

1 Introduction

Standard finance theory suggests that when two portfolios yield identical returns, an investor will be indifferent between investing in either one. However, the notion that financial assets are perfect substitutes when their returns are identical has increasingly been challenged in recent years. For example, Fama and French (2005) suggest that there exists a *taste for assets*, demonstrating that investors select their portfolio based on characteristics other than financial returns alone. In particular, a growing number of mutual funds focuses on so-called Socially Responsible Investing (SRI) – see for example Social Investment Forum (2006). SRI funds acknowledge that certain investors oppose to investing in, for example, alcohol companies, firms that are heavily polluting, or any other enterprises that are somehow perceived to behave “irresponsibly”. The common practice for SRI funds is to simply screen out stocks of companies that behave “irresponsibly” according to some social performance measure – though more sophisticated portfolio selection methods exist.

Heinkel et al. (2001) argue that the consequential drop in demand for stock of companies that are perceived as irresponsible should lead to a premium in their returns. Empirical research supports this claim and shows that so-called “sin stocks” generate an abnormal return of about 2.5 percent annually (see Hong and Kacperczyk, 2009).

To a large extent SRI focuses on environmental issues, a practice sometimes also referred to as *green screening*. This paper relates to this type of SRI and focuses on the environmental and macroeconomic effects of environmental SRI behavior in the presence of pollution due to production. There is a large empirical literature studying the relation between corporate social performance and various financial performance measures (see for a survey Margolis and Walsh, 2001). A few theoretical studies have also incorporated SRI in static portfolio selection models (e.g. Heinkel et al, 2001; Beltratti, 2005). However, the dynamic and macroeconomic effects of SRI are not well understood. On the one hand, it is intuitive that the return premium induced by SRI can have real effects on both investment decisions for (polluting) physical capital and on the level of environmental quality. But there are also potential feedback effects from the level of environmental quality on the associated return premium via its effect on SRI behavior. Since prices adjust immediately, but environmental quality usually adjust slowly, it is not a priori clear what the dynamic effects are, for example, of an abatement shock on output, environmental quality, and financial returns.

To our knowledge, this paper is the first attempt to incorporate the price effects of SRI in a fully fledged macroeconomic general equilibrium model. We follow Turnovsky (1990) by explicitly modeling the household's portfolio investment decisions in a dynamically optimizing setup. We extend his analysis, however, by incorporating SRI behaviour. We use the model to study how a traditional fiscal policy (such as a public abatement program) interacts with SRI. In particular, it is interesting to find out, first, whether public abatement policy and private SRI behaviour are complements or substitutes for each other, and, second, whether socially responsible investment has an effect on the transitional effects triggered by fiscal policy. More precisely, we consider two shocks, namely, first, an increase in the level of public abatement and, second, a boost in social responsibility.

Our analysis relates to Kriström and Lundgren (2003) who present a partial equilibrium model in which profits are affected by *green goodwill*. However, their model is not explicitly on socially responsible investment, since their approach implies that green goodwill is channeled through the consumer goods market. In a related paper, Dam (2006) studies the role of socially responsible investment in a Diamond (1965)-type environmental overlapping generations model to capture the conflict between current and future generations.

The remainder of the paper is structured as follows. In Section 2 we present the model. Households feature a “warm-glow” environmental preservation motive in the sense that they feel partially responsible for the pollution caused by firms in which they hold shares (as in Andreoni, 1990). In order to induce the household-investor to hold shares, these “dirty” securities must yield a higher rate of return than “clean” government bonds. From the point of view of the representative firm, the warm-glow motive of investors acts as an implicit output tax. Through this channel, therefore, socially responsible investment affects the firm's output and capital accumulation decisions. In Section 3, we loglinearize the model and prove existence and saddle-point stability of the macroeconomic equilibrium. In Section 4 we use the loglinearized model to conduct comparative dynamic experiments. The first shock consists of an (unanticipated and permanent) increase in the level of public abatement. Interestingly, this shock weakens (and partially crowds out) the warm-glow motive of socially responsible investors. In the second experiment we study the effects of a permanent increase in the warm-glow parameter, i.e., a strengthening of investors' social responsibility. Finally, in Section 5 we offer some conclusions and possible extensions. All technical issues are found in Dam and

Heijdra (2010) which is available upon request.

2 The model

2.1 Households

There exists a large (and fixed) number, H , of identical, infinitely-lived household-investors. From the perspective of the planning period, t , the representative household possesses a lifetime utility function of the following form:

$$\Lambda(t) \equiv \int_t^{\infty} U(c(\tau), p(\tau), Q(\tau)) e^{\rho(t-\tau)} d\tau, \quad (1)$$

where $c(\tau)$ is consumption, $p(\tau)$ is an index of the responsibility the household feels for the pollution caused by firms that it holds shares in, $Q(\tau)$ is the stock of environmental quality, and ρ is the pure rate of time preference. Consumers do not fully internalize the environmental externality, however, they do experience a *warm glow* from contributing to the public good, as in Andreoni (1990).¹ In equation (1), $Q(\tau)$ represents the traditional external effect on utility whilst $p(\tau)$ denotes the warm-glow effect. The warm glow is channeled through socially responsible investment – see below.

To keep the model as simple as possible, we assume that the felicity function, $U(\cdot)$, is log-linear in its arguments:

$$U(c(\tau), p(\tau), Q(\tau)) \equiv \ln \left[c(\tau) (1 + p(\tau))^{-\beta} Q(\tau)^\zeta \right], \quad \beta > 0, \quad \zeta > 0. \quad (2)$$

This specification of preferences implies that the intertemporal substitution elasticity for private consumption is equal to unity. Furthermore, the felicity function is separable in its arguments, a feature which simplifies the analysis considerably.

We model social responsibility by assuming that the household feels responsible for a proportion of the dirt produced by the firms in which it holds shares. We assume that there are two types of financial claims in the economy, namely “clean” government bonds and “dirty” firm equity.² The household feels responsible for the share-weighted relative pollution

¹Nyborg et al. (2006) provide a detailed discussion of the psychological motivation for this kind of behaviour in the context of green consumption.

²Households assume that the government engages in clean activities, i.e. it screens out all socially irresponsible activities. The assumption of a single dirty asset simplifies the analysis without significant loss of generality.

levels:

$$p(\tau) = \frac{e(\tau)}{\bar{E}} \cdot \frac{\gamma Y(\tau)}{Q(\tau)}, \quad \gamma > 0, \quad (3)$$

where $Y(\tau)$ and \bar{E} are, respectively, aggregate output and the total (fixed) number of outstanding firm shares. Furthermore, $e(\tau)$ is the number of shares the household possesses, so $e(\tau)/\bar{E}$ is the *fraction* of firms owned by the individual household at time τ . Finally, γ is a constant parameter capturing the notion that production generates undesirable side effects, e.g. pollution. The stock of environmental quality, $Q(\tau)$, features in the denominator, i.e. it is the $Y(\tau)/Q(\tau)$ ratio that affects the agent. In its decision making, the household takes as given the paths for $Y(\tau)$ and $Q(\tau)$. Despite the fact that the total number of shares of companies is fixed, the household can still influence $p(\tau)$ by choosing its share holdings, $e(\tau)$.

