

Foundations of Modern Macroeconomics Second Edition

Chapter 13: Exogenous economic growth (sections 13.1 – 13.4)

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Outline

- 1 Introduction and some stylized facts
- 2 The Solow-Swan model
 - A first view
 - Further properties
- 3 Macroeconomic applications
 - Fiscal policy
 - Ricardian non-equivalence

Aims of this chapter

- Stylized facts of economic growth.
- How well does the Solow-Swan model explain these stylized facts?
- Adding human capital to the Solow-Swan model.
- Growth models based on dynamically optimizing consumers.
- Fiscal policy and Ricardian equivalence in various traditional growth models?

Kaldor's stylized facts of economic growth

- (1) (*) Output per worker shows continuing growth “with no tendency for a falling rate of growth of productivity.”
 - (2) Capital per worker shows continuing growth.
 - (3) The rate of return on capital is steady.
 - (4) (*) The capital-output ratio is steady.
 - (5) (*) Labour and capital receive constant shares of total income.
 - (6) (*) There are wide differences in the rate of productivity growth across countries.
- **Note:** Not all these stylized facts are independent:
 - (SF1) and (SF4) imply (SF2).
 - (SF4) and (SF5) imply (SF3).
 - Hence, the starred facts are fundamental.

Romer's additional stylized facts of economic growth

- (7) In cross section, the mean growth rate shows no variation with the level of per capita income.
- (8) The rate of growth of factor inputs is not large enough to explain the rate of growth of output; that is, growth accounting always finds a residual.
- (9) Growth in the volume of trade is positively correlated with growth in output.
- (10) Population growth rates are negatively correlated with the level of income.
- (11) Both skilled and unskilled workers tend to migrate toward high-income countries.

The neoclassical growth model: Solow-Swan (1)

- *Key notion*: capital and labour are substitutable.
- Technology (neoclassical part of the model):

$$Y(t) = F(K(t), L(t), t)$$

- t is time-dependent shift in technology.
- CRTS:

$$F(\lambda K(t), \lambda L(t), t) = \lambda F(K(t), L(t), t), \quad \text{for } \lambda > 0 \quad (\text{P1})$$

- Saving (“Keynesian” part of the model):

$$S(t) = Y(t) - C(t) = sY(t), \quad 0 < s < 1$$

where s is the constant propensity to save (exogenous).

The neoclassical growth model: Solow-Swan (2)

- Goods market (closed economy, no government consumption):

$$Y(t) = C(t) + I(t)$$

- Gross investment:

$$I(t) = \delta K(t) + \dot{K}(t)$$

where $\delta K(t)$ is replacement investment (δ is the constant depreciation rate), and $\dot{K}(t)$ is net addition to the capital stock.

- Labour supply is exogenous but the population grows as a whole at a constant exponential rate n_L :

$$\frac{\dot{L}(t)}{L(t)} = n_L \quad \Leftrightarrow \quad L(t) = L(0)e^{n_L t}$$

where we can normalize $L(0) = 1$.

Case 1: No technical progress (1)

- Time drops out of technology:

$$Y(t) = F(K(t), L(t))$$

- Positive and diminishing marginal products:

$$F_K, F_L > 0, F_{KK}, F_{LL} < 0, F_{KL} > 0 \quad (\text{P2})$$

- Inada conditions:

$$\lim_{K \rightarrow 0} F_K = \lim_{L \rightarrow 0} F_L = +\infty, \quad \lim_{K \rightarrow \infty} F_K = \lim_{L \rightarrow \infty} F_L = 0 \quad (\text{P3})$$

Case 1: No technical progress (2)

- Solve model by writing in per capita form, i.e. $y(t) \equiv Y(t)/L(t)$, $k(t) \equiv K(t)/L(t)$, etcetera. Here are some steps:
 - *Step 1:* Savings equal investment:

$$\begin{aligned} S(t) &= I(t) \\ sY(t) &= \delta K(t) + \dot{K}(t) \\ sF(K(t), L(t)) &= \delta K(t) + \dot{K}(t) \quad \Rightarrow \\ s \frac{F(K(t), L(t))}{L(t)} &= \delta \frac{K(t)}{L(t)} + \frac{\dot{K}(t)}{L(t)} \end{aligned} \quad (S1)$$

- *Step 2:* Since $k(t) \equiv K(t)/L(t)$ it follows that:

$$\begin{aligned} \dot{k}(t) &\equiv \frac{\dot{K}(t)}{L(t)} - \frac{K(t)}{L(t)} \frac{\dot{L}(t)}{L(t)} \quad \Rightarrow \\ \frac{\dot{K}(t)}{L(t)} &= \dot{k}(t) + k(t)n_L \end{aligned} \quad (S2)$$

Case 1: No technical progress (3)

- Continued.

