# Growth effects of consumption and labour income taxation in an overlapping-generations life-cycle model

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**Abstract**

We study labour-income and consumption taxation in an overlapping-generations model featuring endogenous growth due to inter-firm investment externalities. Consumption, saving, and labour supply display life-cycle features because mortality and labour productivity are age dependent and because annuity markets may be imperfect. The government's method of revenue recycling critically affects the growth consequences of taxation. Purely consumptive government spending has a negative impact on growth. Redistribution of tax revenue from dissavers to savers may lead to an increase in growth due to beneficial intergenerational transfer effects.

**JEL codes**: D52, D91, E10, J20.

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# 1 Introduction

In this paper we revisit a classic theme in dynamic public finance theory, namely the effects of consumption and labour-income taxation on the macroeconomic allocation and long-run growth rate. We construct a simple endogenous growth model which builds on an original idea by Romer (1989, pp. 89-90) but extends it by allowing for a non-constant population. Endogenous growth results from a knowledge externality associated with capital accumulation. Knowledge is a public good that is produced as a by-product of physical capital accumulation.

Rather than studying corrective taxation aimed at internalizing the knowledge externality, we focus on a number of traditional public tax policy questions. How do the taxes affect individual decisions regarding consumption, labour supply and saving? And by which mechanisms is the macroeconomic growth rate affected? And to what extent does the method of tax revenue recycling affect the microeconomic decisions and the macroeconomic outcomes?

To answer all these questions, we develop a stylized model of a closed economy. The key features of this model are as follows. As mentioned above, the production side features between-firm externalities resulting in an 'AK'-type endogenous growth model. The consumption side is designed to capture the key life-cycle facts concerning consumption, labour supply, and saving. As was shown by Heijdra and Mierau (2009), a realistic life cycle is attained provided the following three features are taken into account. First, individual agents must experience an age-dependent mortality process, with a low mortality rate early on in life, but a rapidly increasing rate in old age. This results in a shortening of the agent's time horizon during the life cycle. Second, in order to obtain a realistic profile for labour supply, labour productivity of workers must be hump shaped. Third, in order to get a hump-shaped pattern for consumption we assume that annuities are imperfect. In combination with an age-dependent mortality rate, this feature ensures that the agent discounts felicity more heavily as he/she ages.

The government's budgetary policy plays a vital role in our analysis. We analyze three different revenue recycling methods via lump-sum transfers. The first method assumes that these transfers are the same for everybody, regardless of age. The second method gives higher transfers to the young, whereas the third method biases the transfers to favour the old. In order to demonstrate the vital importance of the revenue-recycling mechanism we also discuss the hypothetical scenario in which the government uses its tax revenue for useless government consumption expenditures.

Among other things, we find that tax/transfer combinations that redistribute funds away from the "dissaving elderly" toward the "saving young" leads to higher economic growth. Not surprisingly, wasteful government consumption has a disastrous effect on economic growth. Another finding is that with a realistic life-cycle structure, labour income and consumption taxation are no longer equivalent due to demographics and retirement.

The paper that comes closest to ours is Heijdra and Ligthart (2000) which studies the consequences of capital, labour, and consumption taxes on aggregate capital formation and welfare in a perpetual-youth overlapping-generations model of the Blanchard (1985) type. Their model highlights the importance of intergenerational redistribution effects of taxes. These effects are not present in the infinitely-lived representative-agent model but turn out to play a crucial role. That is, whereas representative-agent models predict an unambiguous negative effect of any form of taxation, Heijdra and Ligthart show that intergenerational transfers may redistribute funds toward individuals with a higher marginal propensity to save, thereby increasing the steady-state capital stock.

In a similar vein, Petrucci (2002) studies the impact of consumption taxation in a model akin to the Heijdra and Ligthart model but with endogenous rather than exogenous growth. He finds that the transfers arising from consumption taxation can induce higher growth due to intergenerational transfers. This result is in stark contrast to the remainder of the literature, which focuses on representative-agent models and finds that taxation, regardless of its base, reduces economic growth (Turnovsky, 2000) or is, at best, neutral (Stokey and Rebelo, 1995).

We thus extend both Petrucci (2002) (by endogenizing the labour supply and retirement decision) and Heijdra and Ligthart (2000) (by endogenizing the long-run growth rate). In addition, our model features realistic life-cycle features as mentioned above. In accordance with Petrucci we find that the intergenerational transfers arising from consumption taxation lead to higher economic growth. In line with Heijdra and Ligthart we find that consumption and labour taxation are not equivalent, as they are in the representative-agent model. In addition, we show that the equivalence breaks down not only because of demographic factors but also because older individuals retire. The non-equivalence between consumption and labour taxation assures that the intergenerational transfer effects of these two taxes differ substantially. In particular, for certain transfer schemes the positive growth effect found for consumption taxation fails to uphold for labour taxes.

The remainder of the paper is set up as follows. The next section outlines the model. Sections 3 and 4 study consumption and labour taxes, respectively. The final section concludes.

# 2 Model

We use the model developed in Heijdra and Mierau (2009) and extend it to include taxes on consumption and labour income and alternative redistribution schemes. In the remainder of this section we discuss the main features of the model. For details the interested reader is referred to our earlier paper.

On the production side the model features inter-firm externalities which constitute the foundation for the endogenous growth mechanism. On the consumption side, the model features age-dependent mortality and labour productivity and allows for imperfections in the annuity market. In combination, these features ensure that the model can capture realistic life-cycle aspects of the consumer-worker's behaviour. Throughout the paper we restrict attention to the steady-state.

## 2.1 Firms

The production side of the model makes use of the insights of Romer (1989, pp. 89-90) and postulates the existence of sufficiently strong external effects operating between private firms in the economy. There is a large and fixed number, $\mathcal{N}$, of identical, perfectly competitive firms. The technology available to firm $i$ is given by:

$$Y_i(t) = \Omega(t) K_i(t)^{\varepsilon_K} N_i(t)^{1-\varepsilon_K}, \qquad 0 < \varepsilon_K < 1, \tag{1}$$

where $Y_i(t)$ is output, $K_i(t)$ is capital use, $N_i(t)$ is the labour input in efficiency units, and $\Omega(t)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$w(t) = (1-\varepsilon_K)\,\Omega(t)\,\kappa_i(t)^{\varepsilon_K}, \tag{2}$$

$$r(t) + \delta = \varepsilon_K \Omega(t)\,\kappa_i(t)^{\varepsilon_K - 1}, \tag{3}$$

where $\kappa_i(t) \equiv K_i(t)/N_i(t)$ is the capital intensity. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity and $\kappa_i(t) = \kappa(t)$ for all $i = 1, \cdots, \mathcal{N}$. This is a very useful property of the model because it enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Paul Romer (1989) to a growing population, we assume that the inter-firm externality takes the following form:

$$\Omega(t) = \Omega_0 \cdot \kappa(t)^{1-\varepsilon_K}, \tag{4}$$

where $\Omega_0$ is a positive constant, $\kappa(t) \equiv K(t)/N(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $N(t) \equiv \sum_i N_i(t)$ is aggregate employment in efficiency units. According to (4), total factor productivity depends positively on the aggregate capital intensity, i.e. if an individual firm $i$ raises its capital intensity, then *all* firms in the economy benefit somewhat as a result because the general productivity indicator rises for all of them.

Using (4), equations (1)–(3) can now be rewritten in aggregate terms:[1]

$$\begin{aligned} Y(t) &= \Omega_0 K(t), & (5)\\ w(t)N(t) &= (1-\varepsilon_K)Y(t), & (6)\\ r(t) &= r = \varepsilon_K \Omega_0 - \delta, & (7) \end{aligned}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $r > \pi$, where $\pi$ is the rate of population growth (see below). The aggregate technology is linear in the capital stock and the interest is constant.[2]

[1]All firms use the same capital intensity ($\kappa_i(t) = \kappa(t)$), so that $Y_i(t) = N_i(t)\Omega(t)\kappa(t)^{\varepsilon_K}$ and $Y(t) = N(t)\Omega(t)\kappa(t)^{\varepsilon_K}$. By using (4) in this expression, we find (5). For the wage we find $w(t) = (1-\varepsilon_K)\Omega(t)\kappa(t)^{\varepsilon_K} = (1-\varepsilon_K)\Omega_0\kappa(t)$, which can be rewritten to get (6). Finally, for the rental rate on capital we find $r(t) + \delta = \varepsilon_K \Omega(t)\kappa(t)^{\varepsilon_K - 1} = \varepsilon_K \Omega_0$.

