The individual life-cycle, annuity market imperfections and economic growth*

Ben J. Heijdra‡
University of Groningen; IHS (Vienna); CESifo; Netspar

Jochen O. Mierau♯
University of Groningen; Netspar

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Abstract: We study the effects of an annuity market imperfection on individual agents' life-cycle decisions and on the macroeconomic growth rate in an overlapping generations model with single-sector endogenous growth. Our model features both age-dependent mortality and labour productivity. We model imperfect annuities by introducing a load factor on the annuity rate faced by finitely-lived agents. Our main finding is that annuity market imperfections decrease economic growth because less assets are necessary for consumption late in life. In addition we find that both the quantitative and qualitative effect of annuity market imperfections are grossly overestimated in a partial equilibrium analysis because it disregards general equilibrium repercussions.

Keywords: Annuity markets, retirement, endogenous growth, overlapping generations, demography.

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‡Faculty of Economics and Business, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-363-7303, Fax: +31-50-363-7337, E-mail: `info@heijdra.org`.

♯Corresponding author. Faculty of Economics and Business, University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-363-8534, Fax: +31-50-363-7337, E-mail: `j.o.mierau@rug.nl`.

1 Introduction

One of the most robust findings in economic theory is that individuals facing an uncertain date of death derive substantial benefits from annuitization of savings. In a seminal paper, Yaari (1965) shows that in the absence of a bequest motive individuals should fully annuitize all of their savings. One of the key assumptions adopted by Yaari concerns the availability of actuarially fair annuities. In a recent paper, Davidoff *et al.* (2005) demonstrate that the full annuitization result holds in a much more general setting than the one adopted by Yaari, i.e. it obtains also when annuities are less than actuarially fair.

The objective of this paper is to study the macroeconomic effects of actuarially unfair annuities. Are the optimal retirement age and the macroeconomic growth rate significantly affected by the degree of actuarial fairness of annuities or is this imperfection quantitatively unimportant? To answer this question we construct a stylized overlapping generations model of a closed economy featuring endogenous growth due to an inter-firm investment externality of the "AK"-type. Our starting point is the celebrated Blanchard (1985) model. We extend this model in five directions. First, we endogenize the individual agent's life-cycle labour supply decision. Second, we introduce an annuity imperfection parameter which allows us to study the cases of actuarially fair and unfair annuities in a single framework. In the latter case annuity firms make profits which are taxed away by the government and transferred to households. Third, we introduce an asset constraint that disallows agents from holding negative asset balances. Fourth, we introduce age-dependent labour efficiency. Finally, we incorporate the insights of Heijdra and Romp (2008) and postulate an age-dependent mortality process.

Our main findings are as follows. First, the imperfection on the annuity market leads individuals to discount future consumption by a term including their mortality rate as well as their pure rate of time preference. Due to the age-dependent mortality rate this leads to a hump-shaped consumption profile. This in turn reduces capital accumulation as less assets are required to finance consumption late in life. The reduction in asset accumulation by the individual agents leads to lower economic growth on the aggregate level because capital accumulation is the driving force behind economic development in our model.

Second, in terms of labour supply we find that individuals supply less labour during their working life but retire slightly later if annuities are less than fair. On the whole this leads to a decrease in aggregate labour supply.

Third, we show that the way in which annuity firms' profits are redistributed plays an important role in the analysis. In the baseline analysis we assume that these profits are transferred equally across all agents. If, however, the profits are redistributed with a skew toward the young, then growth is slightly higher because young agents are savers. On the other hand, if profits are distributed toward the elderly growth is slightly lower because the elderly are dissavers. Finally, if these profits are drained from the economy via wasteful government consumption, growth deteriorates dramatically and the retirement age is reduced substantially.

Fourth, our analysis emphasizes the importance of general equilibrium effects. In a pure partial equilibrium analysis, the impact of imperfect annuities is grossly overestimated because such an analysis ignores both the redistribution of profits and the response in the economic growth rate. This finding shows that, as much as macroeconomics should be microfounded, microeconomics should be macrofounded.

The two papers most closely associated with ours are Bütler (2001) and Hansen and İmrohoroğlu (2008). Both study the role of annuities for individual consumption decisions over the life cycle. In addition, Bütler studies endogenous labour supply decisions, though not in relation to the imperfect annuities market. Hansen and İmrohoroğlu embed their model in a general equilibrium context but disregard the labour supply decisions made by the individual agents. In addition they only study the polar cases of perfect and no-annuities in the general equilibrium model.

We extend the insights of Bütler to the general equilibrium case and explicitly take into account the impact of imperfect annuities on labour supply and retirement decisions (as opposed to Hansen and İmrohoroğlu). Furthermore, we also study imperfect annuities in general equilibrium, not only at the individual level (as in Bütler) or in the absence of annuities (as in Hansen and İmrohoroğlu).

In addition to the papers of Bütler and Hansen and İmrohoroğlu, there is a substantial literature studying the partial equilibrium implications of annuity market imperfections. It is beyond the scope of this paper to give a full review of this literature but is good to alert the interested reader to Chai *et al.* (2011). They study a very detailed partial equilibrium stochastic life-cycle model in which agents feature an endogenous labour supply decision and can choose between a variety of assets to invest in. Their findings regarding the retirement decision fit well with our analysis in the sense that they find that introducing annuities leads to earlier retirement. However, as we show in this paper, the partial equilibrium literature might be strongly misleading due its neglect of general equilibrium repercussions.

