Foundations of Modern Macroeconomics
Second Edition
Chapter 5: The government budget deficit

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Outline

Aims of this lecture

- To discuss the Ricardian equivalence theorem (simple examples and an assessment).
- To give a first presentation of the dynamic consumption theory.
- To look at tax smoothing theory and some of the golden rules of public finance.
- To discover how we should measure the fiscal stance of the government.

The Ricardian equivalence theorem in words

- *For a given path of government spending* the particular method used to finance these expenditures does not matter, in the sense that real consumption, investment, and output are unaffected. Specifically, whether the expenditures are financed by means of taxation or debt, the real consumption and investment plans of the private sector are not influenced. In that sense government debt and taxes are equivalent.
- Government debt is simply viewed as *delayed taxation*: if the government decides to finance its deficit by issuing debt today, private agents will save more in order to be able to redeem this debt in the future through higher taxation levels.

A simple example (1)

- Utility of the representative household:

$$V = U(C_1) + \frac{1}{1+\rho} U(C_2), \quad \rho > 0 \tag{S1}$$

where V is life-time utility, $U(.)$ is instantaneous utility (often called "felicity"), C_τ is consumption in period τ, and ρ is the pure rate of time preference.

- Budget identities (one for each period):

$$A_1 = (1 + r_0)A_0 + (1 - t_1)Y_1 - C_1 \tag{S2}$$

$$A_2 = (1 + r_1)A_1 + (1 - t_2)Y_2 - C_2 \tag{S3}$$

where A_τ is financial assets at the *end* of period τ ($\tau = 0, 1, 2$), Y_τ is exogenous non-interest income in period τ, r_τ is the interest rate, and t_τ is the tax rate in period τ. (Interest income is untaxed.)

A simple example (2)

- Solvency condition:

$$A_2 = 0 \tag{S4}$$

  - Not allowed to "die" indebted by financial markets.
  - Not smart to die with positive assets.

A simple example (3)

- Consolidated life-time budget restriction(use (S2)–(S4)):

$$A_1 = \frac{C_2 - (1-t_2)Y_2}{1+r_1} = (1+r_0)A_0 + (1-t_1)Y_1 - C_1 \;\Rightarrow$$

$$\underbrace{C_1 + \frac{C_2}{1+r_1}}_{(a)} = \underbrace{(1+r_0)A_0 + H}_{(b)}, \qquad \text{(HBC)}$$

where H is called human wealth of the household:

$$H \equiv \underbrace{(1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r_1}}_{(c)} \qquad \text{(S5)}$$

(a) Present value of consumption spending.
(b) Total wealth; the sum of financial and human wealth.
(c) Human wealth; the present value of after-tax non-interest income.

A simple example (4)

- The government budget restriction: government buys goods (G_τ, $\tau = 1, 2$) and finances its spending by means of taxes and/or debt:

$$(D_1 \equiv)\ r_0 B_0 + G_1 - t_1 Y_1 = B_1 - B_0 \qquad \text{(S6)}$$

$$(D_2 \equiv)\ r_1 B_1 + G_2 - t_2 Y_2 = B_2 - B_1 = -B_1 \qquad \text{(S7)}$$

where D_τ is the deficit in period τ. We have imposed the government solvency condition $B_2 = 0$ in the final step.

A simple example (5)

- Consolidated government budget restriction:

$$(1+r_0)B_0 + G_1 - t_1Y_1 = \frac{t_2Y_2 - G_2}{1+r_1} \Rightarrow$$

$$\underbrace{(1+r_0)B_0 + G_1 + \frac{G_2}{1+r_1}}_{(a)} = \underbrace{t_1Y_1 + \frac{t_2Y_2}{1+r_1}}_{(b)} \quad \text{(GBC)}$$

(a) Net liabilities of the government.
(b) Present value of government income (i.e. tax revenues).
→ Note the close correspondence with the household budget restriction.

A simple example (6)

- Since government bonds are the only financial asset in the toy economy, household borrowing (lending) can only take the form of negative (positive) holdings of government bonds. Hence, equilibrium in the financial capital market implies that:

$$A_\tau = B_\tau, \tag{S8}$$

for $\tau = 0, 1, 2$.

