Economic growth and longevity risk with adverse selection*

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Abstract: We study the implications of adverse selection in annuity markets in a general-equilibrium model of the closed economy. Agents differ in their health type and invest their assets in the annuity market. Without informational asymmetries each agent would obtain an actuarially fair insurance. If the individual health types and total annuity purchases are unobservable to the annuity firms then there exists a pooling equilibrium in which all agents annuitize at a common rate. At this pooling rate unhealthy agents would eventually like to borrow but this would reveal their true health type. As a consequence, they rationally drop out of the market. Surprisingly, the welfare and growth effects of the informational asymmetries are rather small.

Keywords: Annuity markets, adverse selection, endogenous growth, overlapping generations, demography.

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1 Introduction

Economic theory suggests that life annuities are very attractive insurance instruments in the presence of longevity risk. This result was first articulated in the seminal paper by Yaari (1965) and was recently recast in a much more general setting by Davidoff, Brown, and Diamond (2005). The intuition behind this result is not very difficult: annuities insure against the risk of outliving one's assets.

Empirical evidence, however, suggests that in reality very few individuals purchase life annuities despite their theoretical attractiveness. Friedman and Warshawsky (1990, pp. 136-7) give the following potential explanations for the low participation in private annuity markets. First, individuals may want to leave bequests to their offspring. Second, individuals may hold other types of annuities, e.g. in the form of social security and private pensions (social annuities). Third, private annuities may be priced unattractively because of transaction costs and taxes, excessive monopoly profits earned by annuity firms, and adverse selection.[1] A fourth explanation is that family risk sharing may act an as incomplete annuity market, a result first proposed by Kotlikoff and Spivak (1981).

The objective of our paper is to study the growth and welfare implications of adverse selection in the annuity market.[2] Intuitively, adverse selection arises because individuals who believe themselves to be healthier than average are more likely to buy annuities, i.e. the high-risk types are overrepresented in the clientele of annuity firms and annuity pricing cannot be based on average population mortality.

Our core model is built on the following assumptions. First, whereas much of the literature is framed in a partial equilibrium setting, we instead postulate a simple general equilibrium model of a closed economy featuring endogenous growth. We choose a dynamic general equilibrium framework because annuity purchases are intimately intertwined with the savings decisions of individuals which in the aggregate give rise to macroeconomic capital accumulation and growth. Second, we assume that the economy is populated by overlapping generations of heterogeneous finitely-lived agents. Individual agents know their own

[1] Following the initial research by Friedman and Warshawsky (1988, 1990), a large subsequent literature has emerged. See for example Mitchell *et al.* (1999), Finkelstein and Poterba (2002, 2004), and Finkelstein *et al.* (2009).

[2] Heijdra and Mierau (2009) study the general equilibrium implications of imperfect annuities under the excess monopoly profit interpretation.

death probability profile, but annuity firms cannot observe an agent's health type. The mortality process is modeled realistically and closely tracks existing demographic data. We distinguish two types of agents, namely healthy and unhealthy, and we restrict their respective population shares to be constant. Third, we assume perfectly competitive annuity markets. Our informational assumptions are consistent with firms offering *linear* annuity contracts, cf. Pauly (1974) and Abel (1986). Under such contracts the insurer can only choose the price of the annuity and cannot achieve complete market separation by offering non-linear price-quantity contracts.[3]

The main findings of our analysis are as follows. First, if health status were observable by insurers then each health type would get actuarially fair insurance against longevity risk. We consider the case of a patient economy in which all types would be net savers during life. In this first-best situation, however, healthy individuals have a huge incentive to misrepresent their health status ("by cheating" and claiming to be a low-risk type) thus destroying market separation. The perfect information equilibrium is therefore a hypothetical case acting as a benchmark.

Second, with asymmetric information regarding health types and annuity purchases, perfect competition in the annuity market will result in a pooling equilibrium. The equilibrium pooling rate is an asset-weighted average of individual mortality rates, a result derived in a partial equilibrium context by Sheshinski (2008). In the pooling equilibrium, the unhealthy (low-risk types) get a less than actuarially fair rate (as stressed in the literature), but the healthy (high-risk types) get a *better than actuarially fair rate*. This result shows that Friedman and Warshawsky (1990, pp. 147-152) only consider one side of the coin by restricting attention to individuals facing less than actuarially fair annuity returns (see their Tables V and VI).

Third, in the pooling equilibrium the unhealthy encounter a "self-imposed borrowing constraint" if they live long enough. Intuitively, as the unhealthy get close to their maximum attainable age, the pooling rate prompts such individuals to become net borrowers. But borrowing would reveal their health status, so the best the unhealthy can do is to impose a

[3]Alternative equilibrium concepts that can be used to deal with adverse selection are the ones suggested by Rothschild and Stiglitz (1976) and Wilson (1977). See Eichenbaum and Peled (1987) for an application of the Rothschild-Stiglitz concept. See Walliser (2000, pp. 376-7) and below for a defense of the linear pricing assumption.

borrowing constraint on themselves during their autumn years. It must be stressed that this asset depletion result is not exogenously imposed (as in the partial equilibrium studies of Friedman and Warshawsky (1990, p. 147) and Walliser (2000, pp. 378-9)) but follows from the internal logic of the model. Hence, our model yields a consistent explanation why not everybody participates in annuity markets in a general equilibrium model with risk pooling. It must be stressed that our model cannot explain the annuity puzzle in the sense of agents rationally holding all or part of their wealth in a non-annuitized form.

Fourth, for a plausibly calibrated version of the core model we find that the first-best is only slightly better in growth and welfare terms than the pooling equilibrium. Hence, the underlying information asymmetry and the resulting adverse selection effects in the annuity market do not seem to cause quantitatively large growth and welfare effects in a general equilibrium setting. We also show that the bulk of the effects on the allocation and welfare is explained by the general equilibrium channel. The macroeconomic adjustments bring about a magnification of the partial equilibrium outcomes.

Fifth, all of these findings are robust to (a) alternative assumptions regarding labour market participation and retirement and (b) to a different specification of the economic growth process.

The structure of our paper is as follows. Section 2 presents the model. Section 3 states the key informational assumptions and studies the balanced growth path for the (hypothetical) perfect information equilibrium and the asymmetric information equilibrium with risk pooling. This section also presents a plausible calibration and visualization of the different equilibria as well as their welfare properties. Section 4 reports some robustness checks. In particular it examines the role of endogenous versus exogenous growth and it incorporates a pay-as-you-go pension system and labour force retirement. Finally, section 5 restates the main results and presents some possible extensions. The paper also contains two brief mathematical appendices.

2 Model

2.1 Consumers

2.1.1 Individual behaviour

Individuals differ according to their health status acquired at birth. This status cannot be changed by the agent and can therefore be interpreted as his or her general 'constitution'. From the perspective of birth, the expected remaining lifetime utility function of a health type j individual is given by:

$$\Lambda_j(v,v) = \int_v^{v+\bar{D}_j} \frac{\bar{c}_j(v,\tau)^{1-1/\sigma}-1}{1-1/\sigma} e^{-\rho(\tau-v)-M_j(\tau-v)} d\tau, \tag{1}$$

where v is the date of birth, $\bar{D}_j$ is the maximum attainable age for this type of agent, $\bar{c}_j(v,\tau)$ is consumption, σ is the intertemporal substitution elasticity ($\sigma > 0$), ρ is the pure rate of time preference, and $e^{-M_j(\tau-v)}$ is the probability that the agent is still alive at some future time τ ($\geq v$).[4] Here, $M_j(\tau-v) \equiv \int_0^{\tau-v} \mu_j(s)ds$ denotes the cumulative mortality rate and $\mu_j(s)$ is the instantaneous mortality rate of an agent of age s, where $0 \leq s \leq \bar{D}_j$. This rate is strictly increasing and convex in age, $\mu_j'(s) > 0$ and $\mu_j''(s) > 0$, and features $\lim_{s\to\bar{D}_j} \mu_j(s) = +\infty$.

The agent's budget identity is given by:

$$\dot{\bar{a}}_j(v,\tau) = \left[r + p_j(\tau-v)\right] \bar{a}_j(v,\tau) + w(\tau) - \bar{c}_j(v,\tau), \tag{2}$$

where $\bar{a}_j(v,\tau)$ is real financial wealth, r is the interest rate (a constant, see below), and $w(\tau)$ is the wage rate. In the spirit of Yaari (1965), we assume that agents can purchase continuous annuities to insure against longevity risk. Annuity contracts are recontracted at each moment in time. Without a bequest motive, financial wealth is fully annuitized so $\bar{a}_j(v,\tau)$ is also the agent's demand for annuities. Below we assume that an agent's age at time τ is directly observable to the insurer so that the net return on annuities, $p_j(\tau-v)$, depends on it. Labour supply is exogenous and each agent supplies a single unit of labour throughout life, i.e. in the main part of the paper we abstract from retirement.[5]

[4] For a detailed derivation of the lifetime utility function in the presence of mortality risk, see Heijdra and Romp (2008, pp. 91–92). The assumption of a maximum attainable age is made for computational convenience only.

[5] In section 4.1, however, we introduce a simple pension system and mandatory retirement.

At time v, the agent chooses paths for consumption and financial assets in order to maximize lifetime utility (1) subject to the flow budget identity (2) and a solvency condition, taking as given the initial level of financial assets, $\bar{a}_j(v,v) = 0$. In the absence of borrowing constraints, the agent's optimal plans for $v \le t \le v + \bar{D}_j$ are fully characterized by:

$$\frac{\dot{\bar{c}}_j(v,t)}{\bar{c}_j(v,t)} = \sigma\left[r + p_j\left(t-v\right) - \mu_j\left(t-v\right) - \rho\right], \tag{3}$$

$$\bar{c}_j(v,v) = \frac{\int_v^{v+\bar{D}_j} w\left(\tau\right) e^{-r(\tau-v)-P_j(\tau-v)} d\tau}{\int_v^{v+\bar{D}_j} e^{-(1-\sigma)\left[r(\tau-v)+P_j(\tau-v)\right]-\sigma\left[\rho(\tau-v)+M_j(\tau-v)\right]} d\tau}, \tag{4}$$

$$\begin{aligned}\bar{a}_j\left(v,t\right) e^{-r(t-v)-P_j(t-v)} &= \int_v^t w\left(\tau\right) e^{-r(\tau-v)-P_j(\tau-v)} d\tau \\ &\quad -\bar{c}_j(v,v) \int_v^t e^{-(1-\sigma)\left[r(\tau-v)+P_j(\tau-v)\right]-\sigma\left[\rho(\tau-v)+M_j(\tau-v)\right]} d\tau,\end{aligned} \tag{5}$$

where $P_j(\tau - v) \equiv \int_0^{\tau-v} p_j(s)ds$ is the cumulative net annuity return factor. Equation (3) is the 'consumption Euler equation', relating the optimal time profile of consumption to the difference between the annuity rate of interest ($r + p_j\left(\tau - v\right)$) and the total rate of felicity discounting due to impatience and mortality ($\rho + \mu_j\left(\tau - v\right)$). Equation (4) shows that consumption at birth is proportional to human wealth (the numerator), consisting of the annuitized value of wages. Finally, the planned path of financial wealth is defined in (5). It is easy to see that financial assets are zero at birth and at the date of certain death, $\bar{D}_j$.

