Optimal pollution taxation and abatement when leisure and environmental quality are complements*

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Abstract: We study optimal environmental policy in a world featuring multiple stable economic-ecological equilibria. There is a two-way interaction between the macro-economy and the environment. We assume that the society under consideration finds itself in a high pollution equilibrium and show that a benevolent social planner is in principle able to engineer substantial welfare gains by choosing the appropriate mix of Pigouvian (capital) taxation and abatement activities. During the initial phase of the policy, abatement is used to reduce the inflow of dirt to zero whereas the tax is employed to bring down the stock of the polluting capital input in an optimal fashion. In the long run abatement is no longer needed and the capital tax settles down at its externality-correcting Pigouvian level. For a plausible parameterization optimal abatement takes place for up to twenty-five years whilst the long-run pollution tax never exceeds seven percent of gross operating surplus. In order to escape out of the pollution trap the full force of the available environmental instruments is only needed temporarily.

Keywords: Leisure-environment complementarity, ecological thresholds, multiple equilibria, nonlinear dynamics, optimal environmental policy, pollution taxes, abatement.

JEL Codes: D60, E62, H23, J22, Q20, Q28, Q50.

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1 Introduction

We study optimal environmental policy using a dynamic model featuring a two-way interaction between the ecological system and the macro-economy. We assume that the process governing pollution dynamics is nonlinear such that there exist two stable economic-ecological equilibria. In additional we postulate that the society under consideration finds itself in what could be labeled a "rat-race pollution trap." People work at a furious pace, the macroeconomic capital stock and output are very high, but the quality of the environment is rather low. Lurking on the horizon, however, there is another steady-state equilibrium in which pollution is low and people live in the leisure society once prophesied by John Maynard Keynes in his marvellous 1930 essay entitled *Economic possibilities for our grandchildren*. We study how a utilitarian policy maker could use the available instruments of environmental policy – pollution taxation and public abatement activities – to engineer a move from the pollution trap to the clean equilibrium.

Our environmental model follows the recent literature by assuming that the ecological process is nonlinear. Just as in our previous work (Heijdra and Heijnen, 2013, 2014) we assume in our base model that (i) ecosystems do not respond smoothly to gradual changes in dirt flows and may display abrupt "catastrophic shifts" in the vicinity of threshold points, (ii) there may be multiple stable equilibria, and (iii) irreversibility and hysteresis are both possible (Scheffer *et al.*, 2001). This type of nonlinear ecological dynamics is referred to as Shallow-Lake Dynamics (SLD hereafter).[1] To demonstrate that our conclusions are not specific to SLD we conduct a robustness check in which we employ a logistic environmental damage function which also gives rise to multiple economic-ecological equilibria (the LOG case hereafter).

To describe the macroeconomic system we use an extended Ramsey-Cass-Koopmans model of a closed economy. Households practice intertemporal consumption smoothing and accumulate capital that is rented out to perfectly competitive firms. Labour supply is endogenous and we assume that leisure and environmental quality are complements. This is in line with the anecdotal evidence that if people have direct preferences over the environment at all, it is through their enjoyment of leisure. Simply put, it is much more fun to stroll through a pristine forest filled with birdsong than it is to wander along the dirty footpaths in some noisy smog-filled metropolis. Following Bovenberg and Heijdra (1998, 2002) we assume that the capital stock is the polluting production factor. Households enjoy living in a clean environment but act as free riders and thus fail to internalize the external effects caused by their capital accumulation decisions.

Our main findings are as follows. First, in the social optimum the two available policy

[1] The details of the ecological SLD system are spelled out in Heijdra and Heijnen (2013, pp. 50-52). For overviews of the SLD literature, see Muradian (2001), Mäler *et al.* (2003), Brock and Starrett (2003), and Wagener (2009). Economic applications of SLD include Heijnen and Wagener (2013), Ranjan and Shortle (2007), and Wirl (2004). A related nonlinear approach is used by Prieur (2009).

instruments each play a distinct role. During the initial phase of the policy, abatement is used to choke off the flow of dirt as much as is feasible whereas the tax is employed to bring down the stock of the polluting capital input at the optimal rate (taking into account that the marginal social product of capital affects the consumption profile of the representative household). In the long run, however, abatement is no longer needed and the capital tax settles down at its externality-correcting Pigouvian level. For a plausible parameterization the long-run pollution tax turns out to be quite low. Hence it is only during transition from the pollution trap to the leisure society that the full force of the available environmental instruments is needed.

Second, the complementarity between leisure and environmental quality plays a major role in the model. By moving from a situation of low to high environmental quality, individuals want to move from a low to a high amount of leisure, i.e. labour supply falls. In the long run, however, the capital intensity of production will be more or less restored to its initial level (because the pollution tax is low) so that the capital stock will also fall drastically. The labour supply effect thus ensures that physical capital and environmental quality are substitutes in the long run.

Third, our numerical results suggest that the policy maker can produce substantial welfare gains by engineering a move from the dirty to the clean equilibrium, e.g. for our parameterization welfare gains range between fifteen to twenty-eight percent of initial steady-state consumption. Of course these results should not be taken too literally in view of the fact that there is little or no empirical information available regarding the magnitude of the structural environmental parameters involved. It does serve to show, however, that in the presence of multiple Pareto-rankable equilibria it is potentially possible to realize large welfare gains by a mostly temporary environmental policy. It is the nudge to get out of the pollution trap that counts in this respect.

The paper is structured as follows. Section 2 presents the base model, consisting of an ecological system featuring SLD and an economic system. Section 3 studies the first-best social optimum. Section 4 discusses a number of robustness scenarios. Finally, in Section 5 we offer a brief summary of the main results.

2 Model

2.1 Households

The representative household is blessed with perfect foresight and lives forever. The lifetime utility function from the perspective of the planning period t is given by:

$$\Lambda(t) = \int_t^\infty U\left(C(\tau), 1 - L(\tau), E(\tau)\right) e^{-\rho(\tau - t)} d\tau, \quad (1)$$

where $\Lambda(t)$ is lifetime utility at time t, $U(\cdot)$ is the felicity function, $C(\tau)$ denotes goods consumption, $L(\tau)$ is labour supply, $E(\tau)$ is environmental quality (a non-excludable and non-rivalrous public good), and ρ is the pure rate of time preference. The household has a unit endowment of time so that $1-L(\tau)$ represents leisure consumption. The felicity function has the usual properties, i.e. marginal utilities are strictly positive ($U_C > 0$, $U_{1-L} > 0$, and $U_E > 0$) though at a diminishing rate ($U_{CC} < 0$, $U_{1-L,1-L} < 0$, and $U_{EE} < 0$). Furthermore, for a given level of environmental quality, indifference curves for consumption and leisure are convex with respect to the origin ($\Delta \equiv U_{CC}U_{1-L,1-L} - U_{C,1-L}^2 > 0$).

The household's budget identity is given by:

$$\dot{A}(\tau) = r(\tau)A(\tau) + w(\tau)L(\tau) - T(\tau) - C(\tau), \tag{2}$$

where $A(\tau)$ are financial assets, $r(\tau)$ denotes the real rate of interest on such assets, $w(\tau)$ represents the wage rate, and $T(\tau)$ are net lump-sum taxes. As usual we define $\dot{A}(\tau) \equiv dA(\tau)/d\tau$. At time t the household owns a predetermined stock of assets, $A(t)$, resulting from past saving decisions.

The representative household chooses paths for $C(\tau)$, $L(\tau)$, and $A(\tau)$ which maximize (1) subject to (2), a solvency requirement, and taking as given both the initial stock of financial assets, $A(t)$, and the rationally anticipated path of environmental quality, $E(\tau)$. The key first-order necessary conditions are given by:

$$U_C(C(\tau), 1-L(\tau), E(\tau)) = \lambda(\tau), \tag{3}$$

$$U_{1-L}(C(\tau), 1-L(\tau), E(\tau)) = \lambda(\tau)w(\tau), \tag{4}$$

$$\frac{\dot{\lambda}(\tau)}{\lambda(\tau)} = \rho - r(\tau), \tag{5}$$

$$\lim_{\tau\to\infty} e^{\rho(t-\tau)}\lambda(\tau) \geq 0, \qquad \lim_{\tau\to\infty} e^{\rho(t-\tau)}\lambda(\tau)A(\tau) = 0, \tag{6}$$

where $\lambda(\tau)$ is the co-state variable representing the shadow price of financial assets.

In our previous paper (Heijdra and Heijnen, 2013) we employed preferences that are weakly separable in consumption, leisure, and environmental quality, i.e. there we imposed that $U_{C,1-L} = U_{CE} = U_{E,1-L} = 0$. In that rather special case environmental quality does not affect the decision rules of households and as a result the economy-ecology interaction is uni-directional. In this paper we generalize our earlier analysis by assuming that preferences are non-separable so that not all cross derivatives of the felicity function are equal to zero. Whilst there are in principle many different ways to model non-separability, we follow Schwartz and Repetto (2000) by assuming that preferences are weakly separable in consumption on the one hand and leisure and environment quality on the other, i.e. $U_{C,1-L} = U_{CE} = 0$ and $U_{E,1-L} \neq 0$.[2] In order to keep the analysis tractable, we employ a specific nested functional

[2]Brock (1977), John and Pecchenino (1994), Michel and Rotillon (1995), Fullerton and Kim (2008), and

form for the household's preferences. In particular, the felicity function takes the logarithmic form:

$$U(\cdot) \equiv \varepsilon_c \ln C(\tau) + (1-\varepsilon_c) \ln M(\tau), \tag{7}$$

where ε_c is a taste parameter $(0 < \varepsilon_c < 1)$ and $M(\tau)$ represents *environmental enjoyment* which is modeled as a CES aggregate of (the flow of) leisure and (the stock of) environmental quality:

$$M(\tau) \equiv \begin{cases} \left[(1-\varepsilon_e)\left[1-L(\tau)\right]^{1-1/\mu} + \varepsilon_e E(\tau)^{1-1/\mu}\right]^{1/(1-1/\mu)} & \text{for } \mu \neq 1, \mu > 0 \\ \left[1-L(\tau)\right]^{1-\varepsilon_e} E(\tau)^{\varepsilon_e} & \text{for } \mu = 1 \end{cases} \tag{8}$$

where μ is the substitution elasticity between leisure and environmental quality, and ε_e is a taste parameter $(0 < \varepsilon_e < 1)$. For future reference we define the partial elasticities of the enjoyment function as $\eta_E \equiv \frac{E}{M}\frac{\partial M}{\partial E}$ and $\eta_{1-L} = 1-\eta_E = \frac{1-L}{M}\frac{\partial M}{\partial(1-L)}$ and note that $0 < \eta_E < 1$. The preference structure (7)–(8) implies that $U_{CE} = U_{C,1-L} = 0$ and $U_{E,1-L} \gtreqless 0 \Leftrightarrow \mu \lesseqgtr 1$. Clearly the substitution elasticity μ plays a crucial role. In order to avoid having to deal with a taxonomy of different case, we focus from here on what we consider to be the intuitively most plausible case, namely with leisure and environmental quality acting as complements in households preferences. This prompts the following assumption.