The Ricardian equivalence theorem (1)

- Substitute the GBC into the HBC and substitute (S8):

$$C_1 + \frac{C_2}{1+r} = (1+r_0)B_0 + (1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r_1}$$
$$= t_1 Y_1 + \frac{t_2 Y_2}{1+r_1} - G_1 - \frac{G_2}{1+r_1} + (1-t_1)Y_1 + \frac{(1-t_2)Y_2}{1+r_1}$$
$$= Y_1 - G_1 + \frac{Y_2 - G_2}{1+r_1} \equiv \Omega$$

  - The tax parameters drop out of the household budget restriction altogether. Hence, the particular timing of taxes does not matter.

- To demonstrate what happens consider the following "toy model". We take a simple felicity function:

$$U(C_\tau) = \ln C_\tau$$

The Ricardian equivalence theorem (2)

- The representative household chooses C_1 and C_2 to maximize lifetime utility subject to its budget constraint:

$$\max_{\{C_1,C_2\}} V \equiv \ln C_1 + \frac{1}{1+\rho} \ln C_2$$

$$\text{subject to:} \quad \Omega = C_1 + \frac{C_2}{1+r_1}$$

- Lagrangian:

$$\mathcal{L} \equiv \ln C_1 + \frac{1}{1+\rho} \ln C_2 + \lambda \left[\Omega - C_1 - \frac{C_2}{1+r_1} \right]$$

where λ is the Lagrange multiplier.

The Ricardian equivalence theorem (3)

- The first-order conditions are the constraint and:

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{1}{C_1} - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{1}{(1+\rho)C_2} - \frac{\lambda}{1+r_1} = 0$$

- Combining the two conditions yields the consumption Euler equation:

$$\lambda = \frac{1}{C_1} = \frac{1+r_1}{(1+\rho)C_2} \Rightarrow \frac{C_2}{C_1} = \frac{1+r_1}{1+\rho}$$

  - Note: This expression plays a vital role in modern macroeconomics!

The Ricardian equivalence theorem (4)

- Combine the Euler equation with the constraint and we get the levels of C_1 and C_2:

$$C_1 = \frac{1+\rho}{2+\rho}\Omega, \quad C_2 = \frac{1+r_1}{2+\rho}\Omega$$

- The implied savings function is:

$$\begin{aligned} S_1 &= A_1 - A_0 \\ &= B_1 - B_0 \\ &= r_0 B_0 + (1-t_1)Y_1 - \underbrace{\frac{1+\rho}{2+\rho}\Omega}_{C_1}, \end{aligned}$$

- Hence, the tax rate does not vanish from the savings function!

The Ricardian tax cut experiment (1)

- Consider the following *Ricardian experiment*: cut t_1, keep G_1 and G_2 the same and raise t_2 such that the GBC is satisfied. The government engages in deficit financing in the first period.
- Household savings increase:

$$dS_1 = -Y_1 dt_1 > 0$$

- The GBC implies that taxes in the second period must satisfy:

$$Y_1 dt_1 + \frac{Y_2}{1+r_1} dt_2 = 0$$

(because B_0, r_1, G_1, G_2, Y_1, and Y_2 are all constant).

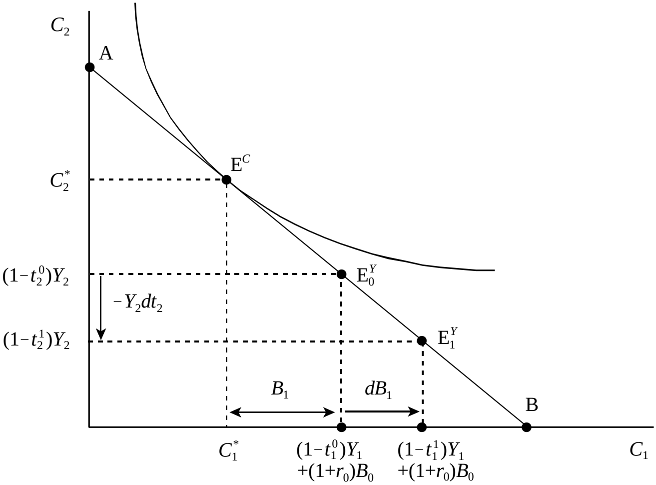
The Ricardian tax cut experiment (2)

- Hence, the tax in the next period goes up:

$$dt_2 = -\frac{(1+r_1)Y_1}{Y_2}dt_1 > 0$$

- We conclude that savings absorb the shock to after-tax income. Nothing happens to C_1 and C_2. This is illustrated in **Figure 5.1**.