Below we encounter equilibria in which type j agents experience a binding borrowing constraint from age $\bar{S}_j$ onward. In that case equations (3) and (5) are valid only for $0 \le t - v \le \bar{S}_j$, $\bar{a}_j\left(v,t\right) = 0$ and $\bar{c}_j\left(v,t\right) = w\left(t\right)$ for $\bar{S}_j \le t - v \le \bar{D}_j$, and $\bar{S}_j$ replaces $\bar{D}_j$ in (4).

2.1.2 Demography

We allow for a non-zero rate of population growth but impose that the relative population proportion of people of different health types is constant over time. Since health groups are distinguished by their mortality process, this requirement furnishes the following condition:

$$\beta_j \int_0^{\bar{D}_j} e^{-ns-M_j(s)} ds = 1, \tag{6}$$

where β_j is the crude birth rate of type j cohorts, and n is the growth rate of the population. For a given value of n and a given mortality process $M_j\left(s\right)$, equation (6) gives the birth rate which is consistent with a constant population share. The newborn cohort of type j at time v is given by $L_j\left(v,v\right) = \pi_j \beta_j L\left(v\right)$ where $L\left(v\right)$ is the total population at time v and π_j is the fraction of type j people in the population ($\sum_j \pi_j = 1$). Finally, the relative cohort size of type j agents of age $t-v$ evolves according to:

$$l_j(v,t) \equiv \frac{L_j(v,t)}{L(t)} = \begin{cases} \beta_j \pi_j e^{-n(t-v)-M_j(t-v)} & \text{for } 0 \le t-v \le \bar{D}_j \\ 0 & \text{for } t-v > \bar{D}_j \end{cases} \tag{7}$$

Intuitively, the *relative* size of the type j cohort declines with age because the aggregate population grows over time (first cause) and cohort members die (second cause).

2.1.3 Aggregate household behaviour

Armed with equation (7), it is possible to compute per capita values for consumption and assets. We restrict attention to the balanced growth path along which wages grow at a constant exponential rate, g (see section 3 below). It follows that:

$$w(t) = w(v)\, e^{g(t-v)}. \tag{8}$$

Allowing for a borrowing constraint at age $\bar{S}_j$ and using (8) we find that per capita consumption of type j agents, $c_j(t) \equiv \int_{t-\bar{D}_j}^{t} l_j(v,t)\, \bar{c}_j(v,t)\, dv$, can be written as:

$$\frac{c_j(t)}{w(t)} = \beta_j \pi_j \left[\frac{\bar{c}_j(v,v)}{w(v)} \int_0^{\bar{S}_j} e^{-(n+g)s-(\sigma+1)M_j(s)+\sigma(r-\rho)s+\sigma P_j(s)} ds + \int_{\bar{S}_j}^{\bar{D}_j} e^{-ns-M_j(s)} ds \right], \tag{9}$$

where $\bar{c}_j(v,v)/w(v)$ is independent of the generations index v. By aggregating over all health types, per capita consumption is obtained, i.e. $c(t) \equiv \sum_j c_j(t)$.

In a similar fashion we find that per capita asset holdings of type j agents, $a_j(t) \equiv \int_{t-\bar{D}_j}^{t} l_j(v,t)\, \bar{a}_j(v,t)\, dv$, evolves over time according to:

$$\dot{a}_j(t) = (r-n)\, a_j(t) + \pi_j w(t) - c_j(t) + \int_{t-\bar{D}_j}^{t} \left[p_j(t-v) - \mu_j(t-v) \right] l_j(v,t)\, \bar{a}_j(v,t)\, dv. \tag{10}$$

It follows that per capita assets, $a(t) \equiv \sum_j a_j(t)$, satisfy the following differential equation:

$$\dot{a}(t) = (r-n)\, a(t) + w(t) - c(t) + \Xi(t), \tag{11}$$

where $\Xi(t)$ is defined as:

$$\Xi(t) \equiv \sum_j \int_{t-\bar{D}_j}^{t} l_j(v,t) \left[p_j(t-v) - \mu_j(t-v) \right] \bar{a}_j(v,t)\, dv. \tag{12}$$

2.2 Firms

In the spirit of Romer (1989), we assume that there exist strong external effects between private firms in the economy. The economy features a large and fixed number, say N_0, of identical, perfectly competitive firms. The technology available to firm i is given by:

$$Y_i(t) = \Omega(t) K_i(t)^{\varepsilon} L_i(t)^{1-\varepsilon}, \qquad 0 < \varepsilon < 1, \tag{13}$$

where $Y_i(t)$ is output, $K_i(t)$ is the capital input, $L_i(t)$ is the labour input, and $\Omega(t)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$w(t) = (1-\varepsilon)\Omega(t) k_i(t)^{\varepsilon}, \tag{14}$$

$$r(t) + \delta = \varepsilon\Omega(t) k_i(t)^{\varepsilon-1}, \tag{15}$$

where $k_i(t) \equiv K_i(t)/L_i(t)$ is the capital intensity. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity such that $k_i(t) = k(t)$ for $i = 1, \ldots, N_0$. This feature enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Romer (1989) to a non-constant population, we assume that the inter-firm externality takes the following form:

$$\Omega(t) = \Omega_0 k(t)^{1-\varepsilon}, \tag{16}$$

where Ω_0 is a positive constant, $k(t) \equiv K(t)/L(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $L(t) \equiv \sum_i L_i(t)$ is aggregate employment. According to (16), total factor productivity depends positively on the economy-wide capital intensity. Hence if an individual firm i raises its capital intensity, then *all* firms in the economy benefit somewhat as a result because the general productivity indicator rises for all of them. Using (16), equations (13)–(15) can now be rewritten in aggregate terms:

$$Y(t) = \Omega_0 K(t), \tag{17}$$

$$w(t) L(t) = (1-\varepsilon) Y(t), \tag{18}$$

$$r(t) = r = \varepsilon\Omega_0 - \delta, \tag{19}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $\varepsilon\Omega_0 > n + \delta$. The macroeconomic technology is linear in the capital stock and the interest rate is constant and exceeds the rate of population growth.

3 Balanced growth path

In this section we study the steady-state features of the general equilibrium growth model. We adopt the following set of assumptions regarding the market for annuities.

Assumption (A1) The annuity market is perfectly competitive. A large number of firms offer annuity contracts to individuals. Firm entry and exit is unrestricted.

Assumption (A2) Annuity firms do not use up any real resources.

Assumption (A3) The annuitant's health status is private information and cannot be observed by the annuity companies. Annuity firms know all the features of the mortality process of each health group.

Assumption (A4) The annuitant's age is public information and can thus be observed by the annuity companies.

Assumption (A5) Annuitants can buy multiple annuities for different amounts and from different annuity firms. Individual annuity firms cannot monitor an annuitant's holdings with their competitors.

These assumptions are consistent with the emergence of a pooling equilibrium. Of critical importance is the joint validity of (A3) and (A5). Together they imply that annuity firms cannot distinguish their customers' health type. Even though healthy annuitants are richer than unhealthy annuitants (both in reality and in our model), and thus feature a higher total demand for annuities, they can nevertheless hide this fact by buying small amounts from several companies. Consequently, annuity firms are forced to apply *linear pricing*: they can only control the rate of return on annuities and cannot set prices and quantities simultaneously in order to induce full information revelation. Intuitively, the annuity market differs from regular insurance markets in that the death of an annuitant ends the insurer's liability instead of creating it. This rules out a system of withholding payments and investigating contract compliance that is often used in markets for fire- or car insurance (Walliser, 2000, pp. 376-7). Note that if (A5) did not hold it might be possible to exploit the information contained in quantities demanded to distinguish between health types and attain a separating equilibrium.

By assumption (A4), annuity firms can observe each annuitant's age so in the pooling equilibrium there is a single pooling rate, $\bar{p}(u)$, for healthy and unhealthy annuitants of age u. There is market segmentation in the sense that the annuity market consists of separate submarkets for each age group or cohort. By assumption (A1), the expected profit in each submarket is zero. With large cohorts, probabilities and frequencies coincide so that actual profit in each submarket is also zero. Finally, assumption (A2) ensures that there is no loading factor on annuities.

Before turning to a detailed study of the pooling equilibrium in subsection 3.2, we first discuss the benchmark case for which assumption (A3) is violated and annuity firms can observe each annuitant's health type. This is the perfect information equilibrium studied in subsection 3.1.

Although the model has been constructed to allow for an arbitrary number of health types, we simplify the discussion from here on by distinguishing only two health groups, namely healthy agents ($j = H$) and unhealthy agents ($j = U$). We furthermore adopt the mortality process of Boucekkine *et al.* (2002) which takes the following form:

$$e^{-M_j(s)} \equiv \frac{\eta_0 - e^{\eta_{1j}s}}{\eta_0 - 1}, \qquad 0 \le s \le \bar{D}_j \equiv \frac{1}{\eta_{1j}} \ln \eta_0, \tag{20}$$

where $\eta_0 > 1$ and $\eta_{1j} > 0$. The implied instantaneous mortality rate is given by:

$$\mu_j(s) \equiv M_j'(s) = \frac{\eta_{1j} e^{\eta_{1j}s}}{\eta_0 - e^{\eta_{1j}s}}. \tag{21}$$

This mortality process satisfies the assumption made in the text below equation (1). To capture the relative health status of the two groups we set $\eta_{1U} = \lambda\eta_{1H}$ with $\lambda > 1$. This parameterization implies that the maximum attainable age for the healthy exceeds the one for the unhealthy, i.e. $\bar{D}_H = \lambda\bar{D}_U$. Furthermore, the instantaneous mortality rate is uniformly higher for the unhealthy, i.e. $\mu_U(u) > \mu_H(u)$ for $0 \le u \le \bar{D}_U$. See Figure 1 below for a visualization of these results. In that figure and throughout the paper an "economic age" of $u = 0$ corresponds to a biological age of 18 years, i.e. we assume that independent economic decision making starts at the age of maturity.