Assumption 1 Consider the preference structure as stated in (7)–(8). The substitution elasticity between leisure and environmental quality in the environmental enjoyment function satisfies $0 < \mu < 1$, i.e. leisure and environmental quality are complementary goods.

2.2 Firms and the government

The production sector of the economy is perfectly competitive. The production function is Cobb-Douglas, with constant returns to scale to the factors of production:

$$Y(\tau) \equiv F(K(\tau), L(\tau)) \equiv \Omega_0 K(\tau)^{\varepsilon_k} L(\tau)^{1-\varepsilon_k}, \tag{9}$$

where $Y(\tau)$ denotes gross output, $K(\tau)$ is the capital stock, $L(\tau)$ is employment, and Ω_0 and ε_k are constants $(\Omega_0 > 0$, and $0 < \varepsilon_k < 1)$. The representative firm chooses its inputs in order to maximize the value of the firm, $V(t)$, which – as of time t – is defined as follows:

$$V(t) = \int_t^{\infty} \left[(1-\theta(\tau))\Big(Y(\tau) - w(\tau)L(\tau)\Big) - I(\tau)\right] \cdot e^{-\int_t^{\tau} r(s)ds} d\tau, \tag{10}$$

Quaas *et al.* (2013) assume that labour supply is exogenous and preferences are nonseparable in consumption and environmental quality.

Table 1: The model

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho \tag{T1.1}$$

$$w(t) = \left(\frac{1-L(t)}{M(t)}\right)^{1-1/\mu} \frac{(1-\varepsilon_e)(1-\varepsilon_c)C(t)}{\varepsilon_c(1-L(t))} \tag{T1.2}$$

$$M(t) = \left[(1-\varepsilon_e)\left[1-L(t)\right]^{1-1/\mu} + \varepsilon_e\left[\bar{E}-P(t)\right]^{1-1/\mu}\right]^{1/(1-1/\mu)} \tag{T1.3}$$

$$\dot{K}(t) = Y(t) - C(t) - G(t) - \delta K(t) \tag{T1.4}$$

$$G(t) = T(t) + \theta(t)\left[Y(t) - w(t)L(t)\right] \tag{T1.5}$$

$$Y(t) = \Omega_0 K(t)^{\varepsilon_k} L(t)^{1-\varepsilon_k} \tag{T1.6}$$

$$r(t) + \delta = [1-\theta(t)]\varepsilon_k\Omega_0\left(\frac{K(t)}{L(t)}\right)^{-(1-\varepsilon_k)} \tag{T1.7}$$

$$w(t) = (1-\varepsilon_k)\Omega_0\left(\frac{K(t)}{L(t)}\right)^{\varepsilon_k} \tag{T1.8}$$

$$D(t) = \kappa K(t) - \gamma G(t) \tag{T1.9}$$

$$\dot{P}(t) = -\Pi(P(t)) + D(t) \tag{T1.10}$$

Variables:		*Parameters:*	
$C(t)$	consumption	ρ	rate of time preference
$M(t)$	environmental enjoyment	Ω_0	productivity parameter
$Y(t)$	aggregate output	ε_k	efficiency parameter of capital
$L(t)$	employment	ε_c and ε_e	preference parameters
$K(t)$	capital stock	μ	leisure-environment substitution elasticity
$w(t)$	wage rate	δ	capital depreciation rate
$r(t)$	rate of interest	$\bar{E}$	pristine environmental quality
$P(t)$	pollution stock	κ	capital dirt parameter
$T(t)$	lump-sum tax	γ	abatement cleanup parameter
$\theta(t)$	capital tax		
$G(t)$	abatement	*Notation:*	
$D(t)$	dirt flow	$\dot{x}(t) \equiv dx(t)/dt$	

where $\theta(\tau)$ is the capital tax (levied on gross operating surplus), $I(\tau)$ is gross investment, and $r(\tau)$ is the real interest rate. Abstracting from adjustment costs of investment, the capital accumulation identity is given by:

$$\dot{K}(\tau) = I(\tau) - \delta K(\tau), \tag{11}$$

where $\dot{K}(\tau) \equiv dK(\tau)/d\tau$ denotes the rate of change in the capital stock and δ is the depreciation rate ($\delta > 0$). The first-order conditions for value maximization imply the usual marginal productivity conditions:

$$F_K(K(\tau), L(\tau)) \equiv \varepsilon_k \Omega_0 \left(\frac{K(\tau)}{L(\tau)} \right)^{-(1-\varepsilon_k)} = \frac{r(\tau) + \delta}{1 - \theta(\tau)}, \tag{12}$$

$$F_L(K(\tau), L(\tau)) \equiv (1 - \varepsilon_k)\Omega_0 \left(\frac{K(\tau)}{L(\tau)} \right)^{\varepsilon_k} = w(\tau). \tag{13}$$

In the absence of adjustment costs of investment, the value of equity corresponds to the replacement value of the capital stock, i.e. $V(t) = K(t)$.

The government levies taxes and spends on environmental abatement. Since the model features Ricardian Equivalence, we can safely ignore government debt and write the periodic budget identity as follows:

$$G(\tau) = T(\tau) + \theta(\tau)\Big(Y(\tau) - w(\tau)L(\tau)\Big), \tag{14}$$

where $G(\tau)$ is public abatement spending. In our policy experiments we assume that the government finances a change in abatement spending or the capital tax by adjusting the lump-sum tax. The capital tax thus serves solely as an instrument of environmental policy.[3] Finally, in the absence of government debt, claims on the capital stock are the only assets available to the households, i.e. $A(t) = K(t)$.

2.3 Ecology

Environmental quality is modeled as a renewable stock of resources which depends negatively on the stock of pollution, $P(\tau)$, according to the following definition:

$$E(\tau) \equiv \bar{E} - P(\tau), \tag{15}$$

where $\bar{E}$ is some time-invariant pristine value attained in a world without any pollution. Pollution itself depends negatively on the flow of dirt, $D(\tau)$, that is generated by economic activities. Following the conventional approach in the literature on environmental macroe-

[3] If lump-sum taxes are not available then θ serves the dual role of corrective tax and revenue raising instrument. See Sandmo (1975) for an overview of environmental Ramsey taxation in a static setting.

conomics (e.g., Bovenberg and Heijdra, 1998, 2002), we assume that capital is the (only) polluting factor of production, that abatement reduces the dirt flow, and that the dirt equation is linear in its arguments:

$$D(\tau) \equiv \kappa K(\tau) - \gamma G(\tau), \tag{16}$$

where κ and γ are positive constants.[4] We interpret $D(\tau)$ as the net flow of dirt and consequently impose a non-negativity constraint on it ($D(\tau) \geq 0$).

Finally, the stock of pollution evolves over time according to the following additively separable expression:

$$\dot{P}(\tau) = -\Pi\left(P(\tau)\right) + D(\tau), \tag{17}$$

where $\Pi\left(P(\tau)\right)$ is a general functional form incorporating all (linear and/or non-linear) interactions between the current pollution stock and its time rate of change. In our base model we employ the functional form giving rise to Shallow Lake Dynamics (SLD) with reversible hysteresis:[5]

$$\Pi(P(\tau)) \equiv \pi P(\tau) - \frac{P(\tau)^2}{P(\tau)^2+1}, \qquad \frac{1}{2} < \pi < \frac{3\sqrt{3}}{8}. \tag{18}$$

The P-isocline for a zero level of abatement has been illustrated in Figure 1(d) – see the dashed line labeled PE.

2.4 Equilibrium

For convenience, the key equations of the full model have been gathered in Table 1. Equation (T1.1) is the consumption Euler equation expressing the optimal growth rate of consumption. It is obtained by using (3) in (5) and incorporating the functional form for felicity given in (7)–(8). As a result of our separability assumptions, optimal consumption growth is independent of both the leisure choice and the quality of the environment.

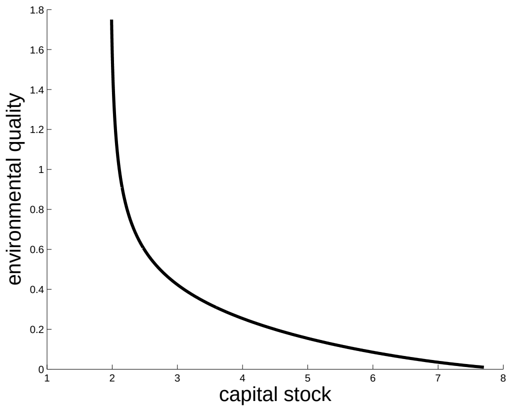
Equation (T1.2) – in combination with (T1.3) – fully characterizes the optimal labour supply choice. It is obtained by substituting (3) into (4) and noting (7)–(8). Intuitively, optimal labour supply is such that the marginal rate of substitution between leisure and consumption (right-hand side of (T1.2)) is equated to the opportunity cost of leisure, i.e. the wage rate (the left-hand side of (T1.2)). As the expression shows, the marginal rate of

[4] Brock (1977) models the gross flow of environmental waste, $D^g(\tau)$, as a joint product and writes the macroeconomic production function (using our notation) as $F(\Phi\left[K(\tau), D^g(\tau)\right], L(\tau))$ where $\Phi\left[\cdot\right]$ is some aggregator function. This captures the idea that the production of one unit of output requires less capital if a more polluting technique is used (1977, p. 443). Our approach is consistent with his provided the aggregator function is of the Leontief type, i.e. $\Phi\left[\cdot\right] \equiv \min\left\{K(\tau), D^g(\tau)/\kappa\right\}$.

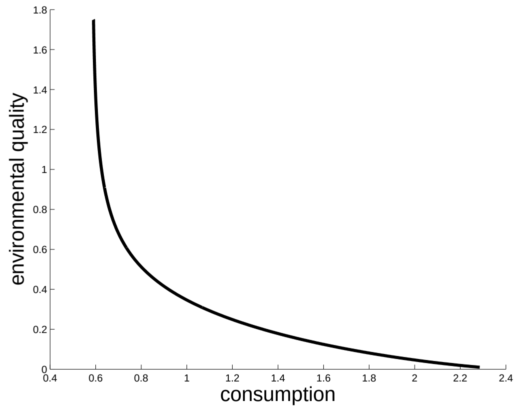
[5] In Section 4.2 below we postulate an alternative functional form for $\Pi\left(P\right)$ consistent with Logistic Dynamics (LOG) and multiple environmental equilibria. We also compare the LOG and SLD cases there.

