The Transitional Dynamics of Fiscal Policy in Small Open Economies*

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Proposed running head: Transitional Dynamics of Fiscal Policy

Abstract

We study the dynamic macroeconomic effects of fiscal shocks under lump-sum tax financing. To this end, we develop an intertemporal macroeconomic model for a small open economy, featuring monopolistic competition in the intermediate goods market, endogenous (intertemporal) labor supply, and finitely lived households. Fiscal shocks are shown to yield endogenously determined (dampened) cycles for a realistic calibration of the model. Impulse response functions of fiscal policy shocks in the finite horizon model differ substantially from those resulting from an infinitely lived representative agent model. This can be explained by the presence of Ethier-productivity effects, which increase the size of long-run output multipliers to a greater extent in the infinite horizon model.

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1 Introduction

In the wake of the Stability and Growth Pact of the European Union,[1] there has been a revival of interest in analyzing the macroeconomic effects of fiscal policy in open economies. This paper contributes to this line of work by analyzing a typical small open economy. More specifically, we investigate the dynamic effects of fiscal policy while allowing for monopolistic competition in the production of intermediate goods and finitely lived households. We investigate whether the assumption of finitely lived households—which we employ to generate an endogenously determined steady state—substantially affects the impulse responses of fiscal policy as found in the standard case of an infinitely lived representative agent (to which we refer as "infinite horizons"). In particular, we would like to know how imperfect competition and the degree of input variety across firms affect our results.

The analysis of fiscal policy in open economy models has received little attention compared with monetary policy. Furthermore, the vast majority of micro-founded literature on fiscal policy assumes perfect competition in the goods market. Early contributions are those by Turnovsky and Sen (1991), Chang (1999), and Karayalçin (1999).[2] More recently, attention has focused on relaxing the assumption of perfectly competitive goods markets, thereby giving rise to a sub-optimal level of output in the decentralized market outcome. Besides providing a rationale for activist government intervention, imperfect competition allows for the explicit modeling of price setting behavior of firms. Dynamic macroeconomic models that introduce some form of imperfect competition in goods or labor markets (without imposing explicit price stickiness) are small in number and are primarily focused on closed economies.[3] A notable exception is the small open economy model of Coto-Martinez and Dixon (2003) to which our work is somewhat related. Coto-Martinez and Dixon (2003) analyze the case of infinite horizons and distinguish between tradables and nontradables.

We develop an intertemporal optimization model, which features two production sectors (final and intermediate goods) and finitely lived households. The small open economy is embedded in a world of a homogeneous final good, which is supplied under perfect competition. Differentiated intermediate goods are produced by firms under internal economies of scale, yielding imperfect competition on the intermediate input market. We assume free entry and exit of firms in the intermediate goods sector, giving rise to endogenous Ethier (1982)-productivity effects. Intuitively, increased input diversity allows firms in the final goods sector to use a more roundabout production technology. The household sector builds on an extended version of the Blanchard-Yaari model (cf. Blanchard, 1985; Yaari, 1965), in which agents face a constant probability of death. In keeping with the literature, there is an internationally traded bond, ensuring that households can use the current account of the balance of payments to smooth private consumption. To avoid trivial capital dynamics and to limit the international mobility of physical capital, we postulate adjustment costs of investment at the level of the portfolio investor.

We employ overlapping generations in the Blanchard-Yaari tradition not only to get a realistic description of the household sector but also to yield an endogenously determined (non-hysteretic) steady state. It is well known that in infinite horizon models of a small open economy the steady state is hysteretic. The dynamic system contains a zero characteristic root in private consumption if the exogenously given world rate of interest equals the constant rate of time preference. This "knife-edge" condition must hold for a steady state to exist.[4] By log-linearizing such a system, one is approximating its dynamics around a hysteretic steady state, (potentially) reducing the reliability of the approximation.[5] To address the hysteresis problem, various authors have employed overlapping generations of households.[6] None of these authors have analyzed the sensitivity of their results to the household's planning horizon, however.

Schmitt-Grohe and Uribe (2003) conclude from a comparison of various stationarity-inducing devices that the comparative dynamic properties of their small open economy RBC model are hardly affected by the type of device.[7] More relevant for our study is their finding that the stationary and non-stationary model feature very similar impulse response functions originating from a technology shock. Schmitt-Grohe and Uribe (2003) neither study domestic demand shocks nor do they employ overlapping generations as a stationarity-inducing device, however. This paper fills that gap for a deterministic model setting. We characterize analytically the transition paths induced by a fiscal impulse in the benchmark overlapping generations model, which we compare with the hysteretic case of infinite horizons. To this end, we apply the Laplace transform technique (Judd, 1982) to a log-linearized version of the model. Numerical examples are used to illustrate the impulse response functions at business cycle frequencies.

We show that finite and infinite horizon versions of our model give rise to very different impulse responses of a fiscal shock. The transition paths in the finite horizon case feature endogenously determined (dampened) cycles for a realistic calibration. Finite horizons together with the interplay of elastic labor supply and external economies of scale generate these cycles.[8] The cycles are of first-order nature and disappear if one of the three factors is eliminated from the analysis. For the benchmark calibration, the infinite horizon model is unstable. By taking an intermediate value of the degree of external economies of scale (which is smaller than the benchmark value), we find smaller cycles in the finite horizon model, whereas the transition becomes monotonic in the infinite horizon case. Thus, we cannot reproduce Schmitt-Grohe and Uribe's (2003) results in our context, indicating that these are not as general as suggested. External economies of scale cause Schmitt-Grohe and Uribe's key result to break down.[9]

Our paper also contributes to the literature on the size of (balanced budget) output multipliers of fiscal policy. Key results are the following. Long-run output multipliers are positive and exceed those in the short run, which are also positive. This stands in sharp contrast to the results of Coto-Martinez and Dixon (2003), who find smaller long-run output multipliers. A more elastic labor supply response and larger external increasing returns to scale increase the size of long-run output multipliers within the parameter range generating a stable outcome. Sufficiently strong Ethier-productivity effects give rise to private consumption and output multipliers that are both positive in the long run, a result which cannot be obtained in the standard framework of an infinitely lived representative household.

The paper is structured as follows. Section 2 sets out the extended Blanchard-Yaari model for a small open economy. Section 3 solves the log-linearized model and studies model stability and calibration issues. Section 4 analyzes the transitional dynamics of a permanent increase in public consumption financed by lump-sum taxes. Section 5 summarizes and concludes.

2 The Model of Perpetual Youth

This section develops a dynamic, micro-founded, macroeconomic model for a small open economy, which features agents endowed with perfect foresight. Subsequently, it discusses decision making by households, firms, and the government.

2.1 Households

The household section of the model builds on Blanchard (1985) and the extension to endogenous intertemporal labor supply by Heijdra and Ligthart (2007). The model features a fixed population of agents (normalized to unity), each facing a constant probability of death ($\beta \geq 0$), which equals the rate at which new agents are born. Labor is assumed to be immobile internationally and is supplied in a perfectly competitive labor market. Households do not leave bequests—implying that generations are disconnected—and do not face liquidity constraints.

During its entire life span, an agent has a time endowment of unity, which it allocates to labor and leisure. The utility functional at time t of the representative agent born at time v is assumed to be weakly separable in private consumption, $C(v,t)$, and leisure, $1 - L(v,t)$:

$$\Lambda(v,t) \equiv \int_t^{\infty} \left[\varepsilon_C \ln C(v,\tau) + (1-\varepsilon_C) \ln(1 - L(v,\tau))\right] e^{(\alpha+\beta)(t-\tau)} d\tau, \tag{1}$$

where $\alpha > 0$ is the (constant) pure rate of time preference and ε_C is the share of private consumption in utility (where $0 < \varepsilon_C < 1$). The agent's budget identity is:

$$\dot{A}(v,t) = (r+\beta)A(v,t) + w(t)L(v,t) - T(t) - C(v,t), \tag{2}$$

where an overdot indicates a time derivative, $A(v,t)$ are financial assets, r is the exogenously given and constant world rate of interest, $w(t)$ is the (age-independent) wage rate, and $T(t)$ are *net* lump-sum taxes (all denoted in real terms). The final good (with price $P(t)$) is used as the numeraire. Despite the constant rate of interest, wages are flexible, reflecting adjustment costs in investment (see below).

The household chooses a time profile for $C(v,t)$ and $L(v,t)$ to maximize $\Lambda(v,t)$ subject to its budget identity (2) and a no-Ponzi-game solvency condition. This yields the optimal time profile of private consumption:

$$\frac{\dot{C}(v,t)}{C(v,t)} = r - \alpha. \tag{3}$$

In the general case of $\beta > 0$, we study a patient nation (i.e., $r > \alpha$), which yields rising individual consumption profiles.[10] Individual labor supply is negatively linked to private consumption (i.e., the wealth effect) and positively associated with wages:

$$L(v,t) = 1 - \frac{(1-\varepsilon_C)C(v,t)}{\varepsilon_C w(t)}. \tag{4}$$

Variables at the aggregate level can be calculated as the weighted sum of the values for different generations. For example, $A(t) \equiv \int_{-\infty}^{t} A(v,t)\beta e^{\beta(v-t)}dv$ is aggregate financial wealth. By aggregating (3), we arrive at the *aggregate* Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = r - \alpha - \beta\varepsilon_C(\alpha+\beta)\frac{A(t)}{C(t)} = \frac{\dot{C}(v,t)}{C(v,t)} - \beta \cdot \frac{C(t) - C(t,t)}{C(t)}. \tag{5}$$

Equation (5) has the same form as the Euler equation for individual households (3), except for a correction term, which captures the wealth redistribution caused by the turnover of generations. Optimal individual consumption *growth* is the same for all generations since they face the same rate of interest. But the consumption *level* of old generations is higher than that of young generations, reflecting the larger stock of financial assets owned by old generations. Because existing generations are continually being replaced by newborns, who are born without financial wealth, aggregate consumption growth falls short of individual consumption growth. The correction term appearing in (5) thus represents the difference in average consumption, $C(t)$, and consumption by newborns, $C(t,t)$.[11]

We model a household-investor, which optimizes its investment portfolio. There are two assets in the economy, that is, claims on domestic capital goods, $V(t)$, and *net* foreign assets, $F(t)$ (which are all measured in real terms). By assuming assets to be perfect substitutes in the household's portfolio, they earn the same real rate of return. The household's cash flow from investing in physical capital is given by:

$$V(t) \equiv \int_t^\infty \left[r^K(\tau)K(\tau) - I(\tau)\right] e^{r(t-\tau)} d\tau, \tag{6}$$

where $K(t)$ denotes physical capital, $r^K(t)$ is the rental rate on capital, and $I(t)$ denotes *gross* investment. We follow Uzawa (1969) by postulating a concave accumulation function, $\Psi(\cdot)$, which links net capital accumulation to gross investment:

$$\dot{K}(t) = \left[\Psi\left(\frac{I(t)}{K(t)}\right) - \delta\right] K(t), \qquad \Psi(0) = 0, \quad \Psi'(\cdot) > 0, \quad \Psi''(\cdot) < 0, \tag{7}$$

where $\delta > 0$ is the constant rate of capital depreciation.