(a) surviving fraction $e^{-M_j(u)}$ (b) instantaneous mortality rate $\mu_j(u)$

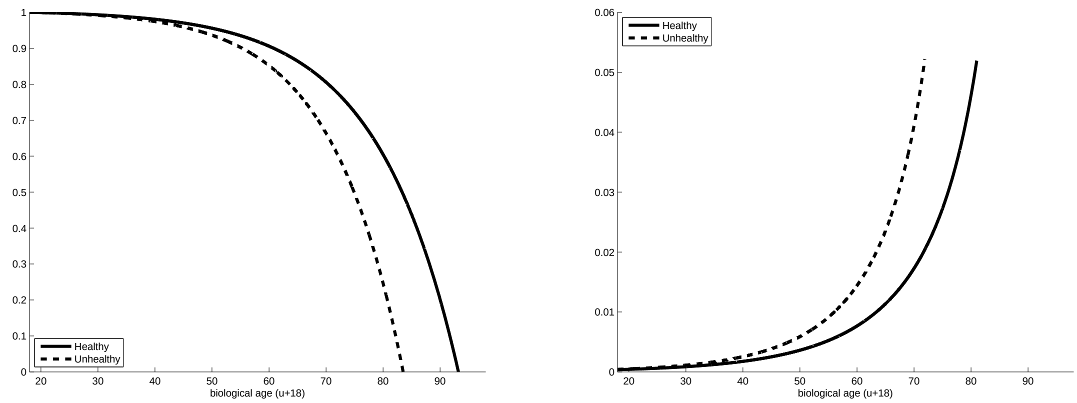


Figure 1: Demographics

3.1 Perfect information

If annuity firms are able to observe an annuitant's health type, then they will set the net return on annuities equal to the relevant mortality rate, $p_j(\tau - v) = \mu_j(\tau - v)$ and thus also $P_j(\tau - v) = M_j(\tau - v)$. It follows from (3), (5), and (8) that:

$$\frac{\dot{\bar{c}}_j(v, v+u)}{\bar{c}_j(v, v+u)} = \sigma[r - \rho], \tag{22}$$

$$\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - M_j(u)} = \int_0^u e^{-(r-g)s - M_j(s)} ds - \frac{\bar{c}_j(v,v)}{w(v)} \int_0^u e^{-(1-\sigma)rs - \sigma\rho s - M_j(s)} ds, \tag{23}$$

where $u \equiv t - v$ is the agent's age at time t. The instantaneous mortality rate does not feature in (22) because households fully insure against the unpleasant effects of lifetime uncertainty (Yaari, 1965). It is easy to see from (23) that financial assets are positive throughout the agent's life.

Proposition 1. *Consider the first-best situation in which annuity firms can observe the health type of annuitants. Provided $\sigma r - g > \rho$, agents of all health types are net savers throughout life, i.e. $\bar{a}_j(v, v) = \bar{a}_j(v, v + \bar{D}_j) = 0$ and $\bar{a}_j(v, v+u) > 0$ for $0 < u < \bar{D}_j$.*

Proof. See Appendix A. ☐

Table 1: Balanced growth with perfect information[a]

(a) Microeconomic relationships:

$$\frac{\bar{c}_j\left(v,v\right)}{w\left(v\right)} = \frac{\int_0^{\bar{D}_j} e^{-(r-g)s-M_j(s)}ds}{\int_0^{\bar{D}_j} e^{-(1-\sigma)rs-\sigma\rho s-M_j(s)}ds}, \qquad j\in\{H,U\} \tag{T1.1}$$

(b) Macroeconomic relationships:

$$\frac{c\left(t\right)}{w\left(t\right)} = \sum_{j\in\{H,U\}} \beta_j\pi_j\frac{\bar{c}_j\left(v,v\right)}{w\left(v\right)}\int_0^{\bar{D}_j} e^{-(n+g)s-M_j(s)+\sigma(r-\rho)s}ds \tag{T1.2}$$

$$g \equiv \frac{\dot{k}\left(t\right)}{k\left(t\right)} = r-n+\left[1-\frac{c\left(t\right)}{w\left(t\right)}\right]\frac{w\left(t\right)}{k\left(t\right)} \tag{T1.3}$$

$$\frac{w\left(t\right)}{k\left(t\right)} = \left(1-\varepsilon\right)\Omega_0 \tag{T1.4}$$

Note. [a]Endogenous are scaled newborn consumption by type j , $\bar{c}_j(v,v)/w(v)$, the steady-state growth rate, g, the wage-capital ratio, $w(t)/k(t)$, and the consumption-wage ratio, $c(t)/w(t)$. There are two types of agents, healthy (subscript H) and unhealthy (subscript U). $M_j(s)$, $\bar{D}_j$, β_j, and π_j stand for, respectively, the cumulative mortality rate at age s, the maximum attainable age, the crude birth rate, and the population fraction of type j agents. n is the population growth rate, ρ is the rate of time preference, ε is the capital coefficient in the technology, σ is the intertemporal substitution elasticity, and Ω_0 is the scale factor in the technology. The interest rate is $r \equiv \varepsilon\Omega_0 - \delta$, where δ is the depreciation rate of capital.

The key equations of the general equilibrium model under perfect information are collected in Table 1. The expressions in (T1.1) follow in a straightforward fashion from (4) and (8) above. Equation (T1.2) is obtained by setting $\bar{S}_j = \bar{D}_j$ in (9) and noting the definition of $c\left(t\right)$. The growth expression, equation (T1.3), follows readily from (11) by noting two features of the model. First, since claims on the capital stock are the only financial assets available, capital market equilibrium ensures that $a\left(t\right) = k\left(t\right)$. Second, since $p_j\left(t-v\right) = \mu_j\left(t-v\right)$ it follows from (12) that $\Xi\left(t\right) = 0$ for all t. There is no redistribution between health groups because each group receives the net return befitting its mortality profile. Finally, equation (T1.4) is obtained by combining equations (17)–(18).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. On the one hand, for a given macroeconomic growth rate g, (T1.1) determines scaled consumption at birth for the two health types. On the other hand, for given values of scaled consumption at birth, (T1.2)–(T1.4) yield general equilibrium solutions for $c(t)/w(t)$, g, and $w(t)/k(t)$.

In order to visualize the properties of the model and to quantify the effects of informational asymmetries and adverse selection on the general equilibrium allocation, we calibrate the model in a plausible fashion. We assume that the rate of population growth is one percent per annum ($n = 0.01$). We use data from biological age 18 onward for the cohort born in the Netherlands in 1960 and estimate the following model:

$$\begin{aligned} SURV_i &= \mathbf{d}_H(u_i)\frac{\beta_H\pi_H}{\beta_H\pi_H+\beta_U(1-\pi_H)}\frac{\eta_0-e^{\eta_{1H}u_i}}{\eta_0-1} \\ &\quad + \mathbf{d}_U(u_i)\frac{\beta_U(1-\pi_H)}{\beta_H\pi_H+\beta_U(1-\pi_H)}\frac{\eta_0-e^{\eta_{1U}u_i}}{\eta_0-1}+\varepsilon_i, \end{aligned} \tag{24}$$

where $SURV_i$ is the actual population fraction surviving up to age u_i, ε_i is the stochastic error term, and $\mathbf{d}_j(u_i)$ is a dummy variable such that $\mathbf{d}_j(u_i) = 1$ for $0 \le u_i \le (1/\eta_{1j})\ln\eta_0$ and $\mathbf{d}_j(u_i) = 0$ otherwise. Equation (24) is estimated with nonlinear least squares under the restrictions that $\eta_{1U} = \lambda\eta_{1H}$ and that the β_j parameters satisfy (6). This gives the following estimates (with robust t-statistic in brackets): $\hat{\eta}_0 = 187.865$ (16.18) and $\hat{\eta}_{1H} = 0.06961$ (79.63), $\hat{\pi}_H = 0.50767$ (15.54), and $\hat{\lambda} = 1.14793$ (177.63). It follows that $\hat{\eta}_{1U} = 0.0799$ (66.14), $\hat{\beta}_H = 0.0221$ (244.30), $\hat{\beta}_U = 0.0244$ (274.55), $\hat{\bar{D}}_H = 75.215$ (194.68), and $\hat{\bar{D}}_U = 65.522$ (176.04). There is a substantial difference between the maximum attainable age for the two health types of about 9.69 years. Similarly, whereas (economic) life expectancy at birth[6] is 61.25 years for the healthy, it is only 53.36 years for the unhealthy types. Figure 1 shows the key features of the mortality processes of the two health types.

The interest rate and capital depreciation rate are set at, respectively, five and seven percent per annum ($r = 0.05$ and $\delta = 0.07$). The efficiency parameter of capital is fixed at $\varepsilon = 0.3$ so that the constant in the production function is equal to $\Omega_0 = (r+\delta)/\varepsilon = 0.4$ and the capital-output ratio attains the plausible value of $K/Y = 2.5$. We postulate that in the perfect information benchmark the economy features a steady-state growth rate of two percent per annum ($g = 0.02$). For the intertemporal substitution elasticity we use $\sigma = 0.7$, a value often reported in empirical studies – see, e.g., Skinner (1985) and Attanasio and Weber

[6]Life expectancy at birth for type j individuals is equal to $\int_0^{\bar{D}_j} e^{-M_j(s)}ds$.

(1995). Using the pure rate of time preference as a calibration parameter we find that it is a shade over half a percent per annum ($\rho = 5.31610^{-3}$). For a summary, see column (a) of Table 2.

Figure 2 visualizes a number of life-cycle features of the perfect information equilibrium. Panel (a) depicts the age profiles for scaled consumption. The paths for the two health types are virtually on top of each other. As is clear from (22), consumption grows exponentially with age at a rate equal to $\sigma [r - \rho]$. Panel (b) of Figure 2 shows the life-cycle pattern of scaled cohort assets (individual assets display a rather similar pattern). As is to be expected, the healthy cohort is also the wealthiest of the two health types. The difference is more pronounced from middle age onward because the instantaneous mortality rates start to deviate strongly (see Figure 1(b)).

3.2 Asymmetric information

If we reinstate assumption (A3) so that annuity firms are not able to observe an annuitant's health type, then the best such a firm can do is to set the net return on annuities equal to a common, age-dependent, *pooling rate* $\bar{p}(u)$. This pooling rate takes the following form:

$$\bar{p}(u) = \begin{cases} \dfrac{\mu_H(u)\, a_H(v, v+u) + \mu_U(u)\, a_U(v, v+u)}{a_H(v, v+u) + a_U(v, v+u)} & \text{for } 0 < u \leq \bar{D}_U \\ \mu_H(u) & \text{for } \bar{D}_U < u \leq \bar{D}_H \end{cases} \tag{25}$$

Annuity firms know that the unhealthy cannot live beyond age $\bar{D}_U$ so for $\bar{D}_U < u \leq \bar{D}_H$ no risk pooling is possible and $\bar{p}(u)$ coincides with the instantaneous mortality rate of the healthy individuals. For $0 < u \leq \bar{D}_U$, however, both health types are alive and (potentially) active on the annuity market. The zero-profit condition for annuity firms furnishes the expression for the pooling rate in that case. It is the cohort-asset weighted sum of instantaneous mortality rates – see Sheshinski (2008, p. 71).