Our model can be seen as a dynamic implementation of the impure altruism (IA) approach developed by Andreoni (1990). In the IA model, an individual's contribution to the pure public good (here consisting of environmental quality, Q) appears *twice* in that person's felicity function, namely as a part of a pure public good and as a private good. In terms of our preference specification (2)–(3), the pure public good aspect is captured by the term $\zeta \ln Q(\tau)$ whilst the private good aspect is captured via the term $-\beta \ln(1 + p(\tau))$, where $p(\tau)$ itself depends on the share-weighted quality of the environment. Note that both appearances of environmental quality in the felicity function feature positive partial derivatives.

We follow Turnovsky (1990) by explicitly modeling the household's portfolio investment decisions in an optimizing setup. We extend his analysis, however, by incorporating SRI behaviour. The household can save by investing in shares or in government bonds (to keep matters simple, there are no corporate bonds³). In this deterministic setting, there is no risk so bonds and shares are perfect substitutes in the household's portfolio. The household budget identity is thus given by:

$$\dot{b}(\tau) + P_e(\tau) \dot{e}(\tau) + c(\tau) = w(\tau) + d(\tau) + r(\tau)b(\tau) - z(\tau), \quad (4)$$

where $b(\tau)$ denotes government bonds, $P_e(\tau)$ is the stock market price of company shares, $w(\tau)$ is the wage rate, $d(\tau)$ is dividends received from firms, $r(\tau)$ is the interest rate, and

The model can be easily generalized by recognizing heterogeneous firms differing in their γ -parameters. In such a setting, the dirtier firms will have a higher rate of return. See also below.

³If there were corporate bonds, we postulate that investors would treat them as dirty assets, i.e. as equivalent to equity. Under this assumption, ignoring corporate bonds entails no loss of generality.

$z(\tau)$ is lump-sum taxes paid to the government. Labour supply is exogenous and equal to unity, so $w(\tau)$ also stands for the household's wage income. As usual, a variable with a dot is that variable's time rate of change, e.g. $\dot{b}(\tau) \equiv db(\tau)/d\tau$. In the planning period, t , the household faces the initial conditions $e(t) = e_0 \equiv \bar{E}/H$ and $b(t) = b_0$.⁴ The dividend payout ratio, π , is defined as follows:

$$\pi(\tau) \equiv \frac{D(\tau)}{P_e(\tau)\bar{E}}, \quad (5)$$

where $D(\tau)$ stands for total dividend payments by firms. The dividend payout ratio is determined by the firm and taken parametrically by the household, i.e. dividend receipts of the individual investor amount to $d(\tau) = \pi(\tau)P_e(\tau)e(\tau)$.

The household chooses time paths for $c(\tau)$, $b(\tau)$, and $e(\tau)$ in order to maximize (1) subject to (4), taking into account (2)–(5), and some transversality conditions. We demonstrate in Dam and Heijdra (2010) that the key expressions characterizing individual household behaviour in an *interior* optimum (with all assets held in positive amounts) are given by:

$$\frac{\dot{c}(\tau)}{c(\tau)} = r(\tau) - \rho, \quad (6)$$

$$r_e(\tau) - r(\tau) = \beta \cdot \frac{c(\tau)}{1+p(\tau)} \cdot \frac{\gamma}{Q(\tau)} \cdot \frac{Y(\tau)}{P_e(\tau)\bar{E}}, \quad (7)$$

$$a(\tau) = b(\tau) + P_e(\tau)e(\tau), \quad (8)$$

where $r_e(\tau) \equiv \dot{P}_e(\tau)/P_e(\tau) + \pi(\tau)$ is the pecuniary rate of return on shares and $a(\tau)$ is total financial wealth. Equation (6) is the conventional Euler equation, equating the growth rate in consumption to the gap between the interest rate and the pure rate of time preference. Equation (7) is the no-arbitrage equation for shares and bonds. Intuitively, since β is positive, the individual investor demands a higher rate of return on shares than on clean bonds ($r_e(\tau) > r(\tau)$), because the former give rise to undesirable side effects in the form of pollution. Ceteris paribus, the excess return depends positively on γ and negatively on $Q(\tau)$ and the equity value per unit of output, $v(\tau) \equiv P_e(\tau)\bar{E}/Y(\tau)$.⁵ Finally, equation (8) shows that total financial wealth consists of bonds plus the market value of shares.

⁴To keep the model symmetric, and thus to be able to employ the notion of a representative agent, we assume that in the initial equilibrium each household has an endowment of shares equal to \bar{E}/H . In the optimum there will be no net trades in shares, i.e. $e(\tau) = \bar{E}/H$ for all τ . As a result, optimal investment behaviour will give rise to an equilibrium price of shares.

⁵Of course, for $\beta = 0$ the agent does not feel a warm glow effect and thus does not demand an excess return on shares, i.e. $r_e(\tau) = r(\tau)$ in that case.

Since agents are identical, aggregate values pertaining to the household sector are defined in a straightforward fashion, i.e. $C(\tau) \equiv Hc(\tau)$, $A(\tau) \equiv Ha(\tau)$, $B(\tau) \equiv Hb(\tau)$, $Z(\tau) \equiv Hz(\tau)$. Each agent holds the same amount of shares, $e(\tau) = \bar{E}/H$ for all τ and experiences the same warm glow effect, $p(\tau) = \gamma Y(\tau)/(HQ(\tau))$. Aggregate assets satisfy $A(\tau) = B(\tau) + K(\tau)$, where $K(\tau)$ is the aggregate capital stock (see below).

2.2 Firms

There are many, perfectly competitive firms, using a constant returns to scale technology to produce a single homogeneous good. We argue on the basis of the representative firm. To keep the model as simple as possible, we abstract from corporate debt (so that financing is by retained earnings only).

Gross operating profit of the firm is denoted by $\Pi(\tau)$ and defined as:

$$\Pi(\tau) \equiv F(K(\tau), L(\tau)) - w(\tau)L(\tau), \quad (9)$$

where $K(\tau)$ is the physical capital stock, $L(\tau)$ is labour demand, and $F(K(\tau), L(\tau)) \equiv \Omega_0 K(\tau)^\alpha L(\tau)^{1-\alpha}$ is a Cobb-Douglas (constant returns to scale) production function. Corporate profit is either paid out to household-investors in the form of dividends, $D(\tau)$, or kept in the form of retained earnings, $RE(\tau)$:

$$\Pi(\tau) = D(\tau) + RE(\tau), \quad (10)$$

The capital accumulation identity is given by:

$$\dot{K}(\tau) = I(\tau) - \delta K(\tau), \quad (11)$$

where $I(\tau)$ is gross investment, δ is the depreciation rate, and $\dot{K}(\tau) \equiv dK/d\tau$ is net investment.

The no-arbitrage equation for household-investors, equation (7), can be written as:

$$r(\tau) = \frac{\dot{P}_e(\tau)}{P_e(\tau)} + \frac{D(\tau) - \theta(\tau)Y(\tau)}{P_e(\tau)\bar{E}}, \quad (12)$$

where $\theta(\tau)$ can be seen as an *implicit tax* that the firm faces as a result of the investors' warm glow motive:

$$\theta(\tau) \equiv \frac{\beta\gamma c(\tau)}{(1+p(\tau))Q(\tau)}. \quad (13)$$

We assume that parameters are such that $\theta(\tau)$ is positive but less than one ($0 < \theta(\tau) < 1$). In equation (12), $\theta(\tau) Y(\tau)$ can be interpreted as a negative dividend (undesirable pollution) resulting from the firm's production activities.