- Step 3:* Since $F(K(t), L(t))$ features CRTS we have:

$$\begin{aligned} Y(t) &= F(K(t), L(t)) = L(t)F\left(\frac{K(t)}{L(t)}, 1\right) && \Rightarrow \\ y(t) &= f(k(t)) && \text{(S3)} \end{aligned}$$

(e.g. Cobb-Douglas $Y = K^\alpha L^{1-\alpha}$ implies $y = k^\alpha$).

- By substituting (S2) and (S3) into (S1) we obtain the *fundamental differential equation* (FDE) for $k(t)$:

$$\dot{k}(t) = \underbrace{sf(k(t))}_{(a)} - \underbrace{(\delta + n_L)k(t)}_{(b)}$$

- (a) Per capita saving.
- (b) To maintain constant $k(t)$ one must replace and expand the *level* of the capital stock.

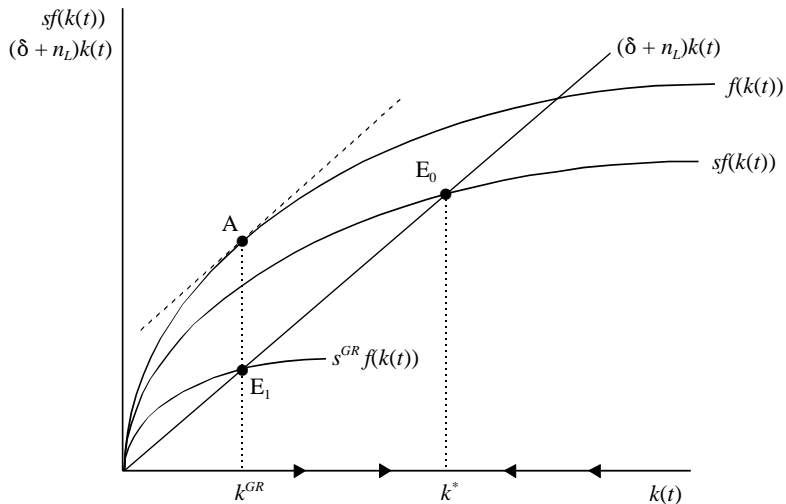
Case 1: No technical progress (4)

- In **Figure 13.1** we illustrate the FDE. The Inada conditions imply:
 - $f(k(t))$ vertical for $k(t) \rightarrow 0$.
 - $f(k(t))$ horizontal for $k(t) \rightarrow \infty$.
 - Unique steady state at E_0 .
 - Stable equilibrium.
- In the balanced growth path (BGP):

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{I}(t)}{I(t)} = \frac{\dot{S}(t)}{S(t)} = \frac{\dot{L}(t)}{L(t)} = n_L$$

Hence the name *exogenous growth*.

Figure 13.1: The Solow-Swan model



Case 2: With technical progress (1)

- Focus on *disembodied* technological progress.

$$Y(t) = F(\underbrace{Z_K(t)K(t)}_{(a)}, \underbrace{Z_L(t)L(t)}_{(b)})$$

- (a) “Effective” capital input.
- (b) “Effective” labour input.
- Three types of progress:
 - Harrod neutral: $Z_K(t) \equiv 1$.
 - Hicks neutral: $Z_K(t) \equiv Z_L(t)$.
 - Solow neutral: $Z_L(t) \equiv 1$.
- Cases are indistinguishable for Cobb-Douglas.

Case 2: With technical progress (2)

- For non-CD case progress must be Harrod neutral to have a steady state with constant growth rate (otherwise one of the shares goes to zero, contra (SF5)).
- Define $N(t) \equiv Z(t)L(t)$ and assume that technical progress occurs at a constant exponential rate:

$$\frac{\dot{Z}(t)}{Z(t)} = n_Z, \quad Z(t) = Z(0)e^{n_Z t}$$

so that the *effective* labour force grows at a constant exponential rate $n_L + n_Z$.