Using (3), (5), and (8) and *ignoring borrowing constraints for the time being*, we obtain the age profiles for consumption and assets for the two health types:

$$\frac{\dot{\bar{c}}_j(v, v+u)}{\bar{c}_j(v, v+u)} = \sigma \left[r + \bar{p}(u) - \mu_j(u) - \rho \right], \tag{26}$$

$$\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - \bar{P}(u)} = \int_0^u e^{-(r-g)s - \bar{P}(s)} ds - \frac{\bar{c}_j(v, v)}{w(v)} \int_0^u e^{-(1-\sigma)[rs + \bar{P}(s)] - \sigma\left[\rho s + M_j(s)\right]} ds, \tag{27}$$

Table 2: Endogenous growth: quantitative effects[a]

	(a) PI	(b) AI	(c) PI	(d) AI
	No retirement[b]		With retirement[c]	
$\dfrac{\bar{c}_H(v,v)}{w(v)}$	0.7668	0.7396	0.8267	0.8147
$\dfrac{\bar{c}_U(v,v)}{w(v)}$	0.7858	0.7775	0.8492	0.8476
$\bar{S}_H$ (years)	$\bar{D}_H$	$\bar{D}_H$	$\bar{D}_H$	$\bar{D}_H$
$\bar{S}_U$ (years)	$\bar{D}_U$	61.63	$\bar{D}_U$	63.93
$\dfrac{c(t)}{w(t)}$	1.0714	1.0753	0.8966	0.8979
g (%year)	2.00	1.89	2.00	1.96
$\dfrac{w(t)}{k(t)}$	0.2800	0.2800	0.3346	0.3346
$\Lambda_H(v_0,v_0)$	27.4619	26.4639	11.9477	11.8281
$\Lambda_U(v_0,v_0)$	21.8047	20.6189	10.3314	9.7873

Notes. [a]PI is the perfect information equilibrium and AI the asymmetric information (pooling) equilibrium. $\bar{S}_j$ is the age from which type j faces a borrowing constraint. Maximum attainable ages are $\bar{D}_H = 75.22$ and $\bar{D}_U = 65.52$. [b]The participation rate is $l = 1$. [c]In this case $\bar{R} = 47$, $\zeta = 0.3952$, $\theta = 0.07$, and $l = 0.8368$. See also Table 1.

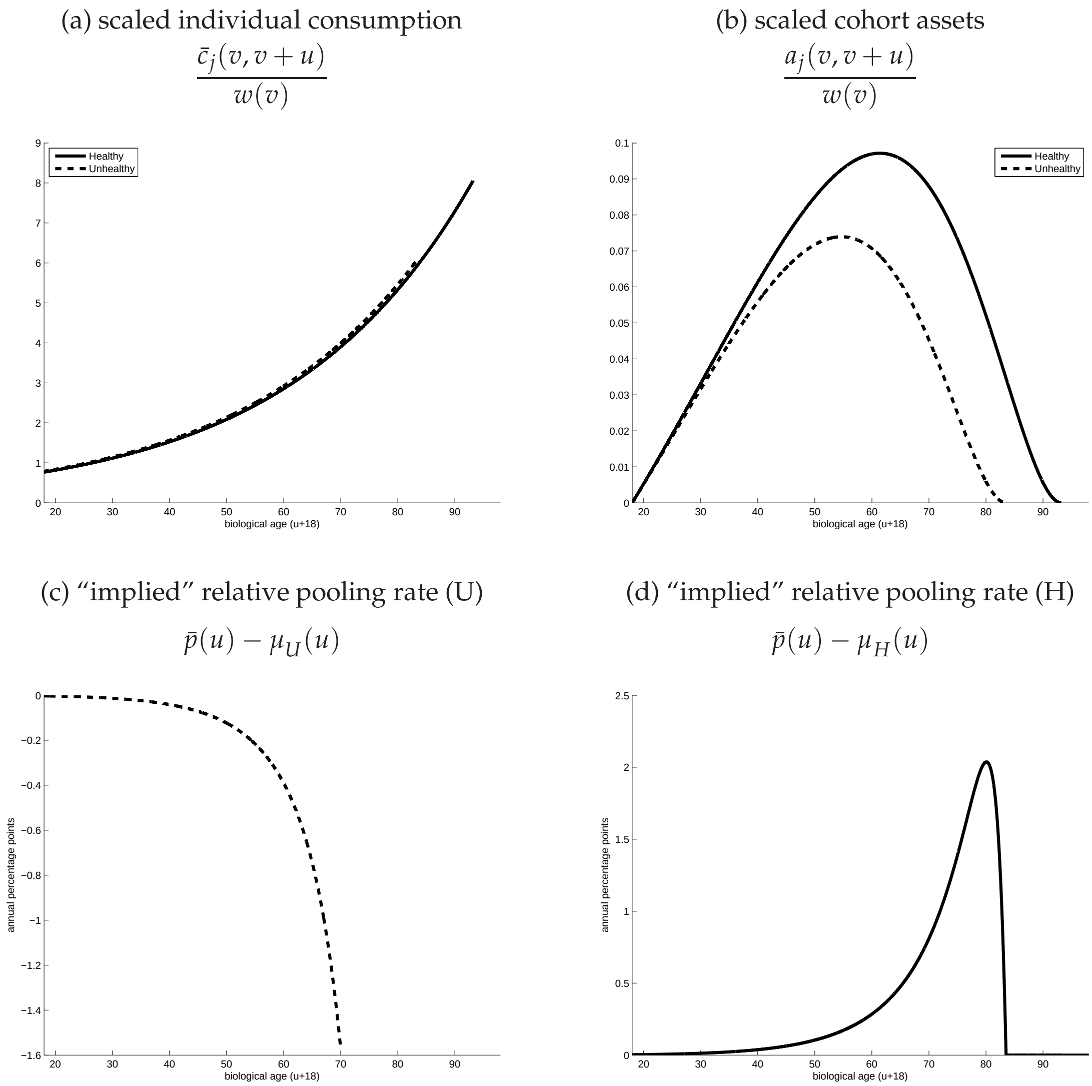


Figure 2: Perfect information equilibrium

where $\bar{P}(u) \equiv \int_0^u \bar{p}(s)\,ds$ for $0 \leq u \leq \bar{D}_U$ and $\bar{P}(u) \equiv \bar{P}(\bar{D}_U) + \int_{\bar{D}_U}^u \mu_H(s)\,ds$ for $\bar{D}_U \leq u \leq \bar{D}_H$. Figure 2(c) provides a strong hint that this is not a complete description of the pooling equilibrium. To construct Figures 2(c)-(d), we use the cohort asset paths for the perfect information equilibrium to compute the "implied" pooling rate $\bar{p}(u)$. Note that this is not an equilibrium rate because it is not consistent with the assumptions under which it has been derived. Panel (d) shows that the healthy benefit from pooling – their excess rate peaks at about 2 percentage points per annum around biological age 78. In contrast, as panel (c) shows, the unhealthy lose out as a result of pooling. For the unhealthy the pooling rate becomes so low relative to their hazard of dying that they want to borrow at that rate. But in doing so, they would reveal their health status to annuity firms who would only be willing to lend them the funds at a punitively high rate equal to their mortality rate.[7] But at that rate they would like to be savers, as the first-best suggests. It follows that the best option for the unhealthy is to impose a binding borrowing constraint on themselves from age $\bar{S}_U < \bar{D}_U$ onward. We summarize as follows.

Proposition 2. *Consider the case in which annuity firms are unable to observe the health type and total annuity purchases of annuitants. Assume that a pooling equilibrium exists and that $\sigma r - g > \rho$. Then: (i) healthy agents are net savers throughout life, i.e. $\bar{a}_H(v,v) = \bar{a}_H(v,v+\bar{D}_H) = 0$ and $\bar{a}_H(v,v+u) > 0$ for $0 < u < \bar{D}_H$; (ii) unhealthy agents are net savers until age $\bar{S}_U < \bar{D}_U$ after which they adopt a self-imposed borrowing constraint, i.e. $\bar{a}_U(v,v) = 0$, $\bar{a}_U(v,v+u) > 0$ for $0 < u < \bar{S}_U$, and $\bar{a}_U(v,v+u) = 0$ and $\bar{c}_U(v,v+u) = w(v+u)$ for $\bar{S}_U \leq u \leq \bar{D}_U$.*

Proof. See Appendix B. □

In the pooling equilibrium, we must redefine $\bar{P}(u) \equiv \bar{P}(\bar{S}_U) + \int_{\bar{S}_U}^u \mu_H(s)\,ds$ for $\bar{S}_U \leq u \leq \bar{D}_H$. Equations (26)–(27) are valid for the healthy throughout life ($0 \leq u \leq \bar{D}_H$), and for the unhealthy only until they hit the self-imposed borrowing constraint ($0 \leq u < \bar{S}_U$). Beyond age $\bar{S}_U$ the unhealthy simply consume their wage income.

The key equations of the general equilibrium model under asymmetric information are collected in Table 3. Equations (T3.1)–(T3.2) are obtained by using (8) in (4) and noting that the integrals only run up to age $\bar{S}_U$ for the unhealthy. Equation (T3.3) is the smooth-

[7] This implies that linear pricing only applies to the positive domain and that the pricing schedule features a kink at zero in equilibrium.

connection condition: consumption at age $\bar{S}_U$ must connect without discontinuity with the level implied by the solved Euler equation under pooling.[8] Equations (T3.4a)–(T3.4c) are the cohort asset paths under pooling, taking account of the self-imposed borrowing constraint for the unhealthy. Equation (T3.5) states the expression for the pooling rate. Equation (T3.6) is obtained from (9) by setting $\bar{S}_H = \bar{D}_H$, $P_j(s) = \bar{P}(s)$, and noting the definition of $c(t)$. The growth expression, equation (T3.7), again follows readily from (11) because $a(t) = k(t)$ and $\Xi(t) = 0$. In the pooling equilibrium, redistribution between health groups takes place, something which affects the equilibrium allocation. However, the redistributive term $\Xi(t)$ nevertheless vanishes from the expression for aggregate growth because the annuity firms break even. Hence, the growth equation is the same as in the first-best situation. Finally, equation (T3.8) is the same as before.

Using the parameter values discussed above, we can compute the pooling equilibrium using an iterative solution algorithm.[9] The results are reported in column (b) of Table 2. Relative to the perfect information benchmark, newborn consumption for both health types is lower in the pooling equilibrium. Similarly, the economic growth rate is somewhat less – 1.89 percent per annum instead of 2 percent. Interestingly, the unhealthy encounter the borrowing constraint fairly early on in old age, namely at economic age 61.63 which is 3.89 years less than their maximum attainable age. Recall, however, that life expectancy at birth of the unhealthy is only 53.36 years, so during youth individual agents are only moderately worried about encountering the borrowing constraint.[10]

It must be stressed that this asset depletion result is not exogenously imposed (as in the partial equilibrium studies of Friedman and Warshawsky (1990, p. 147) and Walliser (2000, pp. 378-9)) but follows from the internal logic of the model. In our model annuitization

[8]Solving equation (26) for the unhealthy gives (for $0 \leq u \leq \bar{S}_U$): $\bar{c}_U(v, v+u) = \bar{c}_U(v,v) e^{\sigma[(r-\rho)u - M_U(u) + \bar{P}(u)]}$. For $\bar{S}_U \leq u \leq \bar{D}_U$ we have: $\bar{c}_U(v, v+u) = w(v) e^{gu}$. For $u = \bar{S}_U$ these two expressions must coincide. This furnishes equation (T3.3) in Table 3.