[2]Romer (1989, p. 90) makes $\Omega(t)$ dependent on the stock of capital $K(t)$, an approach also adopted by Saint-Paul (1992, p. 1247). Romer rationalizes his formulation by appealing to the public good character of knowledge and by assuming that physical capital and knowledge are produced in constant proportions. Both Romer and Saint-Paul assume a constant labour force. In order to accommodate population growth, we make the knowledge spillover dependent on the capital intensity. Note that the original Romer specification would result in $r(t) + \delta = \varepsilon_K \Omega_0 N(t)^{\varepsilon_K - 1}$, i.e. a downward trend in the real interest rate *contra* Kaldor's stylized facts.

## 2.2 Consumers

### 2.2.1 Individual behaviour

We generalize the Heijdra-Romp (2008) model of consumer behaviour by including an endogenous labour-leisure decision and by assuming potentially imperfect annuity markets. Since preferences are dynamically consistent and we restrict attention to the steady-state growth path, we develop the individual's decision rules from the perspective of birth. Expected remaining-lifetime utility of an individual born at time $v$ is given by:

$$\mathbb{E}\Lambda(v,v) \equiv \int_v^{v+\bar{D}} \ln\left[C(v,\tau)^{\varepsilon_C} \cdot [1-L(v,\tau)]^{(1-\varepsilon_C)}\right] \cdot e^{-\rho(\tau-v)-M(\tau-v)} d\tau, \tag{8}$$

where $C(v,\tau)$ is consumption, $L(v,\tau)$ is labour supply (the time endowment is equal to unity), $\rho$ is the pure rate of time preference, $\bar{D}$ is the maximum attainable age for the agent, and $e^{-M(\tau-v)}$ is the probability that the agent is still alive at some future time $\tau$ $(\geq v)$. Here, $M(\tau-v) \equiv \int_0^{\tau-v} \mu(s)ds$ stands for the cumulative mortality rate and $\mu(s)$ is the instantaneous mortality rate of an agent of age $s$.

The agent's budget identity is given by:

$$\dot{A}(v,\tau) = r^A(\tau-v)A(v,\tau) + w(v,\tau)(1-\theta_L)L(v,\tau) - (1+\theta_C)C(v,\tau) + TR(v,\tau), \tag{9}$$

where $A(v,\tau)$ is the stock of financial assets, $r^A(\tau-v)$ is the age-dependent annuity rate of interest rate, $w(v,\tau) \equiv E(\tau-v)w(\tau)$ is the age-dependent wage rate, $E(\tau-v)$ is exogenous labour productivity, $\theta_L$ is the labour income tax, $\theta_C$ is the consumption tax, $TR(v,\tau)$ are lump-sum transfers (see below). Our formulation contains three features in need of further comment.

*Feature 1.* Like Yaari (1965), we postulate the existence of annuity markets, but unlike Yaari we allow the annuities to be less than actuarially fair. Since the agent is subject to lifetime uncertainty and has no bequest motive, he/she will fully annuitize so that the annuity rate of interest facing the agent is given by:

$$r^A(\tau-v) \equiv r + \lambda\mu(\tau-v), \qquad (\text{for } 0 \leq \tau - v < \bar{D}). \tag{10}$$

where $r$ is the real interest rate (see (7)), and $\lambda$ is a parameter $(0 < \lambda \leq 1)$. Our formulation can be rationalized in three ways. First, $1-\lambda$ may be interpreted as a load factor needed to cover the administrative costs of organizing the annuity firm – see Horneff *et al.* (2008,

p. 3595). Second, as Hansen and İmrohoroğlu (2008, p. 569) suggest, $\lambda$ may represent the fraction of assets that are annuitized. Provided $\lambda$ is strictly less that unity, there will be unintended bequests under this interpretation. Third, annuity firms may possess some market power, allowing them to make a profit by offering a less than actuarially fair annuity rate.

In this paper we adopt the market-power interpretation and we assume that the profits of annuity firms are taxed away by the government and redistributed to households in a potentially age-dependent lump-sum fashion (see below). If $\lambda = 1$, annuity markets function perfectly and individuals can fully insure against longevity risk. We shall refer to $1 - \lambda$ as the degree of imperfection in the annuity market.

*Feature 2.* The second distinguishing feature of our model concerns an individual's labour productivity over the life cycle. This feature is needed to obtain a hump-shaped life-cycle profile for labour supply. We assume that:

$$E\left(t-v\right) = \alpha_0 e^{-\zeta_0 (t-v)} - \alpha_1 e^{-\zeta_1 (t-v)}, \qquad (\text{for } 0 \leq t-v \leq \bar{D}), \tag{11}$$

where $\alpha_i$ and $\zeta_i$ are the parameters governing the curvature of the productivity profile. We assume $\alpha_0 > \alpha_1 > 0$, $\zeta_1 > \zeta_0 > 0$, and $\alpha_1\zeta_1 > \alpha_0\zeta_0$ so that labour productivity is non-negative throughout life and hump-shaped over the life-cycle. Using productivity data for male workers between biological age 18 and 70 from Hansen (1993), we obtain the non-linear least squares estimates (with t-statistics in brackets) $\hat{\alpha}_0 = 4.494$ (fixed), $\hat{\alpha}_1 = 4.010$ (71.04), $\hat{\zeta}_0 = 0.0231$ (24.20), and $\hat{\zeta}_1 = 0.050$ (17.81). We visualize the actual and estimated labour efficiency profile in Figure 1(a). The specification (11) fits the data rather well and is quite convenient to work with because it features exponential terms that are relatively easy to integrate (see below). Note finally that along the balanced growth path, labour productivity grows at a constant exponential rate, $\gamma$ (see below), and as a result individual agents face the following path for real wages over their lifetimes:

$$w\left(v,t\right) = w\left(v\right) E\left(t-v\right) e^{\gamma(t-v)}, \qquad (\text{for } 0 \leq t-v \leq \bar{D}). \tag{12}$$

During youth the wage rate rises both because the productivity profile is upward sloping and because of growth in general labour productivity. At higher ages, however, the decline in labour productivity (partially or fully) offsets the growth in general productivity and wage growth falls or even becomes negative. Not surprisingly with positive economic growth, if $w(v,t)$ reaches a peaks, it does so at a later age that $E(t-v)$ does.

*Feature 3.* The third important feature of our model concerns an individual's mortality expectations. Following Boucekkine *et al.* (2002) we assume that $e^{-M(t-v)}$ takes the following rather convenient functional form:

$$e^{-M(t-v)} \equiv \frac{\eta_0 - e^{\eta_1(t-v)}}{\eta_0 - 1}, \qquad (\text{for } 0 \le t-v \le \bar{D}), \tag{13}$$

where $\eta_0 > 1$ and $\eta_1 > 0$ are parameters. The maximum attainable age for this specification is $\bar{D} = (1/\eta_0)\ln\eta_0$, whilst the instantaneous probability of death at age $t-v$ is given by:

$$\mu(t-v) \equiv \frac{\eta_1 e^{\eta_1(t-v)}}{\eta_0 - e^{\eta_1(t-v)}}, \qquad (\text{for } 0 \le t-v \le \bar{D}). \tag{14}$$

It follows that $\mu(t-v)$ is increasing in age and becomes infinite at $t-v=\bar{D}$. Following Heijdra and Romp (2008) we estimate the parameters of equation (13) using Dutch demographic data for the cohort born in 1960. We only use data for people of 18 years and older so the economic age of zero, $u \equiv t-v = 0$, corresponds to a biological age of 18. The non-linear least squares estimates (with t-statistics in brackets) are $\hat{\eta}_0 = 122.643$ (11.14) and $\hat{\eta}_1 = 0.0680$ (48.51), implying an estimate for the maximum attainable economic age of $\bar{D} = 70.75$. In Figure 1(b) we compare the estimated mortality process to the actual data. Up to about age 69 the estimated profile tracks the actual data rather well. At higher ages, however, the fit deteriorates. As we show below, however, for these ages only a small cohort is still alive – see Figure 1(d).