The household-investor chooses paths for gross investment and the capital stock to maximize (6) subject to (7) while taking as given the initial capital stock, $K(0) > 0$. The first-order conditions are:

$$1 = q(t)\Psi'\left(\frac{I(t)}{K(t)}\right), \tag{8}$$

$$\dot{q}(t) = \left[r + \delta - \Psi\left(\frac{I(t)}{K(t)}\right)\right] q(t) - r^K(t) + \frac{I(t)}{K(t)}, \tag{9}$$

where $q(t)$ denotes Tobin's q, which measures the market value of capital relative to its replacement costs. The degree of physical capital mobility is given by $\sigma \equiv -(I/K)\Psi''/\Psi' > 0$, where a small σ characterizes a high degree of capital mobility.[12]

2.2 Firms

Following Hornstein (1993), the production sector consists of two types of firms. The first type concerns monopolistically competitive firms, each of which produces a unique variety of an intermediate input (which are close but imperfect substitutes). The second type are perfectly competitive firms that produce a homogeneous final output using differentiated intermediate goods.

Technology in the final goods sector can be described by a Dixit-Stiglitz (1977) specification:

$$Y(t) = N(t)^{\eta-\mu} \left[\int_0^{N(t)} Z_j(t)^{1/\mu} dj \right]^{\mu}, \ \eta \geq 1, \ \mu > 1, \tag{10}$$

where $Y(t)$ denotes aggregate output of final goods, $Z_j(t)$ is the quantity of variety j of the intermediate good, $N(t)$ denotes the number of input varieties, and μ is a technological parameter measuring the ease with which different varieties can be substituted for each other in production. (In the Chamberlinian equilibrium, the markup of price over marginal cost charged by a firm in the intermediate goods sector will be equal to μ—see below). The parameter η represents the Ethier (1982)-productivity effect. Increased input diversity allows firms to use a more roundabout production technology, giving rise to external economies of scale. For $\eta = 1$, the Ethier-productivity effect is switched off. Following Bénassy (1996), Fatás (1997), and Broer and Heijdra (2001), we parameterize η and μ separately with a view to disentangle the output effect of external economies of scale from that of imperfect competition (see Section 2.4).

The representative producer in the final goods sector minimizes the cost of producing a given quantity of final goods by choosing the optimal mix of input varieties. Input demand functions feature a constant elasticity of demand:

$$Z_j(t) = N(t)^{(\eta-\mu)/(\mu-1)} Y(t) \left(\frac{P_j(t)}{P(t)} \right)^{\mu/(1-\mu)}, \tag{11}$$

where $P_j(t)$ is the price of input variety j and $P(t)$ is the unit cost function corresponding to equation (10):

$$P(t) \equiv N(t)^{\mu-\eta} \left[\int_0^{N(t)} P_j(t)^{1/(1-\mu)} dj \right]^{1-\mu}. \tag{12}$$

The intermediate goods sector features an endogenously determined number of monopolistically competitive firms, each of which produces a single differentiated input. Firm j rents capital and labor from the household sector to produce *gross* output according to:

$$Z_j(t) + f \equiv L_j(t)^{\varepsilon_L} K_j(t)^{1-\varepsilon_L}, \qquad 0 < \varepsilon_L < 1, \tag{13}$$

where f are fixed costs modeled in terms of the output of firm j. Consequently, firms enjoy (local) internal increasing returns to scale, that is, $(Z_j(t) + f)/Z_j > 1$. The representative firm maximizes profits by choosing its price and primary factor demands subject to (11). As a result, the factor demands of firm j are determined by the usual marginal productivity conditions for labor and capital:

$$\frac{\partial Z_j(t)}{\partial L_j(t)} = \mu \frac{w(t)P(t)}{P_j(t)}, \tag{14}$$

$$\frac{\partial Z_j(t)}{\partial K_j(t)} = \mu \frac{r^K(t)P(t)}{P_j(t)}, \tag{15}$$

which feature the firm's markup $\mu > 1$. If $\eta = \mu$, we get the familiar Dixit-Stiglitz case in which primary input use by firms is below its social optimal value.[13] Following Schmitt-Grohe (1997), we assume Chamberlinian monopolistic competition, implying that the instantaneous entry and exit of firms eliminates all *pure* profits for each firm. Accordingly, the intermediate input price equals average cost, which implies a constant equilibrium firm size of $Z_j \equiv f/(\mu - 1)$, where $\mu > 1$ for the equilibrium to exist. A larger markup thus implies a smaller equilibrium firm size. If $\mu \to 1$ and $f \to 0$, then the model converges to a perfectly competitive economy.

2.3 Government and External Sector

The government is assumed to play a rather simple role in our stylized economy. Government spending, $G(t)$, neither yields utility to individuals nor is it productive. We assume that all spending is financed by lump-sum taxes, that is, $G(t) = T(t)$ for all $t \geq 0$.[14]

In the non-degenerate case of $r > \alpha$, households use the current account to smooth consumption (and thus acquire *net* foreign assets). Foreign financial capital is perfectly mobile. The change in net foreign assets equals the current account balance:

$$\dot{F}(t) = rF(t) + [Y(t) - C(t) - I(t) - G(t)], \tag{16}$$

where the term in square brackets is the trade account, showing that domestic output less domestic absorption, $C(t) + I(t) + G(t)$, equals net exports, $X(t)$. National solvency requires: $F(t) = -\int_t^\infty X(\tau)e^{r(t-\tau)}d\tau$, showing that the pre-existing level of net foreign assets (debt) should equal the present value of trade balance deficits (surpluses).

2.4 Symmetric Perfect Foresight Equilibrium

The supply side of the model is symmetric and can thus be expressed in aggregate terms. All existing firms in the intermediate goods sector are of equal size, $\bar{Z}$, and thus charge the same price and demand the same amounts of capital and labor, that is, $K_j(t) = \bar{K}(t)$ and $L_j(t) = \bar{L}(t)$. In view of this, (10) yields aggregate output of final goods as an iso-elastic function of the number of input varieties:

$$Y(t) = N(t)^\eta \bar{Z} = N(t)^\eta \frac{f}{\mu - 1}. \tag{17}$$

A higher level of output thus sustains a larger number of firms in the new equilibrium. Alternatively, by using (17) and (13), we can derive:

$$Y(t) = \Omega_0 L(t)^{\eta\varepsilon_L} K(t)^{\eta(1-\varepsilon_L)}, \qquad \Omega_0 \equiv \left(\frac{1}{\mu}\right)^{\eta} \left(\frac{\mu-1}{f}\right)^{\eta-1} > 0, \tag{18}$$

where $K(t) \equiv N(t)\bar{K}(t)$, $L(t) \equiv N(t)\bar{L}(t)$, and Ω_0 is a constant. To ensure diminishing returns to capital accumulation, we impose Assumption 1. Note that this condition is rather mild. It is easily satisfied for $\varepsilon_L = 2/3$ and typical values of η (see Section 3.3).

Assumption 1 *The Ethier-productivity effects are bounded, that is,* $\chi \equiv 1-\eta(1-\varepsilon_L) > 0$, *implying that* $\partial^2 Y(t)/\partial K(t)^2 = -\chi(1-\varepsilon_L)\eta Y(t)/K(t)^2 < 0$.

Equations (17)–(18) show that η determines the degree of external increasing returns to scale at the aggregate level, whereas μ affects the equilibrium firm size.[15]

The stock market value of the firm, $V(t)$, equals $q(t)K(t)$. Accordingly, portfolio equilibrium amounts to $A(t) = q(t)K(t) + F(t)$. For $r > \alpha$, we assume that there are no net foreign assets in the initial steady state (i.e., $F(0) = 0$) so that the physical capital stock is fully owned domestically.

3 Solving the Model

This section log-linearizes the model around its steady state, analyzes its stability, and discusses calibration issues.

3.1 Log-linearized Model

To solve the model, we log-linearize it around an initial steady state (Appendix Table 1). A tilde (˜) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C^*$, for most variables

(Appendix A.1). The dynamics of the model can be summarized by two predetermined variables (i.e., the physical capital stock and financial assets) and two non-predetermined variables (i.e., Tobin's q and private consumption):

$$\begin{bmatrix} \dot{\tilde{K}}(t) \\ \dot{\tilde{q}}(t) \\ \dot{\tilde{C}}(t) \\ \dot{\tilde{A}}(t) \end{bmatrix} = \Delta \begin{bmatrix} \tilde{K}(t) \\ \tilde{q}(t) \\ \tilde{C}(t) \\ \tilde{A}(t) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_A(t) \end{bmatrix}, \tag{19}$$

where $\gamma_A(t) = r\omega_G \tilde{T}(t)$ is the exogenous policy shock and ω_G is the output share of government spending. The Jacobian matrix (and its typical element δ_{ij}) is:

$$\Delta \equiv \begin{bmatrix} 0 & \frac{\bar{y}\omega_I}{\sigma} & 0 & 0 \\ (1-\varepsilon_L)\,\bar{y}[1-\eta\phi\,(1-\varepsilon_L)] & r & (1-\varepsilon_L)\,\bar{y}\,(\phi-1) & 0 \\ 0 & 0 & r-\alpha & -\frac{r-\alpha}{\omega_A} \\ r\varepsilon_L\eta\phi\,(1-\varepsilon_L) & 0 & -r\,(\omega_C+\varepsilon_L(\phi-1)) & r \end{bmatrix},$$

where $\bar{y} \equiv Y^*/(q^*K^*)$, $\omega_A \equiv r/\bar{y}$, $\omega_C \equiv C^*/Y^*$, and $\omega_I \equiv I^*/Y^*$. The parameter regulating the strength of the intertemporal labor supply effect is given by:

$$\phi \equiv \frac{1+\theta_L}{1+\theta_L(1-\eta\varepsilon_L)} \geq 1, \tag{20}$$

where $\theta_L \equiv (1-L^*)/L^* \geq 0$ is the ratio of leisure to labor, which also represents the intertemporal substitution elasticity of labor supply. Labor supply is exogenous if $\varepsilon_C = 1$, in which case $\phi = 1$ (because $L^* = 1$ and $\theta_L = 0$). For $0 < \varepsilon_C < 1$, labor supply is endogenous and $\phi > 1$ (since $0 < L^* < 1$ and $\theta_L > 0$). We find $\partial\phi/\partial\eta > 0$, implying that the diversity effect magnifies the labor supply effect. To guarantee a positive denominator of (20), we impose:

Assumption 2 *If $\eta\varepsilon_L > 1$, we assume that $0 \leq \theta_L < \bar{\theta}_L \equiv 1/(\eta\varepsilon_L - 1)$.*

If $\eta\varepsilon_L > 1$ (due to a large η), ϕ has a vertical asymptote at $\bar{\theta}_L = 1/(\eta\varepsilon_L - 1)$. On the interval $(0, \bar{\theta}_L)$, ϕ is an increasing function of θ_L, which exceeds unity.