[9]We drop equation (T3.3) and perform a grid search over $\bar{S}_U$ which solves the remaining general equilibrium system. To get the iterations started we use the pooling rate "implied by" the perfect information equilibrium. See Figures 2(c)-(d). The iterations are stopped once the unique value for $\bar{S}_U$ is found which solves (T3.3). In all computations we obtain unique solutions for the pooling equilibrium.

[10]From the perspective of birth, the probability of reaching age $\bar{S}_U = 61.63$ is equal to $e^{-M_U(\bar{S}_U)} = 0.2687$. The discount factor due to pure time preference is given by $e^{-\rho \bar{S}_U} = 0.7206$. It follows that the agent attaches a non-trivial weight of 0.1936 to felicity at age $\bar{S}_U$.

Table 3: Balanced growth with asymmetric information[a]

(a) Microeconomic relationships:

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds} \tag{T3.1}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{\int_0^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds} \tag{T3.2}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = e^{-(\sigma(r-\rho)-g)\bar{S}_U+\sigma[M_U(\bar{S}_U)-\bar{P}(\bar{S}_U)]} \tag{T3.3}$$

$$\frac{a_H(v,v+u)}{w(v)} = \beta_H\pi_H e^{(r-n)u-M_H(u)+\bar{P}(u)}\left[\int_0^u e^{-(r-g)s-\bar{P}(s)}ds - \frac{\bar{c}_H(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds\right], \qquad (0 \le u \le \bar{D}_H) \tag{T3.4a}$$

$$\frac{a_U(v,v+u)}{w(v)} = \beta_U\pi_U e^{(r-n)u-M_U(u)+\bar{P}(u)}\left[\int_0^u e^{-(r-g)s-\bar{P}(s)}ds - \frac{\bar{c}_U(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds\right], \qquad (0 \le u < \bar{S}_U) \tag{T3.4b}$$

$$\frac{a_U(v,v+u)}{w(v)} = 0, \qquad (\bar{S}_U \le u \le \bar{D}_U) \tag{T3.4c}$$

$$\bar{p}(u) = \frac{\mu_H(u)\,a_H(v,v+u)+\mu_U(u)\,a_U(v,v+u)}{a_H(v,v+u)+a_U(v,v+u)} \tag{T3.5}$$

(b) Macroeconomic relationships:

$$\frac{c(t)}{w(t)} = \beta_H\pi_H\frac{\bar{c}_H(v,v)}{w(v)}\int_0^{\bar{D}_H} e^{(r-n-g-\rho^*)s-(1+\sigma)M_H(s)+\sigma\bar{P}(s)}ds + \beta_U\pi_U\left[\frac{\bar{c}_U(v,v)}{w(v)}\int_0^{\bar{S}_U} e^{(r-n-g-\rho^*)s-(1+\sigma)M_U(s)+\sigma\bar{P}(s)}ds + \int_{\bar{S}_U}^{\bar{D}_U} e^{-ns-M_U(s)}ds\right] \tag{T3.6}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r-n+\left[1-\frac{c(t)}{w(t)}\right]\frac{w(t)}{k(t)} \tag{T3.7}$$

$$\frac{w(t)}{k(t)} = (1-\varepsilon)\,\Omega_0 \tag{T3.8}$$

Note. [a]Endogenous are $\bar{c}_j(v,v)/w(v)$, $a_j(v,v+u)/w(v)$, $\bar{p}(u)$, $\bar{S}_U$, g, $w(t)/k(t)$, and $c(t)/w(t)$. $\bar{P}(s)$ is the cumulative pooling rate at age s, $u \equiv t-v$, and $\rho^* \equiv (1-\sigma)r+\sigma\rho$. See also Table 1.

opportunities become prohibitively unattractive for the unhealthy but remain advantageous to the healthy. Hence, our model yields a consistent explanation why not everybody buys positive quantities of annuities in a general equilibrium model with risk pooling. In fact, the unhealthy would like to go short on annuities at the pooling rate (and purchase life-insured loans), but the price they would have to pay for doing so is too high.

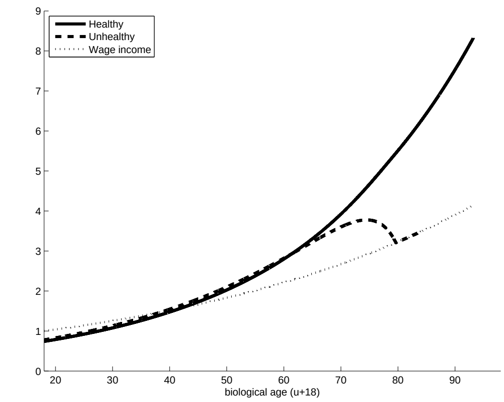
It is not difficult to see that our asset depletion result also holds if there are more than two health types, provided ρ is the same for all types. Indeed, with three types (healthy, normal, N, and unhealthy), $\bar{p}(u)$ is an asset-weighted average of the respective mortality rates, $\mu_H(u)$, $\mu_N(u)$, and $\mu_U(u)$. In this setting the normal group will feature $\bar{D}_H > \bar{S}_N > \bar{S}_U$. It may be the case that the U-types want to borrow at a slightly better rate than without the N-types, but the N-types get a worse rate as a result of the presence of the U-types. So they will not borrow for sure, and the U-types still cannot borrow without revealing themselves. In general, each health type j which encounters the borrowing constraint ends up enforcing the borrowing constraints for all types k that are unhealthier than j.

Figure 3 visualizes the key life-cycle features of the asymmetric information equilibrium for the two health groups. Panel (a) shows that scaled consumption for the unhealthy reaches a local peak just before encountering the borrowing constraint at $\bar{S}_U$. This is because the pooling rate is rather low (due to the predominance of the healthy in the annuity market), and the mortality rate of the unhealthy starts to rise. In terms of (26), consumption falls for a while because the gross annuity rate, $r + \bar{p}(u)$, falls short of the "effective impatience rate" due to time preference and mortality, $\rho + \mu_U(u)$, for u near $\bar{S}_U$. At $u = \bar{S}_U$, the surviving unhealthy reach the Keynesian part of their consumption profile and simply consume their wage income.

Comparing the asset paths for the perfect information and asymmetric information cases in, respectively, Figures 2(b) and 3(b), we observe that the healthy save more and the unhealthy save less in the pooling equilibrium than in the first-best. This is in part because the relative pooling rate, $\bar{p}_j(u) - \mu_j(u)$, is positive for the healthy and is negative for the unhealthy – see panels (c)–(d) in Figure 3. To an extent, the healthy benefit from the presence of the unhealthy in the annuity market as they are able to obtain an annuity rate of interest on their assets that is more than actuarially fair.

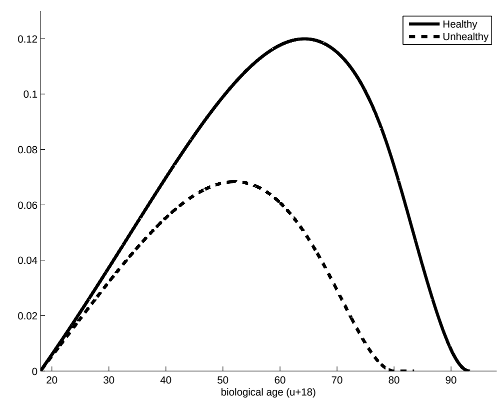
(a) scaled individual consumption

$$\frac{\bar{c}_j(v, v+u)}{w(v)}$$



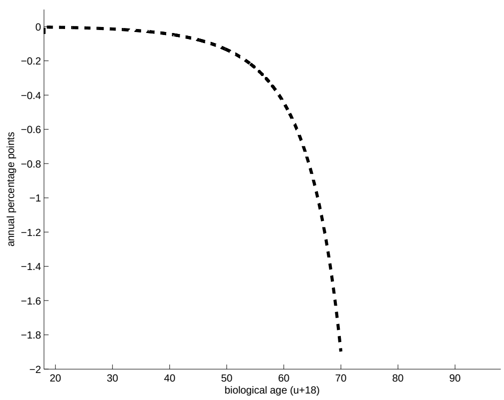
(b) scaled cohort assets

$$\frac{a_j(v, v+u)}{w(v)}$$



(c) relative pooling rate (U)

$$\bar{p}(u) - \mu_U(u)$$



(d) relative pooling rate (H)

$$\bar{p}(u) - \mu_H(u)$$

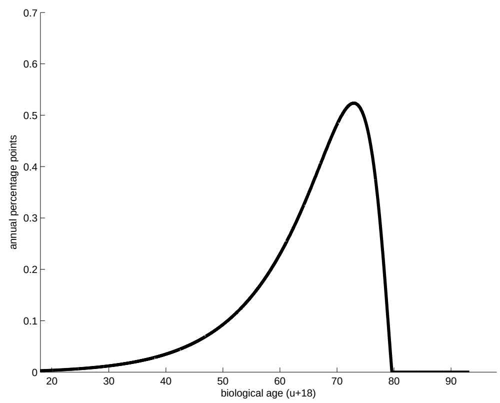


Figure 3: Asymmetric information equilibrium

3.3 Welfare analysis

In the previous subsection we have shown that the growth effects of asymmetric versus perfect information are small even though the difference in mortality risks faced by healthy and unhealthy individuals is rather large, especially at older ages (see Figure 1(b)). But economic growth is not the only relevant indicator. A key question is, to what extent does it matter to individuals whether or not there is adverse selection in the annuity market due to informational imperfections?

To address this question, the last two rows of Table 2 report the lifetime utility scores for newborns (of both health type) at some base year v_0. We normalize the wage rate for that generation to unity, $w(v_0) = 1$. As panels (a)-(b) in Table 2 reveal, welfare is higher under the perfect information than under asymmetric information, i.e. $\Lambda_j^{PI}(v_0, v_0) > \Lambda_j^{AI}(v_0, v_0)$ for $j \in \{H, U\}$. In order to obtain some feel for the significance of these differences, we compute the *lost growth years* in the pooling equilibrium relative to the first-best as follows:

$$LGY_j = \frac{1}{g} \frac{\Lambda_j^{PI}(v_0, v_0) - \Lambda_j^{AI}(v_0, v_0)}{\int_0^{\bar{D}_j} e^{-\rho s - M_j(s)} ds}. \tag{28}$$

Intuitively, LGY_j is equal to $v_1 - v_0$ such that $\Lambda_j^{PI}(v_0, v_0) = \Lambda_j^{AI}(v_1, v_1)$. How far into the future must an economic newborn arrive when there is asymmetric information in order to be equally well off as a base-year newborn in the first-best? Since wage growth explains why newborn lifetime utility increases over time, the macroeconomic growth rate in the asymmetric information equilibrium features in (28).