The agent chooses time profiles for $C(v,\tau)$, $A(v,\tau)$, and $L(v,\tau)$ (for $v \le \tau \le v + \bar{D}$) in order to maximize (8), subject to (i) the budget identity (9), (ii) a NPG condition, $\lim_{\tau\to\infty} A(v,\tau)\cdot e^{-r(\tau-v)-\lambda M(\tau-v)} = 0$, (iii) the initial asset position at birth, $A(v,v)=0$, and (iv) a non-negativity condition for labour supply, $L(v,\tau) \ge 0$. The solution of this optimization problem is fully characterized by the following equations:

$$C(v,t) = C(v,v)\cdot e^{(r-\rho)(t-v)-(1-\lambda)M(t-v)}, \tag{15}$$

$$\frac{(1-\varepsilon_C)/(1-L(v,t))}{\varepsilon_C/C(v,t)} = \frac{1-\theta_L}{1+\theta_C}\cdot w(v,t) \qquad (\text{for } S \le t-v \le R), \tag{16}$$

$$L(v,t) = 0 \qquad (\text{for } 0 \le t-v \le S \text{ and } R \le t-v \le \bar{D}), \tag{17}$$

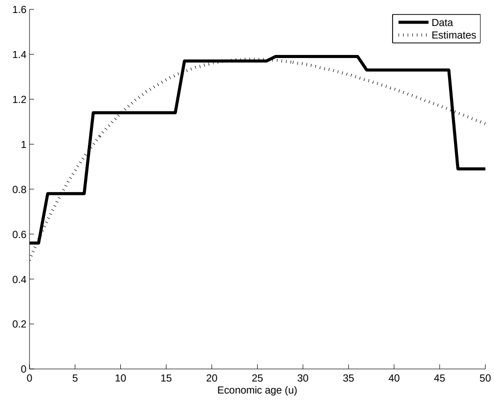
$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(S) \frac{1-\theta_L}{1+\theta_C} e^{-(r-\rho-\gamma)S+(1-\lambda)M(S)}, \tag{18}$$

$$\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R) \frac{1-\theta_L}{1+\theta_C} e^{-(r-\rho-\gamma)R+(1-\lambda)M(R)}, \tag{19}$$

Figure 1: Demographic features and labour productivity

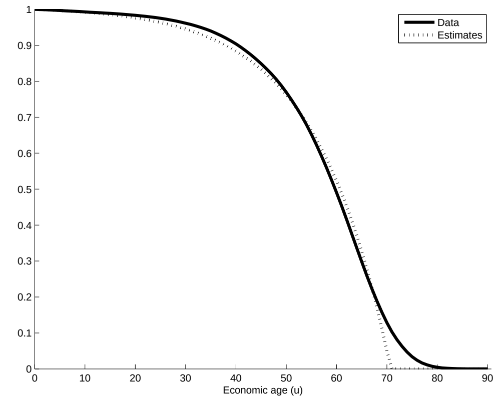
(a) efficiency profile

$E(u)$



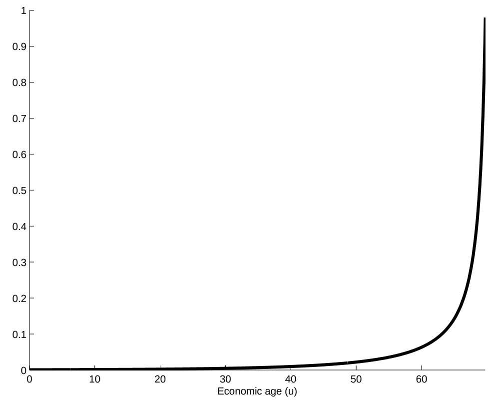

(b) mortality process

$e^{-M(u)}$



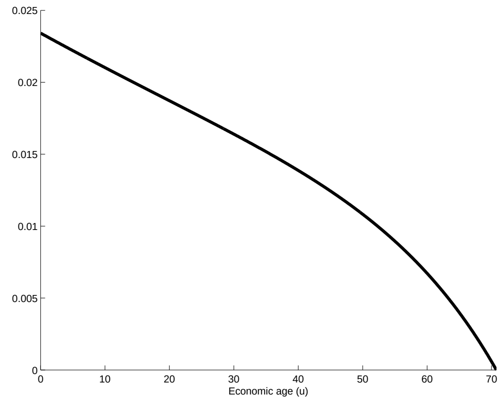

(c) instantaneous mortality rate

$\mu(u)$




(d) relative cohort size

$\beta e^{-\pi u - M(u)}$




**Notes**: $u$ is the agent's age, $\beta$ is the crude birth rate, $\pi$ is the population growth rate, $M(u)$ is the cumulative mortality factor, $\mu(u)$ is the instantaneous mortality rate, and $E(u)$ is labour productivity at age $u$. The maximum attainable age estimated with Dutch data is $\bar{D} = 70.75$.

$$(1+\theta_C)\cdot\frac{C(v,v)}{w(v)} = \frac{\varepsilon_C}{(1-\varepsilon_C)\int_S^R e^{-\rho s-M(s)}ds+\varepsilon_C\int_0^{\bar{D}} e^{-\rho s-M(s)}ds}\cdot\frac{H(v,v)}{w(v)}, \quad (20)$$

$$\frac{H(v,v)}{w(v)} \equiv (1-\theta_L)\int_S^R E(s)\,e^{-(r-\gamma)s-\lambda M(s)}ds + \int_0^{\bar{D}}\frac{TR(v,v+s)}{w(v+s)}e^{-(r-\gamma)s-\lambda M(s)}ds. \quad (21)$$

The intuition behind these expressions is as follows. Equation (15) is best understood by noting the consumption Euler equation resulting from utility maximization:

$$\frac{\dot{C}(v,\tau)}{C(v,\tau)} = r-\rho-(1-\lambda)\,\mu(\tau-v). \quad (22)$$

By using this expression, future consumption can be expressed in terms of consumption at birth as in (15). In the absence of an annuity market imperfection ($\lambda = 1$), consumption growth only depends on the gap between the interest rate and the pure rate of time preference. In contrast, with imperfect annuities, individual consumption growth is negatively affected by the mortality rate, a result first demonstrated for the case with $\lambda = 0$ by Yaari (1965, p. 143).

Equations (16)–(19) characterize the agent's labour supply plans during the life cycle. There are two critical ages in the worker's life cycle, namely the labour market entry age $S$ and the retirement age $R$. During youth, for $0 \le t-v \le S$ the agent has not yet entered the labour market. Toward the end of life, for $R \le t-v \le \bar{D}$, the agent no longer works. During the working period, the agent equates the marginal rate of substitution between leisure and consumption to the wage rate at all times – see equation (16). The optimal labour market entry and retirement points are determined in, respectively (18) and (19).

The consumption-leisure choice over the life cycle is illustrated in Figure 2, where $\frac{C(v,v+u)}{w(v)}$ and $L(u)$ stand for, respectively, consumption (scaled by the wage rate at birth) and labour supply of the agent at age $u \equiv t-v$. To facilitate the discussion we assume that annuities are perfect ($\lambda = 1$) so that consumption grows monotonically over the life cycle. We show four moments in the agent's life. The initial choice at age $u = 0$ is at point $\mathrm{E}_0$. The wage rate is low, leisure is cheap, and the agent faces a binding non-negativity constraint on labour supply. For $0 < u < S$, this constraint remains binding but the agent chooses an increasing path for consumption. This is the gradual move from $\mathrm{E}_0$ to $\mathrm{E}_S$.

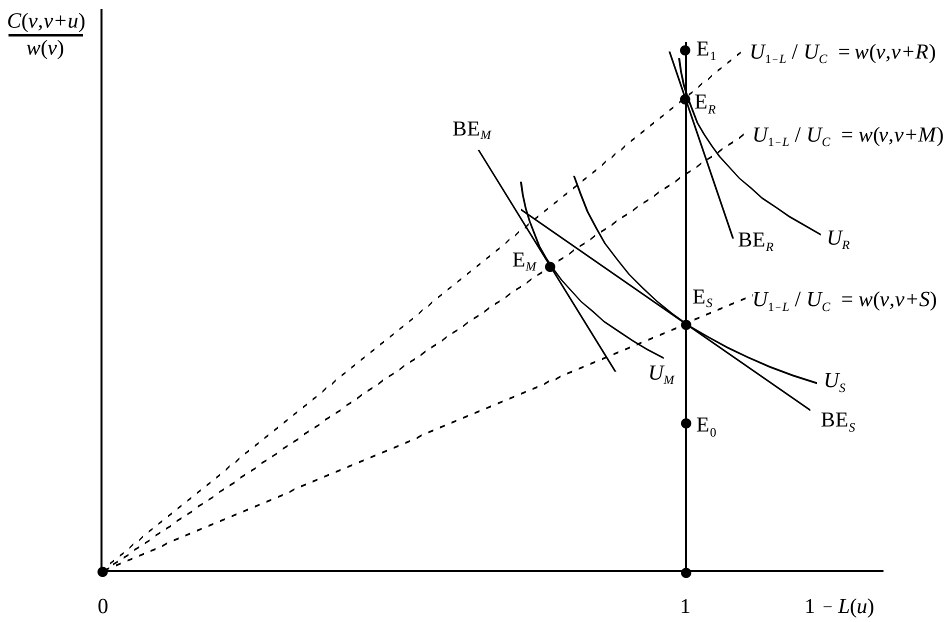



Figure 2: Life-cycle consumption and labour supply

At age $u = S$ the agent achieves a tangency between an indifference curve, $U_S = C(v, v + S)^{\varepsilon_C} \cdot [1 - L(v, v + S)]^{1-\varepsilon_C}$ and a "budget equation" $X(v, v + S) = C(v, v + S) + w(v, v + S) \cdot [1 - L(v, v + S)]$, where $X(v, \tau)$ is full consumption and, of course, $L(v, v + S) = 0$. Equation (18) describes point $\mathrm{E}_S$ in terms of the key economic variables in the model.

