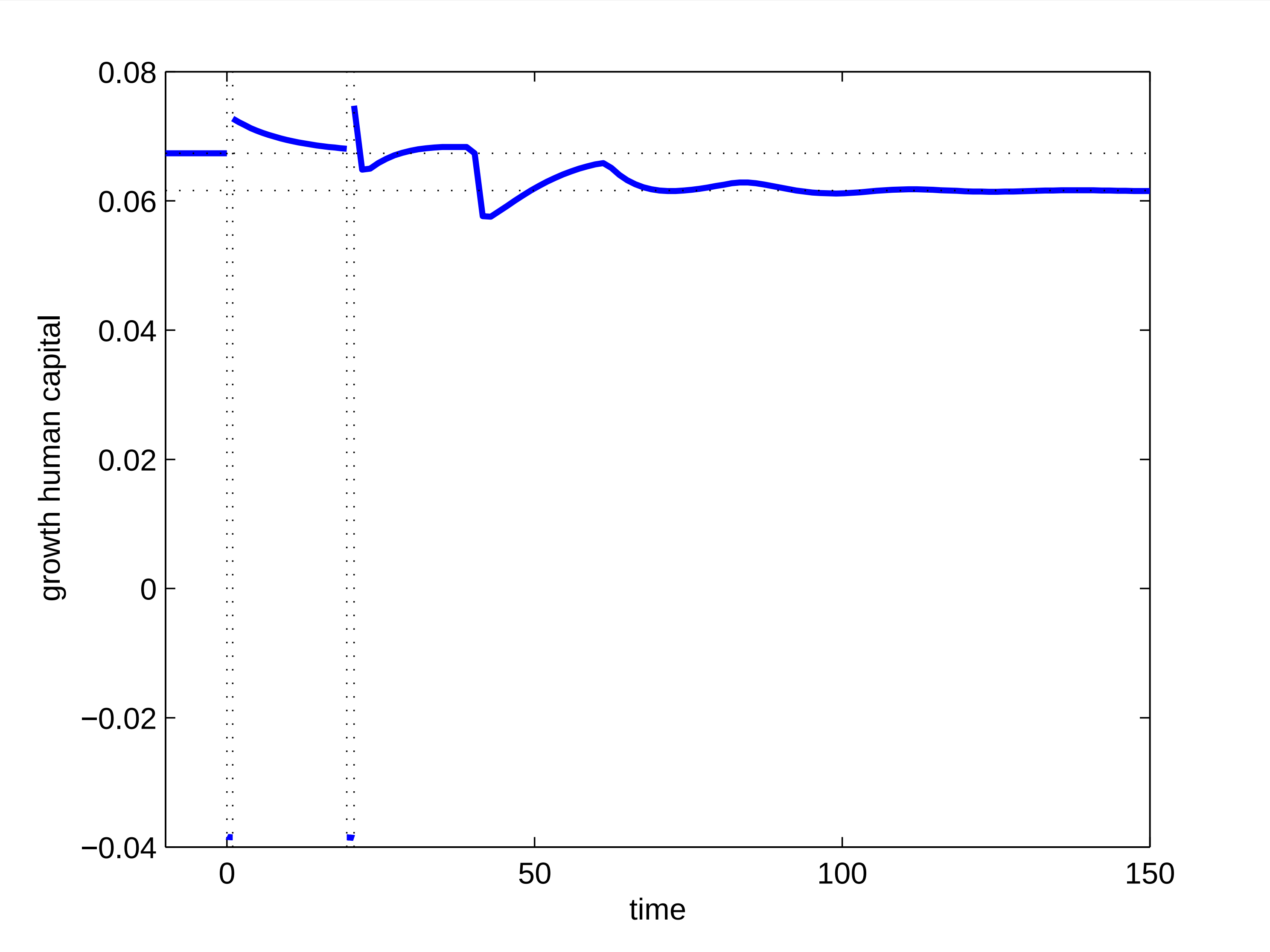


Figure 11: Per Capita Human Capital Growth Under Combined Shock ($\phi = 1$)

5.1. Concrete Plans for 2006-7

- From “almost finished” to “we know what to do, and how to do it”
- Ageing, human capital formation, and economic growth
  - study macroeconomic effects of ageing and educational subsidies in Heijdra-Romp (2006) paper
  - derive more analytical results
  - study feasible model extensions
- Ageing, pensions, and the decision to retire from the labour force
  - introduce pension scheme and endogenous labour force participation in Heijdra-Romp (2005) model
  - study macroeconomic effects of ageing and/or pension reform
  - study feasible model extensions

5.2. Vague Plans for 2006-7 and Beyond

- Increasing order of vagueness
- Ageing and non-traded goods revisited
  - break stranglehold of supply side in Bettendorf-Heijdra (forthcoming 2006)
  - age-dependent demand for non-traded labour-intensive services? (second non-traded good)
  - seek link with endogenous tradeability literature (transport costs; new economic geography). Potential collaboration with Thijs Knaap

- Human capital formation, morbidity, and mortality
  - preliminary discussion with SvW started
  - human capital accumulation with both health risk and mortality risk
  - availability of insurance possibilities
  - endogenous fertility decision
  - HR-approach not feasible: 3-period Diamond-Samuelson model
- Ageing and non-traded production factors