In the next subsection we show that the model is saddle-point stable for ϕ values in the range, $1 \leq \phi \leq \hat{\phi}$.[16] To streamline the discussion to follow, we provide the following definitions regarding the strength of the intertemporal labor supply effect:

Definition 1 *The labor supply effect is small for* $1 < \phi < \bar{\phi} \equiv 1/(\eta(1 - \varepsilon_L))$*, whereas it is large for* $\bar{\phi} < \phi \leq \hat{\phi}$.

3.2 Stability

The dynamics of the finite horizon model depends crucially on the intertemporal labor supply effect. The trivial special case of exogenous labor supply (i.e., $\theta_L = 0$ and $\phi = 1$, so that $\delta_{23} = 0$ in (19)) renders the model recursive, that is, the *investment system* (denoted by $\tilde{q}(t), \tilde{K}(t)$) can be solved completely independently from the *savings system* (denoted by $\tilde{C}(t), \tilde{A}(t)$). This special case always yields a saddle-point stable steady state. For $\theta_L > 0$, however, $\phi > 1$ and $\delta_{23} > 0$, so that the investment system is non-recursive. Provided $\phi < \hat{\phi}$, we find two negative roots and two positive roots that are potentially complex valued (with two negative and two positive real parts). Consequently, the system with endogenous labor supply is also saddle-point stable (Proposition 1). In the stable complex case, the analytical solution for the transition paths of the variables includes cosine and sine terms, which give rise to endogenously determined (dampened) cycles (Appendix A.4).[17] Proposition 1 summarizes the local stability properties of the system.

Proposition 1 *If* $\phi \in [1, \hat{\phi})$*, the overlapping generations model (*$\beta > 0$*) has a unique and locally saddle-point stable steady state, featuring four characteristic roots that are poten-*

tially complex valued. The complex roots have two negative real parts and two positive real parts.

Proof *See Appendix A.2.1.*

Our model nests the infinite horizon case for which $\beta = 0$. To ensure the existence of a steady state for this special case, the knife-edge condition $r = \alpha$ should hold. Notice that the economy would keep accumulating assets (and cease being small in world capital markets) if $r > \alpha$ or be depleting assets if $r < \alpha$. In addition, $\phi < \bar{\phi}$ is a necessary condition for saddle-path stability. As compared to finite horizons, smaller values of η and θ_L are permitted in the infinite horizon framework (see also the discussion in Section 3.3). For infinite horizons, the rate of growth of aggregate consumption does not depend on the holdings of financial assets. Mathematically, in terms of the Jacobian matrix, we have $\delta_{33} = \delta_{34} = 0$ (i.e., the third row of Δ consists of zeros), yielding a singular Jacobian matrix. Thus, the infinite horizon model introduces a zero root in private consumption and labor supply, making the steady-state *levels* of the variables dependent on the initial stock of financial assets (Proposition 2). Provided labor supply is elastic (i.e., $\phi > 1$ so that $\delta_{23} > 0$), there is also hysteresis in the physical capital stock and all variables dependent on it.[18]

Proposition 2 *The infinite horizon model (imposing $\beta = 0$ and $r = \alpha$) features a hysteretic steady state. To guarantee saddle-point stability, it is required that $\phi < \bar{\phi}$. The four characteristic roots are real and distinct: $h_1^* = 0$, $-h_2^* = (r - \sqrt{r^2 + 4\delta_{12}\delta_{21}})/2$, $r_1^* = r$, and $r_2^* = (r + \sqrt{r^2 + 4\delta_{12}\delta_{21}})/2$.*

Proof *See Appendix A.2.2.*

3.3 Calibration

To study the quantitative significance of the comparative dynamics, a numerical treatment is pursued. Table 1 shows the parameter values, which are taken from the literature. The time unit represents a year. We follow Mendoza (1991), who calibrates a dynamic general equilibrium model for the Canadian economy, in assigning values to δ, ε_L, r, and θ_L. In the benchmark model, the intertemporal substitution elasticity of labor supply (θ_L) is set to 2.25. This implies a labor supply effect that is *large* (i.e., $\phi = 2.577$ and $\bar{\phi} \equiv 2.404 < \phi < \hat{\phi} \equiv 2.732$). The value of the external economies of scale parameter ($\eta = 1.30$) is taken from Caballero and Lyons (1992). It gives rise to $\eta\varepsilon_L = 0.88$, which implies a downward sloping labor demand function. We thus do not need to invoke Assumption 2. We also arrive at $\chi = 0.584$, implying that Assumption 1 is easily satisfied. Following Baxter and King (1993), public consumption-to-output ratio (ω_G) is set to 20 percent. Letendre (2004) uses roughly the same value for the Canadian economy. Last but not least, we assume a probability of death (β) of 1.5 percent (cf. Cardia, 1991), so that agents have an expected life span of 67.7 years.

****** INSERT TABLE 1 ABOUT HERE ******

We have chosen a logarithmic specification for the installation function:[19]

$$\Psi(x) \equiv \bar{z} \ln\left(\frac{x+\bar{z}}{\bar{z}}\right), \tag{21}$$

where $\bar{z}$ is an exogenous constant and $x \equiv I/K$. From (21) and the definition of σ, we derive $\sigma = x/(x+\bar{z})$, which features an asymptote at $x = -\bar{z}$. We have set the steady-state value for x at $x^* = 0.11$ and choose $\bar{z} = 0.532$, implying steady-state adjustment costs of about 0.2 percent of output. The latter value is roughly in line with that employed by Mendoza (1991), who calibrates adjustment costs of 0.1 percent of output. The degree of physical capital mobility is $\sigma = 0.17$.

The pure rate of time preference (α) is used as a calibration parameter. Its implied value is 3.91 percent. Once the parameters are set, all other information on output shares, Tobin's q, and the output-capital ratio can be derived.[20] We find an investment-output ratio of 22.2 percent and a consumption-output ratio of 57.8 percent. Given the fixed rate of interest, our calibration yields rising individual consumption profiles in the finite horizon model. All four characteristic roots are complex valued. The roots feature two negative and two positive real parts (i.e., $\nu, \bar{\nu} = -0.0204 \pm 0.0590i$ and $\lambda, \bar{\lambda} = 0.0609 \pm 0.0587i$, where i denotes the imaginary unit). Note that in the special case of infinite horizons, we employ the same fundamental parameters, except that we set $\beta = 0$ and $r = \alpha = 0.04$.

****** INSERT FIGURE 1 ABOUT HERE ******

Figure 1 shows the parameter values for which the model is stable while distinguishing between real roots (yielding non-cyclical dynamics) and complex roots (yielding cyclical dynamics). The dashed line represents the upper bound of the stable region in the finite horizon model (i.e., combinations of η and θ_L for which $\phi = \hat{\phi}$). Panel (a) of Figure 1 shows that it lies above the dotted line (representing the bound $\bar{\phi}$, which also defines the upper stability bound for the infinite horizon model), indicating that the stable region is smaller for infinite horizons. Indeed, the critical η for which the infinite horizon model is still stable at benchmark values (see the asterisk) lies below the calibration point (denoted by

C). The latter is situated in the stable, cyclical region, where the cycles are of a first-order magnitude. The solid line (demarcating the upper bound on the non-cyclical region) shows that a smaller value of θ_L needs to be compensated by a higher η to obtain cycles. In the absence of external increasing returns to scale (i.e., $\eta = 1$), we end up on the horizontal axis of Panel (a), yielding a stable (non-cyclical) outcome. The solid line approaches the θ_L-axis only if $\theta_L \to \infty$, whereas it approaches the η-axis at relatively small values (i.e., $\eta = 3.11$; not drawn). Panel (b) shows that for $\beta = 0$ or $\eta = 1$ we can never end up in the cyclical region, reflecting the real nature of the roots. For the benchmark value of $\eta = 1.30$, the infinite horizon model is unstable. Taking a slightly smaller value (i.e., $\eta = 1.25$), brings us in the saddle-path stable region. The lower part of the figure shows that for smaller values of η, a higher β is needed to take the economy into the cyclical range. In sum, all three elements (i.e., endogenous labor supply, external economies of scale, and finite horizons) are necessary to give rise to cyclical dynamics.

4 The Macroeconomic Effects of Fiscal Policy

This section studies the short-run, transitional, and long-run effects of unanticipated and permanent fiscal policy shocks (i.e., $\tilde{G} > 0$) financed by lump-sum taxes. Unanticipated shocks are defined as shocks for which the dates of announcement and implementation of the policy change coincide. The sensitivity of the results to alternative parameterizations is studied. First, an analytical discussion of the long-run results is provided. Next, the impact and transitional effects are quantified and visualized.

4.1 Long-Run Analytical Results

The analytical expressions for the long-run allocation effects are obtained by solving (19), which yields $[\tilde{K}(\infty), \tilde{q}(\infty), \tilde{C}(\infty), \tilde{A}(\infty)]' = \Delta^{-1}\Gamma$, where Γ is the vector of shock variables, Δ^{-1} is the inverse of Δ, and $t \to \infty$ denotes the long run. The response of the investment system is:

$$\frac{\tilde{K}(\infty)}{\tilde{G}} = \frac{\eta\varepsilon_L\bar{\phi}}{\bar{\phi}-1}\frac{\tilde{L}(\infty)}{\tilde{G}} = -\frac{\delta_{12}\delta_{23}\delta_{34}\omega_G}{|\Delta|} > 0, \qquad \frac{\tilde{q}(\infty)}{\tilde{G}} = 0, \tag{22}$$

and that of the savings system:

$$\frac{dC(\infty)}{dG} = \frac{\delta_{34}\delta_{12}\delta_{21}}{|\Delta|} \lesseqgtr 0, \qquad \frac{\tilde{A}(\infty)}{\tilde{G}} = \frac{\delta_{33}\omega_G}{|\Delta|} > 0, \tag{23}$$

where δ_{12} is unambiguously positive and $|\Delta| > 0$ denotes the determinant of Δ. For the case of finite horizons (i.e., $r > \alpha$), the elements of Δ take on the following signs: $\delta_{33} > 0$ and $\delta_{34} < 0$. We can now demonstrate the importance of the labor supply effect. If labor supply is exogenous, it follows that $\delta_{21} > 0$ and $\delta_{23} = 0$. Consequently, a rise in public spending does not affect the long-run capital stock (see equation (22)). Equation (23) shows that public consumption crowds out private consumption one-for-one in the long run. If labor supply is endogenous ($\phi > 1$ and thus $\delta_{23} > 0$), the long-run capital stock and employment both rise. Interestingly, the sign of the effect on long-run private consumption depends on the size of the labor supply effect. If $1 \leq \phi < \bar{\phi}$ (so that $\delta_{21} > 0$) private consumption falls, whereas for $\bar{\phi} < \phi < \hat{\phi}$ (so that $\delta_{21} < 0$) private consumption rises. To clarify the long-run relationship between private consumption and employment, we introduce the following two expressions:[21]

$$\begin{aligned} \frac{\phi-\bar{\phi}}{\bar{\phi}}\tilde{K}(\infty) &= (\phi-1)\,\tilde{C}(\infty), & (24)\\ \eta\varepsilon_L\tilde{L}(\infty) &= (\phi-1)\left[\frac{1}{\bar{\phi}}\tilde{K}(\infty) - \tilde{C}(\infty)\right]. & (25) \end{aligned}$$