For $S < u < R$ the agent makes interior choices for both consumption and labour supply, and the optimum moves in north-westerly direction from point $\mathrm{E}_S$. The wage increases with age but the substitution effect dominates the wealth effect and labour supply rises initially. During that life phase (and with perfect annuities), full consumption increases exponentially according to $\dot{X}(v, v + u) / X(v, v + u) = r - \rho > 0$. This causes the wealth effect to strengthen.

At some age $u = M$, the wealth effect exactly matches the substitution effect and labour supply reaches its peak. This occurs at point $\mathrm{E}_M$ in Figure 2. The wage at that point exceeds the wage at labour market entry, $w(v, v + M) > w(v, v + S)$, so the budget equation through $\mathrm{E}_M$ is steeper than the one through $\mathrm{E}_S$. Beyond age $M$, the wealth effect dominates and labour supply falls gradually. In the calibrated version of our model, an individual's wage path is uniformly upward sloping (see Figure 3(d)) so the budget equation continues to rotate in a clockwise fashion as the agent gets older.[3]

[3]Equation (12) shows that it is *in principle* possible for the individual's wage to fall after a certain age, namely if the fall in labour productivity exceeds the macroeconomic growth rate ($\dot{E}(u) / E(u) < -\gamma$). This

At age $u = R$, the agent retires from the labour market. Equation (19) describes point $\mathrm{E}_R$ in terms of the key economic variables in the model. The wage rate is high, leisure is expensive, but the agent is rather wealthy and thus faces a binding non-negativity constraint on labour supply, just as at the start of life but for diametrically opposite reasons. Beyond age $R$, consumption continues to increase. The optimum gradually moves from $\mathrm{E}_R$ in the direction of point $\mathrm{E}_1$ in Figure 2.

Equation (20) shows that scaled consumption of a newborn is proportional to human wealth. The marginal propensity to consume out of human wealth at birth is decreasing in the length of the agent's working career. Finally, equation (21) provides the definition of human wealth at birth. The first term on the right-hand side represents the present value of the time endowment during working life, using the growth-corrected annuity rate of interest for discounting. The later one retires, the higher is this term. The second term on the right-hand side of (21) is just the present value of transfers.

In the presence of an age-dependent mortality process, the following demography-dependent function is quite convenient as it shows up in various places in the model characterization:

$$\Xi\left(\xi_1, \xi_2\right)_{u_0}^{u_1} = \int_{u_0}^{u_1} e^{-\xi_1 s} \cdot \left[\frac{\mu_0 - e^{\mu_1 s}}{\mu_0 - 1}\right]^{\xi_2} ds, \tag{23}$$

with $0 \leq u_0 < u_1 \leq \bar{D}$ and $\xi_2 \geq 0$. Provided $\xi_1$ and $\xi_2$ are finite, the integral exists and is strictly positive.

### 2.2.2 Aggregate household behaviour

In this subsection we derive expressions for per-capita average consumption, labour supply, and saving. As is shown in Heijdra and Romp (2008, p. 94), with age-dependent mortality the demographic steady-state equilibrium has the following features:

$$1 = \beta \Xi\left(\pi, 1\right)_0^{\bar{D}}, \tag{24}$$

$$p\left(v, t\right) \equiv \frac{P\left(v, t\right)}{P\left(t\right)} \equiv \beta e^{-\pi(t-v) - M(t-v)}, \tag{25}$$

where $\beta$ is the crude birth rate, $\pi$ is the growth rate of the population, $p\left(v, t\right)$ and $P\left(v, t\right)$ are, respectively, the relative and absolute size of cohort $v$ at time $t \geq v$, and $P\left(t\right)$ is the population size at time $t$. For a given birth rate, equation (24) determines the unique population growth

effect does not occur in our calibrated model so the wage path is monotonically increasing in age.

rate consistent with the demographic steady state or vice versa. The average population-wide mortality rate, $\bar{\mu}$, follows residually from the fact that $\pi \equiv \beta - \bar{\mu}$. Equation (25) shows the two reasons why the relative size of a cohort falls over time, namely population growth and mortality.

Using (25), we can define per-capita average values in general terms as:

$$b(t) \equiv \int_{-\infty}^{t} p(v,t) \, B(v,t) \, dv, \tag{26}$$

where $B(v,t)$ denotes the variable in question at the individual level, and $b(t)$ is the per capita average value of that same variable. Using (15) and (26), we find that per capita average consumption can be written as follows:

$$\frac{c(t)}{w(t)} = \frac{C(v,v)}{w(v)} \cdot \beta \Xi (\pi + \rho + \gamma - r, 2 - \lambda)_0^{\bar{D}} . \tag{27}$$

Efficiency units of labour of vintage $t - v$ are defined as $N(t-v) \equiv E(t-v) \, L(v,t)$. Using this expression, as well as (11), (15)–(16), and (26) we find the per capita average supply of efficiency units of labour:

$$n = \beta \int_{S}^{R} E(s) \, e^{-\pi s - M(s)} ds - \frac{C(v,v)}{w(v)} \frac{1 - \varepsilon_C}{\varepsilon_C} \frac{1 + \theta_C}{1 - \theta_L} \beta \Xi (\pi + \rho + \gamma - r, 2 - \lambda)_S^R , \tag{28}$$

with $0 < n < \bar{n}$, where $\bar{n} \equiv \beta \int_0^{\bar{D}} E(s) \, e^{-\pi s - M(s)} ds$ is the maximum labour potential in the economy. The first term on the right-hand side of (28) provides the first mechanism by which $n$ falls short of $\bar{n}$: agents only work during part of their lives. Prior to labour market entry and after retirement, they consume their unit time endowment in the form of leisure. The second composite term on the right-hand side of (28) represents the second mechanism by which $n$ falls short of $\bar{n}$: during their productive career, workers never supply their full time endowment to the labour market.

Finally, using (26) we observe that per capita average assets are defined as $a(t) \equiv \int_{-\infty}^{t} p(v,t) \, A(v,t) \, dv$ so that its rate of change is:

$$\dot{a}(t) = \int_{t-\bar{D}}^{t} p(v,t) \, \dot{A}(v,t) \, dv - \int_{t-\bar{D}}^{t} [\pi + \mu(t-v)] \, A(v,t) \, dv, \tag{29}$$

where we have incorporated the fact that individual agents have zero financial assets at birth and at the maximum attainable age ($A(v,v) = A\left(v, v + \bar{D}\right) = 0$) and that the relative cohort size evolves over time according to $\dot{p}(v,t) = -[\pi + \mu(t-v)] \, p(v,t)$. Using (9)–(10) in (29) we thus find:

$$\dot{a}(t) = (r-\pi)\,a(t) + (1-\theta_L)\,w(t)\,n - (1+\theta_C)\,c(t) + \int_{t-\bar{D}}^{t} p(v,t)\;TR(v,t)\,dv$$
$$- (1-\lambda)\int_{t-\bar{D}}^{t} \mu(t-v)\,p(v,t)\,A(v,t)\,dv. \tag{30}$$

## 2.3 Government

We assume that the government maintains continuous budget balance and does not use debt financing. Total per capita tax receipts are given by:

$$tax(t) \equiv \theta_C c(t) + \theta_L w(t)\,n + (1-\lambda)\int_{t-\bar{D}}^{t} \mu(t-v)\,p(v,t)\,A(v,t)\,dv, \tag{31}$$

where the last term on the right-hand side represents the excess profits of the annuity industry that are taxed away by the government. We write the per capita government budget constraint as follows:

$$tax(t) = g\cdot k(t) + \int_{t-\bar{D}}^{t} p(v,t)\;TR(v,t)\,dv, \tag{32}$$

where $g\cdot k(t)$ is useless government spending and $g$ is a parameter.