We find that $LGY_H = 1.0166$ years and $LGY_U = 1.3582$ years. By all accounts the annuity market imperfection due to informational asymmetries is rather small in welfare terms. At birth, the unhealthy have an economic life expectancy of 53.36 years and for them the lost growth years amount to about 16.3 months. For the healthy the results are 12.2 months of lost growth on an expected economic lifetime at birth of 61.25 years. Interestingly, the bulk of the welfare effect is accounted for by general equilibrium effects. Indeed, solving the microeconomic household equilibrium under asymmetric information in isolation (keeping the macroeconomic growth rate at two percent per annum) we find that partial equilibrium welfare effects are tiny: $LGY_H = 0.0934$ years and $LGY_U = 0.3898$. Hence, although the *qualitative* effects are similar in partial equilibrium and general equilibrium settings, the former approach grossly underestimates the effects of the information imperfections in a *quantitative* sense.

At first sight it might appear as though the above results imply that the pooling equilibrium does not exist. Both the unhealthy agents and the healthy agents *as a group* are better off by truthfully signaling their health status to the annuity firms. As a perfect information equilibrium gives them higher utility, this announcement would be credible. However, each healthy agent *as an individual* has an incentive to deviate from the optimal group strategy. Once the first-best contracts are available, posing as an unhealthy (low-risk) agent and receiving the higher annuity premium is optimal given that the other agents are honest in their health claim. Indeed, a healthy individual who cheats during the interval $(v_0, v_0 + \bar{T})$ would attain a welfare level of $\Lambda_H^{cheat}(v_0, v_0) = 36.5047$ (for $\bar{T} = 1$) and $\Lambda_H^{cheat}(v_0, v_0) = 37.9725$ (for $\bar{T} = 2$). Indeed, the longer such an individual cheats, the higher is welfare. So cheating clearly dominates the truth-telling strategy for healthy individuals (recall that under truth telling $\Lambda_H^{PI}(v_0, v_0) = 27.4619$). In short, there exists a free-rider problem: as each healthy agent has an incentive to cheat and they cannot coordinate their actions, the pooling equilibrium will be the inevitable, yet suboptimal, outcome.

4 Robustness checks

The analysis conducted thus far has yielded a number of tentative conclusions.

Conclusion (C1) Unhealthy individuals rationally drop out of the annuity market fairly early on during old age.

Conclusion (C2) The welfare effects of the annuity market imperfection due to asymmetric information are rather modest.

In the remainder of this section we study the robustness of conclusions (C1)–(C2) to alternative assumptions regarding labour market participation (in subsection 4.1) and the macroeconomic growth mechanism (in subsection 4.2).

4.1 Retirement and pensions

In the model considered up to this point, agents are assumed to supply one unit of labour in each period until they die. Here we replace this unrealistic assumption by postulating

Table 4: The pooling equilibrium with retirement[a]

(a) Microeconomic relationships:

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{(1-\theta)\int_0^{\bar{R}} e^{-(r-g)s-\bar{P}(s)}ds + \zeta\int_{\bar{R}}^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds} \quad \text{(T4.1)}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{(1-\theta)\int_0^{\bar{R}} e^{-(r-g)s-\bar{P}(s)}ds + \zeta\int_{\bar{R}}^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds}{\int_0^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds} \quad \text{(T4.2)}$$

$$\frac{\bar{c}_U(v,v)}{w(v)} = \zeta e^{-(\sigma(r-\rho)-g)\bar{S}_U+\sigma[M_U(\bar{S}_U)-\bar{P}(\bar{S}_U)]} \quad \text{(T4.3)}$$

$$\begin{aligned}\frac{a_H(v,v+u)}{w(v)} &= \beta_H \pi_H e^{(r-n)u-M_H(u)+\bar{P}(u)}\Bigg[(1-\theta)\int_0^u e^{-(r-g)s-\bar{P}(s)}ds \\ &\quad -\frac{\bar{c}_H(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds\Bigg], \qquad (0 \le u < \bar{R}) \quad \text{(T4.4a)}\end{aligned}$$

$$\begin{aligned}\frac{a_H(v,v+u)}{w(v)} &= \beta_H \pi_H e^{(r-n)u-M_H(u)+\bar{P}(u)}\Bigg[-\zeta\int_u^{\bar{D}_H} e^{-(r-g)s-\bar{P}(s)}ds \\ &\quad +\frac{\bar{c}_H(v,v)}{w(v)}\int_u^{\bar{D}_H} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}ds\Bigg], \qquad (\bar{R} \le u \le \bar{D}_H) \quad \text{(T4.4b)}\end{aligned}$$

$$\begin{aligned}\frac{a_U(v,v+u)}{w(v)} &= \beta_U \pi_U e^{(r-n)u-M_U(u)+\bar{P}(u)}\Bigg[(1-\theta)\int_0^u e^{-(r-g)s-\bar{P}(s)}ds \\ &\quad -\frac{\bar{c}_U(v,v)}{w(v)}\int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds\Bigg], \qquad (0 \le u \le \bar{R}) \quad \text{(T4.4c)}\end{aligned}$$

$$\begin{aligned}\frac{a_U(v,v+u)}{w(v)} &= \beta_U \pi_U e^{(r-n)u-M_U(u)+\bar{P}(u)}\Bigg[-\zeta\int_u^{\bar{S}_U} e^{-(r-g)s-\bar{P}(s)}ds \\ &\quad +\frac{\bar{c}_U(v,v)}{w(v)}\int_u^{\bar{S}_U} e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_U(s)}ds\Bigg], \qquad (\bar{R} \le u < \bar{S}_U) \quad \text{(T4.4d)}\end{aligned}$$

$$\frac{a_U(v,v+u)}{w(v)} = 0, \qquad (\bar{S}_U \le u \le \bar{D}_U) \quad \text{(T4.4e)}$$

$$\bar{p}(u) = \frac{\mu_H(u)\,a_H(v,v+u) + \mu_U(u)\,a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} \quad \text{(T4.5)}$$

(Table 4, continued)

(b) Macroeconomic relationships:

$$\frac{c(t)}{w(t)} = \beta_H \pi_H \frac{\bar{c}_H(v,v)}{w(v)} \int_0^{\bar{D}_H} e^{(r-n-g-\rho^*)s-(1+\sigma)M_H(s)+\sigma\bar{P}(s)} ds$$

$$+\beta_U \pi_U \left[\frac{\bar{c}_U(v,v)}{w(v)} \int_0^{\bar{S}_U} e^{(r-n-g-\rho^*)s-(1+\sigma)M_U(s)+\sigma\bar{P}(s)} ds + \zeta \int_{\bar{S}_U}^{\bar{D}_U} e^{-ns-M_U(s)} ds \right] \quad \text{(T4.6)}$$

$$g \equiv \frac{\dot{k}(t)}{k(t)} = r - n + \left[l - \frac{c(t)}{w(t)} \right] \frac{w(t)}{k(t)} \quad \text{(T4.7)}$$

$$\frac{w(t)\,l}{k(t)} = (1-\varepsilon)\,\Omega_0 \quad \text{(T4.8)}$$

$$l \equiv \sum_j \beta_j \pi_j \int_0^{\bar{R}} e^{-ns-M_j(s)} ds \quad \text{(T4.9)}$$

$$0 = \theta \sum_j \beta_j \pi_j \int_0^{\bar{R}} e^{-ns-M_j(s)} ds - \zeta \sum_j \beta_j \pi_j \int_{\bar{R}}^{\bar{D}_j} e^{-ns-M_j(s)} ds \quad \text{(T4.10)}$$

Note. [a]See also Table 3. The additional endogenous variables are the labour force participation rate, l, and the replacement rate, ζ.

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an exogenously imposed mandatory retirement age, $\bar{R}$, i.e. in the augmented model agents supply one unit of labour for ages u such that $0 \leq u < \bar{R}$ and are fully retired for $\bar{R} \leq u \leq D_j$. The retirement age does not feature a type index because the policy maker, like annuity firms, lacks information about an individual's health status.

In addition we postulate a simple pay-as-you-go (PAYG) pension scheme which imposes a wage-indexed tax of $\theta w(\tau)$ (featuring $0 < \theta < 1$) on workers ($0 \leq \tau - v < \bar{R}$) and provides wage-indexed benefits of $\zeta w(\tau)$ (with $\zeta > 0$) to retirees ($\bar{R} \leq \tau - v \leq D_j$). We refer to ζ as the replacement rate. The PAYG system is run on a balanced budget basis (see below).

Table 4 states the key equations defining the asymmetric information general equilibrium model. The model is based on the presumption – which is verified in the parameterization adopted below – that the retirement age falls well short of the asset depletion age of the unhealthy, i.e. $\bar{R} < \bar{S}_U$. The pension system affects the model via the following channels. First, whilst the tax rate θ reduces the part of human wealth accumulated during the working period, the replacement rate ζ increases human wealth accumulated during the retirement phase. Both affect scaled consumption at birth and the asset accumulation paths – see equations (T4.1)–(T4.2) and (T4.4a)–(T4.4e). Second, the replacement rate also enters the smooth connection condition (T4.3). With the pension system in place, for age $\bar{S}_U$ onward the unhealthy consume their *pension* income $\zeta w\,(v+u)$ rather than their wage income $w\,(v+u)$ as was the case in the base model without mandatory retirement. Third, the macroeconomic labour force participation rate l falls short of unity under mandatory retirement. This rate is defined in equation (T4.9) and also shows up in the growth equation (T4.7) and the expression for the wage-capital ratio (T4.8).

We parameterize the augmented model as follows. For the economic structural parameters (r, n, ε, Ω_0, δ, and σ) and the demographic parameters (η_0, η_{1j}, π_j, β_j and λ) we use the values discussed above (below equation (24)). We set the mandatory retirement age at sixty-five biological years ($\bar{R} = 47$) and assume that the tax rate equals seven percent of wage income ($\theta = 0.07$). It follows that the replacement rate is about forty percent ($\zeta = 0.3952$) and the participation rate is almost eighty-four percent ($l = 0.8368$). These values broadly capture the main features of the Dutch PAYG system. We assume that the macroeconomic growth rate is two percent per annum in the perfect information benchmark ($g = 0.02$) and use the pure rate of time preference as a calibration parameter ($\rho = 0.0179$).[11]

The key features of the different equilibria are reported in columns (c)–(d) of Table 2. It is clear from the table that conclusions (C1)–(C2) still hold under a system of mandatory retirement and PAYG pensions. Comparing columns (b) and (d) we observe that the self-imposed borrowing constraint occurs later on in life under the PAYG system.[12] The intuition behind this result is that with a replacement rate of less than unity, the smooth connection point occurs later on in life. In terms of Figure 4, asset depletion occurs at the point where the consumption Euler path (dashed line) meets the pension income path (dotted line) which lies well below the after-tax wage income path.