Equation (24) describes long-run capital market equilibrium. As a result of openness, the long-run supply of capital is horizontal, that is, $\tilde{r}^K(\infty) = 0$. If $\phi > \bar{\phi}$ ($\phi < \bar{\phi}$), long-run capital demand is upward (downward) sloping. In either case, an increase in private consumption leads to a downward shift in capital demand. Equation (25) describes long-run labor market equilibrium. Intuitively, an increase in $\tilde{K}(\infty)$ boosts equilibrium employment via demand whilst an increase in $\tilde{C}(\infty)$ reduces employment via the fall in the supply of labor. Using (24)–(25), we can eliminate $\tilde{K}(\infty)$ and derive the long-run relationship between private consumption and employment:

$$\frac{\tilde{C}(\infty)}{\tilde{L}(\infty)} = \frac{\eta\varepsilon_L(\phi - \bar{\phi})}{(\phi - 1)(\bar{\phi} - 1)} \gtreqless 0, \qquad \phi \gtreqless \bar{\phi}. \tag{26}$$

As was pointed out above, a fiscal impulse increases both employment and the capital stock in the long run. In contrast to the perfectly competitive model (for which ϕ must be less than $\bar{\phi}$), private consumption rises if the labor supply effect is large (i.e., $\phi > \bar{\phi}$). In this case, equation (24) shows that capital market equilibrium is restored at a higher level of consumption. Intuitively, a given rise in investment yields more output as a result Ethier-productivity effects, thereby creating room for a rise in private consumption.

4.2 Quantitative Short-Run and Long-Run Effects

Table 2 summarizes numerical results for the impact effect (recorded at $t = 0$) and the long-run effect (taken at $t \to \infty$) of a fiscal shock of size $\tilde{G} = 0.1$. We consider three calibrations for the finite horizon case ($\beta > 0$): (i) the benchmark calibration of $\eta = 1.30$ (yielding a large labor supply effect, that is, $\bar{\phi} < \phi < \hat{\phi}$); (ii) the alternative case of $\eta = 1.25$ (generating a small labor supply effect, that is, $1 < \phi < \bar{\phi}$); and (iii) $\eta = 1.00$ (in which case there are no external economies of scale). Under infinite horizons (i.e.,

$\beta = 0$), we consider the latter two calibrations only, owing to the instability of the model for the benchmark value of η. On the size of output multipliers, we find the following two results. First, short-run output multipliers are shown to fall short of those found in the long run irrespective of the type of model. Second, for the benchmark calibration, long-run output multipliers are substantially bigger than unity, reflecting the presence of external economies of scale.

****** INSERT TABLE 2 ABOUT HERE ******

External economies of scale increase long-run output multipliers, implying that input diversity (generated by monopolistic competition) truly matters. But external economies of scale decrease short-run output multipliers (compare columns (1)–(2) and (3)–(5)). Because of the Ethier-productivity effects, employment rises by less than without external economies of scale, thus yielding a smaller output gain. The external economies of scale (if $\eta = 1.3$) are responsible for a rise in long-run private consumption (see also the first expression of (23) and the surrounding discussion). Long-run private consumption and employment both rise (see also (26)). Because of the predetermined capital stock, public consumption crowds out short-run private consumption.

Without Ethier-productivity effects (see columns (1) and (3)),[22] we obtain a long-run output multiplier a little above unity. In this case, the long-run output multipliers of the finite and infinite horizon models are very similar in size.[23] Consumption multipliers are negative both in the short and long run, which is in line with standard findings in the literature. Long-run employment effects are smaller and wage effects are larger than in the benchmark model. Intuitively, without Ethier-productivity effects, the long-run capital-labor ratio is unaffected by the fiscal shock, explaining why steady-state wages do not change.

4.3 Transitional Dynamics

We use the analytical impulse response functions (as derived in Appendix A.4) to plot impulse-response diagrams for the key macroeconomic variables over 200 years. We first discuss the dynamic linkages between variables for the benchmark calibration (see the solid line in Figure 2). On impact, private consumption is crowded out by public consumption, owing to the rise in lump-sum taxes that is required to balance the government's budget. Consequently, households supply more labor (via the negative wealth effect in labor supply), which pushes down wages in the short run (not shown in the figure). Given the predetermined capital stock in the short run, the capital-labor ratio falls and output rises. Tobin's q rises, reflecting a rise in the marginal productivity of capital. Accordingly, private investment rises. The combined increase in investment and public spending exceeds the fall in private consumption thereby boosting domestic absorption. Because the output increase falls short of the rise in domestic absorption in the short run, the trade account swings into deficit. Net foreign debt starts accumulating.

****** INSERT FIGURE 2 ABOUT HERE ******

Private investment increases the physical capital stock over time and pushes up the capital-labor ratio. Since capital and employment are modeled as cooperative factors of production, the demand for labor will increase too. Consequently, wages rise gradually. Capital accumulation induces a fall in the marginal product of capital, gradually pushing down Tobin's q. The increment in the capital-labor ratio falls and thus Tobin's q rises again over time (and even goes through various cycles). In the new steady state, wages have risen, owing to an increased capital-labor ratio. Long-run private consumption rises, a rise in employment induced by capital accumulation. In the new steady state, the current account is balanced again, so that the trade balance surplus offsets interest payments on foreign debt. Because of the cyclical feature of the transitional dynamics, time periods with a negative association between consumption multipliers and output multipliers are followed by time periods with a positive association.

Figure 2 also presents transition paths for alternative values of θ_L. The dotted line shows the value of θ_L for which the real parts of the stable complex roots turn zero, that is, $\theta_L = 2.5355$, in which case we find $\phi = \hat{\phi} \equiv 2.732$. The dynamics of the system can then be characterized as a vortex, which generates cycles with a constant amplitude. Hence, there is no steady state. The dashed lines in Figure 2 represent a small value of θ_L for which the cycles disappear, owing to characteristic roots that are real. It can be seen that the bulk of adjustment toward the new steady state takes place during the first 20 years. The solid line is the benchmark value of $\theta_L = 2.25$, which shows dampened cycles of a first-order nature. Long-run output effects are positive and increasing in θ_L. Furthermore, the amplitude of the cycles increases for larger values of θ_L within the feasible region.

Figure 3 plots the impulse response functions for $\beta = 0$, $\beta = 0.015$, and $\beta = 0.05$. Finite horizons and infinite horizons yield very different transitional dynamics. The transition is monotonic in the infinite horizon case (dashed line), whereas it is non-monotonic for finite horizons (solid and dotted lines). Infinitely lived households face flat individual consumption profiles, represented by the horizontal dashed line in the consumption panel. This economy gradually accumulates domestic capital over time, explaining the smooth rise in output. In the finite horizon model, the long-run output effects of fiscal policy are unaffected by β (see the coinciding dotted and solid lines). Intuitively, the rate of interest is fixed, which pins down the capital-labor ratio and thus the long-run marginal product of capital. The size of β, however, does affect the transitional dynamics. A larger β gives rise to a less pronounced peak in the output path.

****** INSERT FIGURE 3 ABOUT HERE ******

5 Conclusions

The paper has analyzed the dynamic macroeconomic effects of fiscal policy shocks. To this end, a Blanchard-Yaari model for a small open economy is extended to include: (i) monopolistic competition in the intermediate goods sector (yielding Ethier-productivity effects in the final goods sector); and (ii) endogenous intertemporal labor supply. Such a framework gives rise to an endogenously determined (non-hysteretic) steady state, whereas the standard infinite horizon model features hysteresis. The comparative dynamic properties of the finite horizon model are compared with those of an infinitely lived representative agent model.

A number of key results can be extracted from the analysis. The first is that finite and infinite horizons give rise to very different impulse responses of a fiscal shock. The transition paths in the finite horizon case feature endogenously determined (dampened) cycles of a first-order nature. All three elements (i.e., endogenous intertemporal labor supply, Ethier-productivity effects, and finite horizons) are necessary to obtain cyclical dynamics. In the benchmark calibration, the infinite horizon model is unstable, suggesting that finite horizons extend the parameter range for which a stable steady state materializes.

Intermediate values of the Ethier parameter (for which both models are stable) give rise to cycles of a second-order nature in the finite horizon model, whereas the transition is monotonic in the infinite horizon case. The two models deliver virtually identical impulse responses if the Ethier-productivity effect is switched off. Consequently, the often assumed approximate validity of infinite horizon models is tenuous in an environment characterized by Ethier-productivity effects.

A second result is that the *sign* of steady-state output multipliers of fiscal policy shocks is robust to parameter changes. Both long-run and short-run output multipliers are positive, where long-run output multipliers always exceed short-run output multipliers. The *size* of output multipliers, however, is affected by alternative parameterizations. Stronger Ethier-productivity effects boost output multipliers and more so in the infinite horizon model. Note that imperfect competition in itself does not affect the size of output multipliers. Smaller intertemporal substitution elasticities of labor supply reduce output multipliers, possibly below unity.

Another key result is that the sign of steady-state consumption multipliers is not robust to parameter changes. In the benchmark calibration, a fiscal impulse increases private consumption, reflecting strong Ethier-productivity effects. The latter increase the productivity of inputs, implying that a given rise in investment yields a larger increase in the economy's resources. In this context, a rise in public spending and investment does not have to come at the expense of private consumption. For small Ethier-productivity effects, however, we obtain the classic result of a negative private consumption multiplier.

There are of course many aspects of fiscal policy that have not been addressed here, such as the intergenerational welfare effects of fiscal policy, the output effects of anticipated fiscal shocks, and the optimal level of public spending. We leave these extensions for further research.

Appendix

A.1 Log-linearization

Log-linearizing the key expressions of the finite horizon model of Section 2 around an initial steady state (assuming that $F = 0$ initially) yields Appendix Table 1. The following notational conventions are employed. A tilde (˜) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C^*$, for most variables. Financial assets (i.e., $A(t)$ and $F(t)$), however, are scaled by steady-state output and multiplied by r, for example, $\tilde{A}(t) \equiv rdA(t)/Y^*$. Time derivatives are defined as $\dot{\tilde{C}}(t) \equiv d\dot{C}(t)/C^*$, except for financial assets, for example, $\dot{\tilde{A}}(t) \equiv rd\dot{A}(t)/Y^*$.