We show in Dam and Heijdra (2010) that in planning period t the objective function of the firm is given by:

$$V(t) = \int_t^\infty [[1 - \theta(\tau)] F(K(\tau), L(\tau)) - w(\tau) L(\tau) - I(\tau)] e^{R(t,\tau)} d\tau, \quad (14)$$

where $R(t, \tau) \equiv \exp[-\int_t^\tau r(s) ds]$ is the interest factor. The firm chooses optimal time paths for $I(\tau)$ and $L(\tau)$ (and thus implicitly for $K(\tau)$) in order to maximize (14), subject to the accumulation identity (11), the path of implicit taxes, and taking as given the initial capital stock, $K(t)$. The key first-order necessary conditions for an interior solution can be written as follows:

$$w(\tau) = (1 - \theta(\tau)) F_L(K(\tau), L(\tau)), \quad (15)$$

$$r(\tau) + \delta = (1 - \theta(\tau)) F_K(K(\tau), L(\tau)), \quad (16)$$

and it follows that at the optimum solution $V(t) = K(t)$. Equations (15)-(16) are the standard rental expressions for labour and capital.

3 Model summary

The key expressions of the model are collected in Table 1. Equation (T1.1) is the aggregate consumption Euler equation. Equation (T1.2) is the macroeconomic capital accumulation expression, showing that the net change in the capital stock, $\dot{K}(t)$, equals net output, $Y(t) - \delta K(t)$, minus the sum of private consumption and public abatement, $C(t) + G(t)$. Equations (T1.3)-(T1.4) just restate the factor rental expressions (15)-(16), with labour market clearing, $L(t) = H$, imposed. Equation (T1.5) is the aggregate production function. Equation (T1.6) expresses the implicit tax faced by firms, $\theta(t)$, in terms of macro variables, $C(t)$, $Y(t)$, and $Q(t)$. It is obtained by using (3) in (13).⁶ It is easy to verify that $\partial\theta/\partial Q < 0$, $\partial\theta/\partial Y < 0$, $\partial\theta/\partial C > 0$, and $\partial\theta/\partial\gamma > 0$. Equation (T1.7) is the static government budget constraint, showing that the lump-sum tax revenue (left-hand side) equals total government

⁶In the symmetric equilibrium $e/\bar{E} = 1/H$ and $p = \gamma Y/(HQ)$. Using the second expression in (13) and noting that $C \equiv Hc$ we obtain (T1.6).

Table 1: Main Model Equations

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho \tag{T1.1}$$

$$\dot{K}(t) = Y(t) - C(t) - G(t) - \delta K(t) \tag{T1.2}$$

$$w(t) = (1 - \alpha) (1 - \theta(t)) \Omega_0 K(t)^\alpha H^{-\alpha} \tag{T1.3}$$

$$r(t) + \delta = \alpha (1 - \theta(t)) \Omega_0 K(t)^{\alpha-1} H^{1-\alpha} \tag{T1.4}$$

$$Y(t) = \Omega_0 K(t)^\alpha H^{1-\alpha} \tag{T1.5}$$

$$\theta(t) = \frac{\beta \gamma C(t)}{H Q(t) + \gamma Y(t)} \tag{T1.6}$$

$$z(t)H = r(t)B + G(t) \tag{T1.7}$$

$$\dot{Q}(t) = \mu \cdot (-Q(t) + \phi + \xi G(t) - \eta Y(t)) \tag{T1.8}$$

Notes: $C(t)$ is consumption, $r(t)$ is the interest rate, ρ is the rate of time preference, $K(t)$ is the capital stock, H is the fixed labour supply, $G(t)$ is public abatement, δ is the depreciation rate of capital, $\theta(t)$ is the implicit tax (warm glow), $Q(t)$ is environmental quality, $Y(t)$ is output, $z(t)$ is the lump-sum tax per agent, B is the fixed stock of government bonds. Parameters Ω_0 , β , γ , ϕ , μ , ξ , and η are all positive.

spending, consisting of interest payments on existing government debt plus abatement expenditure (right-hand side). Throughout the paper we assume that B is fixed, $G(t)$ is an exogenous policy variable under government control, and the lump-sum tax balances the budget. Finally, equation (T1.8) shows the dynamic expression for the stock of environmental quality. Following Bovenberg and Heijdra (1998, p.7), we assume that nature features a regenerative capacity and, for given values of $Y(t) = \hat{Y}$ and $G(t) = G$, slowly settles into a steady-state quality level, $\hat{Q} = \phi + \xi G - \eta \hat{Y}$, where hats denote steady-state values. We assume that \hat{Q} is positive. The parameter μ measures the speed of regeneration, which we take to be finite in the general case. Occasionally, however, we shall consider the special case of $\mu \rightarrow \infty$, in which case adjustment in environmental quality is instantaneous, i.e. $Q(t)$ has the flow dimension. To summarize, the endogenous variables of the model are $C(t)$, $Y(t)$, $K(t)$, $Q(t)$, $w(t)$, $r(t)$, $\theta(t)$, and $z(t)$. The exogenous variables are H , B , and the path for $G(t)$.

In order to further investigate the model properties and to prepare for the comparative dynamic analyses conducted in the next section, we log-linearize the model around an initial steady state – see Table 2. The definitions of the variables and shares parameters are also stated at the bottom of Table 2. The stability analysis depends on the speed of adjustment of nature, μ .

Q as a flow Under the flow interpretation, we set $\mu \rightarrow \infty$ and find that the quality of nature, $Q(t)$, adjusts immediately. The dynamical system for consumption and the capital stock can be written in a simple matrix expressions as:

$$\begin{bmatrix} \dot{\tilde{C}}(t) \\ \dot{\tilde{K}}(t) \end{bmatrix} = \Delta \cdot \begin{bmatrix} \tilde{C}(t) \\ \tilde{K}(t) \end{bmatrix} + \Gamma(t), \quad (17)$$

where the Jacobian matrix, Δ , possessing typical elements δ_{ij} , is defined as:

$$\Delta \equiv \begin{bmatrix} -\alpha \hat{y} \hat{\theta} & -\alpha \hat{y} \left[(1 - \alpha)(1 - \hat{\theta}) - \alpha \hat{\theta} [1 - \omega_Q (1 + \varepsilon_Y)] \right] \\ -\hat{y} \omega_C & \hat{y} (\alpha - \omega_I) \end{bmatrix}, \quad (18)$$

and the shock vector is given by:

$$\Gamma(t) \equiv \begin{bmatrix} -\alpha \hat{y} \hat{\theta} \omega_Q [\tilde{\gamma} - \varepsilon_G \tilde{G}(t)] \\ -\hat{y} \omega_G \tilde{G}(t) \end{bmatrix} \quad (19)$$