Case 2: With technical progress (3)

- Measuring output and capital per unit of effective labour, i.e. $y(t) \equiv Y(t)/N(t)$ and $k(t) \equiv K(t)/N(t)$, the FDE for $k(t)$ is obtained:

$$\dot{k}(t) = sf(k(t)) - (\delta + n_L + n_Z)k(t)$$

- In the BGP we have:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{I}(t)}{I(t)} = \frac{\dot{S}(t)}{S(t)} = \frac{\dot{N}(t)}{N(t)} = \frac{\dot{L}(t)}{L(t)} + \frac{\dot{Z}(t)}{Z(t)} = n_L + n_Z$$

- Exogenous growth rate now equals $n_L + n_Z$.

Further properties of the Solow-Swan model

- (A) The golden rule of capital accumulation: dynamic inefficiency possible.
- (B) Transitional dynamics: conditional growth convergence seems to hold.
- (C) Speed of adjustment: too fast. Model can be rescued.
- (D) Rescuing the Solow-Swan model.

(A) The golden rule (1)

- Golden rule: maximum steady-state consumption per capita.
- For each savings rate there is a unique *steady-state* capital-labour ratio (assume $n_Z = 0$ for simplicity):

$$k^* = k^*(s)$$

with $dk^*/ds = y^*/[\delta + n - sf'(k^*)] > 0$. The higher is s , the larger is k^* .

- Since $C(t) \equiv (1 - s)Y(t)$ we have for per capita consumption:

$$\begin{aligned}c(s) &= (1 - s)f(k^*(s)) \\ &= f(k^*(s)) - (\delta + n_L)k^*(s)\end{aligned}$$

(A) The golden rule (2)

- The golden-rule savings rate is such that $c(s)$ is maximized:

$$\frac{dc(s)}{ds} = \left[f'(k^*(s)) - (\delta + n_L) \right] \frac{dk^*(s)}{ds} = 0$$

- Since $\frac{dk^*(s)}{ds} > 0$ we get that:

$$f'(k^*(s^{GR})) = \delta + n_L \quad (S4)$$

One interpretation: the produced asset (the physical capital stock) yields an own-rate of return equal to $f' - \delta$, whereas the non-produced primary good (labour) can be interpreted as yielding an own-rate of return n_L . Intuitively, the efficient outcome occurs if the rate of return on the two assets are equalized.

- Recall that in the steady state::

$$s^{GR} f(k^*(s^{GR})) = (\delta + n_L) k^*(s^{GR}) \quad (S5)$$

(A) The golden rule (3)

- By using (S4) we can rewrite (S5) in terms of a national income share:

$$s^{GR} = \frac{(\delta + n_L)k^*(s^{GR})}{f(k^*(s^{GR}))} = \frac{k^*(s^{GR})f'(k^*(s^{GR}))}{f(k^*(s^{GR}))}$$

(e.g. for Cobb-Douglas $f(.) = k(t)^\alpha$, α represents the capital income share so that the golden rule savings rate equals $s^{GR} = \alpha$.)

- In **Figures 13.2-13.3** we illustrate the possibility of *dynamic inefficiency* (oversaving: If $s_0 > s^{GR}$ then a Pareto-improving transition from E_0 to E_1 is possible).