Figure 1: Features of the steady-state laissez-faire equilibrium

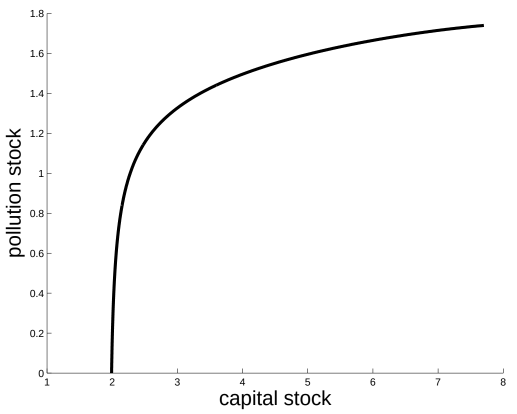
(a) environment-capital tradeoff $(\hat{E}, \hat{K})$



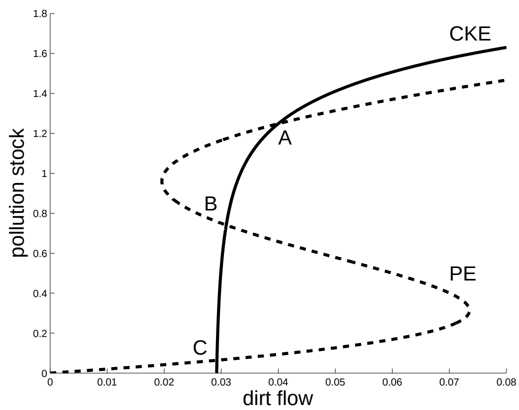
(b) environment-consumption tradeoff $(\hat{E}, \hat{C})$



(c) pollution and capital $(\hat{P}, \hat{K})$



(d) multiple equilibria $(\hat{P}, \hat{D})$



Parameters: see Table 2. The initial steady-state equilibrium is at point A in panel (d).

substitution between leisure and consumption is affected by the stock of pollution provided $\mu \neq 1$, i.e. as long as preferences are non-separable in leisure and environmental quality.

Equation (T1.4) states the dynamic evolution of the macroeconomic capital stock. It is obtained by combining (11) with the goods market clearing condition for a closed economy, i.e. $Y(\tau) = C(\tau) + I(\tau) + G(\tau)$. The remaining equations (T1.5)–(T1.10) in Table 1 just restate, respectively, (14), (9), (12), (13), (16), and (17).

2.4.1 Stability

The model can be condensed into a non-linear system of differential equations in the flow of consumption, $C(t)$, and the stocks of physical capital, $K(t)$, and pollution, $P(t)$. In deriving this system we first use (T1.2)–(T1.3) and (T1.8) to obtain:

$$(1-\varepsilon_k)\Omega_0 \left(\frac{K(t)}{L(t)}\right)^{\varepsilon_k} = \frac{1-\varepsilon_c}{\varepsilon_c} \frac{(1-\varepsilon_e)C(t)(1-L(t))^{-1/\mu}}{(1-\varepsilon_e)(1-L(t))^{1-1/\mu} + \varepsilon_e(\bar{E}-P(t))^{1-1/\mu}}. \tag{19}$$

The left-hand side of this expression is the labour demand function which is downward sloping in employment and depends positively on the capital stock (because labour and capital are cooperative factors of production). The right-hand side of equation (19) implicitly defines a Frisch labour supply function which depends negatively on consumption (which is itself inversely related to the marginal utility of wealth) and positively on the stock of pollution (because leisure and environmental quality are complements –see Assumption 1).[6]

Equation (19) defines an implicit function expressing equilibrium employment in terms of consumption and the stocks of capital and pollution:

$$L(t) \equiv \Psi\left(C(t), K(t), P(t)\right). \tag{20}$$

The partial elasticities around any point (L_0, C_0, K_0, P_0) satisfying (19) are given by:

$$\eta_{LC} \equiv \frac{L_0}{C_0}\frac{\partial \Psi(\cdot)}{\partial C(t)} = -\frac{1}{\varepsilon_k + \xi_L} < 0, \tag{21}$$

$$\eta_{LK} \equiv \frac{K_0}{L_0}\frac{\partial \Psi(\cdot)}{\partial K(t)} = \frac{\varepsilon_k}{\varepsilon_k + \xi_L} > 0, \tag{22}$$

$$\eta_{LP} \equiv \frac{P_0}{L_0}\frac{\partial \Psi(\cdot)}{\partial P(t)} = \frac{1-\mu}{\mu}\frac{\eta_E}{\varepsilon_k + \xi_L}\frac{P_0}{\bar{E}-P_0} > 0, \tag{23}$$

where ξ_L represents the inverse of the Frisch elasticity of labour supply and is defined as:

$$\xi_L \equiv \frac{L_0}{1-L_0}\left(1 + \eta_E \frac{1-\mu}{\mu}\right) > 0. \tag{24}$$

[6] An alternative mechanism by which pollution could affect labour supply operates via the health nexus. In that approach pollution leads to health problems and loss of labour productivity resulting in lower labour supply. In our model this health-based mechanism has been abstracted from.

In the second step we use (T1.1), (T1.4), (T1.7), (T1.9)–(T1.10), and (20) to obtain the system of differential equations describing the dynamic evolutions of the economy and the environment:

$$\frac{\dot{C}(t)}{C(t)} = [1-\theta(t)]\varepsilon_k\Omega_0\left(\frac{\Psi\left(C(t),K(t),P(t)\right)}{K(t)}\right)^{1-\varepsilon_k} - (\rho+\delta)\,, \tag{25}$$

$$\dot{K}(t) = \Omega_0 K(t)^{\varepsilon_k}\Psi\left(C(t),K(t),P(t)\right)^{1-\varepsilon_k} - C(t) - G(t) - \delta K(t), \tag{26}$$

$$\dot{P}(t) = -\Pi(P(t)) + \kappa K(t) - \gamma G(t). \tag{27}$$

Since the stocks of capital and pollution are predetermined (sticky) variables and consumption is a non-predetermined (jumping) variable, the system must be solved with initial conditions for capital and pollution and a terminal condition for consumption. Local saddle-point stability can be ascertained by linearizing the system around a steady-state point $(\hat{C},\hat{K},\hat{P})$. After some manipulation we obtain:

$$\begin{bmatrix} \dot{C}(t) \\ \dot{K}(t) \\ \dot{P}(t) \end{bmatrix} = \Delta \begin{bmatrix} C(t)-\hat{C} \\ K(t)-\hat{K} \\ P(t)-\hat{P} \end{bmatrix} - \begin{bmatrix} (\rho+\delta)\hat{C}\theta(t) \\ G(t) \\ \gamma G(t) \end{bmatrix}, \tag{28}$$

where the Jacobian matrix Δ (with typical element δ_{ij}) is given by:

$$\Delta \equiv \begin{bmatrix} -\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}(\rho+\delta) & -\xi_L\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}(\rho+\delta)\frac{\hat{C}}{\hat{K}} & \eta_E\frac{1-\mu}{\mu}\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}(\rho+\delta)\frac{\hat{C}}{\hat{E}} \\ -\left(\frac{\hat{C}}{\hat{Y}}+\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\right)\frac{\hat{Y}}{\hat{C}} & \left(\frac{\hat{C}}{\hat{Y}}-\frac{\xi_L(1-\varepsilon_k)}{\varepsilon_k+\xi_L}\right)\frac{\hat{Y}}{\hat{K}} & \eta_E\frac{1-\mu}{\mu}\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\hat{Y}}{\hat{E}} \\ 0 & \kappa & -\Pi'(\hat{P}) \end{bmatrix}, \tag{29}$$

and $\hat{C}/\hat{Y} = (\rho+\delta(1-\varepsilon_k))/(\rho+\delta)$ is the steady-state share of consumption in aggregate output. Several things are worth noting. First, if μ were equal to unity, the felicity function would be weakly separable in all its arguments, there would be no complementarity between leisure and environmental quality, and the dynamic system would be recursive in $(C(t),K(t))$ and $P(t)$ (as $\delta_{13}=\delta_{23}=0$ in that case). The sub-determinant of the (C,K) system is given by:

$$|\Delta_{CK}| = -(1-\varepsilon_k)(\rho+\delta)\frac{\hat{C}}{\hat{K}}\frac{1+\xi_L}{\varepsilon_k+\xi_L} < 0, \tag{30}$$

so the characteristic roots alternate in sign implying saddle-point stability. The differential equation for $P(t)$ is stable provided $\Pi'(\hat{P}) > 0$, i.e. the pollution equilibrium must be on an upward-sloping branch of the PE curve in Figure 1(d).

Second, in the general non-separable case (and with leisure-environment complementarity,

$0 < \mu < 1$), the system is non-recursive and there exists a bidirectional interaction between the economic and environmental systems. The determinant of the (C, K, P) system is given by:

$$|\Delta| = -\Pi'(\hat{P})\,|\Delta_{CK}| - \kappa\eta_E(\rho+\delta)\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\hat{C}}{\hat{E}}\frac{1-\mu}{\mu} \gtreqless 0. \tag{31}$$

As $|\Delta|$ equals the product of the characteristic roots, saddle point stability holds provided $|\Delta| > 0$. The first term on the right-hand side of (31) is positive provided $\Pi'(\hat{P}) > 0$. But the second term is negative and thus "endangers" stability of the full system. In the parametric examples discussed below the equilibria are such that the first term dominates the second term so that $|\Delta| > 0$.[7]

2.4.2 Long-run system features

Consider a laissez-faire economy without a pollution tax and government abatement ($\theta(t) = G(t) = 0$). What are the basic trade-offs that exist in such an economy? To answer that question we use Figure 1 which is based on the parameterization discussed more fully below. In panels (a) and (b) we show how environmental quality affects the steady-state levels of, respectively, the capital stock and consumption. Both lines are downward sloping, i.e. a cleaner environment comes at the cost of a lower capital stock and a lower consumption level. Panel (c) re-expresses the graph in panel (a) by noting that $\hat{P} \equiv \bar{E} - \hat{E}$. It visualizes the same message as before: a lower pollution stock comes at the cost of a lower capital stock. Finally, in panel (d) of Figure 1 the solid line (labeled CKE, for consumption-capital equilibrium) re-expresses the graph in panel (c) by noting that – in the unmanaged economy – $\hat{D} = \kappa\hat{K}$. The dashed line (labeled PE for Pollution Equilibrium) is the isocline for $P(t)$ which is obtained from (16)–(18) and setting $G(t) = 0$. Under Shallow Lake Dynamics (SLD) this function is S-shaped and – given the parameterization – PE intersects CKE at three points. The slope of the PE line is equal to $d\hat{P}/d\hat{D} = 1/\Pi(\hat{P})$ so it follows from the stability discussion above that the equilibria at points A and C are saddle-point stable whilst point B is unstable.