Table 2: Log-linearized Model

$$\dot{\tilde{C}}(t) = \rho \tilde{r}(t) \quad (\text{T2.1})$$

$$\dot{\tilde{K}}(t) = \hat{y} \cdot [\tilde{Y}(t) - \omega_C \tilde{C}(t) - \omega_G \tilde{G}(t) - \omega_I \tilde{K}(t)] \quad (\text{T2.2})$$

$$\tilde{w}(t) = \alpha \tilde{K}(t) - \frac{\hat{\theta}}{1 - \hat{\theta}} \tilde{\theta}(t) \quad (\text{T2.3})$$

$$\frac{\rho}{\rho + \delta} \tilde{r}(t) = -(1 - \alpha) \tilde{K}(t) - \frac{\hat{\theta}}{1 - \hat{\theta}} \tilde{\theta}(t) \quad (\text{T2.4})$$

$$\tilde{Y}(t) = \alpha \tilde{K}(t) \quad (\text{T2.5})$$

$$\tilde{\theta}(t) = \tilde{C}(t) - (1 - \omega_Q) \tilde{Y}(t) - \omega_Q \tilde{Q}(t) + \omega_Q \tilde{\gamma} \quad (\text{T2.6})$$

$$\tilde{Z}(t) = \omega_B \tilde{r}(t) + \omega_G \tilde{G}(t) \quad (\text{T2.7})$$

$$\dot{\tilde{Q}}(t) = \mu \cdot [-\tilde{Q}(t) + \varepsilon_G \tilde{G}(t) - \varepsilon_Y \tilde{Y}(t)] \quad (\text{T2.8})$$

Notes. (i) Variables are defined as follows: $\dot{\tilde{x}}(t) \equiv dx(t)/\hat{x}$ and $\tilde{x}(t) \equiv dx(t)/\hat{x}$. Exception: $\tilde{Z}(t) \equiv Hdz(t)/\hat{Y}$. (ii) Steady-state shares are defined as: $\omega_C \equiv \hat{C}/\hat{Y}$, $\omega_G \equiv \hat{G}/\hat{Y}$, $\hat{y} \equiv \hat{Y}/\hat{K}$, $\omega_B \equiv \rho B/\hat{Y}$, $\omega_Q \equiv H\hat{Q}/[H\hat{Q} + \gamma\hat{Y}]$, $\varepsilon_G \equiv \xi G/\hat{Q}$, $\varepsilon_Y \equiv \eta\hat{Y}/\hat{Q}$. (iii) Relationship between shares: $\omega_I = \delta/\hat{y} = 1 - \omega_C - \omega_G$, $\rho + \delta = \alpha(1 - \hat{\theta})\hat{y}$, $\hat{\theta} = \beta\omega_C(1 - \omega_Q)$, $\alpha - \omega_I = (\rho + \delta\hat{\theta})/[(1 - \hat{\theta})\hat{y}] > 0$.

The stability analysis proceeds as follows. It is easy to show that the trace of Δ is positive:

$$\text{tr}\Delta = \hat{y} \left[(1 - \hat{\theta})\alpha - \omega_I \right] = \rho > 0, \quad (20)$$

suggesting that there is at least one positive characteristic root. The determinant of Δ is equal to:

$$|\Delta| = -\alpha \hat{y}^2 \left[\hat{\theta}(\alpha - \omega_I) + \omega_C \left(1 - \alpha - \hat{\theta} + \alpha \hat{\theta} \omega_Q (1 + \varepsilon_Y) \right) \right]. \quad (21)$$

In the absence of the warm-glow effect ($\hat{\theta} = 0$), the determinant is negative and the model is saddle-point stable, i.e. it possesses one positive (unstable) root, say $\lambda_1 > 0$, and one negative (stable) root, say $-\lambda_2 < 0$. The roots satisfy the usual relationships, i.e. $\lambda_1 = \rho + \lambda_2$ and $|\Delta| = -\lambda_1 \lambda_2$. With an operative warm-glow effect the implicit tax is positive, and saddle-point stability is not guaranteed for all parameter values. Since $\alpha > \omega_I$, however, a very mild *sufficient* condition for saddle-point stability is that $\alpha + \hat{\theta} < 1$, which we assume from here on.⁷ To summarize, the model is saddle-point stable, with consumption and the capital stock acting as, respectively, the non-predetermined (“jumping”) variable and the predetermined (“sticky”) variable.

Q as a stock Under the stock interpretation, μ is finite and the quality of nature only changes gradually over time. As a result, there are now three dynamic variables, namely $C(t)$, $K(t)$, and $Q(t)$. Saddle-point stability now requires there to be two stable roots, and one unstable root. The matrix expression for the dynamical system is given by:

$$\begin{bmatrix} \dot{\tilde{C}}(t) \\ \dot{\tilde{K}}(t) \\ \dot{\tilde{Q}}(t) \end{bmatrix} = \bar{\Delta} \cdot \begin{bmatrix} \tilde{C}(t) \\ \tilde{K}(t) \\ \tilde{Q}(t) \end{bmatrix} + \bar{\Gamma}(t), \quad (22)$$

⁷Recall that α represents the capital share of national income, for which $\alpha = 0.3$ is a plausible value. The implicit tax, though positive, is likely to be quite small, easily satisfying the sufficient condition.

where $\bar{\Delta}$ and $\bar{\Gamma}$ are defined as follows:

$$\bar{\Delta} \equiv \begin{bmatrix} -\alpha\hat{y}\hat{\theta} & -\alpha\hat{y} \left[(1-\alpha)(1-\hat{\theta}) - \alpha\hat{\theta}(1-\omega_Q) \right] & \alpha\hat{y}\hat{\theta}\omega_Q \\ -\hat{y}\omega_C & \hat{y}(\alpha - \omega_I) & 0 \\ 0 & -\alpha\mu\varepsilon_Y & -\mu \end{bmatrix}, \quad (23)$$

$$\bar{\Gamma} \equiv \begin{bmatrix} -\alpha\hat{y}\hat{\theta}\omega_Q\tilde{\gamma} \\ -\hat{y}\omega_G\tilde{G}(t) \\ \mu\varepsilon_G\tilde{G}(t) \end{bmatrix}. \quad (24)$$

We find that $\text{tr}\bar{\Delta} = \rho - \mu$ and $|\bar{\Delta}| = -\mu \cdot |\Delta| > 0$, where we have used the fact that $|\Delta| < 0$. We conclude that the model is saddle-point stable and write the characteristic roots as $\bar{\lambda}_1 > 0$, $-\bar{\lambda}_2 < 0$, and $-\bar{\lambda}_3 < 0$.⁸

4 Comparative Dynamics

In this section we use the loglinearized model of Table 2 to investigate the impact, transitional, and long-run effects of two environmental shocks. In the first subsection we study a tax-financed unanticipated and permanent increase in the level of public abatement, whilst in the second subsection we demonstrate what happens if there is an once-off increase in the warm-glow parameter, γ . For both shocks, we show the effects under both the flow and the stock interpretation of environmental quality. The time at which the shock occurs is normalized to zero.

4.1 Public abatement

Starting from an initial steady-state equilibrium, the economy is perturbed by an unanticipated and permanent increase in the level of public abatement, i.e. $\tilde{G}(t) = \tilde{G} > 0$ for $t \geq 0$ and $\tilde{G}(t) = 0$ otherwise. The government uses the lump-sum tax to balance its budget.

⁸See Dam and Heijdra (2010) for details. The Routh-Hurwitz condition ensures that the alternative case, with three positive characteristic roots, is impossible in our model. In the absence of the warm-glow effect ($\hat{\theta} = 0$), the system is recursive in $(\tilde{C}(t), \tilde{K}(t))$ and $\tilde{Q}(t)$, and the roots simplify to $\bar{\lambda}_1 = \lambda_1$, $\bar{\lambda}_2 = \lambda_2$ and $\bar{\lambda}_3 = \mu$.