Figure 13.2: Per capita consumption and the savings rate

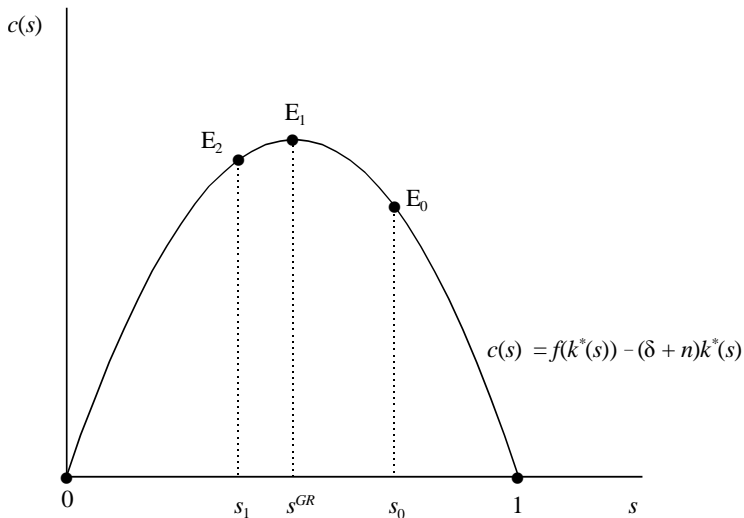
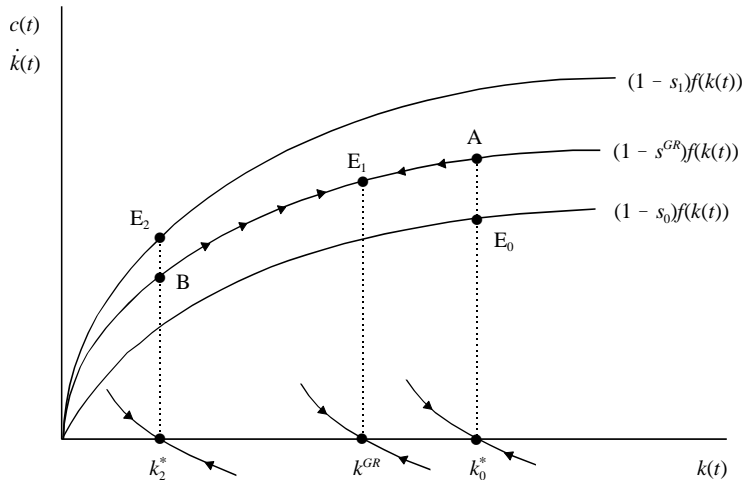


Figure 13.3: Per capita consumption during transition to its golden rule level



(B) Transitional dynamics towards the steady state (1)

- Defining the growth rate of $k(t)$ as $\gamma_k(t) \equiv \dot{k}(t)/k(t)$, we derive from the FDE:

$$\gamma_k(t) \equiv s \frac{f(k(t))}{k(t)} - (\delta + n)$$

where $n \equiv n_L + n_Z$.

- In **Figure 13.4** this growth rate is represented by the vertical difference between the two lines. (The Inada conditions ensure that $\lim_{k \rightarrow 0} sf(k)/k = \infty$ and $\lim_{k \rightarrow \infty} sf(k)/k = 0$.)
- Countries with little capital (in efficiency units) grow faster than countries with a lot of capital. In other words, poor and rich countries should converge! (Link between $\gamma_k(t)$ and $\gamma_y(t)$ is easily established, especially for the CD case.)

(B) Transitional dynamics towards the steady state (2)

- This suggests that there is a simple empirical test of the Solow-Swan model which is based on the convergence property of output in a cross section of many different countries.
 - *Absolute convergence hypothesis* (ACH) then suggests that poor countries should grow faster than rich countries. Barro and Sala-i-Martin (1995, p. 27) show the results of regressing of $\gamma_y(t)$ on $\log y(t)$ for a sample of 118 countries. The results are dismal: instead of finding a negative effect as predicted by the ACH, they find a slight positive effect. Absolute convergence does not seem to hold and (Romer's) stylized fact (SF7) is verified by the data.
 - More refined test: *Conditional convergence hypothesis* (CCH) according to which *similar* countries should converge. In **Figure 13.5** we show case where poor country is closer to its steady state than the rich country is to its own steady state. Hence, rich country grows at a faster rate. According to Barro and Sala-i-Martin the data confirm the CCH.