The (approximate) long-run effects of pollution taxation and government abatement can be computed with the aid of (28). Indeed, changing the policy variables from $(\theta(t), G(t)) = (0, 0)$ to $(\theta(t), G(t)) = (\theta, G)$ and imposing the steady state we find:

$$\Delta \begin{bmatrix} \hat{C}^n - \hat{C} \\ \hat{K}^n - \hat{K} \\ \hat{P}^n - \hat{P} \end{bmatrix} = \begin{bmatrix} (\rho+\delta)\hat{C}\theta \\ G \\ \gamma G \end{bmatrix},$$

where the superscript n denotes new steady-state values. For the pollution tax the long-run

[7] In terms of the graphical aparatus developed below, this condition implies that the CKE curve must be steeper than the PE locus at stable equilibrium points – see Figure 1(d).

effects are:

$$\frac{\hat{C}^n-\hat{C}}{\theta} = -\frac{(\rho+\delta)\hat{C}}{|\Delta|}\left[\left(\rho\frac{\hat{K}}{\hat{Y}}+\frac{\varepsilon_k\,(1-\varepsilon_k)}{\varepsilon_k+\xi_L}\right)\frac{\hat{Y}}{\hat{K}}\Pi'(\hat{P})+\kappa\eta_E\frac{1-\mu}{\mu}\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\hat{Y}}{\hat{E}}\right]<0,$$

$$\frac{\hat{K}^n-\hat{K}}{\theta} = -\frac{(\rho+\delta)\hat{Y}\Pi'(\hat{P})}{|\Delta|}\left[\frac{\hat{C}}{\hat{Y}}+\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\right]<0,$$

$$\frac{\hat{P}^n-\hat{P}}{\theta} = -\frac{\kappa(\rho+\delta)\hat{Y}}{|\Delta|}\left[\frac{\hat{C}}{\hat{Y}}+\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\right]<0,$$

where we recall that $|\Delta|>0$ at points A and C in Figure 1(d). The introduction of a pollution tax shifts the CKE locus to the left resulting in a decrease in equilibrium pollution in both saddle-point stable equilibria. In addition, the tax leads to a reduction in both consumption and the capital stock.

For government abatement the long-run effects are given by:

$$\frac{\hat{C}^n-\hat{C}}{G} = -\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\rho+\delta}{|\Delta|}\left[\xi_L\frac{\hat{C}}{\hat{K}}\Pi'(\hat{P})+\eta_E\frac{\hat{C}}{\hat{E}}\frac{1-\mu}{\mu}\left(\gamma\frac{\hat{C}}{\hat{K}}-\kappa\right)\right]\lesseqgtr 0,$$

$$\frac{\hat{K}^n-\hat{K}}{G} = \frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\rho+\delta}{|\Delta|}\left[\Pi'(\hat{P})-\gamma\eta_E\frac{\hat{C}}{\hat{E}}\frac{1-\mu}{\mu}\right]\gtreqless 0,$$

$$\frac{\hat{P}^n-\hat{P}}{G} = -\frac{1-\varepsilon_k}{\varepsilon_k+\xi_L}\frac{\rho+\delta}{|\Delta|}\left[\gamma(1+\xi_L)\frac{\hat{C}}{\hat{K}}-\kappa\right]\lesseqgtr 0.$$

In terms of Figure 1(d), an increase in government abatement causes two shifts in the CKE locus. The direct effect of abatement on the pollution stock leads to a leftward shift in the CKE line. The induced effect, however, leads to a rightward shift. The intuition behind this capital-induced shift is as follows. The increase in abatement necessitates an increase in lump-sum taxes which makes the representative agent poorer. This prompts an increase in labour supply followed in the long run by an equiproportional increase in the capital stock because the equilibrium capital intensity of production is unaffected by the abatement shock. For plausible parameter values (see below) we find that the direct effect dominates the induced effect so that consumption and pollution fall whilst the capital stock rises if abatement is increased.

3 Optimal environmental policy

In the previous section we have shown that there typically exist two saddle-point stable steady-state equilibria if the environmental dynamics is of the SLD form and environmental policy instruments are not utilized. Indeed, in Figure 1(d) there is a “clean” steady state at point C in which pollution is low, environmental quality is high, and individuals enjoy a high level of leisure. There also is a "polluted" steady state at point A in which people work hard and environmental quality is low. Since pollution is a public bad – an external effect resulting from the decentralized decisions of households and firms – there is a strong presumption that the clean equilibrium must somehow be better than the dirty equilibrium. In order to investigate this intuition more formally, we consider the optimal environmental policy that is available to a benevolent policy maker facing the following objective function and conditions.

1. The social planner aims to maximize lifetime utility of the representative agent.
2. Both the economic and ecological systems are in a steady-state equilibrium, there is no pollution tax, and environmental abatement is zero (the unmanaged economy).
3. There exist two saddle-point stable steady-state ecological equilibria, namely the polluted equilibrium $(\hat{K}_D, \hat{P}_D)$ and the clean steady state at $(\hat{K}_C, \hat{P}_C)$.
4. The ecological system has settled down at the dirty equilibrium featuring a high stock of pollution and high economic activity.

In Figure 1(d) the dirt flow equals $\kappa\hat{K}_D$ and the ecological equilibrium is located at point A. Given this initial condition, can the policy maker bring about substantial welfare gains by choosing the appropriate mix of capital taxation and abatement activities? In the planning period $t = 0$, the planner chooses paths for $C(t)$, $L(t)$, $P(t)$, and $K(t)$ (for $t \geq 0$) in order to maximize (1) subject to the resource constraint (T1.4), the dirt flow definition (T1.9), and the emission equation (T1.10). The initial conditions are:

$$K(0) = \hat{K}_D, \qquad P(0) = \hat{P}_D. \tag{32}$$

Abatement and the dirt flow must remain non-negative:

$$G(t) \geq 0, \qquad [D(t) \equiv]\ \kappa K(t) - \gamma G(t) \geq 0. \tag{33}$$

The current-value Hamiltonian can be written as:

$$\begin{aligned}\mathcal{H}(t) \equiv\ & \varepsilon_c \ln C(t) + (1-\varepsilon_c) \ln M(1-L(t), \bar{E} - P(t)) \\ & + \lambda_K(t) \Big[F\left(K(t), L(t)\right) - C(t) - G(t) - \delta K(t) \Big] \\ & + \lambda_P(t) \Big[-\Pi\left(P(t)\right) + \kappa K(t) - \gamma G(t) \Big] + \lambda_D(t) \Big[\kappa K(t) - \gamma G(t) \Big].\end{aligned}$$

The control variables for this optimization problem are $C(t)$, $L(t)$, and $G(t)$ (and thus implicitly $D(t)$), the state variables are $K(t)$ and $P(t)$, the co-state variables are $\lambda_K(t)$ and $\lambda_P(t)$, and $\lambda_D(t)$ is the Lagrange multiplier for the dirt constraint. The first-order conditions are:

$$\frac{\varepsilon_c}{C(t)} = \lambda_K(t), \tag{34}$$

$$\lambda_K(t) F_L(K(t), L(t)) = (1-\varepsilon_c)\frac{M_{1-L}(1-L(t), \bar{E}-P(t))}{M(1-L(t), \bar{E}-P(t))}, \tag{35}$$

$$0 \leq \lambda_K(t) + \gamma\left(\lambda_P(t) + \lambda_D(t)\right),\ G(t) \geq 0,\ G(t)\left[\lambda_K(t) + \gamma\left(\lambda_P(t) + \lambda_D(t)\right)\right] = 0, \tag{36}$$

$$0 \leq \kappa K(t) - \gamma G(t), \qquad \lambda_D(t) \geq 0, \qquad \lambda_D(t)\left[\kappa K(t) - \gamma G(t)\right] = 0, \tag{37}$$

$$\dot{\lambda}_K(t) = -\kappa\left(\lambda_P(t) + \lambda_D(t)\right) + \left[\rho + \delta - F_K\left(K(t), L(t)\right)\right]\lambda_K(t), \tag{38}$$

$$\dot{\lambda}_P(t) = (1-\varepsilon_c)\frac{M_E(1-L(t), \bar{E}-P(t))}{M(1-L(t), \bar{E}-P(t))} + \lambda_P(t)\left[\rho + \Pi'\left(P(t)\right)\right]. \tag{39}$$

The social optimum is characterized by (T1.4), (T1.9)–(T1.10), (32), (34)–(39) and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = \lim_{t\to\infty} e^{-\rho t}\lambda_P(t)P(t) = 0. \tag{40}$$

Since the current-value Hamiltonian is linear in abatement, it follows that the optimal solution for $G(t)$ is of the "bang-bang" (all or nothing) type – see, for example, Clark and Munro (1975) and Spence and Starrett (1975).

3.1 Long-run optimum

We first study the long-run properties of the social optimum. In terms of notation, hatted variables denote steady-state values and the subscript "s" denotes socially optimal. Depending on the structural parameters and the resulting magnitude of $\hat{G}_s$ two cases are possible.

3.1.1 Case 1: With long-run abatement

Assume that $\hat{\lambda}_D > 0$ so that $\hat{D}_s = 0$, $\hat{G}_s = (\kappa/\gamma)\,\hat{K}_s$, and $\gamma(\hat{\lambda}_P + \hat{\lambda}_D) = -\hat{\lambda}_K < 0$. It follows that the steady-state first-best equilibrium is characterized by:

$$F_K(\hat{k}_s, 1) = \rho + \delta + \frac{\kappa}{\gamma}, \tag{41}$$

$$\hat{\lambda}_D = \left(\hat{q}_s - \frac{1}{\gamma}\right)\frac{\varepsilon_c}{\hat{C}_s}, \tag{42}$$

$$F_L(\hat{k}_s, 1) = \left(\frac{1-\hat{L}_s}{\hat{M}_s}\right)^{1-1/\mu}\frac{(1-\varepsilon_e)(1-\varepsilon_c)\hat{C}_s}{\varepsilon_c(1-\hat{L}_s)}, \tag{43}$$

$$\hat{M}_s \equiv \left[(1-\varepsilon_e)\left[1-\hat{L}_s\right]^{1-1/\mu} + \varepsilon_e[\bar{E}-\hat{P}_s]^{1-1/\mu}\right]^{1/(1-1/\mu)}, \tag{44}$$

$$\hat{q}_s\left[\rho + \Pi'_C(\hat{P}_s)\right] = F_L(\hat{k}_s, 1)\left(\frac{1-\hat{L}_s}{\bar{E}-\hat{P}_s}\right)^{1/\mu}\frac{\varepsilon_e}{1-\varepsilon_e}, \tag{45}$$

$$\hat{C}_s = \hat{L}_s \left[F(\hat{k}_s, 1) - \left(\delta + \frac{\kappa}{\gamma} \right) \hat{k}_s \right], \tag{46}$$

$$\Pi_C(\hat{P}_s) = 0, \tag{47}$$

where $\hat{q}_s \equiv -\hat{\lambda}_P / \hat{\lambda}_K$ is the shadow price of environmental quality, $\hat{k}_s \equiv \hat{K}_s / \hat{L}_s$ is the optimal capital intensity of production, and $\Pi_C(x)$ is the function representing the lower upward-sloping branch of the P-isocline. The system of first-order conditions is block recursive in that equation (41) pins down the capital intensity, $\hat{k}_s$, (42)–(46) jointly determine $\hat{\lambda}_D$, $\hat{q}_s$, $\hat{C}_s$, $\hat{L}_s$, and $\hat{M}_s$, whilst (47) determines $\hat{P}_s$. A solution to the system (41)–(47) is internally consistent if and only if $\hat{q}_s > 1/\gamma$ so that $\hat{\lambda}_D > 0$ and it is indeed socially optimal to reduce the dirt flow to zero.

