Foundations of Modern Macroeconomics
Second Edition
Chapter 8: Search in the labour market

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Outline

Aims of this lecture

- How can we explain unemployment *duration*?
- What policies can be used to reduce equilibrium unemployment?
- Can the search model explain the persistence in the unemployment rate?

Searching and matching (1)

- Matching function:

$$XN = G(\underset{+}{UN}, \underset{+}{VN}),$$

  - X is the matching rate.
  - N is the number of workers.
  - U is the unemployment rate.
  - V is the vacancy rate.
  - $G(.,.)$ features CRTS (i.e. $G(UN, VN) = NG(U, V) = NVG(U/V, 1)$. Example: Cobb-Douglas matching function: $XN = (UN)^{\alpha}(VN)^{1-\alpha}$.
  - Further properties: $G_U, G_V > 0$; $G_{UU}, G_{VV} < 0$; $G_{UU}G_{VV} - G_{UV}^2 > 0$.

Searching and matching (2)

- Instantaneous probability of a vacancy being filled:

$$\begin{aligned} q &\equiv \frac{\text{number of matches}}{\text{number of vacancies}} = \frac{G(UN, VN)}{VN} \\ &= \frac{VN \cdot G(UN/VN, 1)}{VN} = G(U/V, 1) \equiv q(\underset{-}{\theta}), \end{aligned}$$

where θ is the indicator for labour market pressure:

$$\theta \equiv \frac{V}{U}$$

  - If θ is high then there are relatively many vacancies so firms with a vacancy find it hard to get a match with an unemployed job seeker (q is low).
  - If θ is low then there are relatively few vacancies so firms with a vacancy find it easy to get a match with an unemployed job seeker (q is high).

Searching and matching (3)

- Continued.
  - For later use: the elasticity of the $q(\theta)$ function:

$$\eta(\theta) \equiv -\frac{\theta}{q}\frac{dq}{d\theta} = \frac{G_U}{\theta q} \Rightarrow \quad 0 < \eta(\theta) < 1,$$

- Instantaneous prob. of an unemployed job seeker finding a job:

$$\begin{aligned} f &\equiv \frac{\text{number of matches}}{\text{number of unemployed}} = \frac{G(UN, VN)}{UN} \\ &= \frac{VN \cdot G(UN/VN, 1)}{UN} = \theta q(\theta) \equiv f(\underset{+}{\theta}), \end{aligned}$$

  - If θ is high then there are relatively few unemployed workers so unemployed job seekers find it easy to locate a firm with a vacancy (f is high).
  - If θ is low then there are relatively many unemployed workers so unemployed job seekers find it hard to locate a firm with a vacancy (f is low).

Searching and matching (4)

- Continued.
  - For later use: the elasticity of the $f(\theta)$ function:

$$\frac{\theta}{f}\frac{df}{d\theta} = \left[q(\theta) + \theta\frac{dq}{d\theta}\right]\frac{\theta}{\theta q(\theta)} = 1 + \frac{\theta}{q}\frac{dq}{d\theta} = 1 - \eta(\theta) > 0$$

- Note the intimate link between the probabilities facing the two searching parties, i.e. firms with a vacancy and unemployed job seekers. [Two sides of the same coin.]
- We now already have some duration definitions:
  - Expected duration of a job vacancy:

$$\frac{1}{q(\theta)}$$

  - Expected duration of unemployment spell:

$$\frac{1}{f(\theta)}$$

Searching and matching (5)

- Inflow/outflow equilibrium

$$\underbrace{s(1-U)Ndt}_{(a)} = \underbrace{\theta q(\theta)UNdt}_{(b)}, \tag{S1}$$

where s is the (exogenous) job destruction rate (due to idiosyncratic match-productivity shocks.