By using (31)–(32) in (30) and noting that the capital market equilibrium condition is given by $a(t) = k(t)$, we find the macroeconomic accumulation equation for the per capita capital stock:

$$\dot{k}(t) = (r-\pi-g)\,k(t) + w(t)\,n - c(t). \tag{33}$$

In the remainder of the paper we consider two financing scenarios.

- *Transfer scenario.* Government consumption is zero, and the entire tax revenue is transferred to households, i.e. $g = 0$ and $TR(v,t) > 0$ for all $v$ and $t$. Within this scenario we consider three prototypical modes of transfer redistribution. To capture these three options we assume that government transfers are set according :

  $$\frac{TR(v,t)}{w(t)} = z\cdot e^{\phi(t-v)}, \tag{34}$$

  where $z$ is the policy tool assuring budget balance and $\phi$ is the parameter governing the government's choice of redistribution scheme. The government either gives the same

Table 1: The model

---

*(a) Microeconomic relationships:*

$$
\begin{aligned}
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{(1-\theta_L)\,\varepsilon_C\left[\alpha_0\Xi(r+\zeta_0-\gamma,\lambda)_S^R-\alpha_1\Xi(r+\zeta_1-\gamma,\lambda)_S^R\right]}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R+\varepsilon_C\Xi(\rho,1)_0^{\bar{D}}} \\
&\quad+\frac{\varepsilon_C\Xi(z,r-\phi-\gamma,\lambda)_0^{\bar{D}}\cdot z}{(1-\varepsilon_C)\,\Xi(\rho,1)_S^R+\varepsilon_C\Xi(\rho,1)_0^{\bar{D}}} \qquad \text{(T1.1)} \\
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{\varepsilon_C}{1-\varepsilon_C}E(S)\,(1-\theta_L)\,e^{-(r-\gamma-\rho)S+(1-\lambda)M(S)} \qquad \text{(T1.2)} \\
\frac{(1+\theta_C)\,C(v,v)}{w(v)} &= \frac{\varepsilon_C}{1-\varepsilon_C}E(R)\,(1-\theta_L)\,e^{-(r-\gamma-\rho)R+(1-\lambda)M(R)} \qquad \text{(T1.3)} \\
\frac{A(v,v+u)}{w(v)}\cdot e^{-ru-\lambda M(u)} &= -\frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\Xi(\rho,1)_0^u+z\cdot\Xi(r-\phi-\gamma,\lambda)_0^u \qquad \text{(T1.4a)} \\
&= -\frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\left[\Xi(\rho,1)_0^S+\frac{1}{\varepsilon_C}\Xi(\rho,1)_S^u\right]+z\cdot\Xi(r-\phi-\gamma,\lambda)_0^u \\
&\quad+(1-\theta_L)\left[\alpha_0\Xi(r+\zeta_0-\gamma,\lambda)_S^u-\alpha_1\Xi(r+\zeta_1-\gamma,\lambda)_S^u\right] \qquad \text{(T1.4b)} \\
&= \frac{(1+\theta_C)\,C(v,v)}{w(v)}\cdot\Xi(\rho,1)_u^{\bar{D}}-z\cdot\Xi(r-\phi-\gamma,\lambda)_u^{\bar{D}} \qquad \text{(T1.4c)}
\end{aligned}
$$

*(b) Macroeconomic relationships:*

$$
\begin{aligned}
0 &= g\cdot k(t)+z\cdot w(t)\,\beta\Xi(\pi-\phi,1)_u^{\bar{D}}-\theta_C c(t)-\theta_L w(t)\,n \\
&\quad-(1-\lambda)\cdot w(t)\int_0^{\bar{D}}\beta e^{-(\pi+\gamma)u-M(u)}\mu(u)\,\frac{A(v,v+u)}{w(v)}du \qquad \text{(T1.5)} \\
\gamma &\equiv \frac{\dot{k}(t)}{k(t)}=r-\pi-g+\left[n-\frac{c(t)}{w(t)}\right]\cdot\frac{w(t)}{k(t)} \qquad \text{(T1.6)} \\
\frac{w(t)\,n}{k(t)} &= (1-\varepsilon_K)\,\Omega_0 \qquad \text{(T1.7)} \\
n &= \beta\cdot\left[\alpha_0\Xi(\pi+\beta_0,1)_S^R-\alpha_1\Xi(\pi+\beta_1,1)_S^R\right. \\
&\quad\left.-\frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v)}{w(v)}\frac{1+\theta_C}{1-\theta_L}\cdot\Xi(\pi+\rho+\gamma-r,2-\lambda)_S^R\right] \qquad \text{(T1.8)} \\
\frac{c(t)}{w(t)} &\equiv \frac{C(v,v)}{w(v)}\cdot\beta\Xi(\pi+\rho+\gamma-r,2-\lambda)_0^{\bar{D}} \qquad \text{(T1.9)}
\end{aligned}
$$

**Note**: The expressions (T1.4a)–(T1.4c) are valid for, respectively, $0\le u\le S$, $S\le u\le R$, and $R\le u\le\bar{D}$. Either $g$ or $z$ balances the government budget.

---

lump-sum transfer to everyone (captured by setting $\phi = 0$), redistributes with a bias toward the young ($\phi = -1/\bar{D}$), or redistributes with a bias toward the elderly ($\phi = +1/\bar{D}$). The equilibrium value for $z$ follows from the government budget constraint:

$$z \cdot w(t) \cdot \beta \Xi (\pi - \phi, 1)_0^{\bar{D}} = \theta_C c(t) + \theta_L w(t) n + (1-\lambda) \int_{t-\bar{D}}^{t} \mu(t-v) p(v,t) A(v,t) \, dv.$$

- *Wasteful scenario.* Government tax revenue is entirely spent on wasteful government consumption and transfers are zero, i.e. $g > 0$ and $TR(v,t) = 0$ for all $v$ and $t$. The spending parameter $g$ is endogenously determined by the government budget constraint:

$$g \cdot k(t) = \theta_C c(t) + \theta_L w(t) n + (1-\lambda) \int_{t-\bar{D}}^{t} \mu(t-v) p(v,t) A(v,t) \, dv.$$

## 2.4 Balanced growth path

We need to tidy up some loose ends. From (5)–(6) we easily find:

$$y(t) = \Omega_0 k(t), \qquad w(t) n = (1-\varepsilon_K) y(t), \tag{35}$$

where $k(t) \equiv K(t)/P(t)$ is the per capita stock of capital and $y(t) \equiv Y(t)/P(t)$ is per capita output. The macroeconomic growth model has been written in a compact format in Table 1. In various places the demographic function (23) has been applied. Equation (T1.1) is obtained by using (20)–(21), (11), and (34). Equations (T1.2)–(T1.3) follow directly from (18)–(19). The scaled asset path, (T1.4), has been derived by solving (9) and features three segments, depending on the agent's life-cycle phase. Equation (T1.5) is the government budget equation in its most general form. Equation (T1.6) is a slightly rewritten version of (33), whilst (T1.7) follows from the expressions in (35). Equation (T1.8) is obtained by using (11) in (28). Finally, equation (T1.9) is the same as (27).

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Equations (T1.1)–(T1.4) determine scaled newborn consumption, $C(v,v)/w(v)$, the optimal labour market entry and retirement ages, $S$ and $R$, and the life-cycle path for assets as a function of the key macroeconomic variables ($\gamma$ and $z$) and the tax rates ($\theta_C$ and $\theta_L$). In their turn, equations (T1.5)–(T1.9) determine equilibrium transfers or wasteful spending, $z$ or $g$, the macroeconomic growth rate, $\gamma$, the overall wage-capital ratio, $w(t)/k(t)$, aggregate labour supply in efficiency units, $n$, and the $c(t)/w(t)$ ratio as a function of the microeconomic variables.