A (Pigouvian) capital tax is an essential instrument with which the social optimum can be decentralized. Indeed, equation (T1.7) shows that in the decentralized economy the steady-state capital intensity is such that $F_K(\hat{k}, 1) = (\rho + \delta) / (1 - \hat{\theta})$. By comparing this expression to (41) we find that $\hat{k} = \hat{k}_s$ if and only if the steady-state capital tax is set equal to:

$$\hat{\theta}_s = \frac{\kappa/\gamma}{\rho + \delta + \kappa/\gamma}. \tag{48}$$

The optimal Pigouvian capital tax is feasible (as it satisfies $0 < \hat{\theta}_s < 1$) and is increasing in κ/γ. Intuitively, the more polluting is capital (κ up) and the less potent is abatement (γ down), the higher is the optimal environmental tax.

3.1.2 Case 2: Without long-run abatement

Assume that $\hat{\lambda}_K > -\gamma \hat{\lambda}_P$ so that $\hat{q}_s < 1/\gamma$ and $\hat{G}_s = 0$. Since $\hat{K}_s > 0$ it follows that $\hat{D}_s > 0$ and thus $\hat{\lambda}_D = 0$. The first-best steady-state equilibrium can now be written as:

$$F_K(\hat{k}_s, 1) = \rho + \delta + \kappa \hat{q}_s, \tag{49}$$

$$F_L(\hat{k}_s, 1) = \left(\frac{1 - \hat{L}_s}{\hat{M}_s} \right)^{1-1/\mu} \frac{(1-\varepsilon_e)(1-\varepsilon_c)\hat{C}_s}{\varepsilon_c (1 - \hat{L}_s)}, \tag{50}$$

$$\hat{M}_s \equiv \left[(1-\varepsilon_e) \left[1 - \hat{L}_s\right]^{1-1/\mu} + \varepsilon_e [\bar{E} - \hat{P}_s]^{1-1/\mu} \right]^{1/(1-1/\mu)}, \tag{51}$$

$$\hat{q}_s \left[\rho + \Pi'_C(\hat{P}_s) \right] = F_L(\hat{k}_s, 1) \left(\frac{1 - \hat{L}_s}{\bar{E} - \hat{P}_s} \right)^{1/\mu} \frac{\varepsilon_e}{1 - \varepsilon_e}, \tag{52}$$

$$\hat{C}_s = \hat{L}_s \left[F(\hat{k}_s, 1) - \delta \hat{k}_s \right], \tag{53}$$

$$\Pi_C(\hat{P}_s) = \kappa \hat{L}_s \hat{k}_s. \tag{54}$$

Unlike the previous case, here the system of long-run first-order conditions is non-recursive, i.e. (49)–(54) jointly determine $\hat{k}_s$, $\hat{C}_s$, $\hat{L}_s$, $\hat{M}_s$, $\hat{P}_s$, and $\hat{q}_s$. Of course, just as for Case 1, a capital tax is needed to decentralize the first-best optimum:

$$\hat{\theta}_s = \frac{\kappa \hat{q}_s}{\rho + \delta + \kappa \hat{q}_s}, \tag{55}$$

where the optimal Pigouvian capital tax is again feasible, i.e. $0 < \hat{\theta}_s < 1$. Since $\hat{q}_s < 1/\gamma$ we find that the capital tax is lowest in Case 2.

3.1.3 Parameterization

In order to avoid having to deal with a taxonomy of possible cases, we use a parameterized version of the model to illustrate its main dynamic properties. Table 2 states the selected values for the structural parameters ρ, ε_k, ε_c, ε_e, μ, $\bar{E}$, δ, Ω_0, π, κ, and γ. Our calibration approach is as follows. We take as our point of departure the unmanaged economy, in which $\theta = G = 0$. Next we postulate some plausible values for a subset of the structural parameters, namely $\rho = 0.04$, $\varepsilon_k = 0.3$, $\delta = 0.07$. For these parameters we find that the steady-state real interest rate is four percent per annum ($\hat{r} = \rho = 0.04$), the labour share of output is seventy percent ($\hat{w}\hat{L}/\hat{Y} = 1 - \varepsilon_k = 0.7$), and the capital-output ratio attains a plausible value ($\hat{K}/\hat{Y} = \varepsilon_k/(\hat{r}+\delta) = 2.7273$). An important feature of the labour supply decision is summarized by the adopted value for $\omega_{LL} \equiv \hat{L}/(1-\hat{L})$. If people typically work for eight hours per twenty-four-hour day, then a reasonable value for this parameter is $\omega_{LL} = 8/(24-8) = 0.5$, or $\hat{L} = 1/3$. This value for steady-state employment is obtained by choosing ε_c appropriately (see below). We normalize steady-state output to unity ($\hat{Y} = 1$) by setting $\Omega_0 = \hat{K}^{-\varepsilon_k}\hat{L}^{-(1-\varepsilon_k)} = 1.5969$ and note that $\hat{C} = \hat{Y} - \delta\hat{K} = 0.8091$ in that case.

Since the labour supply decision is non-separable with environmental quality we need to postulate values of ε_e, $\bar{E}$, and μ about which little or no direct information is available. We set these ("free") parameters equal to $\varepsilon_e = 0.20$, $\bar{E} = 1.75$, and $\mu = 0.25$.[8] To characterize the SLD dynamics (as embodied in (18)) we need to select a value for π. Again this parameter is hard to pin down empirically, so instead we follow our earlier papers by setting $\pi = 0.52$ which ensures that the P-isocline is S-shaped and lies entirely in the first quadrant – see the PE curve in Figure 1(d). Next we postulate a steady-state dirt level, $\hat{D}_0$, and find from the PE curve that point A is associated with $\hat{P}_D = 1.2482$ so that $\hat{E} = \bar{E} - \hat{P}_D = 0.5018$. By setting $\kappa = \hat{D}_0/\hat{K}$ the capital stock gives rise to the targeted dirt level. The taste parameter ε_c is set at such a value that the labour market is in equilibrium:

$$(1-\varepsilon_k)\Omega_0 \left(\frac{\hat{K}}{\hat{L}}\right)^{\varepsilon_k} = \frac{1-\varepsilon_c}{\varepsilon_c} \frac{(1-\varepsilon_e)\hat{C}(1-\hat{L})^{-1/\mu}}{(1-\varepsilon_e)(1-\hat{L})^{1-1/\mu} + \varepsilon_e \hat{E}^{1-1/\mu}}. \tag{56}$$

For the base model we find $\varepsilon_c = 0.2670$. At the calibration values, the Frisch elasticity of

[8] This μ-value implies strong complementarity between leisure and environmental quality. In subsection 4.1 below we conduct a robustness analysis of the model by considering alternative values for μ.

labour supply equals $1/\xi_L = 0.948$ whilst the elasticities of the employment function (20) are equal to $\eta_{LC} = -0.738$, $\eta_{LK} = 0.222$, and $\eta_{LP} = 5.510$.[9]

The final parameter that must be pinned down is γ. Our approach is as follows. First we assume that abatement is relatively efficient in the sense that the move from the polluted to the clean equilibrium is in principle feasible for a relatively modest abatement level of ten percent of output, i.e. we set $G_0 = 0.1$. Denoting the dirt level at the left-hand kink point of the PE curve in Figure 1(d) by D_L, we next postulate $\gamma G_0 = \kappa \hat{K} - (1-\xi) D_U$, where ξ determines the amount by which the dirt flow resulting from abatement level G_0 overshoots D_L $(0 \leq \xi \leq 1)$. For $\xi = 0$ there is no overshoot at all whilst for $\xi = 1$ there is maximum overshoot as the dirt level would be zero. In the model we set $\xi = 1/2$ which results in $\gamma = 0.302$.

3.2 Optimal dynamic allocation

For the chosen parameter values reported in Table 2 we find that $\hat{q}_s = 0.0488$. Since $\gamma = 0.302$ it follows that $\hat{q}_s < 1/\gamma$ so that Case 2 is the relevant one and abatement is not needed in the long run, i.e. $\hat{G}_s = 0$. We furthermore compute $\hat{K}_s = 1.9735$, $\hat{L}_s = 0.2435$, $\hat{C}_s = 0.5902$, $\hat{Y}_s = 0.7283$, $\hat{P}_s = 0.0634$, and $\hat{D}_s = 0.0290$. For ease of comparison, we report these values in column (b) in Table 3. The long-run Pigouvian capital tax, $\hat{\theta}_s$, is 0.65 percent of gross operating surplus. Despite this minute tax, consumption, employment, output, and the capital stock are all substantially lower than in the initial steady-state equilibrium, the key features of which have been reported in column (a) of Table 3.

The dynamic properties of the first-best optimum are illustrated in Figure 2. Details of the computations are found in Heijdra *et al.* (2014). There is one critical date characterizing the optimal solution, namely the earliest time at which the dirt constraint becomes slack, $t_E = 20$. The optimal paths thus evolve through two distinct regimes. In either regime optimal labour supply follows from:

$$L_s(t) = \Psi(C_s(t), K_s(t), P_s(t)), \tag{57}$$

where $\Psi(\cdot)$ is defined in (20) above.

[9] Reichling and Whalen (2012) survey the empirical literature and argue that studies based on micro-data typically find estimates for the Frisch elasticity in the range of 0.1 to more than 1. In contrast, the macro-based estimates usually fall in the range from 2 to 4. Interestingly, in the clean equilibrium of the unmanaged economy we find $1/\xi_L = 2.910$.