(a) (expected) flow into unemployment.
(b) (expected) flow out of unemployment.
NB 1 Note: Large numbers, so frequencies and probabilities coincide.
NB 2 Equation (S1) implies equilibrium unemployment rate:

$$U = \frac{s}{s+\theta q(\theta)} = \frac{s}{s+f(\theta)}$$

Remainder of the model solved as follows

(A) Firm behaviour.
(B) Worker behaviour.
(C) Wage setting.
(D) Market equilibrium.

(A) Firm behaviour (1)

- Analyze single-job firms (risk-neutral owner).
- Focus on intuitive "derivation".
- Firms with a vacancy have the following arbitrage equation:

$$\underbrace{rJ_V}_{(a)} = \underbrace{-\gamma_0 + q(\theta)\left[J_O - J_V\right]}_{(b)}$$

  - J_V is the value of a (firm with a) vacancy; r is the interest rate
  - γ_0 is the search cost of the firm with a vacancy
  - J_O is the value of (a firm with) an occupied job

(a) capital cost of the asset.

(b) return on the asset: "dividend" [search costs] plus expected capital gain [finding a worker, upgrading from vacancy to a filled job].

(A) Firm behaviour (2)

- Assumption: free entry of firms with a vacancy:

$$\begin{aligned} J_V &= 0 \;\Rightarrow\; 0 = -\gamma_0 + q(\theta) J_O \;\Rightarrow \\ J_O &= \frac{\gamma_0}{q(\theta)} \end{aligned}$$

  Hence, the value of a filled job equals the expected cost of creating it [i.e. the cost of filling a vacancy].

- Firms with an occupied job have the following arbitrage equation:

$$\underbrace{rJ_O}_{(a)} = \underbrace{[F(K,1) - (r+\delta)K - w] - sJ_O}_{(b)} \qquad \text{(S2)}$$

  - $F(K,1)$ is the output of the single-job firm (note $L = 1$ substituted).
  - Firm rents capital at rental rate $r + \delta$.
  - Firm hires labour at wage rate w [to be determined below].

(A) Firm behaviour (3)

- Continued.
  - (a) Capital cost of the asset.
  - (b) Return on the asset, consisting of the "dividend" [profit, i.e. output left over after capital and labour have been paid] plus the expected capital gain [experiencing a shock by which the match is destroyed: downgrading from filled job to vacancy].
  - → The firm hires capital such that J_O is maximized:

$$\begin{aligned} \max_{\{K\}} (r+s) J_O &\equiv F(K,1) - (r+\delta)K - w \quad \Rightarrow \\ F_K(K,1) &= r + \delta \qquad \text{(S3)} \end{aligned}$$

(A) Firm behaviour (4)

- Since $J_O = \gamma_0 / q(\theta)$ and $F(K,1) = F_K K + F_L$ we can combine (S2) and (S3):

$$\frac{(r+s)\gamma_0}{q(\theta)} = F(K,1) - F_K(K,1)K - w \quad \Rightarrow$$

$$\underbrace{\frac{F_L(K,1) - w}{r+s}}_{(a)} = \underbrace{\frac{\gamma_0}{q(\theta)}}_{(b)} \qquad \text{(ZP condition)}$$

(a) The value of an occupied job, equalling the present value of rents (accruing to the firm during the job's existence) using the risk-of-job-destruction-adjusted discount rate, $r+s$, to discount future rents.

(b) Expected search costs.

NB Since firm search costs are positive ($\gamma_0 > 0$) it follows that $w < F_L$ (workers do not get their marginal product!).

(B) Worker behaviour (1)

- Risk-neutral / infinitely-lived worker.
- Cares only for the present value of present and future income stream
- Receives wage w when employed and "unemployment benefit" z when unemployed.
- Unemployed worker's arbitrage equation is:

$$\underbrace{rY_U}_{(a)} = \underbrace{z + \theta q(\theta) \left[Y_E - Y_U \right]}_{(b)} \tag{S4}$$

  - Y_U is the human wealth of the unemployed worker (who is looking for a job).
  - Y_E is the human wealth of the employed worker.