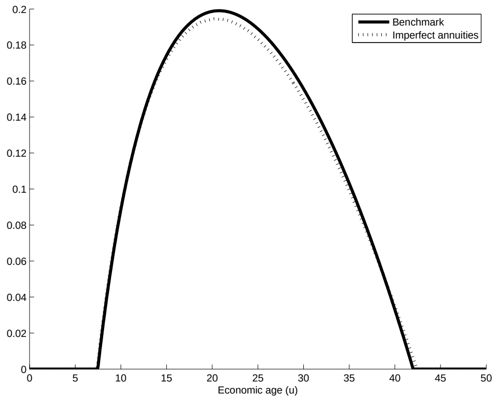
We calibrate the model to capture the key features of an advanced economy. The productivity and demographic parameters are taken from the estimates mentioned above. For the set of structural parameters we assume that $r = 0.06$, $\rho = 0.035$, $\pi = 0.01$, and $\delta = 0.10$. Using $\pi$ in combination with the demographic parameters in (24) we find that $\beta = 0.0234$. The remaining parameters are used for calibration purposes. The utility parameter, $\varepsilon_C$, has been chosen so as to induce retirement at model age $u = 42$ (i.e., 60 years in biological age), the capital share, $\varepsilon_K$, has been chosen to assure a growth rate of 2% per annum in the initial calibration, and $\Omega_0$ is set to produce the right interest rate. We find $\Omega_0 = 0.6661$, $\varepsilon_K = 0.2402$, and $\varepsilon_C = 0.0935$. The policy parameters $\{\theta_C, \theta_L, \phi, z, g\}$ and the wedge factor, $\lambda$, are set according to need in the separate calibrations below. In the initial calibration annuities are perfect, i.e. $\lambda = 1$.

The main features of the benchmark calibration are reported in column (a) of Table 2. Figure 3 shows some life-cycle features of the initial model calibration with lump-sum transfers but without consumption and labour income taxes. The solid line indicates the scenario with a perfect annuity market ($\lambda = 1$) whereas the dotted line indicates the scenario with an imperfect annuity market ($\lambda = 0.7$). Figure 3(a) shows that individuals enter the labour market at age 7.47, quickly increase labour supply to full-time levels, about twenty percent of the total time endowment, and then smoothly ease into retirement at age 42. Figure 3(b) shows that, in the presence of perfect annuity markets, individuals opt for an ever increasing consumption level whereas, in the presence of imperfect annuity markets, they choose a hump-shaped consumption profile. The latter effect is a direct consequence of equation (22) where we see that $\lambda < 1$ causes individuals to discount future consumption by a term proportional to their instantaneous probability of death, $\mu(t - v)$. As $\mu(t - v)$ is increasing in age, future consumption is discounted more heavily. In terms of assets, the hump-shaped consumption profile implies a lower demand for assets to finance old-age consumption. Hence, the imperfect (dotted) annuity path lies below the perfect (solid) annuity path in Figure 3(c). The decrease in capital accumulation translates into lower growth (see the column (b) in Table 2) because capital accumulation – including the externalities associated with it – constitutes the engine of endogenous growth. Finally, Figure 3(d) shows that the individual's scaled wage path is monotonically increasing over the life-cycle. Hence, the macroeconomic growth effect dominates the reduction in labour productivity for ageing workers.

Figure 3: General equilibrium in the core model

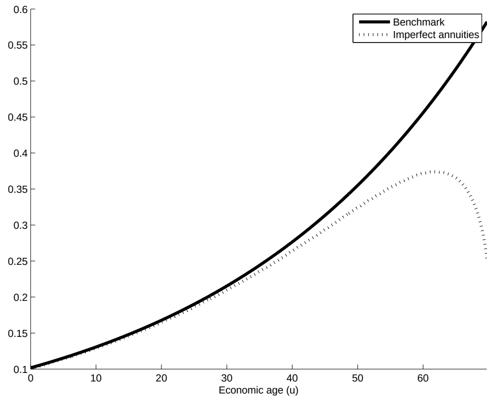
(a) labour supply

$$L(u)$$



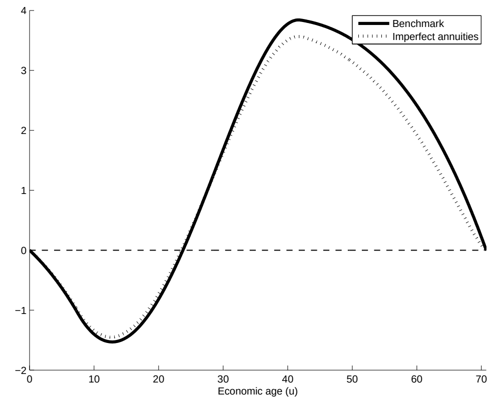

(b) scaled consumption newborns

$$\frac{C(v, v+u)}{w(v)}$$




(c) scaled financial assets

$$\frac{A(v, v+u)}{w(v)}$$




(d) scaled wage rate

$$\frac{w(v, v+u)}{w(v)}$$

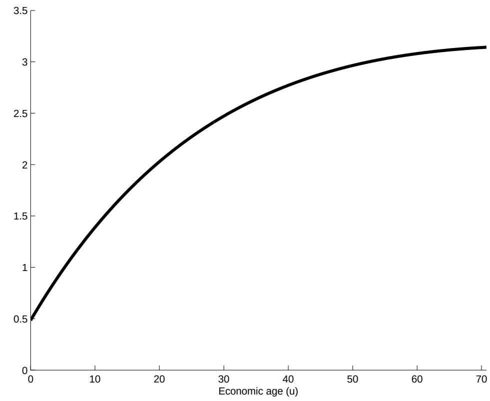


Table 2: Taxation, retirement, and growth: quantitative effects

| | *Benchmark* | | *Consumption Tax* | | *Labour Tax* | |
|---|---|---|---|---|---|---|
| | (a) | (b) | (c) | (d) | (e) | (f) |
| | $\lambda = 1.0$ | $\lambda = 0.7$ | $\lambda = 1.0$ | $\lambda = 0.7$ | $\lambda = 1.0$ | $\lambda = 0.7$ |
| $\frac{C(v,v)}{w(v)}$ | 0.1017 | 0.1007 | 0.0878 | 0.0867 | 0.0648 | 0.0638 |
| $S$ (years) | 7.47 | 7.40 | 8.22 | 8.11 | 9.00 | 8.81 |
| $R$ (years) | 42.00 | 42.40 | 41.05 | 41.27 | 38.15 | 38.01 |
| $z$ | 0 | 0.0025 | 0.0199 | 0.0216 | 0.0281 | 0.0285 |
| $\gamma$ (%) | 2 | 1.89 | 2.05 | 1.93 | 1.99 | 1.86 |
| $n$ | 0.1101 | 0.1079 | 0.0939 | 0.0920 | 0.0703 | 0.0688 |
| $\frac{c(t)}{w(t)}$ | 0.1167 | 0.1145 | 0.0994 | 0.0976 | 0.075 | 0.073 |
| $\frac{w(t)}{k(t)}$ | 4.60 | 4.69 | 5.39 | 5.50 | 7.20 | 7.36 |

From the initial calibration in Figure 3 we see that the core model with imperfect annuities captures the basic features of the empirically observed life-cycle (see Heijdra and Mierau, 2009, p. 20). In particular, individuals exhibit a hump-shaped profile in consumption (although peaking somewhat late in life), labour supply, and assets. Furthermore, from a macroeconomic perspective, we see that the model captures the broad features of the aggregate economy. That is, the model exhibits an economic growth rate of 2% and implies a capital efficiency parameter of 0.24. In combination the realistic life-cycle and macroeconomic features allow us to analyze how public policy influences the intergenerational allocation of resources and affects economic growth. In the following sections we study two classical public policies, namely consumption and labour income taxation.

# 3 Consumption tax

Figure 4 shows the effects of a twenty percent tax on consumption expenditures ($\theta_C = 0.2$). The labour income tax is assumed to be zero ($\theta_L = 0$). The solid line is the benchmark calibration (featuring $\theta_C = \theta_L = 0$), the dashed line is the benchmark calibration with the consumption tax imposed, and the thin dotted line is the model with the consumption tax and imperfect annuities. Figure 4(a) reveals that the consumption tax prompts individuals to enter the labour market later, to spend fewer hours working and to retire earlier. See columns (a) and (c) in Table 2 for the quantitative effects. Figure 4(b) shows that consumption decreases at all ages in the presence of taxes. Intuitively, the consumption tax increases the retail price of consumption goods. Furthermore, the consumption tax acts as an implicit labour tax, hence individuals decrease their supply of labour. As before, imperfections on the annuity market lead to a hump shaped profile in consumption.