Figure 13.4: Growth convergence

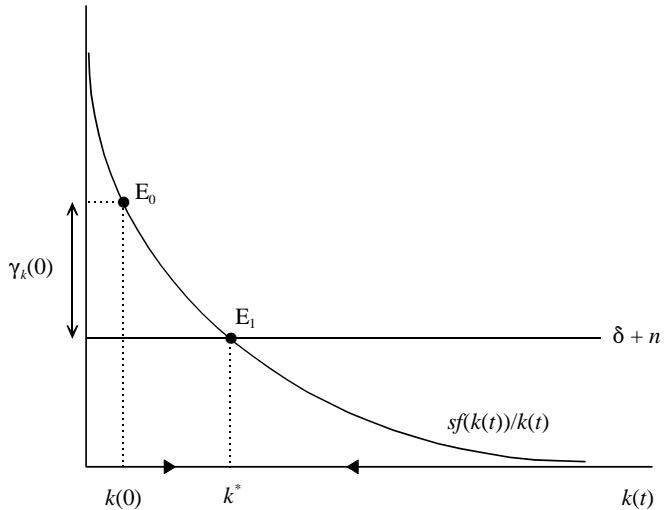
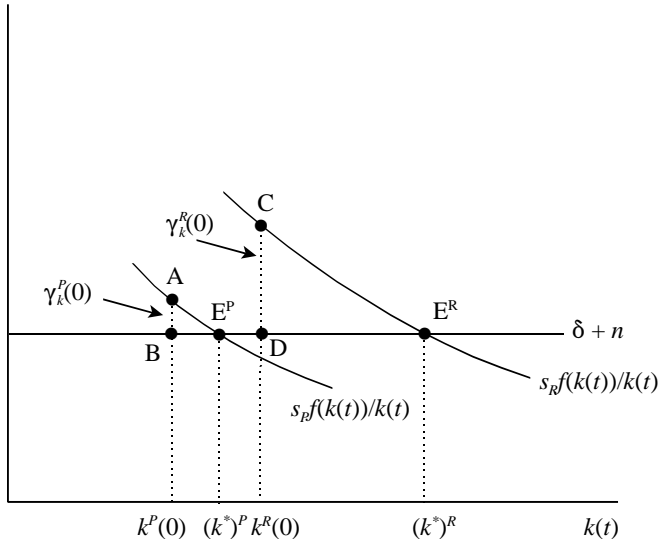


Figure 13.5: Conditional growth convergence



(C) Speed of adjustment (1)

- How *fast* is the convergence in a Solow-Swan economy?
- Focus on the Cobb-Douglas case for which $f(\cdot) = k(t)^\alpha$, and the FDE is:

$$\dot{k}(t) = sk(t)^\alpha - (\delta + n)k(t) \quad (\text{S6})$$

- First-order Taylor approximation around k^* :

$$\begin{aligned} sk(t)^\alpha &\approx s \cdot (k^*)^\alpha + s\alpha \cdot (k^*)^{\alpha-1} \cdot [k(t) - k^*] \\ &= (\delta + n) \cdot k^* + \alpha(\delta + n) \cdot [k(t) - k^*] \end{aligned} \quad (\text{S7})$$

- Using (S7) in (S6) we obtain the *linearized* differential equation for $k(t)$:

$$\dot{k}(t) = -\beta \cdot [k(t) - k^*], \quad \beta \equiv (1 - \alpha)(\delta + n) > 0 \quad (\text{S8})$$

(C) Speed of adjustment (2)

- Solving (S8) with initial condition $k(0)$, we find:

$$k(t) = k^* + [k(0) - k^*] \cdot e^{-\beta t} \quad (\text{S9})$$

where β measures the speed of convergence / adjustment.

- Speed of adjustment in the growth rate of output for the Cobb-Douglas case. Divide both sides of (S8) by $k(t)$, note that $\dot{k}(t)/k(t) = d \ln k(t)/dt$, $d \ln y(t)/dt = \alpha d \ln k(t)/dt$, and use the approximation $\ln(k(t)/k^*) = 1 - k^*/k(t)$:

$$\frac{d \ln y(t)}{dt} = -\beta \cdot [\ln y(t) - \ln y^*] \quad (\text{S10})$$

(C) Speed of adjustment (3)

- Solving (S10) with initial condition $y(0)$, we find:

$$\ln y(t) = \ln y^* + [\ln y(0) - \ln y^*] \cdot e^{-\beta t} \quad (\text{S11})$$

- $\beta \equiv (1 - \alpha)(\delta + n)$ is the **common** (approximate) adjustment speed for $k(t)$, $\ln k(t)$, $y(t)$, and $\ln y(t)$ toward their respective steady-states
- Interpretation of β : $\zeta \times 100\%$ of the difference between, say, $y(t)$ and y^* is eliminated after a time interval of t_ζ :

$$t_\zeta \equiv -\frac{1}{\beta} \cdot \ln(1 - \zeta)$$

For example, the **half-life** of the convergence ($\zeta = \frac{1}{2}$) equals $t_{1/2} = \ln 2 / \beta = 0.693 / \beta$.