Figure 5.1: Ricardian equivalence experiment



Objections to Ricardian equivalence

(a) Distorting taxes are changed in the Ricardian experiment.

(b) Borrowing restrictions.

(c) Finite lives ("apres nous le deluge").

(d) New agents/population growth.

(e) Informational problems.

(f) Imperfect insurance markets and the "bird-in-the-hand" issue.

→ Note: We study (a)-(c) in some detail here; (c)-(d) are studied further in Chapters 13 and 16–17.

(a) Distorting taxes destroy RET

- What if non-interest income is not exogenous?
- With endogenous labour supply choice we can derive that Y_1 and Y_2 depend on the tax rates.
- Write:

$$Y_1 = Y_1(t_1, t_2), \qquad Y_2 = Y_2(t_1, t_2) \tag{S9}$$

- The household budget constraint becomes:

$$\begin{aligned} C_1 + \frac{C_2}{1+r_1} &= (1+r_0)A_0 + (1-t_1)Y_1(t_1,t_2) \\ &\quad + \frac{(1-t_2)Y_2(t_1,t_2)}{1+r_1} \end{aligned} \tag{HBC}$$

(a) Distorting taxes destroy RET

- The government budget constraint is:

$$(1+r_0)\,B_0+G_1+\frac{G_2}{1+r_1}=t_1Y_1\,(t_1,t_2)+\frac{t_2Y_2\,(t_1,t_2)}{1+r_1} \quad \text{(GBC)}$$

- Substituting GBC into HBC and noting (S8) we get:

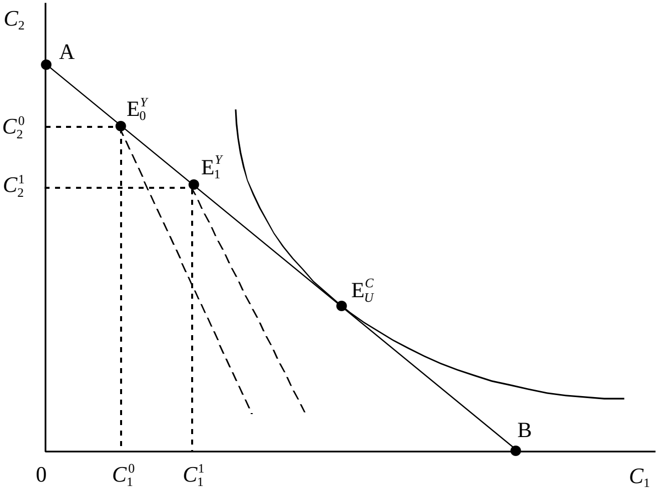
$$C_1+\frac{C_2}{1+r_1}=Y_1\,(t_1,t_2)-G_1+\frac{Y_2\,(t_1,t_2)-G_2}{1+r_1} \quad \text{(S10)}$$

- The particular tax mix (t_1, t_2) will now affect $Y_1\,(t_1, t_2)$ and $Y_2\,(t_1, t_2)$ and a Ricardian tax cut experiment will not be neutral!

(b) Borrowing restrictions destroy RET

- Suppose that the agent cannot borrow at all (borrowing constraint) or must pay a higher interest rate than the government.
- This case is illustrated in **Figure 5.2**.
- If the income endowment point lies to the north-west from the preferred consumption point there is a problem: the borrowing restrictions constrain the choice set of the household.
- RET will fail because the Ricardian experiment affects the choice set of the household.