Table 2: Structural parameters and steady-state features

Economic system:					
$\rho = 0.04$	$\delta = 0.07$	$\varepsilon_k = 0.30$	$\varepsilon_c = 0.2670$	$\Omega_0 = 1.5969$	$\theta = G = 0$
$\hat{r} = 0.04$	$\hat{K} = 2.7273$	$\hat{Y} = 1.000$	$\hat{C} = 0.8091$	$\hat{I} = 0.1909$	$\hat{L} = 0.3333$
Ecological system:					
$\pi = 0.52$	$\kappa = 0.0147$	$\gamma = 0.302$	$\varepsilon_e = 0.20$	$\mu = 0.25$	$\bar{E} = 1.75$
$D_L = 0.0196$	$D_U = 0.0735$	$\hat{D}_0 = 0.04$	$\hat{P}_D = 1.2482$	$\hat{P}_C = 0.0641$	$P_E = 0.7408$

3.2.1 Regime 1

For $0 \leq t < t_E$ it is optimal to reduce the dirt flow to zero, $D_s(t) = 0$, and to set abatement at its maximum feasible level, $G_s(t) = (\kappa/\gamma)\, K_s(t)$. The stock of pollution falls according to:

$$\dot{P}_s\,(t) = -\Pi(P_s\,(t)), \qquad P_s(0) = \hat{P}_D, \tag{58}$$

where $\hat{P}_D$ is the initial steady-state level of pollution. The optimal paths for consumption and capital are given by:

$$\frac{\dot{C}_s(t)}{C_s(t)} = F_K\left(K_s(t), L_s(t)\right) - \left(\rho + \delta + \frac{\kappa}{\gamma}\right), \tag{59}$$

$$\dot{K}_s(t) = F\left(K_s(t), L_s(t)\right) - C_s(t) - \left(\delta + \frac{\kappa}{\gamma}\right) K_s(t), \tag{60}$$

and the optimal shadow price of environmental quality satisfies $q_s(t) > 1/\gamma$ and evolves according to:

$$\frac{\dot{q}_s(t)}{q_s(t)} = \rho + \Pi_C'(P_s(t)) + \frac{\dot{C}_s(t)}{C_s(t)} - \frac{F_L(K_s(t), L_s(t))}{q_s(t)} \left(\frac{1 - L_s(t)}{\bar{E} - P_s(t)}\right)^{1/\mu} \frac{\varepsilon_e}{1 - \varepsilon_e}. \tag{61}$$

The transition paths for $G_s\,(t)$, $C_s\,(t)$, $L_s\,(t)$, $K_s\,(t)$, and $P_s\,(t)$ have all been depicted in Figure 2 whilst the impact effects on the non-predetermined variables ($C_s(0)$, $L_s(0)$, and $G_s(0)$) have been reported in column (b) of Table 3.

At impact the commencement of abatement activities coincides with a sharp drop in investment resulting from the large increase in the capital tax ($\theta(0) = 0.4496$). The reduction in investment even allows for a small increase in consumption at impact. Since the stocks of capital and pollution are both predetermined at impact, equation (57) implies that equilibrium employment is slightly lower. By discouraging capital formation the policy maker thus engineers a short-run increase in both consumption and leisure as is illustrated in Figure 2(c).

During transition the stocks of physical capital and pollution decline sharply – see panels (e) and (f) in Figure 2. To keep the dirt flow equal to zero a lower abatement level suffices and since capital is falling the capital tax is lowered – see panels (a) and (b). As the pollution stock falls, the shadow price of environmental quality, $q(t)$, gradually declines as is shown in

Table 3: Quantitative effects of optimal taxation and abatement‡

	BM	SO				
			Complementarity			*Logistic*
	(a)	(b)	(c)	(d)	(e)	(f)
$Y(0)$		0.9820	0.9576	0.9486	0.9441	0.9804
$\hat{Y}$	1.0000	0.7283	0.8646	0.9245	0.9622	0.7197
$C(0)$		0.8378	0.8553	0.8599	0.8620	0.8405
$\hat{C}$	0.8091	0.5902	0.7043	0.7567	0.7903	0.5857
$L(0)$		0.3248	0.3133	0.3091	0.3070	0.3240
$\hat{L}$	0.3333	0.2435	0.2919	0.3149	0.3300	0.2425
$\hat{K}$	2.7273	1.9735	2.2890	2.3971	2.4555	1.9136
$\hat{P}$	1.2482	0.0634	0.0754	0.0798	0.0821	0.0411
$q(0)$		8.4967	7.4130	7.3178	7.4109	8.9067
$\hat{q}$		0.0488	0.2257	0.3890	0.5152	0.1925
$\theta(0)$		0.4496	0.4610	0.4654	0.4676	0.4503
$\hat{\theta}$		0.0065	0.0292	0.0493	0.0643	0.0250
$G(0)$		0.1324	0.1324	0.1324	0.1324	0.1324
$\hat{G}$		0.0000	0.0000	0.0000	0.0000	0.0000
t_E		20.0	23.0	25.0	25.0	8.0
$\Lambda(0)$		−8.9490	−8.6970	−8.4919	−8.3573	−8.6288
$\bar{\Lambda}$	−10.2998	−10.2998	−9.7821	−9.6422	−9.5779	−10.2998
$EV(0)$		0.2243	0.1543	0.1589	0.1670	0.2844
μ	0.25	0.25	0.50	0.75	1.00	0.25
ε_c	0.2670	0.2670	0.3026	0.3119	0.3162	0.2670

‡BM: parameterized base model of the polluted unmanaged economy. SO: social optimum. Notation: $x(0)$ and $\hat{x}$ denote, respectively, the impact- and long-run (steady-state) value of the variable $x(t)$. In columns (c)–(e) alternative values for μ are used and ε_c is recomputed such that the unmanaged economies are observationally equivalent to the base scenario.

panel (d). Note that during transition the two private components of household felicity, consumption and leisure, move in opposite directions. Intuitively, since environmental enjoyment and leisure are complements they move together over time.

3.2.2 Regime 2

At time $t = t_E$ abatement is permanently reduced to zero ($G_s(t) = 0$ for $t \geq t_E$) and the dirt flow becomes positive (as $D_s(t) = \kappa K_s(t)$). The value of t_E is such that $q_s(t_E) = 1/\gamma$. This is the point in Figure 2(d) where the solid line intersects the dashed line. Like the stocks of capital and pollution, consumption, employment, and the shadow price of environmental quality are all continuous at time t_E. For $t \geq t_E$ the expressions in (57) and (61) are still relevant but the optimal time profiles for pollution, consumption, and the capital stock are changed to:

$$\dot{P}_s(t) = -\Pi(P_s(t)) + \kappa K_s(t), \tag{62}$$

$$\frac{\dot{C}_s(t)}{C_s(t)} = F_K(K_s(t), L_s(t)) - (\rho + \delta + \kappa q_s(t)), \tag{63}$$

$$\dot{K}_s(t) = F(K_s(t), L_s(t)) - C_s(t) - \delta K_s(t). \tag{64}$$

The abrupt cessation of abatement at time t_E results in a discrete upward jump in investment – see Figure 2(a). As a result the capital stock recovers somewhat over time (panel (e)) as does consumption (panel (c)). Despite the fact that the dirt flow features a discrete upward jump at time t_E, pollution continues to decline – though at a slightly retarded rate at first – because the environmental system is now safely in the basin of attraction described by the lower branch of the PE locus in Figure 1(d).

In passing through the two regimes, the first-best social optimum is decentralized by means of a tax on capital, $\theta_s(t)$, which takes the following form:

$$\theta_s(t) = \begin{cases} \dfrac{\kappa/\gamma}{F_K(K_s(t), L_s(t))} & 0 \leq t < t_E \\ \dfrac{\kappa q_s(t)}{F_K(K_s(t), L_s(t))} & t \geq t_E \end{cases} \tag{65}$$

The system consisting of (57)–(64) converges to the steady state given in (49)–(54) above. Similarly, the optimal pollution tax (65) converges in the long run to the value given in (55).

3.2.3 Discussion

Several features of the optimal environmental policy are worth noting. First, as is clear from the numerical results in column (b) of Table 3, the policy maker can produce welfare gains by engineering a move from the dirty to the clean equilibrium, i.e. for our parameterization lifetime utility increases from $\bar{\Lambda} = -10.2998$ (its value in the initial polluted steady state described in column (a)) to $\Lambda(0) = -8.9490$ (see column (b)). In order to get a better understanding of the size of this welfare effect we compute an equivalent-variation welfare measure which is given by:

$$EV(0) \equiv \frac{\hat{C}^c - \hat{C}}{\hat{C}} = e^{\frac{\rho}{\varepsilon_c}\left[\Lambda(0)-\bar{\Lambda}\right]} - 1, \tag{66}$$

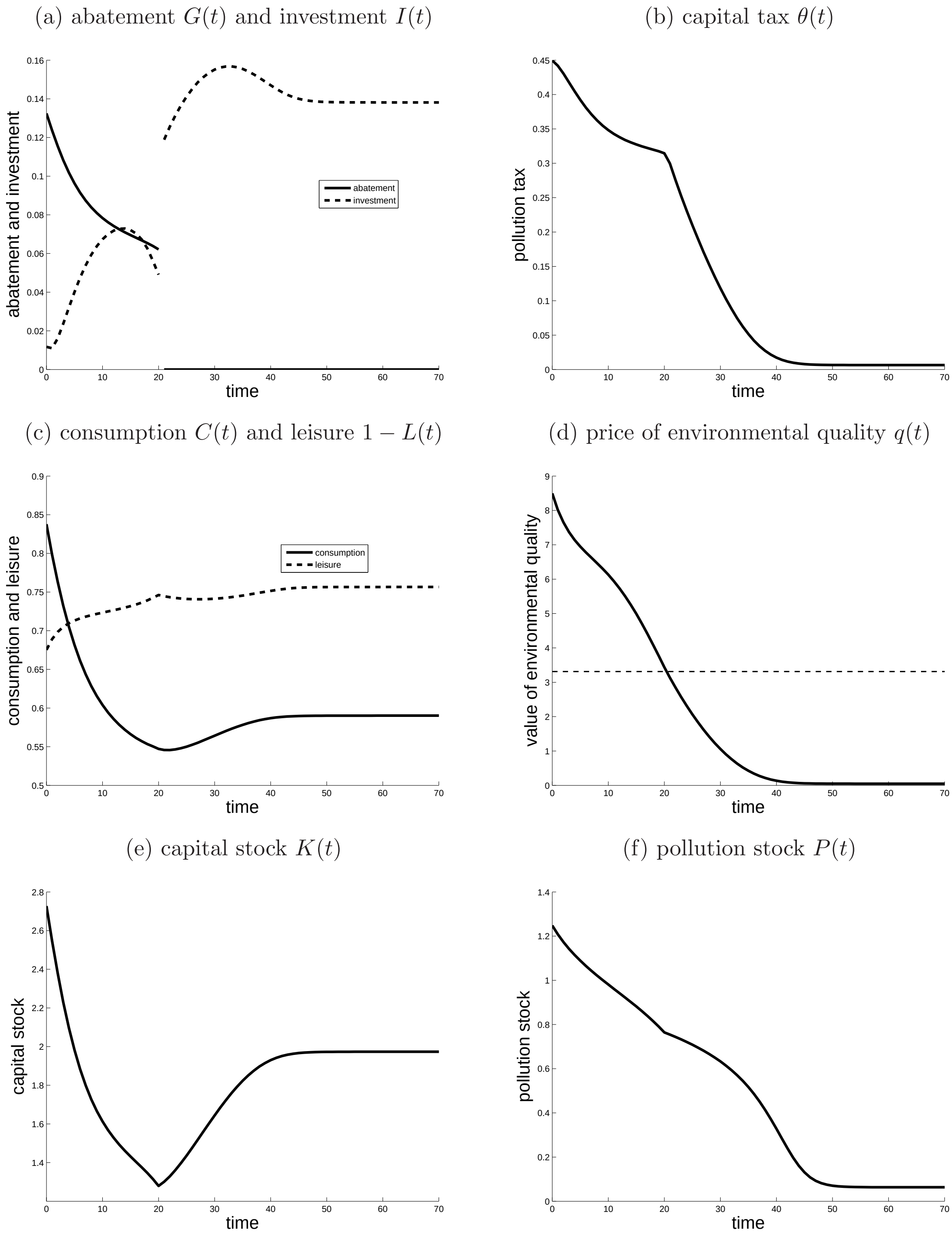
where $\hat{C}$ and $\bar{\Lambda}$ are, respectively, the initial steady-state consumption and utility level in the unmanaged economy, $\Lambda(0)$ is utility under the first-best social optimum, and $\hat{C}^c$ is the hypothetical steady-state consumption level that would render people indifferent between the unmanaged economy and the social optimum. Note that we hold leisure and environmental quality constant under the compensated scenario. We find that the welfare effect of the first-best optimal policy is quite substantial, $EV(0) = 0.2243$ in column (b) of Table 3, i.e. compensated consumption is almost 23 percent higher than actual consumption. Despite the fact that consumption is much lower that its initial level during much of the transition, the gradual improvement in environmental quality and the increase in leisure more than compensate for this.