(a) Capital cost of the asset.

(b) Return on the asset: "dividend" [unemployment benefits] plus expected capital gain [finding a job and upgrading from unemployment to being employed].

(B) Worker behaviour (2)

- Employed worker's arbitrage equation is:

$$\underbrace{rY_E}_{(a)} = \underbrace{w - s\left[Y_E - Y_U\right]}_{(b)} \tag{S5}$$

  - Capital cost of the asset.
  - Return on the asset, consisting of the "dividend" [the wage] plus the expected capital gain [losing one's job due to a shock and downgrading from being employed to being unemployed].
- Combining (S4) and (S5) yields:

$$rY_U = \frac{(r+s)z + \theta q(\theta) w}{r + s + \theta q(\theta)},$$

$$rY_E = \frac{sz + \left[r + \theta q(\theta)\right] w}{r + s + \theta q(\theta)} = \frac{r(w-z)}{r + s + \theta q(\theta)} + rY_U$$

(C) Wage setting (1)

- Generalized wage bargaining over the wage between the firm and the worker.
- Expected gain from striking a deal.
  - To the firm:

$$rJ_O^i = F(K_i, 1) - (r+\delta)K_i - w_i - sJ_O^i \quad \Rightarrow$$
$$J_O^i = \frac{F_L(K_i, 1) - w_i}{r+s}$$

  - To the worker:

$$r\left(Y_E^i - Y_U\right) = w_i - s\left[Y_E^i - Y_U\right] - rY_U$$

(C) Wage setting (2)

- Bargaining is over a wage, w_i, which maximizes Ω:

$$\max_{\{w_i\}} \Omega \equiv \beta \ln \left[Y_E^i - Y_U \right] + (1-\beta) \ln \left[J_O^i - \underbrace{J_V}_{=0} \right]$$

  where $0 < \beta < 1$ represents the (relative) bargaining power of the worker and Y_U and $J_V = 0$ are the threat points of, respectively the worker and the firm.

- Maximization yields the *rent sharing rule*:

$$Y_E^i - Y_U = \frac{\beta}{1-\beta} \left[J_O^i - J_V \right] \qquad \text{(S6)}$$

(C) Wage setting (3)

- There are two ways to turn the rent sharing rule into a wage equation [details in the book].
  1) After some substitutions we get:

$$w_i = (1-\beta) r Y_U + \beta F_L(K_i, 1)$$

  - Worker gets a weighted average of the reservation wage (rY_U) and the marginal product of labour (F_L).

(C) Wage setting (4)

- Continued.
  2) In symmetric situation we have $K_i = K$ and $w_i = w$ for all firm/worker pairs:

$$w = (1-\beta)z + \beta \left[F_L(K,1) + \theta\gamma_0 \right] \qquad \text{(WS curve)}$$

  - Worker gets a weighted average of the unemployment benefit (z) and the match surplus ($F_L + \gamma_0\theta$).
  - The match surplus consists of the marginal product of labour plus the expected search costs that are saved if the deal is struck [$\theta \equiv V/U$ so that $\gamma_0\theta \equiv \gamma_0 V/U$ represents the average hiring costs per unemployed worker].

(D) Market equilibrium

- Summary of the model

$$F_K(K,1) = r+\delta \tag{T1}$$

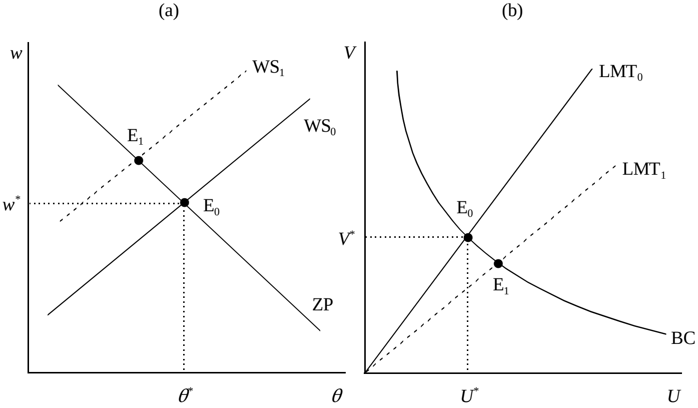
$$\frac{\gamma_0}{q(\theta)} = \frac{F_L\left[K(r+\delta),1\right]-w}{r+s} \tag{T2}$$