4.2 Endogenous versus exogenous growth

In this section we consider the role of the economic growth process. Are conclusions (C1)–(C2) still valid if growth is exogenous rather than endogenous? To study this issue we postulate an alternative model featuring exogenous labour-augmenting technological change

[11] In the interest of brevity we do not provide a summary table for the perfect information model with the pension system incorporated. It is easily deduced from Table 4, however, by comparing Tables 2 and 3.

[12] Of course, the results in columns (a)–(c) in the table are strictly speaking not directly compatible to those in columns (d)–(f) because they are based on a different value of ρ, and thus on structurally different individuals.

Table 5: Exogenous growth: quantitative effects[a]

	(a) PI	(b) AI	(c) PI	(d) AI
	No retirement[b]		With retirement[c]	
$\frac{\bar{c}_H(v,v)}{w(v)}$	0.7668	0.7428	0.8267	0.8157
$\frac{\bar{c}_U(v,v)}{w(v)}$	0.7858	0.7803	0.8492	0.8485
$\bar{S}_H$ (years)	$\bar{D}_H$	$\bar{D}_H$	$\bar{D}_H$	$\bar{D}_H$
$\bar{S}_U$ (years)	$\bar{D}_U$	61.60	$\bar{D}_U$	63.93
$\frac{c(t)}{w(t)}$	1.0714	1.0755	0.8966	0.8980
$\frac{k(t)}{Z(t)}$	2.5000	2.4593	2.5000	2.4827
r (%year)	5.00	5.14	5.00	5.06
$\frac{w(t)}{Z(t)}$	0.7000	0.6966	0.8365	0.8348
$\Lambda_H(v_0, v_0)$	27.4619	27.6194	11.9477	12.1563
$\Lambda_U(v_0, v_0)$	21.8047	21.5796	10.3314	10.0692

Notes. [a]See the notes in Tables 2 and 3. [b]The participation rate is $l = 1$. [c]In this case $\bar{R} = 47$, $\zeta = 0.3952$, $\theta = 0.07$, and $l = 0.8368$.

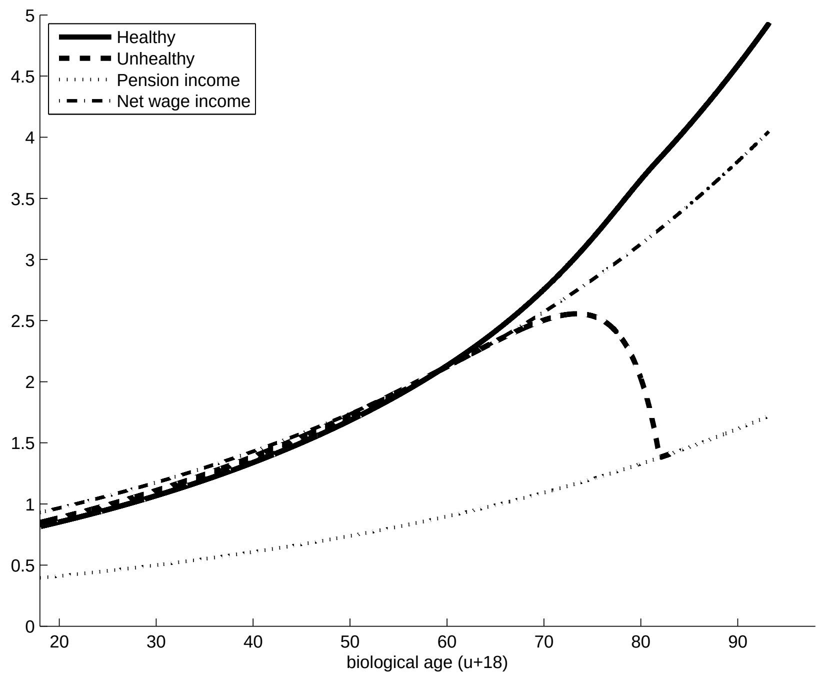

Figure 4: Self-imposed borrowing constraint under PAYG pensions

yielding a long-run growth rate of g. Of course, in a closed-economy exogenous growth model the interest rate $r(t)$ is an endogenous variable. In the model employed here, the representative firm faces an intensive-form production function of the form $y(t) = \Omega_0 k(t)^{\varepsilon}$ $[lZ(t)]^{1-\varepsilon}$, where $\dot{Z}(t)/Z(t) = g$ is the exogenous rate of change in labour-augmenting technology. Instead of (18)–(19) the marginal productivity conditions are given by:

$$\frac{w(t)}{Z(t)} = (1-\varepsilon)\,\Omega_0 \left(\frac{k(t)}{lZ(t)}\right)^{\varepsilon}, \qquad r(t) + \delta = \varepsilon\Omega_0 \left(\frac{k(t)}{lZ(t)}\right)^{\varepsilon-1}. \tag{29}$$

These expressions replace equation (T4.8) in Table 4. The remainder of the model is unchanged except for the fact that $r(t)$ is endogenous and g is exogenous. In the steady state, $k(t)/Z(t)$ and $r(t)$ are time-invariant constants, and $c(t)$, $w(t)$, $k(t)$, and $y(t)$ all grow at the exponential rate g.

We analyze the model both without and with a pension system. We parameterize the two versions of the model as follows. For the economic structural parameters (n, ε, δ, σ, $\bar{R}$, θ, and ζ) and the demographic parameters (η_0, η_{1j}, π_j, β_j and λ) we use the values discussed above (below equation (24)). We set $g = 0.02$, assume that the steady-state capital-output ratio equals $K(t)/Y(t) = 2.5$, and postulate that the interest rate equals five percent per annum in the perfect information benchmark ($r = 0.05$). We use the scale parameter in production and the pure rate of time preference as calibration parameters and find $\Omega_0 = 0.7597$ and $\rho = 5.31610^{-3}$ for the case without retirement, and $\Omega_0 = 0.8606$ and $\rho = 0.0179$ for the case with retirement.

In Table 5 columns (a)–(b) and (c)–(d) present the quantitative results for, respectively, the model without and with retirement. Apart from the changed roles played by r and g, columns (a) and (c) in Table 5 by construction coincide with, respectively columns (a) and (c) in Table 2.

Two main conclusions can be drawn. First, it is clear from Table 5 that conclusions (C1)–(C2) still hold in an exogenous growth setting. Second, the comparison between Tables 2 and 5 reveals that the quantitative effects are virtually identical for the endogenous and exogenous growth models. Whereas the difference between perfect and asymmetric information results in different growth rates in Table 2, it shows up in the form of different interest rates in Table 5. But the growth-corrected interest rates ($r - g$) for the two growth models are very close for the different equilibria.

5 Conclusions

We have constructed a dynamic general equilibrium model featuring overlapping generations of heterogeneous agents distinguished by health status. Under our set of assumptions about the annuity market, competitive firms offer linear contracts so that a risk pooling equilibrium emerges. In this equilibrium the healthy (high-risk) individuals benefit from the market presence of unhealthy (low-risk) annuitants in the sense that they obtain a better than actuarially fair return on their annuities. The model explains why not everybody participates in annuity markets. In particular, at high ages, low-risk individuals cease to purchase annuities and impose a "borrowing constraint" on themselves.

Interestingly, the growth and welfare effects of the annuity market imperfection due to adverse selection are rather small. The fact that information is asymmetric in this market may thus be quantitatively unimportant after all.

In future work we hope to pursue the following extensions. First, we wish to endogenize the labour supply decision in order to investigate the retirement effects of annuity market imperfections. In that context we will also introduce social annuity schemes such as funded and PAYG pension systems. Second, we wish to model the optimal schooling decision by individuals in an adverse selection setting and study the effects on aggregate human capital formation and macroeconomic growth. Finally, we want to extend the model to include agents who differ both in health type and labour productivity. In this context we will study the emergence of joint pooling equilibria for annuities and life-insurance.

Appendix A: Proof of Proposition 1

In a full information equilibrium we have $M_j(u) = P_j(u)$ for all $0 \leq u \leq \bar{D}_U$ as $p_j(u) = \mu_j(u)$ for all $0 \leq u \leq \bar{D}_j$. Define $\rho^* \equiv (1-\sigma)\, r + \sigma\rho$. For $\sigma r - g > \rho$ it follows that $r - g > \rho^*$ and thus:

$$\frac{\bar{c}_j(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_j} e^{-(r-g)u - M_j(u)}\, du}{\int_0^{\bar{D}_j} e^{-\rho^* u - M_j(u)}\, du} < 1.$$

Let $u \in [0, \bar{D}_j]$ be the age of the consumer. Then we can write:

$$\frac{\bar{a}_j(v, v+u)}{w(v)} e^{-ru - M_j(u)} = \Gamma_j(u),$$

where $\Gamma_j : [0, \bar{D}_j] \to \mathbb{R}$ is defined by:

$$\Gamma_j(u) = \int_0^u e^{-(r-g)s - M_j(s)} ds - \frac{\bar{c}_j(v,v)}{w(v)} \int_0^u e^{-\rho^* s - M_j(s)} ds.$$

As Γ_j is a continuous function defined on a closed and bounded interval $[0, \bar{D}_j]$, we know that Γ_j has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_j(0) = \Gamma_j(\bar{D}_j) = 0$ as $\bar{a}_j(v,v) = \bar{a}_j(v, v+\bar{D}_j)$ by the initial condition and the property of non-saturation.