Second, in the social optimum the two available policy instruments each play a distinct role. During the initial phase of the policy, abatement is used to choke off the flow of dirt as much as is feasible whereas the tax is employed to bring down the stock of the polluting capital input at the optimal rate (taking into account that the marginal social product of capital affects the consumption profile of the representative household). In the long run, however, abatement is no longer needed and the capital tax settles down at its externality-correcting Pigouvian level. As the results in column (b) of Table 3 confirm, the long-run pollution tax is quite low. It is only during transition that the full force of the available environmental instruments is needed.

Third, the complementarity between leisure and environmental quality plays a major role in the model. Intuitively, by moving from a situation of low to high environmental quality, individuals will want to move from a low to a high amount of leisure, i.e. labour supply will fall. In the long run, however, the capital intensity of production will be more of less restored to its initial level (because the pollution tax is low) so that the capital stock will also fall drastically. The labour supply effect thus ensures that physical capital and environmental quality are substitutes in the long run.[10]

[10] Heijdra and Heijnen (2014) study optimal environmental policy in a model with exogenous labour supply. Their model is a special case of the model employed here – it is obtained by setting $\varepsilon_e = 1$, $L(t) = 1$, and dropping equation (T1.2). With exogenous labour supply the steady-state reduction of the capital stock is quite small.

Figure 2: The first-best optimal policy



4 Extensions

In this section we consider a number of robustness scenarios. In subsection 4.1 we look more closely at the core mechanism of our base model, namely the complementarity between leisure and environmental quality. In subsection 4.2 we zoom in on another crucial component in our model, namely the functional form of the pollution dynamics equation.

4.1 Strength of the complementarity effect

In our model the parameter μ is the substitution elasticity between leisure and environmental quality in the environmental enjoyment function (8). In the base parametrization we set $\mu = 0.25$ which implies strong complementarity between leisure and environmental quality. In this subsection we investigate to what extent our qualitative and quantitative conclusions are affected if the complementarity effect is less extreme. Indeed, in columns (c)-(e) of Table 3 we provide some quantitative results for the cases $\mu \in \{0.5, 0.75, 1\}$ whereas in Figure 3 we depict the time profiles of some key variables for these three alternative cases.

In recomputing the social optimum we adopt the following strategy. For each alternative value of μ we recalibrate the ε_c parameter in such a way that for output, consumption, employment, the capital and pollution stocks, and the value of environmental quality the steady-state equilibrium values reported in column (a) are found for the unmanaged laissez-faire equilibrium at the alternative (μ, ε_c) values. The resulting values for ε_c are reported in the bottom row of Table 3. By adopting this approach we retain compatibility between the different allocations. Of course, since μ and ε_c are preference parameters, the welfare indicator $\Lambda(0)$ cannot be compared between columns (b)-(e). The equivalent-variation measures, however can still be used to compare the social optimum to the unmanaged equilibrium in each case. Specifically, the reported values for $\bar{\Lambda}$ in columns (c)–(e) are based on the steady-state level of environmental enjoyment, $\hat{M}$, recomputed with the new ε_c value. Within the same columns, therefore, $\Lambda(0)$ and $\bar{\Lambda}$ are directly compatible.

Several features of the optimal environmental policy are worth noting. First, as is clear from Table 3, the long-run reductions in output, consumption, employment, and the stocks of capital and pollution are smaller the larger is μ, i.e. the weaker is the complementarity between leisure and environmental quality. In the most extreme case considered, $\mu = 1$ the felicity function is loglinear (and thus separable) in its arguments, labour supply does not depend on the pollution stock (as $\eta_{LP} = 0$ in (23)), and the link between the economy and the environment is again unidirectional as in our earlier papers (Heijdra and Heijnen, 2013, 2014). In this case the long-run effects on $\hat{Y}$, $\hat{C}$, $\hat{L}$, and $\hat{K}$ are small both because there is no abatement in the long run and the long-run capital tax is quite small. For lower values of μ the long-run reduction in the pollution stock exerts an additional direct effect – via labour supply – on equilibrium employment and the capital stock.

Second, the pattern of the optimal environmental policy is very similar for the different μ values considered – see panels (a)-(b) in Figure 3. Government abatement occurs at full throttle during the early part of the transition and is reduced to zero thereafter whilst the capital tax is set at a high level initially and falls gradually over time to settle at a fairly low externality-correcting level in the long run. Interestingly, both the length of the abatement period and the level of the long-run capital tax are increasing in μ. Hence, the weaker is the complementarity effect the more vigorously the policy instruments have to be applied.

Finally, the welfare conclusions are very similar for the different μ values. The $EV(0)$ measure suggests welfare gains due to the optimal environmental policy in the range of 15 to 17 percent of initial steady-state consumption.

4.2 Logistic versus shallow lake dynamics

In our base model environmental dynamics is characterized by SLD with reversible hysteresis – see equation (18). The thinly dotted line labeled PE-SLD in Figure 4 is the P-isocline for this type of environmental dynamics when abatement is absent. As the solid line – labeled CKE – represents the (P, K) combinations for which the unmanaged economy is in a steady-state equilibrium, it follows that in the base model (featuring $\mu = 0.25$) there are two saddle-point stable equilibria, namely a polluted one at A and a clean one at C.

In this subsection we investigate to what extent our qualitative and quantitative conclusions are affected by the specific functional form of the P-isocline. In particular we study the case in which environmental dynamics is of the logistic form, i.e. we assume that (18) is replaced by:

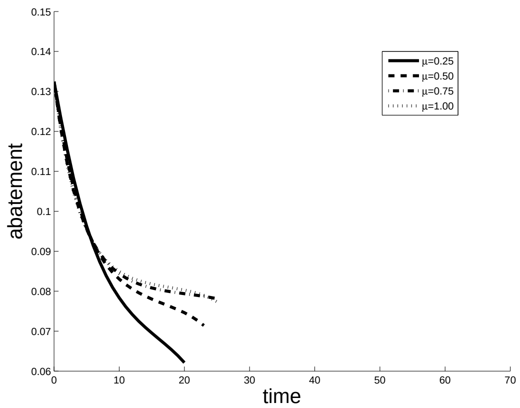
$$\Pi(P(t)) \equiv \frac{1}{\pi_0} \log\left(\frac{\bar{E} + \pi_1 P(t)}{\bar{E} - P(t)}\right), \qquad \text{for } 0 \leq P(t) \leq \bar{E}, \tag{67}$$

where π_0 and π_1 are positive parameters. This function features $\Pi(0) = 0$ so a pristine environment is not precluded, it is upward-sloping for $0 \leq P(t) < \bar{E}$, and features a horizontal asymptote at $P(t) = \bar{E}$. We parameterize the logistic equation in such a way that it intersects the CKE locus at the same stable equilibrium points A and C – see the dashed line labeled PE-LOG in Figure 4. This gives $\pi_0 = 3.8884\,10^2$ and $\pi_1 = 2.2861\,10^6$.

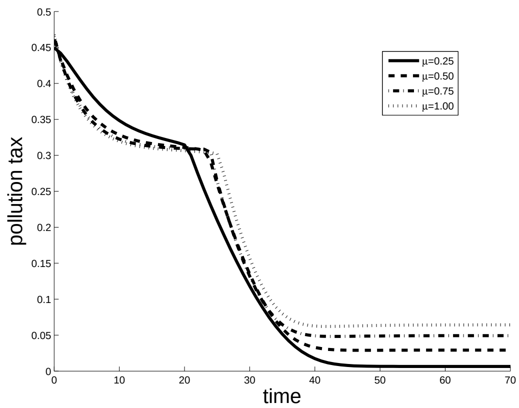
In column (f) of Table 3 we report some quantitative results for the logistic case and in Figure 5 we depict the corresponding time profiles for some key variables. The main features of the optimal environmental policy are as follows. First, as the comparison between columns (a) and (f) in Table 3 reveals, the long-run effects on output, consumption, employment, and the capital stock are very similar in magnitude. The long-run reduction in the pollution stock is, however, much larger for the logistic case because PE-LOG lies below PE-SLD at low pollution levels – see Figure 4. As a result of this, though still modest in size, the long-run capital tax is substantially higher for the logistic specification.