4.1.1 Flow interpretation

Under the flow interpretation of environmental quality, there are only two state variables and a convenient graphical representation of the model is available. Indeed, by using equations (17)–(19) and setting $\tilde{G} > 0$ and $\tilde{\gamma} = 0$ we find two equilibrium loci:

$$\tilde{C}(t) = -\frac{\delta_{12}}{\delta_{11}}\tilde{K}(t) + \varepsilon_G\omega_Q\tilde{G}, \quad (25)$$

$$\tilde{C}(t) = -\frac{\delta_{22}}{\delta_{21}}\tilde{K}(t) - \frac{\omega_G}{\alpha - \omega_I}\tilde{G}, \quad (26)$$

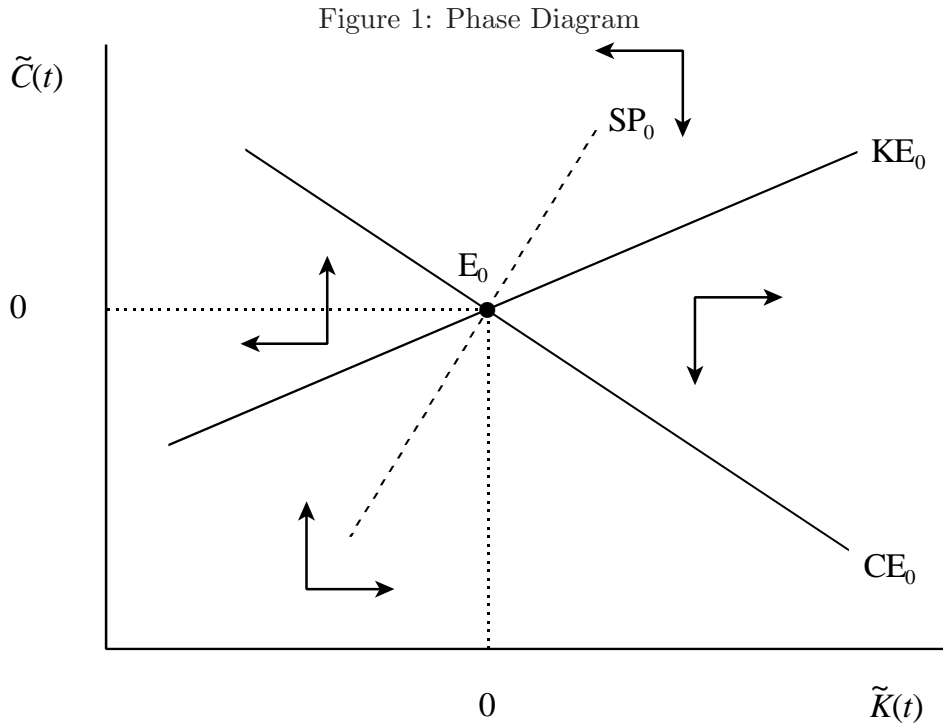
where δ_{ij} are the typical elements of Δ (given in (18) above) and we recall that $\delta_{11} < 0$, $\delta_{12} < 0$, $\delta_{21} < 0$, $\delta_{22} > 0$, $\alpha > \omega_I$, and $|\Delta| \equiv \delta_{11}\delta_{22} - \delta_{12}\delta_{21} < 0$. Equation (25) depicts combinations for $\tilde{C}(t)$ and $\tilde{K}(t)$, for which $\dot{\tilde{C}}(t) = 0$. This is the consumption equilibrium line, CE_0 , in Figure 1. This line is downward sloping, and points to the right (left) of the line are consistent with a falling (rising) consumption profile, i.e. $\dot{\tilde{C}}(t) < 0$ (> 0) – see the vertical arrows in Figure 1.

Equation (26) gives combinations of $\tilde{C}(t)$ and $\tilde{K}(t)$, for which $\dot{\tilde{K}}(t) = 0$. This is the capital stock equilibrium line, KE_0 , in Figure 1. This line is upward sloping, and points above (below) the line are consistent with a falling (rising) capital stock, i.e. $\dot{\tilde{K}}(t) < 0$ (> 0) – see the horizontal arrows in Figure 1. The configuration of arrows confirms saddle-point stability: the initial equilibrium is at E_0 and the saddle path is denoted by SP_0 .

An increase in public abatement shifts both curves in Figure 2. First, the CE curve shifts to the right from CE_0 to CE_1 . Intuitively, following the abatement shock consumption equilibrium ($\hat{r} = \rho$) is attained at a higher level of the capital stock and a lower implicit tax, $\hat{\theta}$.⁹ Second, the abatement shock reduces the amount of resources available for private consumption and capital accumulation, thus shifting the KE line down, say from KE_0 to KE_1 in Figure 2. In the long run, the equilibrium shifts from E_0 to E_1 , consumption decreases and the capital stock increases.¹⁰ The effect on environmental quality is ambiguous, because the capital stock (and thus output) increases the flow of pollution which may dominate the abatement effect.

⁹Recall that $\hat{r} = \rho$ implies that $\rho + \delta = \alpha(1 - \hat{\theta})(H/\hat{K})^{1-\alpha}$, from which the result mentioned in the text follows readily.

¹⁰We show in Dam and Heijdra (2010) that consumption must fall in the long run for small values of the implicit tax rate.

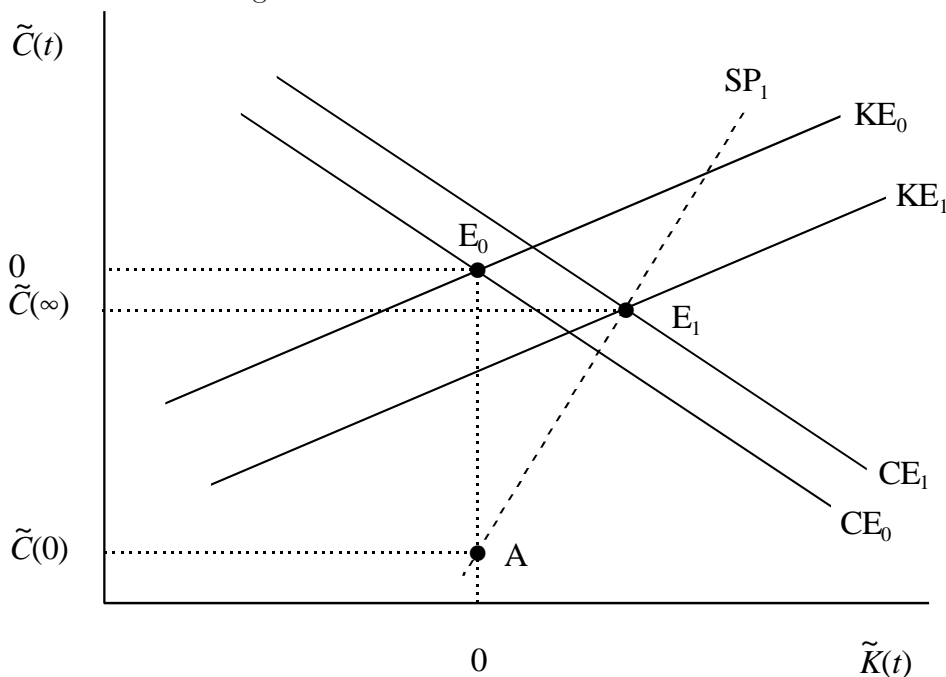


At impact, the capital stock is predetermined and consumption falls as a result of the tax increase. This is the move from E_0 to A on the new saddle path, SP_1 . At point A, the interest rate exceeds the rate of time preference because the increase in public abatement decreases the implicit warm-glow tax. This means that consumption follows an upward sloping time profile during transition. At the same time, the reduction in consumption more than compensates for the increase in public abatement, thus resulting in net capital accumulation. During transition, the economy proceeds along the saddle path from point A to the new equilibrium at E_1 .