$$w = (1-\beta)z + \beta\left[F_L\left(K(r+\delta),1\right)+\theta\gamma_0\right] \tag{T3}$$

$$U = \frac{s}{s+\theta q(\theta)} \tag{T4}$$

- Endogenous: K, w, θ, and U. Exogenous: r, s, γ_0, and δ.
- Model is recursive and can thus be solved sequentially:
  - (T1) yields K^* as a function of $r+\delta$ $[K^* = F_K^{-1}(r+\delta)]$.
  - (T2)-(T3) with $K = K^*$ inserted only depend on (and determine) w^* and θ^*.
  - Once θ^* is known equation (T4) determines U^*.

Figure 8.1: Search equilibrium in the labour market



Graphical analysis (1)

- The model can be represented graphically in **Figure 8.1**.
- ZP curve: [equation (T2)] supply of vacancies under free entry/exit of firms.
  - Slopes downwards in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{ZP} = \frac{(r+s)\gamma_0}{q(\theta)^2} q'(\theta) < 0.$$

    Intuition: $w \downarrow$ increases the value of an occupied job [raises the right-hand side of (T2)]. To restore the zero-profit equilibrium the expected search cost for firms (the left-hand side of (T2) must also increase, i.e. $q(\theta) \downarrow$ and $\theta \uparrow$.
  - Shifts up as $\gamma_0 \downarrow$ or as $s \downarrow$.

Graphical analysis (2)

- WS curve: [equation (T3)] wage setting curve.
  - Upward sloping in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{WS} = \beta\gamma_0 > 0$$

    Intuition: the worker receives part of the search costs that are foregone when he strikes a deal with a firm with a vacancy.
  - Shifts up as $z \uparrow$ or $\gamma_0 \uparrow$
- In panel (a) the intersection of ZP and WS yields the equilibrium (w^*, θ^*) combination. This is the ray from the origin in panel (b).

Graphical analysis (3)

- The Beveridge curve (BC) is given by equation (T4). It can be linearized in (V, U) space as follows:

$$\tilde{V} = \frac{1}{1-\eta}\tilde{s} - \frac{s+f\eta}{f\,(1-\eta)}\tilde{U}$$

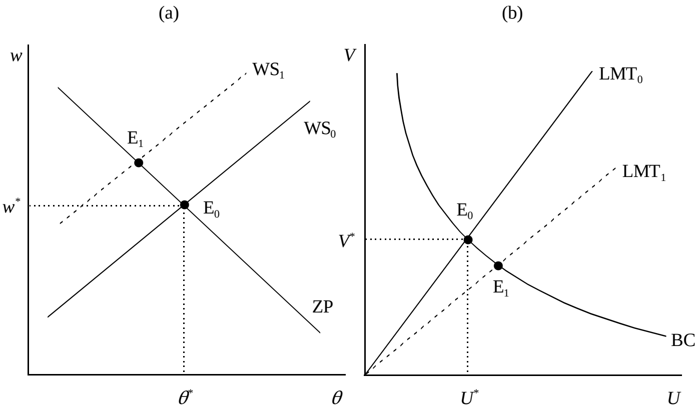
where $\tilde{U} \equiv dU/U$, $\tilde{V} \equiv dV/V$, and $\tilde{s} \equiv ds/s$.