Using Leibnitz' rule, we find that the first order derivative of Γ_j is given by:

$$\Gamma_j'(u) = e^{-M_j(u)} \left[e^{-(r-g)u} - \frac{\bar{c}_j(v,v)}{w(v)} e^{-\rho^* u} \right].$$

The unique interior root of this equation is:

$$u^* \equiv -\frac{1}{r-g-\rho^*} \ln \left(\frac{\bar{c}_j(v,v)}{w(v)} \right),$$

where $u^* > 0$ as $\bar{c}_j(v,v)/w(v) < 1$ and $r - g > \rho^*$ by assumption. We find that $\Gamma_j'(u) > 0$ for $0 \leq u < u^*$ and $\Gamma_j'(u) < 0$ for $u^* < u < \bar{D}_j$. We conclude that Γ_j has a global maximum at u^* and a global minimum at 0 and $\bar{D}_j$. As this global minimum equals zero, we find $\bar{a}_j(v, v+u) > 0$ for all $u \in (0, \bar{D}_j)$. □

Appendix B: Proof of Proposition 2

We assume that there exists a pooling equilibrium in the annuity market. This is only possible if the asset holdings of both health groups have the same sign everywhere. Hence, the equilibrium price must lie somewhere between the fair prices for the two types. But at that price the healthy wish to save. It follows that the asset holdings of the healthy and unhealthy agents will both have to be nonnegative: $\bar{a}_H(v, v+u) \geq 0$ and $\bar{a}_U(v, v+u) \geq 0$ for $0 \leq u \leq \bar{D}_U$. The corresponding pooling premium is given by:

$$\bar{p}(u) = \begin{cases} \dfrac{\mu_H(u) a_H(v,v+u) + \mu_U(u) a_U(v,v+u)}{a_H(v,v+u) + a_U(v,v+u)} & \text{for } 0 \leq u \leq \bar{D}_U \\ \mu_H(u) & \text{for } \bar{D}_U < u \leq \bar{D}_H \end{cases}.$$

Write $\bar{P}(u) \equiv \int_0^u \bar{p}(s)\, ds$. It follows that:

$$\mu_H(u) \leq \bar{p}(u) \leq \mu_U(u), \qquad \text{for } 0 \leq u \leq \bar{D}_U,$$
$$M_U(u) \geq \bar{P}(u), \qquad \text{for } 0 \leq u \leq \bar{D}_U,$$
$$M_H(u) \leq \bar{P}(u), \qquad \text{for } 0 \leq u \leq \bar{D}_H.$$

Now consider the two statements made in the proposition.

Item (i). Take a healthy agent. Define $f : [0, \bar{D}_H] \rightarrow \mathbb{R}$ by:

$$f(u) = e^{\sigma[M_H(u) - \bar{P}(u)]}.$$

It follows that f is a differentiable function, that $f(0) = 1$ and that $f(u) \leq 1$ for all $u \in (0, \bar{D}_H]$. The first-order derivative of f is given by:

$$f'(u) = [\mu_H(u) - \bar{p}(u)] f(u) = \begin{cases} \sigma[\mu_H(u) - \bar{p}(u)] f(u) & \text{for } 0 \leq u \leq \bar{D}_U \\ 0 & \text{for } \bar{D}_U < u \leq \bar{D}_H \end{cases},$$

such that $f'(u) \leq 0$ for all $u \in [0, \bar{D}_H]$. Using the function f, we can write consumption at birth as:

$$\frac{\bar{c}_H(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_H} f(u) e^{-(r-g)u-(1-\sigma)\bar{P}(u)-\sigma M_H(u)}\, du}{\int_0^{\bar{D}_H} e^{-\rho^* u-(1-\sigma)\bar{P}(u)-\sigma M_H(u)}\, du},$$

where $\rho^* \equiv (1-\sigma) r + \sigma\rho$. By the properties of f and the assumption that $r - g > \rho^*$ it immediately follows that:

$$\frac{\bar{c}_H(v,v)}{w(v)} < 1.$$

We now write:

$$\frac{\bar{a}_H(v, v+u)}{w(v)} e^{-ru-\bar{P}(u)} = \Gamma_H(u),$$

where $\Gamma_H : [0, \bar{D}_H] \to \mathbb{R}$ is defined by:

$$\Gamma_H(u) = \int_0^u f(s) e^{-(r-g)s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}\, ds - \frac{\bar{c}_H(v,v)}{w(v)} \int_0^u e^{-\rho^* s-(1-\sigma)\bar{P}(s)-\sigma M_H(s)}\, ds.$$

As Γ_H is a continuous function defined on a closed and bounded interval we know that Γ_H has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_H(0) = \Gamma_H(\bar{D}_H) = 0$ as $\bar{a}_H(v,v) = \bar{a}_H(v, v+\bar{D}_H) = 0$.

The first-order derivative of Γ_H is given by:

$$\Gamma_H'(u) = e^{-(1-\sigma)\bar{P}(u)-\sigma M_H(u)} \left[f(u) e^{-(r-g)u} - \frac{\bar{c}_H(v,v)}{w(v)} e^{-\rho^* u} \right].$$

It follows that a stationary point u_H^* of Γ_H satisfies:

$$\frac{\bar{c}_H(v,v)}{w(v)} e^{(r-g-\rho^*)u_H^*} = f(u_H^*).$$

Since $r - g > \rho^*$, the left-hand side of this equation is increasing in u. Combined with the fact that $\bar{c}_H(v,v)/w(v) < 1$, $f(0) = 1$ and $f'(u) \leq 0$ for all $u \in [0, \bar{D}_H]$ we find that the stationary point is unique. The second-order derivative of Γ_H evaluated in u_H^* is:

$$\Gamma_H''(u_H^*) = -e^{-(r-g)u_H^*-(1-\sigma)\bar{P}(u_H^*)-\sigma M_H(u_H^*)} \left[(r-g-\rho^*) f(u_H^*) - f'(u_H^*) \right].$$

As $f'(u_H^*) \leq 0$, it follows that $\Gamma_H''(u_H^*) < 0$. We conclude that Γ_H has a global maximum at u_H^* and a global minimum at 0 and $\bar{D}_H$. As this global maximum is strict and equals zero, we find $\bar{a}_H(v, v+u) > 0$ for all $u \in (0, \bar{D}_H)$.

Item (ii). Take an unhealthy agent. Define $h : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$h(u) = e^{\sigma[M_U(u) - \bar{P}(u)]}.$$

It follows that h is a differentiable function, that $h(0) = 1$ and that $h(u) \geq 1$ for all $u \in (0, \bar{D}_U]$. The first-order derivative of h is given by:

$$h'(u) = \sigma[\mu_U(u) - \bar{p}(u)]h(u).$$

Since we have shown above that $\bar{a}_H(v, v+u) > 0$ for all $u \in (0, \bar{D}_H)$ and $\mu_U(u) > \mu_H(u)$ for all $u \in [0, \bar{D}_U]$ by assumption, it follows that $\bar{p}(u) < \mu_U(u)$ for all $u \in (0, \bar{D}_U]$. As a consequence, we find that $h'(u) > 0$ on its domain. Using the function h, we can write consumption at birth as:

$$\frac{\bar{c}_U(v,v)}{w(v)} = \frac{\int_0^{\bar{D}_U} h(u) e^{-(r-g)u - (1-\sigma)\bar{P}(u) - \sigma M_U(u)}\, du}{\int_0^{\bar{D}_U} e^{-\rho^* u - (1-\sigma)\bar{P}(u) - \sigma M_U(u)}\, du}.$$

If $\bar{c}_U(v,v)/w(v) > 1$ then there exists $\varepsilon > 0$ such that $\bar{a}_U(v, v+\varepsilon) < 0$, which contradicts the assumption that a pooling equilibrium exists. Hence:

$$\frac{\bar{c}_U(v,v)}{w(v)} < 1.$$

It follows that $\bar{a}_U(v, v+u)$ is positive for small values of u. Now suppose to the contrary that there does not exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v+u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$. Then we would have $\bar{a}_U(v, v+u) > 0$ for $u \in (0, \bar{D}_U)$. In that case we can write:

$$\frac{\bar{a}_U(v, v+u)}{w(v)} e^{-ru - \bar{P}(u)} = \Gamma_U(u),$$

where $\Gamma_U : [0, \bar{D}_U] \to \mathbb{R}$ is defined by:

$$\Gamma_U(u) = \int_0^u h(s) e^{-(r-g)s - (1-\sigma)\bar{P}(s) - \sigma M_U(s)}\, ds - \frac{\bar{c}_U(v,v)}{w(v)} \int_0^u e^{-\rho^* s - (1-\sigma)\bar{P}(s) - \sigma M_U(s)}\, ds.$$

As Γ_U is a continuous function defined on a closed and bounded interval we know that Γ_U has a global maximum and a global minimum on its domain. Candidates for these extreme points are the boundaries of the domain and the interior critical points. For the boundary points we find $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ as $\bar{a}_U(v,v) = \bar{a}_U(v, v+\bar{D}_U) = 0$.

The first-order derivative of Γ_U is given by:

$$\Gamma_U'(u) = e^{-(1-\sigma)\bar{P}(u) - \sigma M_U(u)} \left[h(u) e^{-(r-g)u} - \frac{\bar{c}_U(v,v)}{w(v)} e^{-\rho^* u} \right].$$

It follows that a stationary point u_U^* of Γ_U satisfies:

$$\frac{\bar{c}_U(v,v)}{w(v)} e^{(r-g-\rho^*)u_U^*} = h(u_U^*).$$

Both the left-hand side and the right-hand side of this equation are increasing and convex in u, opening the possibility of multiple stationary points. Define $g : [0, \bar{D}_U] \to \mathbb{R}$ by:

$$g(u) = \frac{\bar{c}_U(v,v)}{w(v)} e^{(r-g-\rho^*)u}.$$

As $\bar{c}_U(v,v)/w(v) < 1$ it follows that $g(0) < h(0)$. Since both functions are strictly increasing and $\lim_{u \to \bar{D}_U} h(u) > g(\bar{D}_U)$, it follows that if g and h cross on $[0, \bar{D}_U]$ then they cross exactly twice. Hence, we conclude Γ_U has two critical points on its domain.

The second-order derivative of Γ_U evaluated in u_U^* is:

$$\begin{aligned}\Gamma_U''(u_U^*) &= -e^{-(r-g)u_U^* - (1-\sigma)\bar{P}\left(u_U^*\right) - \sigma M_U(u_U^*)} \left[(r-g-\rho^*)h(u_U^*) - h'(u_U^*)\right] \\ &= -h(u_U^*) e^{-(r-g)u_U^* - (1-\sigma)\bar{P}\left(u_U^*\right) - \sigma M_U(u_U^*)} \left[(r-g-\rho^*) - [\mu_U(u_U^*) - \bar{p}(u_U^*)]\right] \gtreqless 0.\end{aligned}$$

where we have used the fact that $h'(u_U^*) = [\mu(u_U^*) - \bar{p}(u_U^*)]h(u_U^*)$. Since $r - g > \rho^*$ and $\mu_U(u) - \bar{p}(u) \approx 0$ for low values u, we find that the first stationary point is a maximum. As $[\mu_U(u) - \bar{p}(u)] \to \infty$ for $u \to \bar{D}_U$, we find that the second stationary point is a minimum. As $\Gamma_U(0) = \Gamma_U(\bar{D}_U) = 0$ and there are exactly two interior stationary points it follows that the minimum is associated with negative asset holdings. This is a contradiction to the assumption that $\bar{a}_U(v, v+u) > 0$ for all $u \in (0, \bar{D}_U)$. Hence we conclude that there does exist an age $\bar{u} \in (0, \bar{D}_U)$ such that $\bar{a}_U(v, v+u) > 0$ for $u \in (0, \bar{u})$ and $\bar{a}_U(v, v+u) = 0$ for $u \in [\bar{u}, \bar{D}_U]$. □

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