4.1.2 Stock interpretation

Under the stock interpretation, the long-run effects are exactly the same as under the flow interpretation. There are, however, nontrivial differences in the adjustment paths toward the new equilibrium. Dam and Heijdra (2010) contains analytical expressions for the transition paths. In order to visualize the transition paths for the different variables, we calibrate the model using plausible parameter values. We set the scaling parameter in the Cobb-Douglas production function equal to $\Omega_0 = 0.808$, normalize the number of households to unity,

Figure 2: Increase in Public Abatement



$H = 1$, and set the capital share in national income at $\alpha = 0.3$. The pure rate of time preference is set at four percent per annum, $\rho = 0.04$, and annual capital depreciation rate is ten percent $\delta = 0.1$. The rate of natural regeneration is equal to $\mu = 0.05$, and the remaining parameters of the ecological equation (T1.8) are chosen such that plausible values for the elasticities ε_G and ε_Y are obtained. We find that $\phi = 12.605$ and $\xi = \eta = 6.464$. Finally, we set $\beta = 0.5$, $\gamma = 1$ and assume that the initial share of abatement equals $\omega_G = 0.05$. Using these values in Table 1, and solving for the steady state yields: $\hat{\theta} = 0.05$, $\hat{Y} = 1$, $\hat{r} = 0.04$, $\hat{C} = 0.746$, $\hat{K} = 2.036$, and $\hat{Q} = 6.464$, where hats denote steady-state values.¹¹ For convenience Table 3(a) summarizes the main features of the initial steady state. At this steady state, the environmental elasticities are equal to $\varepsilon_G = 0.05$ and $\varepsilon_Y = 1$, and $\omega_Q = 0.866$. The characteristic roots are real: $\lambda_1 = 0.212$, $-\lambda_2 = -0.171$, and $-\lambda_3 = -0.051$.

¹¹The parameters characterizing the macroeconomy (α , ρ , r , and δ) are quite standard. Ω_0 is just a scaling parameter. We obtain reasonable values for the capital-output ratio and the income shares of consumption and investment. In contrast, the parameters relating to the environment (ε_G , ε_Y , β , μ , and γ) are much harder to come by. We have chosen these parameters in such a way that the resulting implicit tax attains a plausible steady-state value of five percent ($\hat{\theta} = 0.05$). Hong and Cacperczyk (2009) estimate the excess return on “sin stocks” to be in the order of 2.5 percentage points per annum. In our model the steady-state excess return is $\hat{r}_e - \hat{r} = \hat{\theta}\hat{Y}/\hat{K} = 0.0246$, i.e. 2.46% per annum.

Table 3: Quantitative effects of abatement and socially responsibility*

	(a)	(b)	(c)
\hat{Y}	1.0000	1.0023	0.9832
\hat{C}	0.7464	0.6972	0.7408
\hat{I}	0.2036	0.2052	0.1924
G	0.0500	0.1000	0.0500
\hat{K}	2.0357	2.0515	1.9241
\hat{r}	4.00	4.00	4.00
\hat{w}	0.6650	0.6686	0.6393
$\hat{r}^e - \hat{r}$	2.46	2.19	4.43
$\hat{\theta}$	5.00	4.48	8.68
\hat{Q}	6.4643	6.7725	6.5727

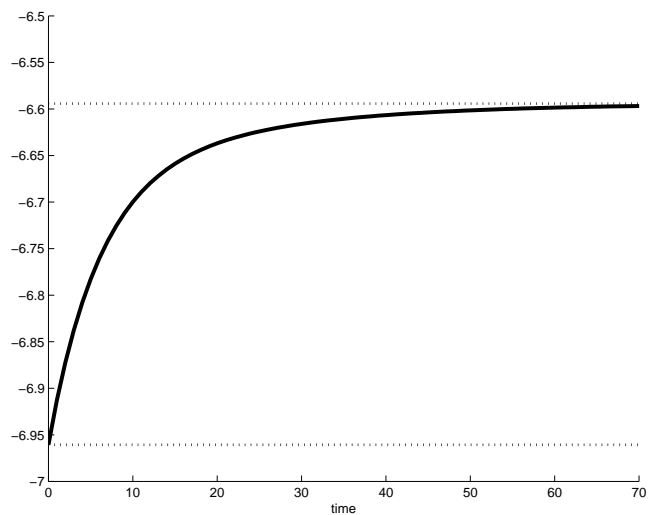
*Hats denote steady-state values. To facilitate interpretation, r and $r^e - r$ are reported as annual percentage rates of return, and θ is reported in percentage points. Column (a) is the base calibration. Column (b) reports on the abatement shock, whilst column (c) shows the results for the responsibility shock.

Figure 3 illustrates the adjustment paths for consumption, the capital stock, environmental quality, and the warm-glow tax following a doubling in public abatement, from $G_0 = 0.05$ to $G_1 = 0.10$. All paths are monotonic and qualitatively the same as under the flow interpretation. The warm-glow tax falls at impact because of the downward jump in consumption. It continues to fall during transition because both output and environmental quality increase.

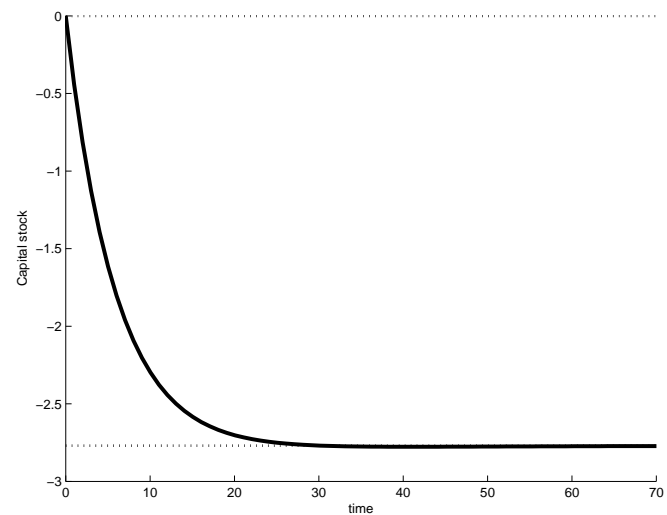
Comparing columns (a) and (b) in Table 3 shows that output and the capital stock both increase slightly thus partially offsetting the positive effect of abatement on environmental quality. Public abatement leads to a reduction in both the implicit tax and in the excess return on dirty equity.