  - BC slopes down: for a given unemployment rate, $V \downarrow$ leads to a fall in the instantaneous probability of finding a job ($f \downarrow$), i.e. for points below the BC curve the unemployment rate is less than the rate required for flow equilibrium in the labour market ($U < s/(s+f)$). To restore flow equilibrium the $U \uparrow$.
  - Shifts to the right as $s \uparrow$.

Shock 1: Increase in the unemployment benefit

- Suppose that $z \uparrow$.
- In Figure 8.1 this shock is illustrated.
  - WS curve to the left.
  - Equilibrium from E_0 to E_1.
  - $w^* \uparrow$ and $\theta^* \downarrow$.
  - In panel (b) the LMT ratio rotates clockwise.
  - $V \downarrow$ and $U \uparrow$.

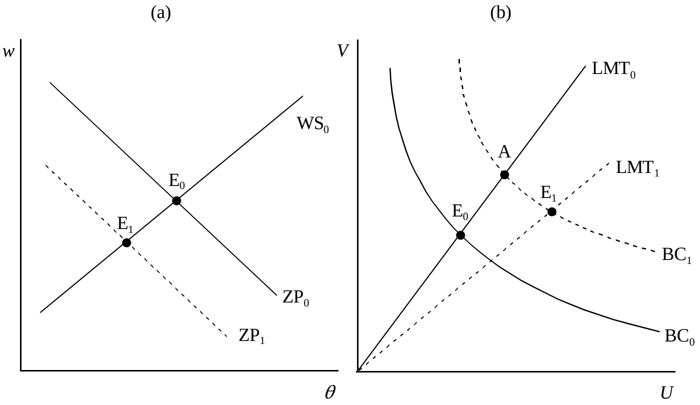
Figure 8.1: Search equilibrium in the labour market



Shock 2: Increase in the job destruction rate

- Suppose that $s\uparrow$.
- ZP curve down in panel (a) of **Figure 8.2**.
- Equilibrium from E_0 to E_1.
- $w^*\downarrow$ and $\theta^*\downarrow$.
- In panel (b) the LMT ratio rotates clockwise **and** BC shifts outwards [dominant effect].
- $V\uparrow$ and $U\uparrow$.

Figure 8.2: The effects of a higher job destruction rate



Labour taxes

- The effects of labour taxes; t_E levied on firms t_L levied on households.
- The model becomes:

$$\frac{\gamma_0}{q(\theta)} = \frac{F_L\left(K(r+\delta),1\right) - w(1+t_E)}{r+s}$$

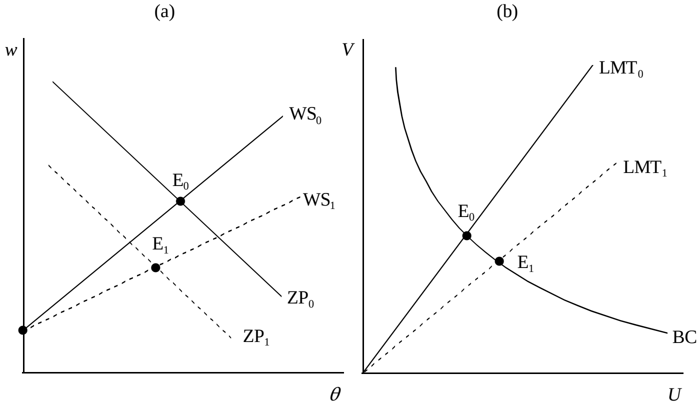
$$w = (1-\beta)\frac{z}{1-t_L} + \beta\frac{F_L\left(K(r+\delta),1\right) + \theta\gamma_0}{1+t_E}$$

$$U = \frac{s}{s+\theta q(\theta)}$$

Labour taxes

- In Figure 8.3 the effects of the payroll tax increase are analyzed ($t_E \uparrow$).
    - WS curve to the right.
    - ZP curve to the left.
    - equilibrium from E_0 to E_1 and $w^* \downarrow$ and $\theta^* \downarrow$.
    - In panel (b) the LMT ratio rotates clockwise.
    - $V \downarrow$ and $U \uparrow$.