(C) Speed of adjustment (4)

- Back-of-the-envelope computations: $n_L = 0.01$ (per annum), $n_Z = 0.02$, $\delta = 0.05$, and $\alpha = 1/3$ yield the value of $\beta = 0.0533$ (5.33 percent per annum) and an estimated half-life of $t_{1/2} = 13$ years. Fast transition.
- Estimate is way too high to accord with empirical evidence: actual β is in the range of 2 percent per annum (instead of 5.33 percent).
- Problem with the Solow-Swan model. Solutions:
 - Assume high capital share (for $\alpha = \frac{3}{4}$ we get $\beta = 0.02$)!
 - Assume a broad measure of capital to include human as well as physical capital (Mankiw, Romer, and Weil (1992)).

(D) Rescuing the Solow-Swan model (1)

- *Key idea*: add human capital to the model.
- Technology:

$$Y(t) = K(t)^{\alpha_K} H(t)^{\alpha_H} [Z(t)L(t)]^{1-\alpha_K-\alpha_H}, \quad 0 < \alpha_K + \alpha_H < 1$$

where $H(t)$ is the stock of human capital and α_K and α_H are the efficiency parameters of the two types of capital ($0 < \alpha_K, \alpha_H < 1$).

- In close accordance with the Solow-Swan model, productivity and population growth are both exponential ($\dot{Z}(t)/Z(t) = n_Z$ and $\dot{L}(t)/L(t) = n_L$).

(D) Rescuing the Solow-Swan model (2)

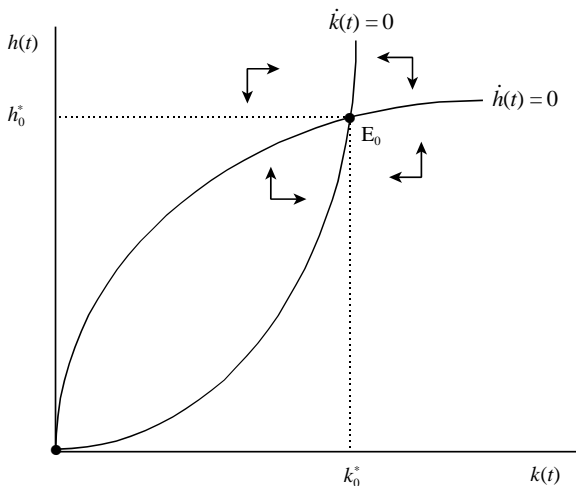
- The accumulation equations for the two types of capital can be written in effective labour units as:

$$\begin{aligned}\dot{k}(t) &= s_K y(t) - (\delta_K + n)k(t) \\ \dot{h}(t) &= s_H y(t) - (\delta_H + n)h(t)\end{aligned}$$

where $h(t) \equiv H(t)/[Z(t)L(t)]$, $n \equiv n_Z + n_L$, and s_K and s_H represent the propensities to accumulate physical and human capital, respectively. The depreciation rates are δ_K and δ_H .

- Phase diagram in **Figure 13.6**.

Figure 13.6: Augmented Solow-Swan model



(D) Rescuing the Solow-Swan model (3)

- Since there are decreasing returns to the two types of capital in combination ($\alpha_K + \alpha_H < 1$) the model possesses a steady state for which $\dot{k}(t) = \dot{h}(t) = 0$, $k(t) = k^*$, and $h(t) = h^*$:

$$k^* = \left(\left(\frac{s_K}{\delta_K + n} \right)^{1-\alpha_H} \left(\frac{s_H}{\delta_H + n} \right)^{\alpha_H} \right)^{1/(1-\alpha_K-\alpha_H)}$$

$$h^* = \left(\left(\frac{s_K}{\delta_K + n} \right)^{\alpha_K} \left(\frac{s_H}{\delta_H + n} \right)^{1-\alpha_K} \right)^{1/(1-\alpha_K-\alpha_H)}$$