Figure 3: Transition Path for the Abatement Shock (increase in G)

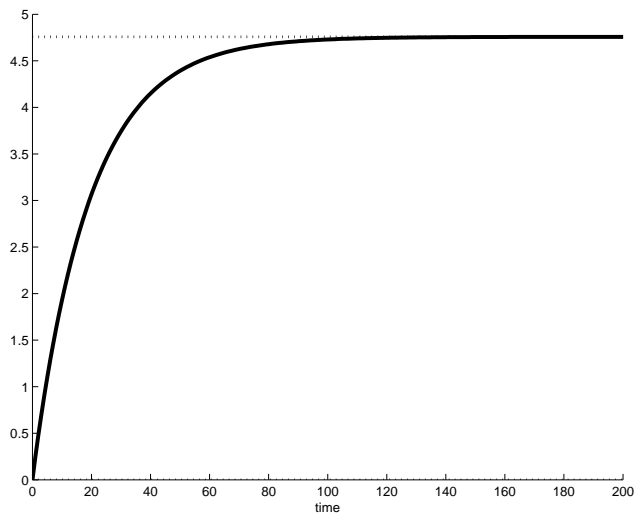
Consumption $\tilde{C}(t)$



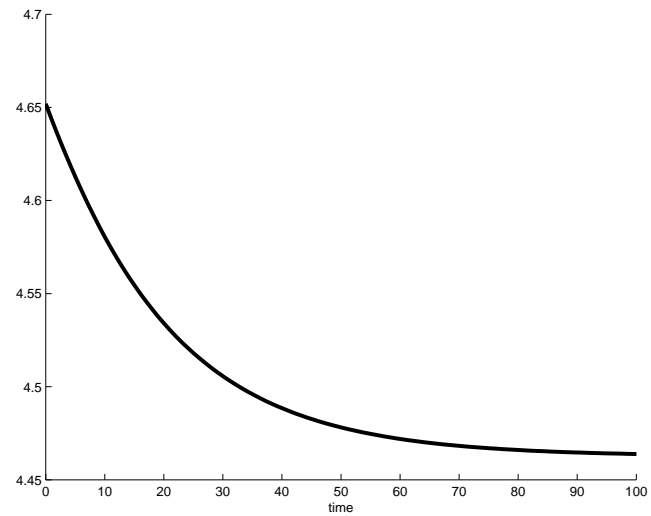
Capital stock $\tilde{K}(t)$



Environmental quality $\tilde{Q}(t)$



Implicit tax $\theta(t)$



4.2 Stronger warm-glow effect

In this subsection we study the effects of a stepwise increase in the warm-glow parameter, γ .

4.2.1 Flow interpretation

Under the flow interpretation, the CE and KE lines are given by, respectively,

$$\tilde{C}(t) = -\frac{\delta_{12}}{\delta_{11}}\tilde{K}(t) - \omega_Q\tilde{\gamma}, \quad (27)$$

and:

$$\tilde{C}(t) = -\frac{\delta_{22}}{\delta_{21}}\tilde{K}(t). \quad (28)$$

An increase in γ shifts the CE curve down, say from CE_0 to CE_1 in Figure 4. The KE line is unaffected because the shock does not affect resources available for private consumption and investment and for public abatement. At impact, the increase in γ gives rise to an upward jump in consumption– the economy moves from E_0 to point A on the new saddle path. At point A, the interest rate exceeds the rate of time preference thus causing a downward sloping time profile for consumption along the saddle path. Intuitively, the shock leads to an increase in the warm-glow tax, both because γ rises (direct effect) and because consumption increases (indirect effect). Point A lies above the KE line, which implies that capital decumulation takes place during transition. In the long run, both consumption and the capital are reduced, and the environmental quality is increased.¹²

4.2.2 Stock interpretation

In Figure 5 we illustrate the adjustment paths for the key variables following a doubling of the warm-glow parameter, from $\gamma_0 = 1$ to $\gamma_1 = 2$. The paths for consumption, the capital stock, the warm-glow tax, and environmental quality are all monotonic, as under the flow interpretation. Comparing columns (a) and (c) in Table 3 shows that output and the capital stock both decrease in the long run. This has a positive effect on environmental quality. Increased social responsibility leads to an increase in both the implicit tax and the excess return on dirty equity.

¹²We show in Dam and Heijdra (2010) that the long-run effect on consumption is unambiguously negative.

Figure 4: Increase in the Warm-Glow Motive

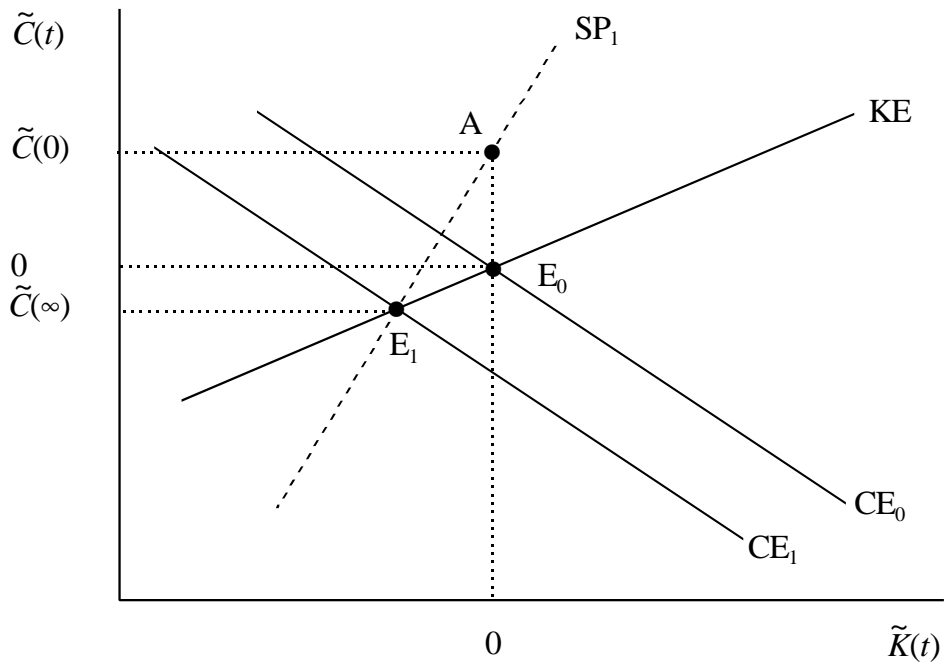
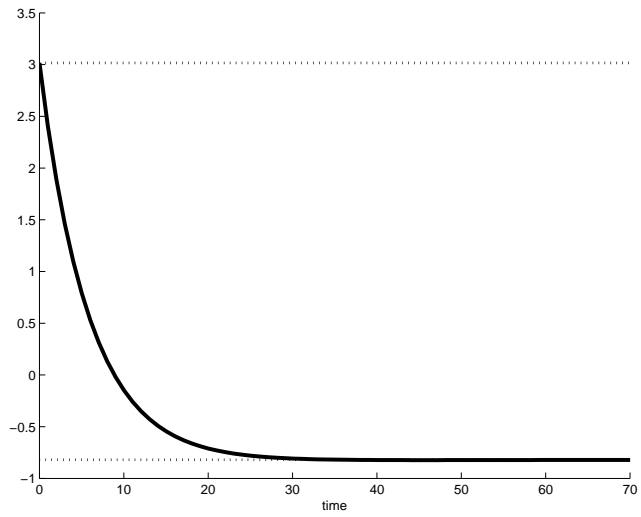
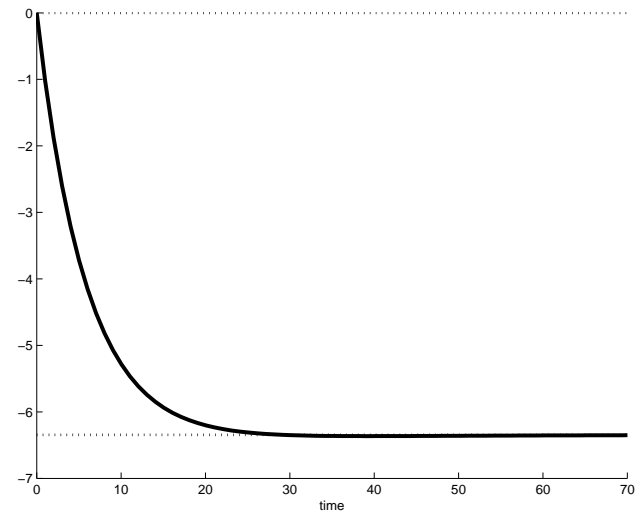