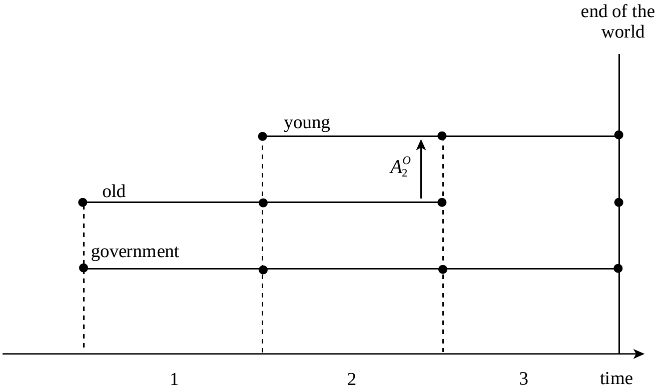
Figure 5.2: Liquidity restrictions and the Ricardian experiment



(c) Finite lives (may) destroy RET

- *Intuition*: if Ricardian experiment involves changing taxes after one's death then surely the currently living generations are better off and will consume more. Yes or no?
- We consider a simple model.
- Economy exists for three periods.
- Two types of households (overlapping generations).
  - (a) "old" who are alive in periods 1 and 2.
  - (b) "young" who are alive in periods 2 and 3.
- Government exists in all three periods.
- See **Figure 5.3** for the representation.

Figure 5.3: Overlapping generations in a three-period economy



Old households (1)

- Lifetime utility function:

$$V^O = \ln C_1^O + \frac{1}{1+\rho} \ln C_2^O + \alpha V^Y, \quad \alpha \geq 0 \qquad \text{(S11)}$$

where the superscript "O" designates the old generation, and "Y" the young generation.

  - The equation says that if $\alpha > 0$, the old generation cares for the young generation (e.g. because they are related to each other).

- The lifetime budget restriction:

$$A_1^O = (1+r_0)A_0^O + (1-t_1)Y_1^O - C_1^O \qquad \text{(S12)}$$

$$A_2^O = (1+r_1)A_1^O + (1-t_2)Y_2^O - C_2^O \qquad \text{(S13)}$$

Old households (2)

- We eliminate A_1^O from these expressions to get:

$$C_1^O + \frac{C_2^O + A_2^O}{1+r_1} = (1+r_0)A_0^O + \underbrace{(1-t_1)Y_1^O + \frac{(1-t_2)Y_2^O}{1+r_1}}_{H^O} \equiv \Omega^O$$

  where A_2^O is the inheritance given to the young at the end of period 2 (when the old meet their maker) and H^O is human wealth of the old.

- Negative inheritances are not allowed:

$$A_2^O \geq 0 \tag{S14}$$

- We conjecture that the young like to receive an inheritance, i.e. we wish to find:

$$V^Y = \Phi(\underset{+}{A_2^O}) \tag{S15}$$

Young households (1)

- Lifetime utility function:

$$V^Y = \ln C_2^Y + \frac{1}{1+\rho} \ln C_3^Y \tag{S16}$$

- Budget identities:

$$A_2^Y = (1-t_2)Y_2^Y - C_2^Y \tag{S17}$$
$$A_3^Y = (1+r_2)\left[A_2^O + A_2^Y\right] + (1-t_3)Y_3^Y - C_3^Y = 0 \tag{S18}$$

- We eliminate A_2^Y to get the consolidated budget restriction:

$$C_2^Y + \frac{C_3^Y}{1+r_2} = A_2^O + \underbrace{(1-t_2)Y_2^Y + \frac{(1-t_3)Y_3^Y}{1+r_2}}_{H^Y} \equiv \Omega^Y$$

Young households (2)

- Our usual tricks yield the solutions for consumption in the two periods:

$$C_2^Y = \frac{1+\rho}{2+\rho}\Omega^Y$$
$$C_3^Y = \frac{1+r_2}{2+\rho}\Omega^Y$$

and for (indirect) utility:

$$V^Y = \ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{1}{1+\rho}\ln\left(\frac{1+r_2}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\ln\Omega^Y$$
$$= \Phi_0 + \frac{2+\rho}{1+\rho}\ln\left(A_2^O + H^Y\right) \equiv \Phi(A_2^O)$$

- In the last step we have found our result: the young indeed like to get a bequest.