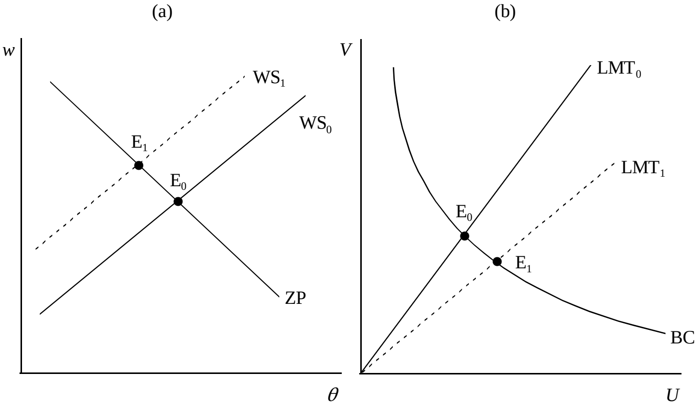
Figure 8.3: The effects of a payroll tax



Labour taxes

- In **Figure 8.4** the effects of the labour income tax increase are analyzed ($t_L \uparrow$).
  - WS curve to the left [z untaxed!].
  - Equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \downarrow$.
  - In panel (b) the LMT ratio rotates clockwise.
  - $V \downarrow$ and $U \uparrow$.

Figure 8.4: The effects of a labour income tax



Deposits on labour

- Workers as empty pop bottles.
- Deposit scheme: firm pays a deposit b to the government when it fires a worker, to be refunded b when it (re-) hires that (or another) worker.
- Model becomes:

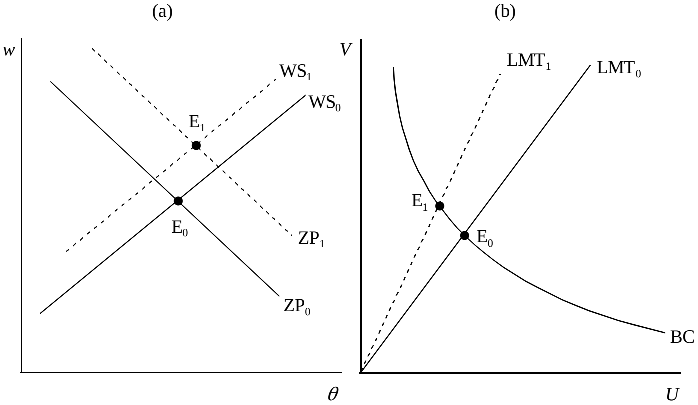
$$
\begin{aligned}
\frac{F_L(K,1) - w + rb}{r+s} &= \frac{\gamma_0}{q(\theta)} \\
w &= (1-\beta)z + \beta\left[F_L(K,1) + rb + \theta\gamma_0\right] \\
U &= \frac{s}{s+\theta q(\theta)}
\end{aligned}
$$

Hence, the capital value of the deposit (rb) acts as a subsidy on the use of labour!

Deposits on labour

- In Figure 8.5 we show the effects of $b \uparrow$.
  - ZP curve to the right.
  - WS curve up.
  - Equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \uparrow$.
  - In panel (b) the LMT ratio rotates counterclockwise.
  - $V \uparrow$ and $U \downarrow$.
- The system works to combat unemployment!

Figure 8.5: The effects of a deposit on labour



Encore: Unemployment persistence in the search model

- One of the stylized facts of the labour market: high persistence in the unemployment rate.
- Pissarides argues that loss of skills during unemployment can explain this phenomenon.
  - Unemployed lose human capital ["skills"].
  - Are thus less attractive to firms, vacancy supply falls.
  - More long-term unemployment.

Punchlines

- Central elements of the search model:
    - Search frictions.
    - Matching function.
    - Wage negotiations.
    - Beveridge curve.
- Attractive model which abandons notion of the aggregate labour market.
- Holds up well empirically.