Figure 5: Transition Path for the Warm-Glow Shock (increase in γ)

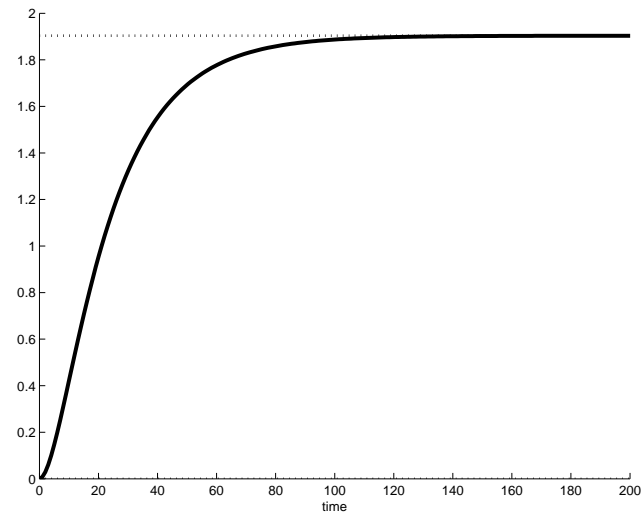
Consumption $\tilde{C}(t)$



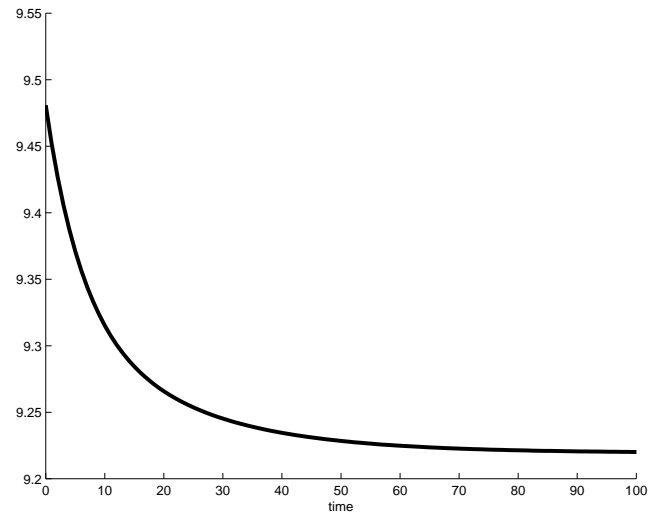
Capital stock $\tilde{K}(t)$



Environmental quality $\tilde{Q}(t)$



Implicit tax $\theta(t)$



5 Concluding remarks

In this paper we explore the effects of socially responsible investment and public abatement on environmental quality and the economy. An important question we address is whether environmental policy is effective when consumers themselves have an incentive to (at least partially) internalize the environmental externality due to a “warm glow” motive. We show that socially responsible investment behaviour by households partially offsets the positive effects on environmental quality of public abatement policies. The “warm glow” motive results in socially responsible investment in the equity market. This in turn imposes an *implicit tax* on the value of the polluting firm. Abatement policy reduces resources available for consumption, which in turn lowers the implicit tax, leading to a larger capital stock and higher pollution. As a consequence, the abatement policy is (partly) offset via the implicit tax mechanism.

The main contribution of this paper is to propose an analytical framework in which the rather vague notion of social responsibility can be captured in a general equilibrium framework. The household-investor model developed by Brock and Turnovsky (1981) and Turnovsky (1990) is in our view ideally suited for this purpose because it allows us to translate social responsibility into an implicit tax on factors of production thus allowing the use of all the standard tools of modern macroeconomics and public finance theory.

In future work we hope to utilize our theoretical framework to study a number of further issues. First, we want to endogenize the labour supply decision thus increasing the potential effects of social responsibility on output, employment, and welfare. Second, we want to study the interaction between social responsibility and traditional environmental policy tools such as Pigouvian dirt taxes. Third, we want to model an endogenous private abatement decision by firms who can thus hope to lower the cost of funds in a setting with socially responsible household-investors.

References

- Andreoni, J. (1990). Impure altruism and donations to public goods: A theory of warm-glow giving. *Economic Journal*, 100:464–477.
- Beltratti, A. (2005). Capital market equilibrium with externalities, production and heterogeneous agents. *Journal of Banking and Finance*, 29:3061–3073.
- Bovenberg, A. L. and Heijdra, B. J. (1998). Environmental tax policy and intergenerational distribution. *Journal of Public Economics*, 67:1–24.
- Brock, W. A. and Turnovsky, S. J. (1981). The analysis of macroeconomic policies in perfect foresight equilibrium. *International Economic Review*, 22(1):179–209.
- Dam, L. (2006). Firm valuation in an environmental overlapping generations model. *Resource and Energy Economics*. Forthcoming.
- Dam, L. and Heijdra, B. J. (2010). The environmental and macroeconomic effects of socially responsible investment: Mathematical appendix. Faculty of Economics and Business, University of Groningen, July. Download from <http://www.heijdra.org/Lam01MA.pdf>.
- Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review*, 55:1126–1150.
- Fama, E. F. and French, K. R. (2005). Disagreement, tastes and asset prices. CRSP Working Paper 552, CRSP. Available at SSRN: <http://ssrn.com/abstract=502605>.
- Heinkel, R., Kraus, A., and Zechner, J. (2001). The effect of green investment on corporate behavior. *Journal of Financial and Quantitative Analysis*, 36:431–449.
- Hong, H. and Kacperczyk, M. (2009). The price of sin: The effects of social norms on markets. *Journal of Financial Economics*, 93:15–36.
- Kriström, B. and Lundgren, T. (2003). Abatement investment and green goodwill. *Applied Economics*, 13:1915–1921.
- Margolis, J. D. and Walsh, J. P. (2001). *People and Profits? The Search for a Link Between a Company's Social and Financial Performance*. Erlbaum, Greenwich, CT.

- Nyborg, K., Howarth, R. B., and Brekke, K. A. (2006). Green consumers and public policy: On socially contingent moral motivation. *Resource and Energy Economics*, 28:351–366.
- Social Investment Forum (2006). *2005 Report on Socially Responsible Investing Trends in the United States: 10-Year Review*. Social Investment Forum Foundation, Washington, DC.
- Turnovsky, S. J. (1990). The effects of taxes and dividend policy on capital accumulation and macroeconomic behavior. *Journal of Economic Dynamics and Control*, 14:491–521.