Figure 3: Optimal policy and complementarity

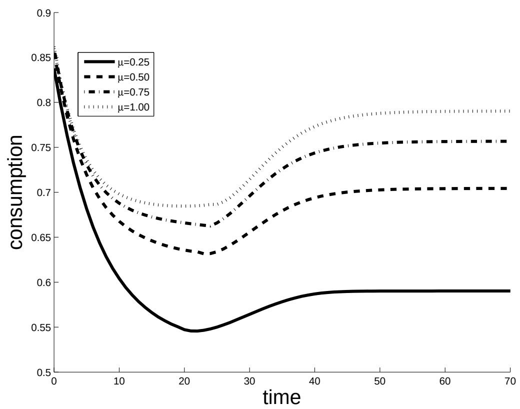
(a) government abatement $G(t)$



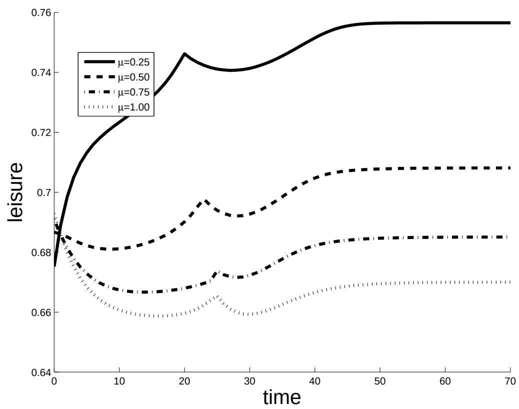
(b) capital tax $\theta(t)$



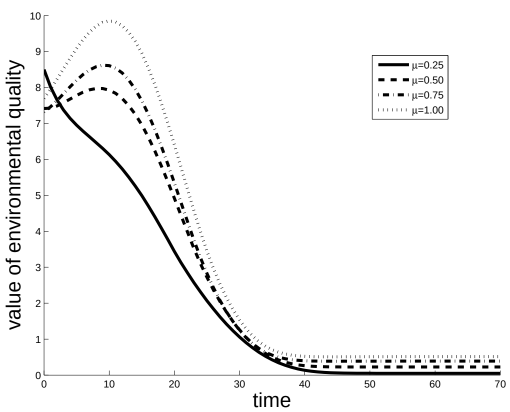
(c) consumption $C(t)$



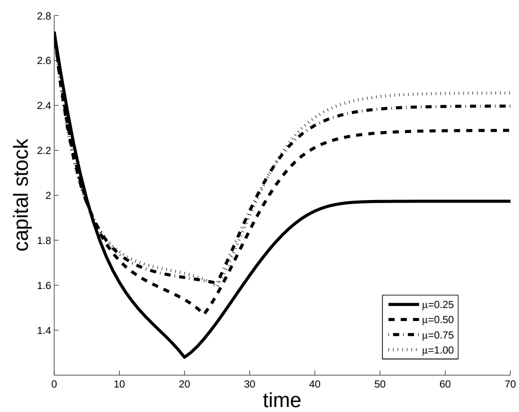
(d) leisure $1 - L(t)$



(e) price of environmental quality $q(t)$



(f) capital stock $K(t)$



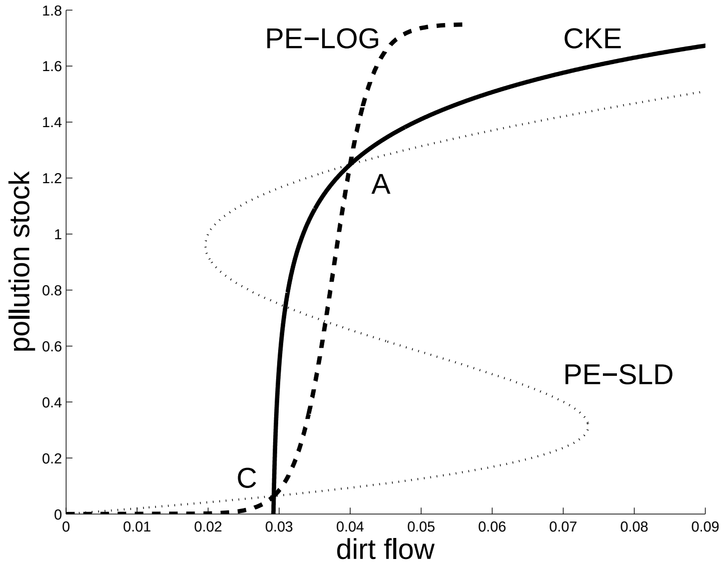


Figure 4: Logistic environmental dynamics

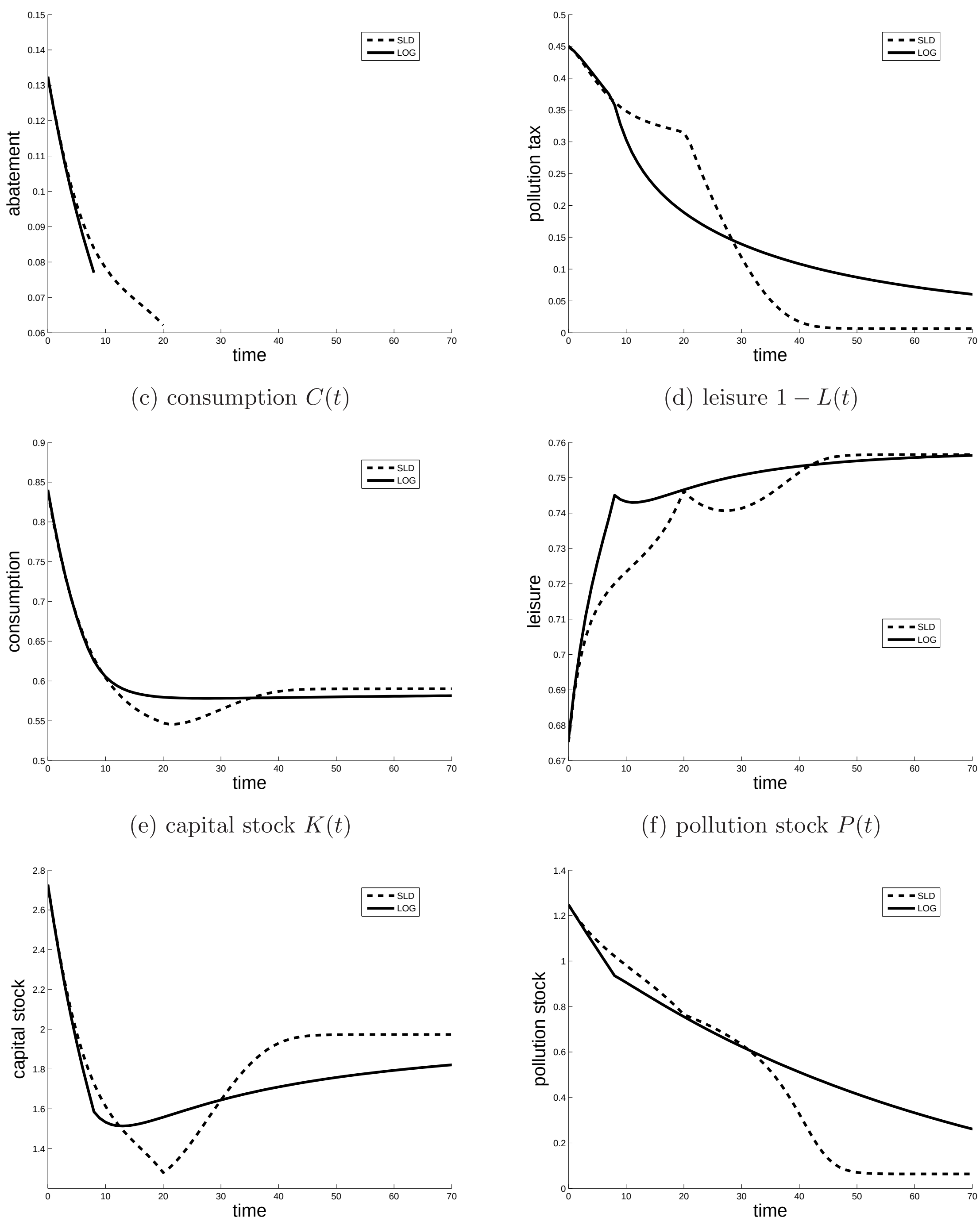
Second, the pattern of the optimal environmental policy is very similar for the logistic and shallow-lake cases – see panels (a)-(b) in Figure 5. Government abatement takes place at its maximum feasible level early on and then drops to zero whilst the capital tax is set at a high level initially and falls gradually over time to settle at a relatively low level in the long run. Interestingly, the optimal duration of abatement and the speed of transition in the stocks of capital and pollution are much lower for the logistic case. Intuitively, once the critical (K, P) point has been passed under SLD (i.e. point B in Figure 1(d)), adjustment to the clean steady-state equilibrium is quite rapid (compared to convergence under the logistic formulation).

Finally, the welfare conclusions are very similar for the logistic and shallow-lake cases. Indeed, for the former the welfare gain is about 28 percent of initial steady-state consumption whilst it is about 22 percent for the SLD case.

5 Conclusions

In this paper we have studied optimal environmental policy in a world featuring multiple stable economic-ecological equilibria. There is a two-way interaction between the macro-economy and the environment because, on the one hand, the capital input gives rise to pollution and, on the other hand, leisure and environmental quality are complementary inputs in the household's felicity function. We assume that there exist two stable economic-ecological equilibria and that the society under consideration finds itself in a high-output *cum* high-pollution equilibrium which we call the "rat-race pollution trap." We show that a benevolent social planner is

Figure 5: Optimal policy and logistic dynamics



(a) government abatement $G(t)$

(b) capital tax $\theta(t)$

(c) consumption $C(t)$

(d) leisure $1 - L(t)$

(e) capital stock $K(t)$

(f) pollution stock $P(t)$

in principle able to engineer substantial welfare gains by choosing the appropriate mix of Pigouvian (capital) taxation and abatement activities. In the first-best economic-ecological equilibrium that is attained in the long run, the stock of pollution is low, environmental quality is high, and people enjoy a lot of leisure.

In the social optimum the two available policy instruments each play a distinct role. During the initial phase of the policy, abatement is used to reduce the stock of pollution as quickly as feasible whereas the tax is employed to bring down the stock of the polluting capital input in an optimal fashion. In the long run, however, abatement is no longer needed and the capital tax settles down at its externality-correcting Pigouvian level. For a plausible parameterization optimal abatement takes place for up to twenty-five years whilst the long-run pollution tax never exceeds seven percent of gross operating surplus. Hence, in order to escape out of the pollution trap the full force of the available environmental instruments is only needed temporarily.

In this paper we present a purely theoretical description of optimal environmental policy in the presence of multiple Pareto-rankable equilibria. While the theoretical results are clear, our model is far too stylized to yield concrete policy recommendations. It must be stressed that we deliberately focus on the closed-economy case because we agree with Nordhaus (2008) and consider pollution and environmental degradation to be a truly global phenomenon. But this is also where our approach fails to connect to reality. Whereas in our model the benevolent social planner can put in place the socially optimal allocation hindered only by resource constraints, matters are much more complicated in the decentralized and heterogeneous world that we live in. As the spectacular failure of the 2009 Copenhagen Summit reveals, it is quite easy for world leaders to agree that climate change is a big problem but very hard for them to agree on how to solve it in an equitable way. If ever a coordinated global approach becomes feasible then the insights from our paper may be of some use to the policy maker.

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