Back to the old households (1)

- The old household knows the relationship $V^Y \equiv \Phi(A_2^O)$ and takes it into account when forming its own optimal plans.
- The old households chooses C_1^O, C_2^O, and A_2^O to maximize lifetime utility subject to the budget constraint and the non-negativity constraint on bequests.
- The Lagrangian is:

$$\begin{aligned} \mathcal{L} \equiv & \ln C_1^O + \frac{1}{1+\rho} \ln C_2^O + \alpha \Phi(A_2^O) \\ & + \lambda \left[\Omega^O - C_1^O - \frac{C_2^O + A_2^O}{1 + r_1} \right] \end{aligned}$$

Back to the old households (2)

- The first-order conditions are the budget constraint and:

$$\frac{\partial \mathcal{L}}{\partial C_1^O} = \frac{1}{C_1^O} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_2^O} = \frac{1}{(1+\rho)C_2^O} - \frac{\lambda}{1+r_1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial A_2^O} = \underbrace{\alpha \frac{dV^Y}{dA_2^O}}_{(a)} - \underbrace{\frac{\lambda}{1+r_1}}_{(b)} \leq 0, \; A_2^O \geq 0, \; A_2^O \frac{\partial \mathcal{L}}{\partial A_2^O} = 0 \quad \text{(S19)}$$

(a) Marginal benefit to the old of leaving one additional unit of output to the young in the form of an inheritance.
(b) Marginal cost of leaving the additional unit of output to the young (instead of consuming it him/herself).

Back to the old households (3)

- Let us look at two cases of (S19):
  - If $\alpha = 0$ (unloved offspring) then:

$$\frac{\partial \mathcal{L}}{\partial A_2^O} = -\frac{\lambda}{1+r_1} < 0 \text{ and thus } A_2^O = 0$$

    It is optimal not to leave any inheritance at all! Same conclusion would be obtained if α is positive but low (mildly loved offspring): "Non-operative bequests".
  - If a positive bequest is to be optimal it must be the case that a and Φ' are so high as to render $\partial \mathcal{L}/\partial A_2^O = 0$ (interior solution for A_2^O): "Operative bequests".

Back to the old households (4)

- With operative bequests we can solve the first-order conditions for C_1^O, C_2^O, and A_2^O:

$$C_1^O = \frac{(1+\rho)\left[\Omega^O + H^Y/(1+r_1)\right]}{(2+\rho)(1+\alpha)}$$

$$C_2^O = \frac{(1+r_1)\Omega^O + H^Y}{(2+\rho)(1+\alpha)}$$

$$A_2^O = \frac{\alpha(1+r_1)\Omega^O - H^Y}{1+\alpha}$$

Insights from this model

(1) Provided the bequest motive remains operative during the Ricardian experiment (and A_2^O stays positive) the RET holds despite the fact that lives of agents are finite. Intuitively, if $t_1 \downarrow$ and $t_3 \uparrow$ (for given G_1, G_2, and G_3) then the old leave larger bequest to the young to exactly compensate them for the higher taxes during their life. No effect on anybody's consumption: C_1^O, C_2^O, C_2^Y, and C_3^Y all unchanged.

(2) With non-operative bequest motive ($\alpha = 0$ or too low) RET fails as the old do **not** compensate the young and increase consumption in the Ricardian experiment.

Test your understanding

****** Self Test ******

Study how these two insights work in the model. Make sure that demand and supply on the bond market do not call for an interest rate change during the Ricardian experiment.

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The theory of public debt creation: Tax smoothing

- *Basic idea*: if taxes are distortionary and households like to smooth consumption over time then it is smart for the government to use debt to smooth tax rates over time rather than letting them fluctuate freely.
- We consider a simple model.
- Welfare loss associated with taxation:

$$L_G \equiv \tfrac{1}{2} t_1^2 Y_1 + \tfrac{1}{2} \frac{t_2^2 Y_2}{1+\rho_G}$$

where ρ_G is the policy maker's time preference.