In terms of growth we see in column (c) of Table 2 that steady state growth is slightly *higher* in the presence of consumption taxes. As can be seen in Figure 4(d) this is a direct consequence of intergenerational redistribution effects arising from the recycling of tax revenues. Indeed, scaled *net* transfers to individuals are defined as follows:

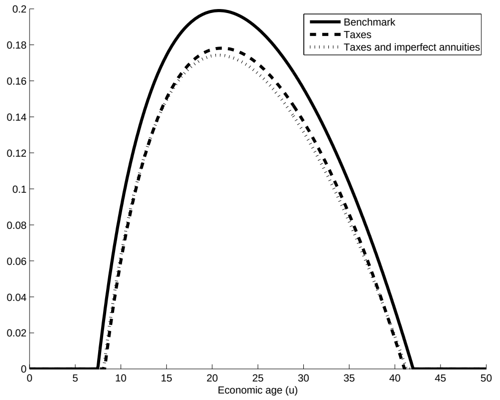
$$\frac{NTR\,(v,t)}{w\,(v)} \equiv \frac{TR\,(v,t)}{w\,(v)} - \theta_L \frac{w(v,t)}{w\,(v)} L\,(v,t) - \theta_C \frac{C(v,t)}{w\,(v)} - (1-\lambda)\,\mu\,(t-v)\,\frac{A\,(v,t)}{w\,(v)}, \quad (36)$$

where the first element of right-hand side are the transfers received from the government, the second and the third elements are the taxes paid, and the final term represents the part of

Figure 4: Consumption Taxation

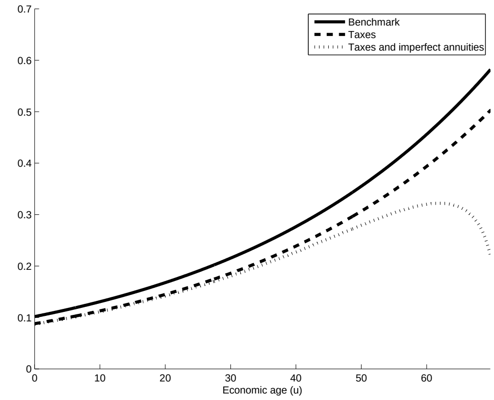
(a) labour supply

$$L(u)$$



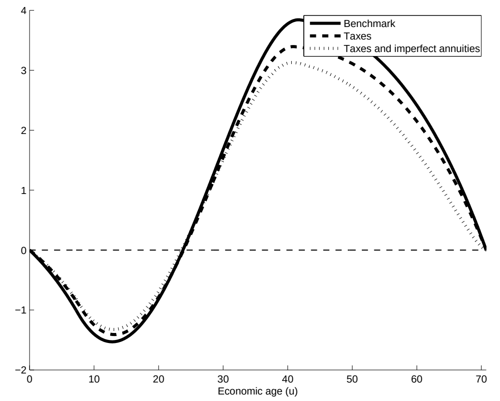

(b) scaled consumption newborns

$$\frac{C(v, v+u)}{w(v)}$$



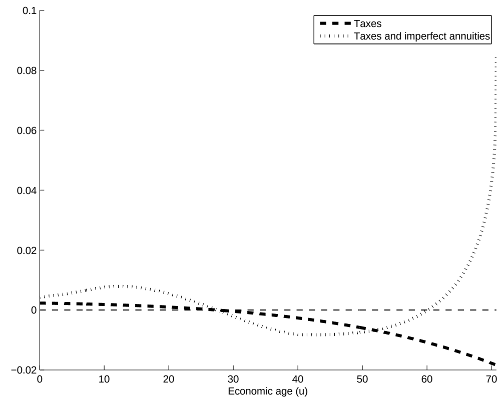

(c) scaled financial assets

$$\frac{A(v, v+u)}{w(v)}$$




(d) net transfers

$$\frac{NTR(v, v+u)}{w(v)}$$



annuity income that is lost due to the imperfection on the annuity market. Over the life-cycle an individual may alternate between being a net recipient of transfers and a net donor of transfers because the four elements in equation (36) exhibit different life-cycle patterns – see Figure 4.[4]

In Figure 4(d) we see that under consumption taxation, individuals are net recipients of transfers when young whilst they are net donors when old. Figure 4(d) shows that this result follows from the rising profile of consumption. As the young are accumulators of assets and the elderly are decumulators of assets this leads to an increase in aggregate capital accumulation, and hence an increase in economic growth.

If annuity market imperfections are also taken into account, we find that individuals start out as net recipients, are donors during mid-life, and become recipients again later in life. Again from Figure 4(d) we see that part of the different transfer profile is due to the hump-shape in consumption induced by the annuity market imperfection. Furthermore, from Figure 4(c) we see that the implicit tax on annuity income befalls especially on the resourceful middle-aged so that the elderly and the young benefit from the transfers whereas the middle aged pay. The growth effect, however, remains positive as a comparison between columns (b) and (d) in Table 2 reveals.

To highlight the impact of intergenerational redistribution we study the consequences of alternative redistribution schemes in Figure 5. In the solid line the transfers are spread evenly ($\phi = 0$, the case discussed above), in the dashed line we skew the distribution of transfers toward the elderly ($\phi = +1/\bar{D}$) and in the dotted line we skew the transfers toward the young ($\phi = -1/\bar{D}$). In Table 3 these cases corresponds to the second, third, and fourth column, respectively. Comparing rows (a) and (c) (or indeed (b) and (d)) we find that the positive impact on growth disappears for the regime in which transfers are skewed toward the elderly. In contrast, the positive impact is enhanced if transfers are skewed toward the young. This allows us to conclude that the increase in growth arises from the intergenerational redistribution effect that channels funds from the decumulating elderly to the accumulating

[4] Notice that because the profits made by annuity firms are taxed away by the government, the annuity market imperfection acts as an implicit tax on the annuity premium. Furthermore, in the initial phase of the life-cycle the annuity market imperfection may act as a subsidy on loans. Although this is a somewhat troubling feature of the model, the effect of this feature is negligible because the absolute magnitude of loans as well as $\mu(t-v)$ are low for the young.

Table 3: Economic growth*

| | $\{\lambda, \theta_C, \theta_L\}$ | $\phi = 0$ | $\phi = +\frac{1}{\bar{D}}$ | $\phi = -\frac{1}{\bar{D}}$ | $z = 0$ |
|---|---|---|---|---|---|
| (a) | $\{1, 0, 0\}$ | 2.00 | 2.00 | 2.00 | 2.00 |
| (b) | $\{0.7, 0, 0\}$ | 1.89 | 1.88 | 1.90 | 1.69 |
| (c) | $\{1, 0.2, 0\}$ | 2.05 | 1.95 | 2.14 | −1.20 |
| (d) | $\{0.7, 0.2, 0\}$ | 1.93 | 1.81 | 2.03 | −1.53 |
| (e) | $\{1, 0, 0.4\}$ | 1.99 | 1.72 | 2.24 | −2.60 |
| (f) | $\{0.7, 0, 0.4\}$ | 1.86 | 1.57 | 2.13 | −2.96 |

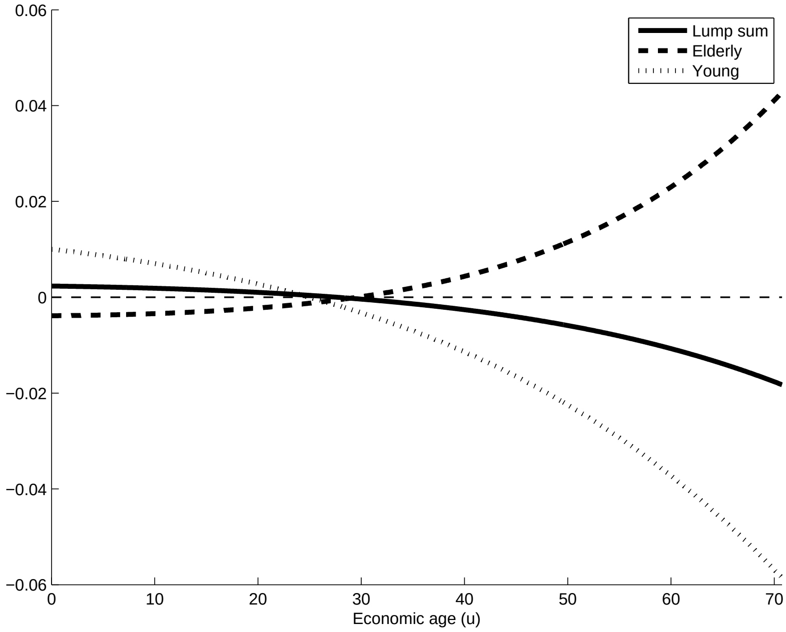
*Cell entries show the percentage of economic growth per annum in the respective scenario.

young.

A similar effect is found in Heijdra and Ligthart (2000, p. 697) where it is shown that a consumption tax may lead to a higher steady-state capital stock; the exogenous growth equivalent of a higher growth rate. However, Heijdra and Ligthart dismiss this possibility as empirically unlikely because it would require an unreasonably high instantaneous probability of death. The discrepancy between their results and ours arises from the fact that they assume a constant instantaneous probability of death whereas we assume an age-dependent one.