- By substituting k^* and h^* into the (logarithm of the) production function we obtain an estimable expression for per capita output along the balanced growth path:

$$\begin{aligned} \ln \left(\frac{Y(t)}{L(t)} \right)^* &= \ln Z(0) + n_Z t - \frac{\alpha_K \ln(\delta_K + n_Z + n_L) + \alpha_H \ln(\delta_H + n_Z + n_L)}{1 - \alpha_K - \alpha_H} \\ &\quad + \frac{\alpha_K}{1 - \alpha_K - \alpha_H} \ln s_K + \frac{\alpha_H}{1 - \alpha_K - \alpha_H} \ln s_H \end{aligned}$$

(D) Rescuing the Solow-Swan model (4)

- Continued.
 - Mankiw et al. (1992, p. 417) suggest approximate guesses for $\alpha_K = \frac{1}{3}$ and α_H between $\frac{1}{3}$ and $\frac{4}{9}$.
 - The extended Solow-Swan model is much better equipped to explain large cross-country income differences for relatively small differences between savings rates (s_K and s_H) and population growth rates (n). (Multiplier factor is $\frac{1}{1-\alpha_K-\alpha_H}$ instead of $\frac{1}{1-\alpha_K}$.)
 - The inclusion of a human capital variable works pretty well empirically; the estimated coefficient for α_H is highly significant and lies between 0.28 and 0.37.

(D) Rescuing the Solow-Swan model (5)

- Continued.
 - The convergence property of the augmented Solow-Swan model is also much better. For the case with $\delta_K = \delta_H = \delta$, the convergence speed is defined as $\beta \equiv (1 - \alpha_K - \alpha_H)(n + \delta)$ which can be made in accordance with the observed empirical estimate of $\hat{\beta} = 0.02$ without too much trouble.
- Hence, by this very simple and intuitively plausible adjustment the Solow-Swan model can be salvaged from the dustbin of history. The speed of convergence it implies can be made to fit the real world.

Macroeconomic applications of the Solow-Swan

- (A) Fiscal policy: long-run crowding out of private by public consumption?
- Balanced-budget: without government debt
 - Deficit financing: with government debt
- (B) Ricardian non-equivalence: government debt is not neutral.

(A) Fiscal policy in the Solow-Swan model (1)

- The government consumes $G(t)$ units of output so that aggregate demand in the goods market is:

$$Y(t) = C(t) + I(t) + G(t)$$

- Aggregate saving is proportional to after-tax income:

$$S(t) = s [Y(t) - T(t)]$$

where $T(t)$ is the lump-sum tax.

- Since $S(t) \equiv Y(t) - C(t) - I(t)$ any primary government deficit must be compensated for by an excess of private saving over investment, i.e.

$$G(t) - T(t) = S(t) - I(t)$$

(A) Fiscal policy in the Solow-Swan model (2)

- The government budget identity is given by:

$$\dot{B}(t) = r(t)B(t) + G(t) - T(t)$$

where $B(t)$ is public debt and $r(t)$ is the real interest rate.

- Under the competitive conditions the interest rate equals the net marginal productivity of capital (see also below):

$$r(t) = f'(k(t)) - \delta$$

- By writing all variables in terms of effective labour units, the model can be condensed to the following two equations:

$$\begin{aligned}\dot{k}(t) &= f(k(t)) - (\delta + n)k(t) - c(t) - g(t) \\ &= sf(k(t)) - (\delta + n)k(t) + (1 - s)\tau(t) - g(t),\end{aligned}\quad (\text{S12})$$

$$\dot{b}(t) = \left[f'(k(t)) - \delta - n \right] b(t) + g(t) - \tau(t), \quad (\text{S13})$$

with: $\tau(t) \equiv T(t)/N(t)$, $g(t) \equiv G(t)/N(t)$, $b(t) \equiv B(t)/N(t)$.