Bookkeeping (1)

- Government budget restriction:

$$(D_1 \equiv) \; r_0 B_0 + G_1^C + G_1^I - t_1 Y_1 = B_1 - B_0$$
$$(D_2 \equiv) \; r_1 B_1 + G_2^C - R_2^I - t_2 Y_2 = B_2 - B_1 = -B_1$$

where R_2^I is the gross return on public investment obtained in period 2, so that the *rate* of return r_1^G on public investment can be written as:

$$R_2^I = (1 + r_1^G) G_1^I$$

  - Government spending is divided into "consumption" and "investment" spending.

Bookkeeping (2)

- Consolidated GBC is:

$$(1+r_0)B_0 + G_1^C + G_1^I - t_1Y_1 = \frac{t_2Y_2 + (1+r_1^G)G_1^I - G_2^C}{1+r_1} \Rightarrow$$

$$\Xi_1 \equiv (1+r_0)B_0 + G_1^C + \frac{G_2^C}{1+r_1} + \underbrace{\frac{(r_1 - r_1^G)G_1^I}{1+r_1}}_{(a)} = t_1Y_1 + \frac{t_2Y_2}{1+r_1}$$

where Ξ_1 is the present value of the net liabilities of the government.

(a) Part 1 of the golden rule of public finance: if $r_1^G = r_1$ then public investment (G_1^I) can be debudgeted. It does not form part of the net liabilities of the government. It is OK to finance public investment with debt provided it makes the market rate of return.

→ If $r_1^G > r_1$ then public investment is a source of revenue to the government!

A simple demonstration (1)

- Suppose the growth rate of Y is defined as:

$$\gamma = \frac{Y_2 - Y_1}{Y_1} \quad \Leftrightarrow \quad Y_2 = (1+\gamma) Y_1$$

- Then the GBC can be rewritten as:

$$\xi_1 \equiv \frac{\Xi_1}{Y_1} = t_1 + \frac{1+\gamma}{1+r} t_2 \tag{S20}$$

- Taking as given ξ_1 and Y_1, the government chooses the tax rates to minimize the welfare loss, L_G:

$$L_G \equiv \tfrac{1}{2} t_1^2 Y_1 + \tfrac{1}{2} \frac{t_2^2 Y_2}{1+\rho_G}$$

subject to the GBC in (S20).

A simple demonstration (2)

- The Lagrangian is:

$$\mathcal{L} \equiv \tfrac{1}{2}t_1^2 Y_1 + \tfrac{1}{2}t_2^2 \frac{1+\gamma}{1+\rho_G} Y_1 + \lambda \left[\xi_1 - t_1 - \frac{1+\gamma}{1+r_1} t_2 \right]$$

so that the first-order conditions are the GBC and:

$$\frac{\partial \mathcal{L}}{\partial t_1} = t_1 Y_1 - \lambda = 0$$

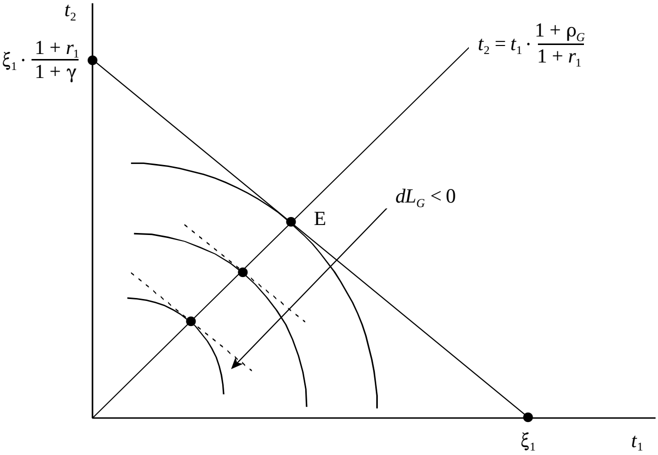
$$\frac{\partial \mathcal{L}}{\partial t_2} = t_2 \frac{1+\gamma}{1+\rho_G} Y_1 - \lambda \frac{1+\gamma}{1+r_1} = 0$$

- Combining the two first-order conditions yields the tax smoothing line (like an Euler equation for tax rates):

$$\lambda = t_1 Y_1 = \frac{1+r_1}{1+\rho_G} t_2 Y_1 \quad \Rightarrow t_1 = \frac{1+r_1}{1+\rho_G} t_2$$

- Graphical representation of the theory: **Figure 5.4**.