Conditional on the state variables and the policy shocks, the static part of the model can be condensed to the following quasi-reduced form expressions:

$$\tilde{Y}(t) = \eta\phi\,(1-\varepsilon_L)\,\tilde{K}(t) - (\phi-1)\tilde{C}(t), \tag{A.1}$$

$$\eta\varepsilon_L\tilde{L}(t) = (\phi-1)\left[\eta(1-\varepsilon_L)\tilde{K}(t) - \tilde{C}(t)\right], \tag{A.2}$$

$$\eta\varepsilon_L\tilde{w}(t) = (\eta\varepsilon_L-1)\tilde{Y}(t) + \eta(1-\varepsilon_L)\tilde{K}(t), \tag{A.3}$$

$$\tilde{r}^K(t) = [\eta\phi(1-\varepsilon_L)-1]\tilde{K}(t) - (\phi-1)\tilde{C}(t). \tag{A.4}$$

Equation (A.1) is obtained by using (AT1.6), (AT1.9), and (AT1.10). Using (A.1) and (AT1.6) we can also solve for $\tilde{L}(t)$, which yields (A.2). Equation (A.3) is derived from (AT1.6) and (AT1.10). The expression for the rental rate follows from combining (AT1.7) and (A.1). Given the level of private consumption, the capital demand curve slopes downward if $1 < \phi < \bar{\phi}$, but slopes upward for $\bar{\phi} < \phi < \hat{\phi}$ (Definition 1).

A.2 Stability

Using (A.1)–(A.2) and the expressions in Appendix Table 1, the system of equations can be written as in (19). Proposition 1 pertains to the general finite horizon case, whereas Proposition 2 considers infinite horizons.

A.2.1 Finite Horizons (Proposition 1)

For the general case of finite horizons, the following non-zero elements of Δ have an unambiguous sign:

$$\delta_{12}, \delta_{22}, \delta_{33}, \delta_{41}, \delta_{44} > 0, \tag{A.5}$$

$$\delta_{34}, \delta_{43} < 0. \tag{A.6}$$

The sign of δ_{21} depends on the strength of the labor supply effect:

$$\delta_{21} \lesseqgtr 0, \qquad \phi \gtreqless \bar{\phi} \equiv \frac{1}{\eta(1-\varepsilon_L)}. \tag{A.7}$$

If labor supply is endogenous (i.e., $\phi > 1$), we can also determine that $\delta_{23} > 0$.

Solving the dynamic system (19) gives rise to a characteristic polynomial of the fourth order:

$$P(s) \equiv |s\mathrm{I} - \Delta| = \varphi(s)\,\psi(s) - \delta_{12}\delta_{23}\delta_{34}\delta_{41} = 0, \tag{A.8}$$

where I is the identity matrix and $\varphi(s)$ and $\psi(s)$ are:

$$\begin{aligned} \varphi(s) &\equiv (s-\delta_{33})(s-\delta_{22}) - \delta_{34}\delta_{43}, && \text{(A.9)} \\ \psi(s) &\equiv s(s-\delta_{22}) - \delta_{12}\delta_{21}. && \text{(A.10)} \end{aligned}$$

We can rewrite $P(s)$ as:

$$P(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0, \tag{A.11}$$

where the a_i's are:

$$
\begin{aligned}
a_3 &\equiv -\mathrm{tr}(\Delta) = -(2\delta_{22} + \delta_{33}) = -(3r - \alpha) < 0, \\
a_2 &= \delta_{22}^2 - \delta_{12}\delta_{21} + 2\delta_{22}\delta_{33} - \delta_{34}\delta_{43} \gtrless 0, \\
a_1 &= \delta_{12}\delta_{21}(\delta_{22} + \delta_{33}) + \delta_{22}\left[\delta_{34}\delta_{43} - \delta_{22}\delta_{33}\right] \gtrless 0, \\
a_0 &\equiv |\Delta| = -\delta_{12}\left[\delta_{12}(\delta_{22}\delta_{33} - \delta_{34}\delta_{43}) + \delta_{23}\delta_{34}\delta_{41}\right] \\
&= \frac{r}{\omega_A}\frac{\bar{y}^2\omega_I}{\sigma}\left(1 - \varepsilon_L\right)(r - \alpha)\left[\omega_G\left(\phi - 1\right) + \phi\chi\left(\omega_C - \omega_A\right)\right] > 0.
\end{aligned}
$$

Note that we have made use of $-\mathrm{tr}(\Delta) = -\left(\nu + \bar{\nu} + \lambda + \bar{\lambda}\right)$ and $|\Delta| = \nu\bar{\nu}\lambda\bar{\lambda}$, where $\bar{\nu}$ and $\bar{\lambda}$ are the characteristic roots of the investment system and ν and λ are the roots of the savings system. If all roots are complex, we find:

$$\nu \equiv -h^* + \theta_\nu i, \quad \bar{\nu} \equiv -h^* - \theta_\nu i, \quad \lambda \equiv r^* + \theta_\lambda i, \quad \bar{\lambda} \equiv r^* - \theta_\lambda i, \tag{A.12}$$

where an overbar denotes its complex conjugate and i is the imaginary unit. We define h^* and r^* to be positive. The first terms of the roots in (A.12) represent the real parts.[24] If the roots are real, the cyclical terms, θ_k, disappear from (A.12).

The positive determinant (see $a_0 > 0$) may either indicate two positive roots and two negative roots or four positive roots (in which case the system is unstable). The case of four negative roots—giving rise to an indeterminate steady state (see Benhabib and Farmer, 1994, p. 30)—is excluded because of the positive trace of Δ (i.e., $\mathrm{tr}(\Delta) > 0$). To prove local stability, we can use Routh's criterion (cf. Shi and Epstein, 1993), which considers the number of sequential sign changes in the first column of the Routh scheme as an indicator of the number of unstable roots. The first column of the Routh scheme corresponding to (A.11) is:

$$1, \quad a_3, \quad b_1, \quad c_1, \quad a_0, \tag{A.13}$$

where we have used that $a_4 = 1$. The coefficients b_1 and c_1 are defined as:

$$b_1 \equiv a_2 - \frac{a_1}{a_3}, \quad c_1 \equiv a_1 - \frac{b_2}{b_1}a_3, \quad b_2 = a_0. \tag{A.14}$$

We note from (A.11) that $a_3 < 0$, generating already one sign change when we move from the first to the second element on the left-hand side of (A.13). The signs of b_1 and c_1 are not determined yet, because we do not know the signs of a_1 and a_2. This gives rise to four possible cases: (i) $a_1 > 0$ and $a_2 > 0$; (ii) $a_1 < 0$ and $a_2 < 0$; (iii) $a_1 > 0$ and $a_2 < 0$; and (iv) $a_1 < 0$ and $a_2 > 0$. It is immediately evident that cases (i)–(ii) yield unambiguously two sign changes in (A.13). As a result, we find two stable roots and two unstable roots. The sign changes in case (iii) are as follows. If $b_1 > 0$, we find $c_1 > 0$, giving rise to two sign changes. If $b_1 < 0$ then $c_1 \lesseqgtr 0$. In either case, there are just two sign changes. This leaves us with case (iv) for which further restrictions have to be imposed to determine the number of sign changes. If $b_1 < 0$, we find two sign changes. Conditional on $b_1 > 0$ and $\phi < \hat{\phi}$, we get $c_1 > 0$, thus yielding two sign changes. Thus, for $1 \leqslant \phi < \hat{\phi}$, the equilibrium is unique and saddle-path stable.

A.2.2 Infinite Horizons (Proposition 2)

For infinite horizons, we find $\delta_{33} = \delta_{34} = 0$, so that the third row of Δ consists of zeros. The polynomial takes the form:

$$P(s) \equiv s(s - \delta_{22}) \left[s^2 - \delta_{22}s - \delta_{12}\delta_{21}\right] = 0. \tag{A.15}$$

The roots are thus real (assuming that $\phi < \bar{\phi}$) and distinct:

$$h_1^* = 0, \tag{A.16}$$

$$-h_2^* = \frac{\delta_{22} - \sqrt{\delta_{22}^2 + 4\delta_{12}\delta_{21}}}{2}, \tag{A.17}$$

$$r_1^* = \delta_{22}, \tag{A.18}$$

$$r_2^* = \frac{\delta_{22} + \sqrt{\delta_{22}^2 + 4\delta_{12}\delta_{21}}}{2}, \tag{A.19}$$

yielding $\mathrm{tr}(\Delta) = 2r$. If the labor supply effect is small (i.e., $\phi < \bar{\phi}$), it follows readily that $\delta_{21} \equiv (1-\varepsilon_L)\,\bar{y}[1-\eta\phi\,(1-\varepsilon_L)] > 0$. Because $\delta_{12} > 0$ and $\delta_{22} > 0$, the discriminant in (A.17) and (A.19) is positive so that $\sqrt{\delta_{22}^2 + 4\delta_{12}\delta_{21}} > \delta_{22} > 0$. We thus find that $-h_2^* < 0$ and $r_2^* > 0$. The steady state is well defined because the number of non-predetermined variables equals the number of roots with strictly positive real parts (cf. Giavazzi and Wyplosz, 1985). For $\phi \geq \bar{\phi}$, the discriminant may turn negative, so that $-h_2^*$ and r_2^* are complex conjugates with real part $\delta_{22} > 0$. Because the real part is positive, the steady state is unstable.