To emphasize the importance of government transfers in general we conclude this section with a study of the case in which the government pursues the wasteful scenario, i.e. tax revenue is spent entirely on wasteful government expenditure (see above). The growth effects are reported in the final column of Table 3. The comparison of row (a) and (c) (or (b) and (d)) immediately reveals the detrimental growth effects of wasteful government expenditures. For consumption taxation and annuity market imperfection the growth rate drops from 2% in the initial calibration to −1.20% and −1.53% for the pure taxation and the combined case, respectively. These detrimental effects derive from the fact that the government now not only distorts the market through taxation but also drains resources from the economy for wasteful purposes.

Figure 5: Alternative transfers




## 4 Labour income tax

In Figure 6 we study the impact of a forty percent tax on labour income ($\theta_L = 0.4$). The consumption tax is assumed to be zero ($\theta_C = 0$). As before, the solid lines represent the benchmark case, the dashed line the scenario with the labour income tax alone, and the dotted line the taxes and imperfect annuities case. Figure 6(a) shows that the labour tax induces individuals to postpone labour market entry, to work less during their active career and to retire early. Compare the columns (a) and (e) in Table 2 for the quantitative effects. In Figure 6(b) we see that the tax decreases initial consumption and that the annuity market imperfection causes the familiar hump-shaped profile. Finally, Figure 6(c) reveals that agents accumulate less debt early on in life but also substantially fewer assets later on in life.

From an intuitive point of view the labour tax decreases the benefits from work, so that individuals reduce their labour effort, both on the intensive and the extensive margin. The labour tax also acts as an implicit tax on consumption, hence consumption is reduced in a fashion akin to the imposition of the explicit consumption tax studied above. Even though the consumption and labour tax act as each others implicit equivalents, the standard labour-consumption tax equivalence result (Atkinson and Stiglitz, 1980) no longer holds. The failure of the equivalence results is also derived in Heijdra and Ligthart (2000) where it is shown that the life-cycle paths of consumption and labour induced by finite lives lead to asymmetric tax incidence of labour and consumption. In our model the equivalence result also fails because individuals retire.

In terms of growth we find an asymmetry between consumption and labour taxes, cf. columns (c) and (e) in Table 2. Whereas consumption taxes lead to a slight *increase* in growth, labour taxes lead to a slight *decrease* in growth. From Figure 6(d) we see that this is a consequence of the different intergenerational redistribution structure. Where the consumption tax induced redistribution from the decumulating elderly to the accumulating young, the labour tax induces redistribution from the working to the idle. Because idleness is an attribute of the elderly, the labour tax causes a redistribution from the accumulating workers to the decumulating retired, hence depressing growth.

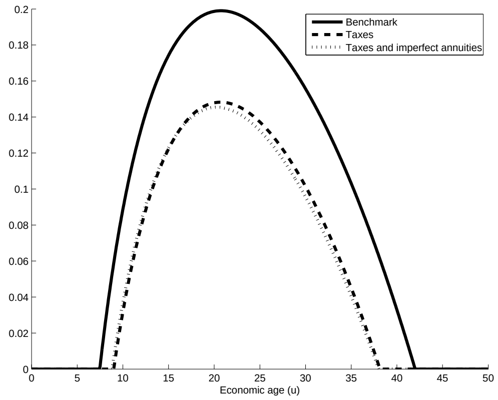
In rows (e)–(f) of Table 3 we study the consequences of alternative redistributions schemes once more. In accordance with the variational exercise depicted in Figure 5, we find that a redistribution scheme skewed toward the elderly has detrimental effects on economic growth, whereas a redistribution scheme skewed toward the young has advantageous effects on economic growth. Notice especially, that the positive growth effect of a redistribution skewed toward the young outpaces the positive effect found under consumption taxes. From the entries for $z$ in Table 2 we see that is because the pie to be redistributed is larger under labour taxation.

Finally, in rows (e)–(f) of the last column of Table 3 we repeat the analysis of a wasteful government for the case of labour income taxes. As before we find that wasteful government expenditure causes huge detrimental effects on economic growth. In particular, for labour income taxes and annuity market imperfection the growth rate drops from 2% to $-2.60\%$ and $-2.96\%$ for the pure taxation and combined case, respectively. In accordance with the consumption taxation case the detrimental effect is driven by the government's drain of productive resources. The absolute magnitude of the effect is bigger than in the consumption taxation case because government revenue derived from labour taxation is larger.

Figure 6: Labour Taxation

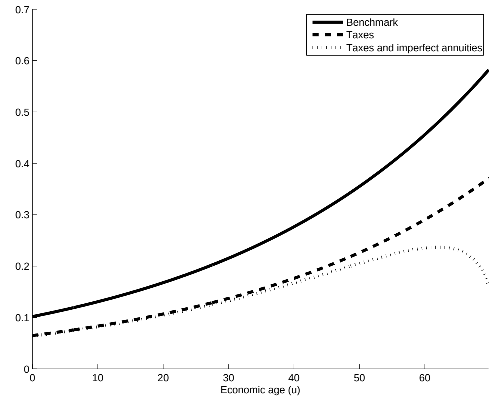
(a) labour supply

$$L(u)$$



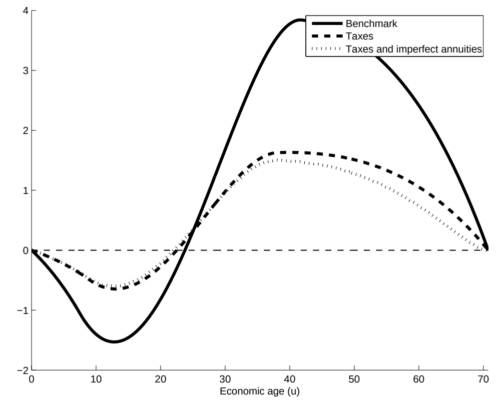

(b) scaled consumption newborns

$$\frac{C(v, v+u)}{w(v)}$$



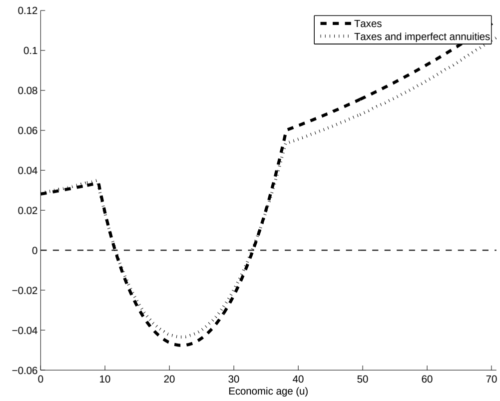

(c) scaled financial assets

$$\frac{A(v, v+u)}{w(v)}$$




(d) net transfers

$$\frac{NTR(v, v+u)}{w(v)}$$



## 5 Conclusions

In this paper we have studied the effects of consumption and labour income taxation in a life-cycle overlapping generations model. On the individual level we have replicated the basic features of the life-cycle by including age-dependent mortality and productivity. Furthermore, in order to replicate the hump-shaped profile in consumption, we have introduced an imperfection in the annuity market. On the macroeconomic level we have applied the insights of Romer (1989) and postulated sufficiently strong externalities between firms so that growth is endogenous and remains positive even in the long-run.

Using this model we have studied two classical themes in dynamic public finance theory; consumption and labour income taxation. In the analysis we have paid special attention to alternative modes of redistributing the proceeds of taxation. That is we have analyzed three systems that either redistribute government revenue evenly across all individuals or with a bias toward the young or the old. Finally, to emphasize the importance of redistribution of government income in general we have also analyzed a wasteful government that uses the proceeds from taxation for useless consumptive purposes.

We have found that both consumption and labour income taxation lead to an increase in economic growth if the proceeds are redistributed with a bias toward the young. This is due to the beneficial intergenerational redistribution effects that channel resources from the dissaving elderly to the saving young. In addition we have found that the two taxes lead to lower growth if proceeds are redistributed toward the elderly. In this case the intergenerational redistribution effect is negative because resources flow from the saving young to the dissaving elderly. For the case of lump-sum transfers we have found that growth increases slightly for consumption taxation and decreases slightly for labour income taxation. The differing consequences of the two taxes are, again, due to intergenerational redistribution effects. The consumption tax redistributes funds from the elderly, who are strong consumers and thus pay the lion's share of tax, to the young, who barely consume but save a lot. The labour income tax, on the other hand, redistributes funds between the working and the idle. Idleness being an attribute of the retired, the tax induces redistribution from saving workers to consuming retirees. Finally, we have found that wasteful government expenditures have strong detrimental effects on growth. Besides distorting the economy through the taxation, the government also drains productive resources.

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