Figure 5.4: Optimal taxation



A simple demonstration (3)

- By using the tax smoothing line in the GBC we get the levels of the tax rates:

$$t_1 = \frac{(1+r_1)^2 \xi_1}{(1+r_1)^2 + (1+\gamma)(1+\rho_G)}$$
$$t_2 = \frac{(1+\rho_G)(1+r_1)\xi_1}{(1+r_1)^2 + (1+\gamma)(1+\rho_G)}$$

- The key thing to note is that if $r_1 = \rho_G$ then tax rate are equalized (hence the name tax smoothing):

$$t_1 = t_2 = \frac{1+r_1}{2+r_1+\gamma}\xi_1$$

A simple demonstration (4)

- Now rewrite the net liabilities of the government, ξ_1:

$$\xi_1 \equiv \frac{G_1^C}{Y_1} + \frac{1}{1+r_1}\frac{G_2^C}{Y_1} + \frac{r_1 - r_1^G}{1+r_1}\frac{G_1^I}{Y_1} + (1+r_0)\frac{B_0}{Y_1}$$
$$= g_1^C + \frac{1+\gamma}{1+r_1} g_2^C + \frac{r_1 - r_1^G}{1+r_1} g_1^I + (1+r_0) b_0$$

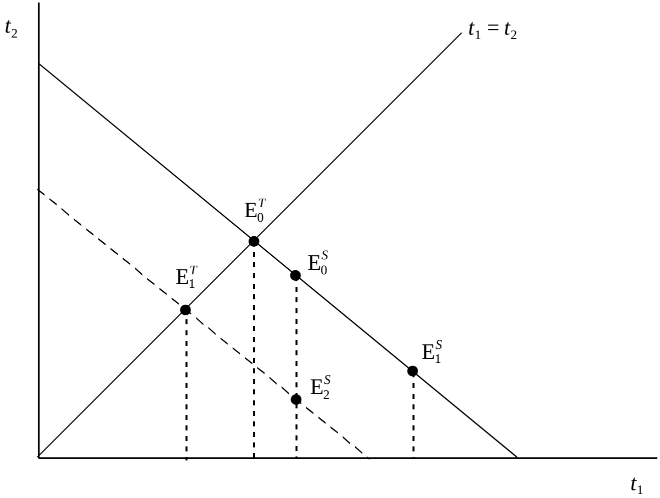
where $g_t^C \equiv G_t^C / Y_t$, $g_1^I \equiv G_1^I / Y_1$, and $b_0 \equiv B_0 / Y_1$.

- Furthermore, the deficit in period 1 can also be written in terms of national income in period 1:

$$d_1 \equiv \frac{D_1}{Y_1} = \frac{r_0 B_0 + G_1^C + G_1^I - t_1 Y_1}{Y_1} = r_0 b_0 + g_1^C + g_1^I - t_1$$

- The smoothing model can be illustrated with **Figure 5.5**

Figure 5.5: Optimal taxation and tax smoothing



Some further golden rules of public finance

- A temporary rise in government spending can be financed by means of debt. Intuition: $g_1^C \uparrow$ and $g_2^C \downarrow$ (such that ξ_1 is unchanged) only changes the spending point E_0^S but not the optimal taxation point E_0^T in Figure 5.5. It is optimal to leave tax rates unchanged and to let the deficit absorb the shock
- A permanent rise in public spending ($\xi_1 \uparrow$) should be tax financed. The iso-revenue curve shifts out as does the optimal taxation point. Both tax rates should be increased.
- A credible announcement by the government that its future spending will fall ($g_2^C \downarrow$) should lead to an immediate tax cut in both periods. The shock reduces ξ_1.

Punchlines

- We have given a simple demonstration of the Ricardian equivalence theorem.
- A side benefit of the exercise is that we now know what the forward-looking theory of consumption looks like.
- There are many reasons to disbelieve strict validity of RET.
- Nevertheless RET may be approximately valid.
- Even if RET holds, public debt may be a useful policy instrument (tax smoothing).