A.3 Solving for the Comparative Dynamics

The Laplace transform method of Judd (1982) is used to solve the model. By taking the Laplace transform of (19), and noting that $\tilde{K}(0) = 0$ and $\tilde{A}\,(0) = \omega_A \tilde{q}\,(0)$, we obtain:

$$\Lambda(s) \begin{bmatrix} \mathcal{L}\{\tilde{K}, s\} \\ \mathcal{L}\{\tilde{q}, s\} \\ \mathcal{L}\{\tilde{C}, s\} \\ \mathcal{L}\{\tilde{A}, s\} \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{q}(0) \\ \tilde{C}(0) \\ \omega_A \tilde{q}\,(0) - \mathcal{L}\{\gamma_A, s\} \end{bmatrix}, \tag{A.20}$$

where $\Lambda(s) \equiv |s\mathrm{I}-\Delta|$ and $\mathcal{L}$ denotes the Laplace transform operator.[25] By pre-multiplying both sides of (A.20) by

$$\Lambda(s)^{-1} \equiv \frac{1}{(s-\nu)(s-\bar{\nu})(s-\lambda)(s-\bar{\lambda})}\operatorname{adj}\Lambda(s), \tag{A.21}$$

and rearranging we find the following expression in Laplace transforms:

$$(s-\nu)(s-\bar{\nu})\begin{bmatrix} \mathcal{L}\{\tilde{K},s\} \\ \mathcal{L}\{\tilde{q},s\} \\ \mathcal{L}\{\tilde{C},s\} \\ \mathcal{L}\{\tilde{A},s\} \end{bmatrix} = \frac{\operatorname{adj}\Lambda(s)\begin{bmatrix} 0 \\ \tilde{q}(0) \\ \tilde{C}(0) \\ \omega_A\tilde{q}\left(0\right)-\mathcal{L}\{\gamma_A,s\} \end{bmatrix}}{(s-\lambda)(s-\bar{\lambda})}. \tag{A.22}$$

The adjoint matrix is equal to:

$$\operatorname{adj}\Lambda(s) \equiv \begin{bmatrix} \left(s-\delta_{22}\right)\varphi\left(s\right) & \delta_{12}\varphi\left(s\right) & \delta_{12}\delta_{23}\left(s-\delta_{22}\right) & \delta_{12}\delta_{23}\delta_{34} \\ \delta_{21}\varphi\left(s\right)+\delta_{23}\delta_{34}\delta_{41} & s\varphi\left(s\right) & \delta_{23}s\left(s-\delta_{22}\right) & \delta_{23}\delta_{34}s \\ \delta_{34}\delta_{41}\left(s-\delta_{22}\right) & \delta_{12}\delta_{34}\delta_{41} & \left(s-\delta_{22}\right)\psi\left(s\right) & \delta_{34}\psi\left(s\right) \\ \delta_{41}\left(s-\delta_{22}\right)\left(s-\delta_{33}\right) & \delta_{12}\delta_{41}\left(s-\delta_{33}\right) & \delta_{43}\psi\left(s\right)+\delta_{12}\delta_{23}\delta_{41} & \left(s-\delta_{33}\right)\psi\left(s\right) \end{bmatrix}.$$

A.4 Analytical Impulse Responses

This section derives analytical impulse response functions of fiscal shocks. The mathematical expressions pertain to the case of complex roots. We can easily show that both the impact and long-run results are still valid even if the stable roots are real and distinct (the unstable roots can be complex or real). In the latter case, the expressions for the transition terms differ from those under complex roots because the cyclical terms disappear.[26]

A.4.1 Impact Effects

The jumps in $\tilde{C}(0)$ and $\tilde{q}\,(0)$ can be derived from (A.22). Because the rank of $\operatorname{adj}\Lambda(s)$ equals 1 (for $s = \lambda, \bar{\lambda}$) either row of the matrix can be used. Using the first row of $\operatorname{adj}\Lambda(s)$, for example, we get a system of two equations in $\tilde{C}(0)$ and $\tilde{q}(0)$, which can be solved to yield:

$$\begin{bmatrix} \tilde{q}\,(0) \\ \tilde{C}\,(0) \end{bmatrix} = \delta_{23}\delta_{34} \begin{bmatrix} \varphi\,(\lambda) + \delta_{23}\delta_{34}\omega_A & \delta_{23}\,(\lambda - \delta_{22}) \\ \varphi\left(\bar{\lambda}\right) + \delta_{23}\delta_{34}\omega_A & \delta_{23}\left(\bar{\lambda} - \delta_{22}\right) \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{L}\{\gamma_A, \lambda\} \\ \mathcal{L}\{\gamma_A, \bar{\lambda}\} \end{bmatrix}. \tag{A.23}$$

A.4.2 Transition Paths

The analytical expressions for the transition paths (see (A.24)–(A.27) below), feature temporary transition terms and a general adjustment term, which are specified in Definitions 2–3. The transition and adjustment terms consist of exponential functions weighted by functions generating periodic cycles:

Definition 2 *The first temporary transition term,*

$$\mathbf{T}_1\,(h^*, \theta_\nu, t) \equiv \frac{1}{\theta_\nu} e^{-h^* t} \sin\theta_\nu t,$$

has properties: (i) $\mathbf{T}_1\,(h^*, \theta_\nu, 0) = 0$*; and (ii)* $\lim_{t\to\infty} \mathbf{T}_1\,(h^*, \theta_\nu, t) = 0$.

Definition 3 *The second temporary transition term,*

$$\mathbf{T}_2\,(h^*, \theta_\nu, t) \equiv e^{-h^* t} \left[\cos\theta_\nu t - \frac{h^*}{\theta_\nu}\sin\theta_\nu t\right],$$

has properties: (i) $\mathbf{T}_2\,(h^*, \theta_\nu, 0) = 1$*; and (ii)* $\lim_{t\to\infty} \mathbf{T}_2\,(h^*, \theta_\nu, t) = 0$.

Definition 4 *The general adjustment term is given by:*

$$\mathbf{A}\,(h^*, \theta_\nu, t) \equiv \frac{1}{h^{*2} + \theta_\nu^2}\left[1 - e^{-h^* t}\left(\cos\theta_\nu t + \frac{h^*}{\theta_\nu}\sin\theta_\nu t\right)\right],$$

which has properties: (i) $\mathbf{A}(h^*, \theta_\nu, 0) = 0$*; (ii)* $\lim_{t\to\infty} \mathbf{A}(h^*, 0, \theta_\nu, t) = 1/\left[(h^*)^2 + \theta_\nu^2\right]$*; and (iii)* $\lim_{t\to\infty} \mathbf{A}(h^*, \theta_\nu, t) = 0$*.*

We first study transition in the investment system. The transition path for the capital stock is derived by taking the inverse Laplace transform of the first row of (A.22):

$$\tilde{K}(t) = \delta_{12}\tilde{q}(0)\,\mathbf{T}_1(h^*, \theta_\nu, t) - \frac{\delta_{12}\delta_{23}\delta_{34} r\omega_G \tilde{G}}{\lambda\bar{\lambda}}\mathbf{A}(h^*, \theta_\nu, t). \tag{A.24}$$

Similarly, we can derive the path for Tobin's q:

$$\tilde{q}(t) = \tilde{q}(0)\,\mathbf{T}_2(h^*, \theta_\nu, t) + \left[\left(\lambda + \bar{\lambda} - \delta_{22} - \delta_{33}\right)\tilde{q}(0) + \delta_{23}\tilde{C}(0)\right]\mathbf{T}_1(h^*, \theta_\nu, t). \tag{A.25}$$

The second term in (A.24) drops out for exogenous labor supply (i.e., $\delta_{23} = 0$) or for infinite horizons (i.e., $\delta_{34} = 0$) or both. In addition, the cosine and sine terms disappear from the transition terms for these cases.[27] In this context, the adjustment speed to the new steady state is driven by h^*.

We now turn to the savings system. The paths for private consumption and financial capital are:

$$\begin{aligned}
\tilde{C}(t) &= \left[\delta_{34}\omega_A\tilde{q}(0) + \left(\lambda + \bar{\lambda} - 2\delta_{22}\right)\tilde{C}(0)\right]\mathbf{T}_1(h^*, \theta_\nu, t) \\
&\quad + \tilde{C}(0)\,\mathbf{T}_2(h^*, \theta_\nu, t) + \frac{\delta_{34}\delta_{12}\delta_{21} r\omega_G \tilde{G}}{\lambda\bar{\lambda}}\mathbf{A}(h^*, \theta_\nu, t),
\end{aligned} \tag{A.26}$$

$$\begin{aligned}
\tilde{A}(t) &= \left[\omega_A\left(\lambda + \bar{\lambda} - \delta_{22} - \delta_{33}\right)\tilde{q}(0) + \delta_{43}\tilde{C}(0)\right]\mathbf{T}_1(h^*, \theta_\nu, t) \\
&\quad + \omega_A\tilde{q}(0)\,\mathbf{T}_2(h^*, \theta_\nu, t) - \frac{\delta_{33}\delta_{12}\delta_{21} r\omega_G \tilde{G}}{\lambda\bar{\lambda}}\mathbf{A}(h^*, \theta_\nu, t).
\end{aligned} \tag{A.27}$$

Note that equations (A.1)–(A.3) can be used to derive the transition paths for $Y(t)$, $L(t)$, and $w(t)$. The paths for $F(t)$ and $I(t)$ follow from (AT1.12) and (AT1.8), respectively.

Appendix Table 1: The Log-linearized Model

$$\dot{\tilde{K}}(t) = \bar{y}\omega_I[\tilde{I}(t) - \tilde{K}(t)] \tag{AT1.1}$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - (1-\varepsilon_L)\,\bar{y}\left(\tilde{Y}(t) - \tilde{K}(t)\right) \tag{AT1.2}$$

$$\dot{\tilde{C}}(t) = (r-\alpha)\left[\tilde{C}(t) - (1/\omega_A)\tilde{A}(t)\right] \tag{AT1.3}$$

$$\dot{\tilde{A}}(t) = r\left[\tilde{A}\,(t) + \varepsilon_L(\tilde{w}(t) + \tilde{L}(t)) - \omega_T\tilde{T}(t) - \omega_C\tilde{C}(t)\right] \tag{AT1.4}$$

$$0 = \omega_G\tilde{G}(t) - \omega_T\tilde{T}(t) \tag{AT1.5}$$

$$\tilde{L}(t) = \tilde{Y}(t) - \tilde{w}(t) \tag{AT1.6}$$

$$\tilde{r}^K(t) = \tilde{Y}(t) - \tilde{K}(t) \tag{AT1.7}$$

$$\tilde{q}(t) = \sigma[\tilde{I}(t) - \tilde{K}(t)] \tag{AT1.8}$$

$$\tilde{L}(t) = \theta_L\left[\tilde{w}(t) - \tilde{C}(t)\right] \tag{AT1.9}$$

$$\tilde{Y}(t) = \eta\left[\varepsilon_L\tilde{L}(t) + (1-\varepsilon_L)\tilde{K}(t)\right] \tag{AT1.10}$$

$$\left(\frac{\eta-1}{\eta}\right)\tilde{Y}(t) = \varepsilon_L\tilde{w}(t) + (1-\varepsilon_L)\tilde{r}^K(t) \tag{AT1.11}$$

$$\tilde{A}(t) = \omega_A\left[\tilde{q}(t) + \tilde{K}(t)\right] + \tilde{F}(t) \tag{AT1.12}$$

Notes: The following definitions are used: $\varepsilon_L \equiv w^*L^*/Y^*$, $\bar{y} \equiv Y^*/(q^*K^*)$, $\omega_A \equiv r/\bar{y}$, $\omega_C \equiv C^*/Y^*$, $\omega_I \equiv I^*/Y^*$, $\omega_G \equiv G^*/Y^*$, $\omega_T \equiv T^*/Y^*$, and $\sigma \equiv -(I/K)^*(\Psi''/\Psi') > 0$. Asterisks indicate steady-state values of variables. A tilde ($\tilde{}$) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C^*$ for most variables. Financial assets (i.e., $A(t), F(t)$), however, are scaled by steady-state output and multiplied by r, for example, $\tilde{A}(t) \equiv rdA(t)/Y^*$.