(A) Fiscal policy in the Solow-Swan model (3)

- Under pure tax financing we have $\dot{b}(t) = b(t) = 0$ so that government budget identity (b) reduces to $g(t) \equiv \tau(t)$. The FDE becomes:

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) - sg(t)$$

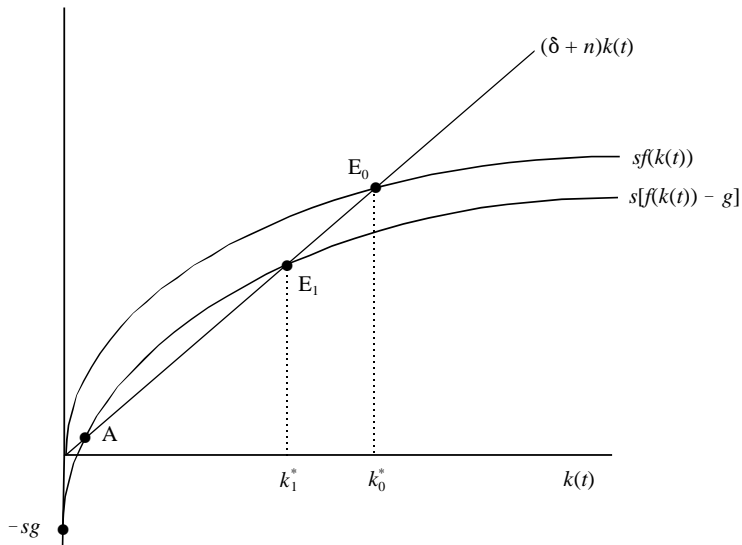
- In **Figure 13.7** we show stability and illustrate the effects of an increase in government consumption.
- Qualitative effects:

$$\frac{dy(\infty)}{dg} = \frac{f' dk(\infty)}{dg} = \frac{sf'}{sf' - (\delta + n)} < 0$$

$$\frac{dc(\infty)}{dg} = (1 - s) \left[\frac{dy(\infty)}{dg} - 1 \right] = \frac{(1 - s)(\delta + n)}{sf' - (\delta + n)} < 0$$

Capital and consumption are both crowded out in the long run! Model is Classical in the long run (despite its Keynesian consumption function).

Figure 13.7: Fiscal policy in the Solow-Swan model



(B) Ricardian non-equivalence in the Solow-Swan model (1)

- If the economy is dynamically efficient we have:

$$r(t) \equiv f'(k(t)) - \delta > n$$

- This means that debt process in (S13) is inherently unstable (explosive); an economically uninteresting phenomenon.
- *Buiter rule* ensures that debt is stabilized:

$$\tau(t) = \tau_0 + \xi b(t), \quad \xi > r - n$$

- The system of fundamental differential equations becomes:

$$\dot{b}(t) = \left[f'(k(t)) - \delta - n - \xi \right] b(t) + g(t) - \tau_0$$

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) + (1 - s) [\tau_0 + \xi b(t)] - g(t)$$

(B) Ricardian non-equivalence in the Solow-Swan model (2)

- The model can be analyzed graphically with the aid of **Figure 13.8**.
- The $\dot{k} = 0$ line:
 - Upward sloping in (k, b) space.
 - Points above (below) the line are associated with positive (negative) net investment, i.e. $\dot{k} > 0$ (< 0).
- The $\dot{b} = 0$ line:
 - Downward sloping in (k, b) space.
 - For points above (below) the $\dot{b} = 0$ line there is a government surplus (deficit) so that debt falls (rises).
- Equilibrium at E_0 is inherently stable.

(B) Ricardian non-equivalence in the Solow-Swan model (3)

- *Ricardian experiment*: postponement of taxation.
 - In the model this amounts to a reduction in τ_0 . This creates a primary deficit at impact ($g(t) > \tau_0$) so that government debt starts to rise.
 - In terms of **Figure 13.8**, both the $\dot{k} = 0$ line and the $\dot{b} = 0$ line shift up, the latter by more than the former.
 - In the long run, government debt, the capital stock, and output (all measured in efficiency units of labour) rise as a result of the tax cut.

$$\frac{dy(\infty)}{d\tau_0} = \frac{f' dk(\infty)}{d\tau_0} = -\frac{(1-s)(r-n)f'}{|\Delta|} < 0$$

$$\frac{db(\infty)}{d\tau_0} = \frac{sf' - (\delta+n) + (1-s)bf''}{|\Delta|} < 0$$

- Ricardian equivalence does not hold in the Solow-Swan model. A temporary tax cut boosts consumption, depresses investment and thus has real effects.

Figure 13.8: Ricardian non-equivalence in the Solow-Swan model