Table 1: The Model Parameters

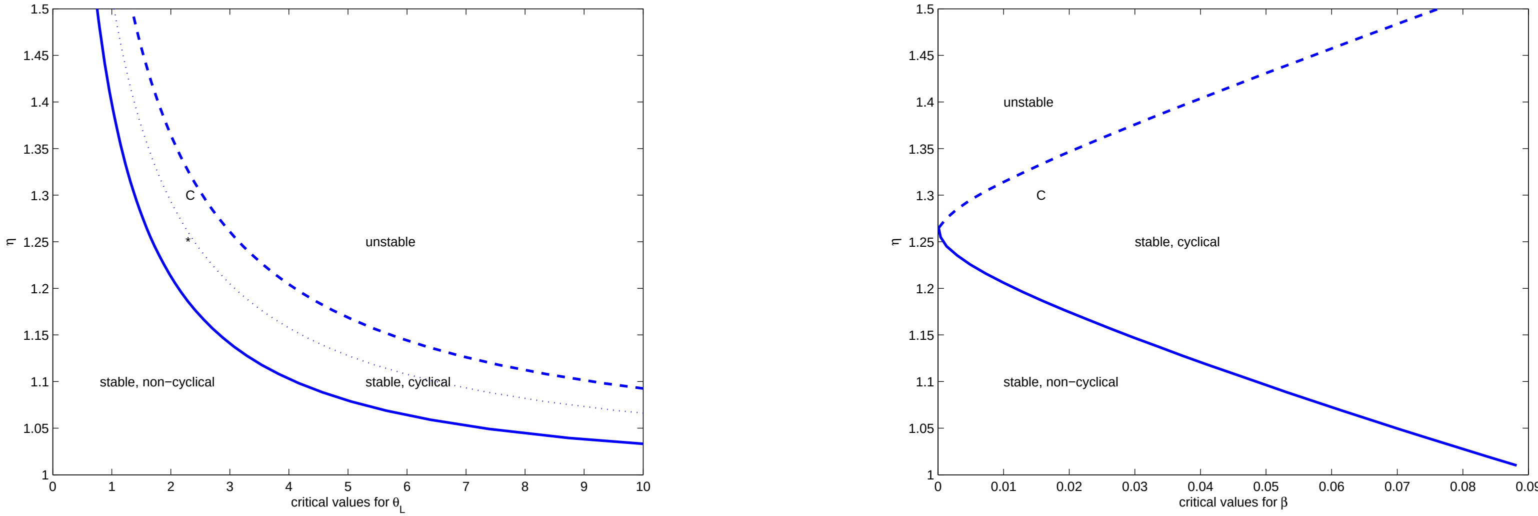
Parameter	β	δ	ε_L	η	ω_G	r	θ_L	$\bar{z}$
Value	0.015	0.100	0.680	1.300	0.200	0.040	2.250	0.532

Table 2: Allocation Effects in the Finite and Infinite Horizon Model

	Infinite horizons ($\beta = 0$)		Finite horizons ($\beta > 0$)		
	$\eta = 1.00$	$\eta = 1.25$	$\eta = 1.00$	$\eta = 1.25$	$\eta = 1.30$
	(1)	(2)	(3)	(4)	(5)
$\frac{dY(0)}{dG}$	0.4514	0.0902	0.4503	0.1625	0.0342
$\frac{dY(\infty)}{dG}$	1.1417	3.2168	1.1194	1.4505	1.5197
$\frac{dC(0)}{dG}$	−0.2931	−0.0364	−0.2924	−0.0656	−0.0125
$\frac{dC(\infty)}{dG}$	−0.2931	−0.0364	−0.2874	−0.0164	0.0402
$\frac{dI(0)}{dG}$	0.4103	0.2240	0.4108	0.3352	0.2507
$\frac{dI(\infty)}{dG}$	0.2539	0.7154	0.2490	0.3226	0.3380
$\frac{dX(0)}{dG}$	−0.0053	−0.0088	−0.6682	−1.1070	−1.2040
$\frac{dX(\infty)}{dG}$	0.0014	0.0123	0.1578	0.1443	0.1415
$\frac{\tilde{L}(0)}{\tilde{G}}$	0.0374	0.0212	0.1324	0.0382	0.0077
$\frac{\tilde{L}(\infty)}{\tilde{G}}$	0.2283	0.4541	0.2239	0.2048	0.2008
$\frac{\tilde{K}(0)}{\tilde{G}}$	0	0	0	0	0
$\frac{\tilde{K}(\infty)}{\tilde{G}}$	0.2283	0.6434	0.2239	0.2901	0.3004
$\frac{\tilde{w}(0)}{\tilde{G}}$	−0.0425	−0.0032	−0.0424	−0.0057	−0.0009
$\frac{\tilde{w}(\infty)}{\tilde{G}}$	0	0.1892	0	0.0853	0.1031

Notes: Unless indicated otherwise, all parameters are set at their benchmark values (see Table 1). The infinite horizon model sets $r = \alpha$ and $\beta = 0$. The finite horizon model is represented by $r > \alpha$ and $\beta = 0.015$. The benchmark calibration of $\eta = 1.30$ yields an unstable outcome in the infinite horizon model (see also Figure 1), explaining why this column is not presented.

Figure 1: Stability Regions for Various Values of η, θ_L, and β



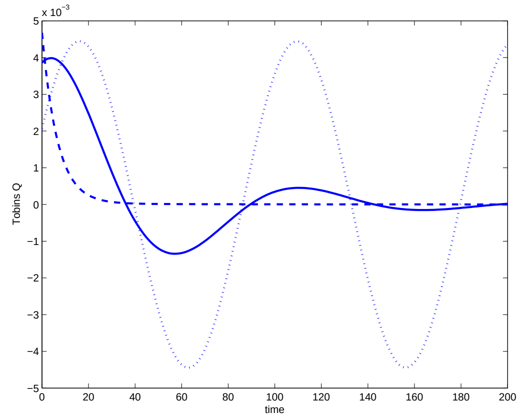
Panel (a)

Panel (b)

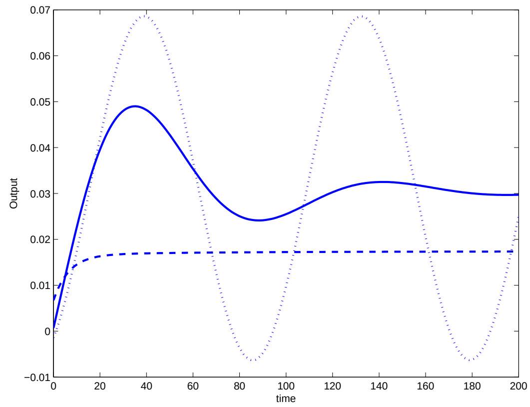
Notes: The solid line represents the upper bound on the stable, non-cyclical region for the finite horizon model, the dotted line demarcates the upper bound on the stable region in the infinite horizon model, and the dashed line denotes the lower bound of the unstable region in the finite horizon model. The area in between the solid line and the dashed line represents parameter combinations for which the finite horizon model yields stable, cyclical dynamics. C denotes the calibration point and $*$ indicates a value of η (given $\theta_L = 2.25$) for which the infinite horizon model is still stable.

Figure 2: Permanent Spending Shock ($\eta = 1.30$, Various Values of θ_L)

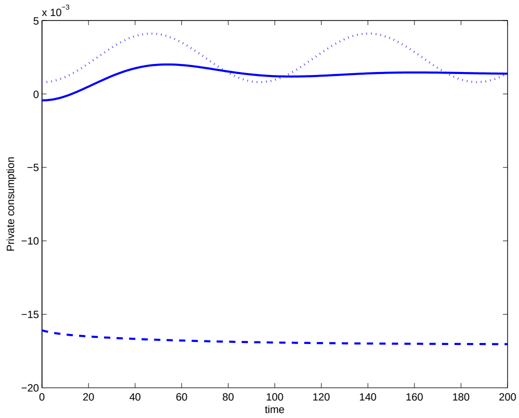
(a) $\tilde{q}(t)$



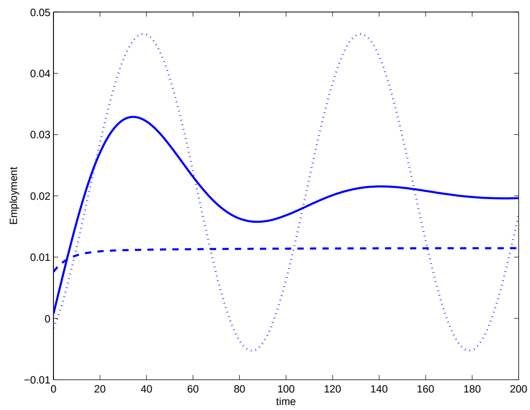
(b) $\tilde{Y}(t)$



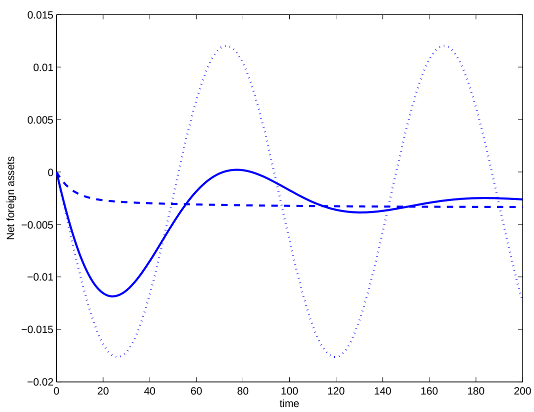
(c) $\tilde{C}(t)$



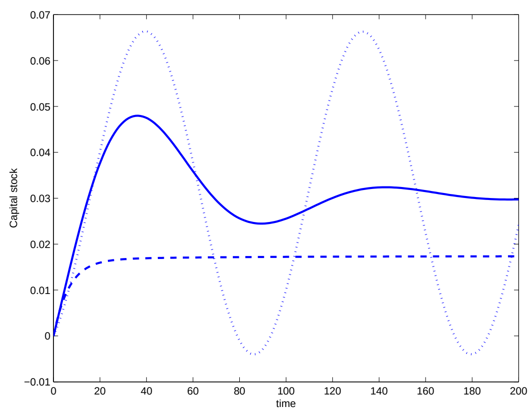
(d) $\tilde{L}(t)$



(e) $\tilde{F}(t)$



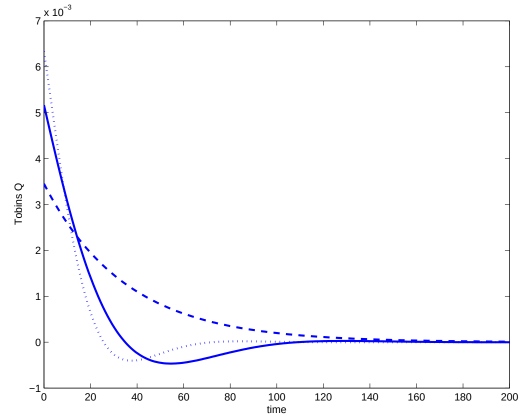
(f) $\tilde{K}(t)$



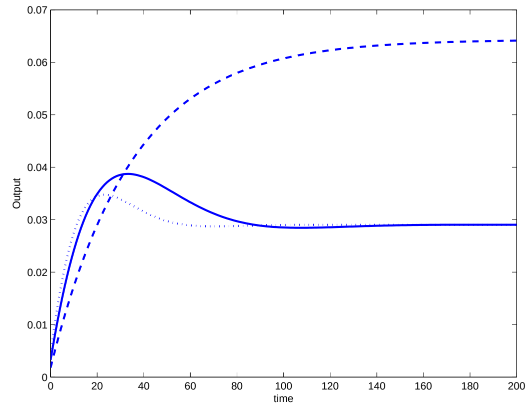
Notes: θ_L takes on the values 0.50 (dashed line), 2.25 (solid line), and 2.5355 (dotted line), respectively. The other parameters are set at their benchmark values.

Figure 3: Permanent Spending Shock ($\eta = 1.25$, Various Values of β)

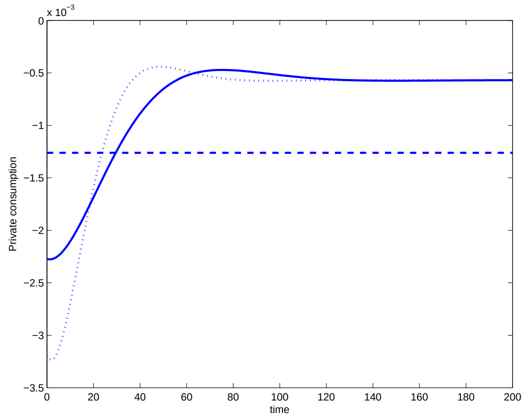
(a) $\tilde{q}(t)$



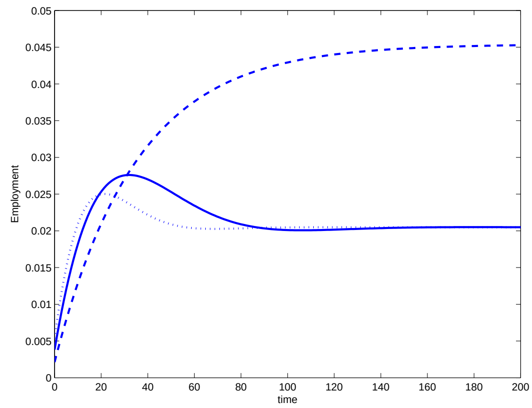
(b) $\tilde{Y}(t)$



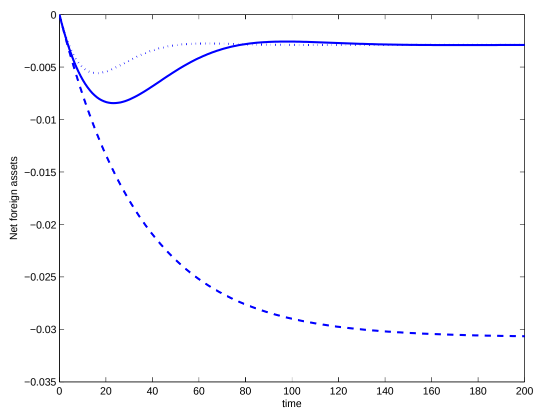
(c) $\tilde{C}(t)$



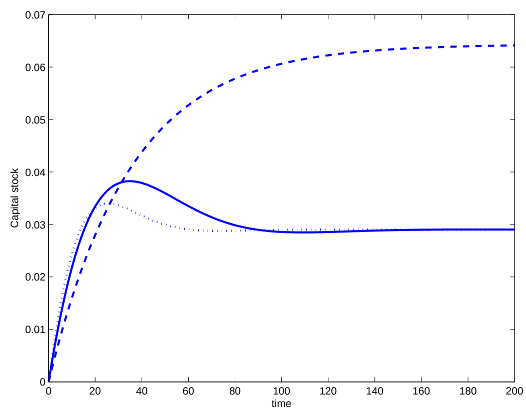
(d) $\tilde{L}(t)$



(e) $\tilde{F}(t)$



(f) $\tilde{K}(t)$



Notes: η is set to 1.25 and β takes on the values 0 (dashed line), 0.015 (solid line), and 0.050 (dotted line), respectively. The other parameters are set at their benchmark values.

Notes

1. The Stability and Growth Pact applied to the third stage of Economic and Monetary Union, which began on January 1, 1999. The Stability and Growth Pact was implemented to ensure that EU member states maintain budgetary discipline after the introduction of the euro.

2. There are many remotely connected contributions. Buiter (1981), Frenkel and Razin (1987), and Buiter and Kletzer (1991) study public spending policy in two-country models. Cardia (1991) and Mendoza (1991) analyze fiscal policy in a real business cycle (RBC) model. Giovannini (1988), Sen and Turnovsky (1989, 1990), and Bovenberg (1993) also employ small open economy models, but focus on tax or tariff policy rather than on public spending policy.

3. Devereux et al. (1996), Heijdra (1998), and Heijdra and Ligthart (2007) analyze fiscal policy in a closed economy in which goods markets are imperfectly competitive. The first paper takes a stochastic RBC approach, whereas the latter two assume a deterministic setting.

4. If the rate of interest exceeds (falls short of) the pure rate of time preference, households permanently accumulate (deplete) foreign assets. To obtain a steady state, the exogenous world rate of interest should equal the pure rate of time preference. See Turnovsky (2002).

5. The steady state of the infinite horizon model depends on the initial conditions. This implies that temporary shocks will have permanent effects on the economy, that is, the equilibrium dynamics possess hysteresis (or non-stationarity in the stochastic

environment of an RBC model).

6. Authors have used various specifications of overlapping generations. Giovannini (1988), Bovenberg (1993), and Cardia (1993) also use the Blanchard-Yaari framework. Buiter (1981) and Buiter and Kletzer (1991) employ a Diamond (1965)-style life-cycle model.

7. Schmitt-Grohe and Uribe (2003) consider four instruments to arrive at an endogenously determined steady state: (i) an endogenous discount factor; (ii) a debt-elastic interest premium; (iii) convex portfolio adjustment costs; and (iv) complete asset markets.

8. Employing habit formation as a stationarity-inducing device in a framework of infinitely lived households, Karayalçin (2003) also finds cycles of a first-order magnitude.

9. The existence of imperfect competition is a necessary but not a sufficient condition for this result. Indeed, in the absence of Ethier-productivity effects, finite and infinite horizons yield very similar impulse responses originating from a fiscal shock, supporting the widely held view that finite horizons can be approximated by infinite horizons (see Bernheim, 1987).

10. Rising individual consumption profiles imply a positive stock of financial assets in the initial equilibrium. By using (5) in steady state, we arrive at $(r-\alpha)C^* = \beta\varepsilon_C(\alpha+\beta)A^*$, where asterisks indicate steady-state values of variables. For the general case of $\beta > 0$ and $r - \alpha > 0$, we find $A^* > 0$.

11. We use $C(t) = \varepsilon_C(\alpha+\beta)[A(t) + H(t)]$ and $C(t,t) = \varepsilon_C(\alpha+\beta)H(t)$, where $H(t)$ is "full" human wealth, that is, the after-tax value of the household's time endowment:

$H(t) \equiv \int_t^{\infty} [w(\tau) - T(\tau)]\, e^{(r+\beta)(t-\tau)} d\tau.$

12. Without adjustment costs we have $\Psi(\cdot) = I(t)/K(t)$ (and thus $\sigma = 0$), which implies $q = 1$ (from (8)). As a result, $q(t)$ and $K(t)$ adjust instantaneously to their steady-state levels, reflecting an infinite rate of investment in an infinitesimal small time period (i.e., perfect physical capital mobility). Consequently, (9) reduces to $r^K = r + \delta$, which is the familiar rental rate derived in a static framework.

13. See Broer and Heijdra (2001) for an analysis of the case in which $\eta \neq \mu$. They show that if $\eta > \mu$, it is socially optimal for society to produce many varieties. In that case, lump-sum subsidies to firms are required to take the decentralized market equilibrium to the social optimal outcome.

14. We do not explicitly distinguish between lump-sum tax and debt financing. See Heijdra and Ligthart (2006, 2007) for analyses pertaining to debt financing in a small open economy and the closed economy, respectively.

15. Coto-Martinez and Dixon (2003) do not distinguish between internal and external economies of scale.

16. The bound $\hat{\phi}$ is discussed in Appendix A.2.1 and numerically determined in Figure 1.

17. If $\phi > \hat{\phi}$, the real parts of the complex roots turn positive, thus yielding an outright unstable solution.

18. In their analysis of the current account effects of tariff policy, Sen and Turnovsky (1990) also find hysteresis in the capital stock.

19. It is easy to see that $\lim_{\bar{z}\to\infty} \Psi(x) = x$, that is, the installation function is linear

(and adjustment costs are zero) for large $\bar{z}$.

20. Equations (7) and (8) are solved, using (21), to yield $(I/K)^* = \bar{z}\left[e^{(\delta/\bar{z})} - 1\right]$ and $q^* = e^{(\delta/\bar{z})}$.

21. Equations (24) and (25) are obtained from Appendix equations (A.4) and (A.2), respectively.

22. The case without Ethier-productivity effects is represented by $\eta = 1$, which implies $\phi = 1.89 \; < \; \bar{\phi} = 2.404$.

23. The transition paths are also virtually identical explaining why we will not cover the special case of $\eta = 1$ in Section 4.3.

24. Recall that the exponential form of any complex number is $e^{(b \pm \theta_k i)t} = e^{bt}\left[\cos\theta_k t \pm i \sin\theta_k t\right]$, where $k = \{\nu, \lambda\}$ and $b = \{-h^*, r^*\}$. It follows that the sign of the real part (denoted by b) dictates stability.

25. $\mathcal{L}\{G, s\}$ is the Laplace transformation of $G(t)$ evaluated at s, which is given by $\mathcal{L}\{G, s\} \equiv \int_0^\infty G(t)e^{-st}\mathrm{d}t$. Intuitively, $\mathcal{L}\{G, s\}$ represents the present value of $G(t)$ using s as the discount rate.

26. The details of the derivations are more straightforward than for complex roots and can be found in Heijdra and Ligthart (2008).

27. The cyclical terms also drop out from the finite horizon model if θ_L is sufficiently low such that the stable roots are real and distinct. See Heijdra and Ligthart (2008) for a derivation of the expressions.

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