The remainder of the paper is structured as follows. Section 2 sets out the model, whilst section 3 discusses how empirical observations are fed into the model. Section 4 studies the relationship between the annuity market imperfection, the individual life-cycle, and the macroeconomic growth rate and contains some sensitivity analysis. Section 5 concludes.

2 Model

2.1 Firms

The production side of the model makes use of the insights of Romer (1989) and postulates the existence of sufficiently strong external effects operating between private firms in the economy. There is a large and fixed number, $\mathcal{N}$, of identical, perfectly competitive firms. The technology available to firm i is given by:

$$Y_i\left(t\right) = \Omega\left(t\right) K_i\left(t\right)^{\varepsilon_K} N_i\left(t\right)^{1-\varepsilon_K}, \qquad 0 < \varepsilon_K < 1, \tag{1}$$

where $Y_i\left(t\right)$ is output, $K_i\left(t\right)$ is capital use, $N_i\left(t\right)$ is the labour input measured in efficiency units, and $\Omega\left(t\right)$ represents the general level of factor productivity which is taken as given by individual firms. The competitive firm hires factors of production according to the following marginal productivity conditions:

$$w\left(t\right) = \left(1-\varepsilon_K\right)\Omega\left(t\right)\kappa_i\left(t\right)^{\varepsilon_K}, \tag{2}$$

$$r\left(t\right) + \delta = \varepsilon_K \Omega\left(t\right)\kappa_i\left(t\right)^{\varepsilon_K - 1}, \tag{3}$$

where $\kappa_i\left(t\right) \equiv K_i\left(t\right)/N_i\left(t\right)$ is the capital intensity. The rental rate on each factor is the same for all firms, i.e. they all choose the same capital intensity and $\kappa_i\left(t\right) = \kappa\left(t\right)$ for all $i = 1, \cdots, \mathcal{N}$. This is a very useful property of the model because it enables us to aggregate the microeconomic relations to the macroeconomic level.

Generalizing the insights of Saint-Paul (1992, p. 1247) and Romer (1989, p. 90) to a growing population, we assume that the inter-firm externality takes the following form:

$$\Omega\left(t\right) = \Omega_0 \kappa\left(t\right)^{1-\varepsilon_K}, \tag{4}$$

where Ω_0 is a positive constant, $\kappa(t) \equiv K(t)/N(t)$ is the economy-wide capital intensity, $K(t) \equiv \sum_i K_i(t)$ is the aggregate capital stock, and $N(t) \equiv \sum_i N_i(t)$ is aggregate employment in efficiency units. According to (4), total factor productivity depends positively on the aggregate capital intensity, i.e. if an individual firm i raises its capital intensity, then *all* firms in the economy benefit somewhat as a result because the general productivity indicator rises for all of them. Using (4), equations (1)–(3) can be rewritten in aggregate terms:

$$Y(t) = \Omega_0 K(t), \tag{5}$$

$$w(t) L(t) = (1-\varepsilon_K) Y(t), \tag{6}$$

$$r(t) = r = \varepsilon_K \Omega_0 - \delta, \tag{7}$$

where $Y(t) \equiv \sum_i Y_i(t)$ is aggregate output and we assume that capital is sufficiently productive, i.e. $\varepsilon_K \Omega_0 - \delta > 0$. The aggregate technology is linear in the capital stock and the interest is constant.

2.2 Consumers

From a modeling perspective it is interesting to briefly reflect on the life-cycle elements contained in our model. In Heijdra and Mierau (2009) we used the perpetual youth model with constant productivity as a starting point. From there onward we introduced the age-dependent mortality and productivity first separately and then, as in the current paper, simultaneously. Using that approach, we could highlight that both life-cycle elements are critical for the analysis. For instance, in the presence of only imperfect annuities, an age-dependent mortality profile would lead agents to re-enter the labour market very late in life. This is because the labour supply profile is the inverse of consumption profile. If age-dependent productivity is also added to the model, labour market re-entry will no longer occur because the wage against which agents are giving up leisure late in life is too low. In addition, we found that the perpetual youth model substantially overestimates the impact of annuity market imperfections. This is due to that fact that it gives a strong weight to old agents who, in that model, posses a substantial amount of assets. In contrast, in a model with age-dependent mortality the elderly are decumulating assets and only relatively few of them are around.

2.2.1 Individual behaviour

We generalize the model by Heijdra and Romp (2008) by including an endogenous labour-leisure decision, by recognizing age-dependent productivity, and by assuming potentially imperfect annuity markets. At time t, expected remaining-lifetime utility of an individual born at time v ($v \leq t$) is given by:

$$\mathbb{E}\Lambda\left(v,t\right) \equiv \int_{t}^{v+\bar{D}} \ln\left[C(v,\tau)^{\varepsilon_C} \cdot \left[1-L(v,\tau)\right]^{(1-\varepsilon_C)}\right] \cdot e^{-\rho(\tau-t)+M(t-v)-M(\tau-v)} d\tau, \tag{8}$$

where $C\left(v,\tau\right)$ is consumption, $L\left(v,\tau\right)$ is labour supply (the time endowment is equal to unity), ρ is the pure rate of time preference, $\bar{D}$ is the maximum attainable age, and $e^{M(t-v)-M(\tau-v)}$ is the conditional probability that the agent of age $t-v$ is still alive at age $\tau-v$ (with $\tau \geq t$). Note that $M\left(x\right) \equiv \int_0^x \mu\left(s\right) ds$ is the cumulative mortality rate whilst $\mu\left(s\right)$ is the instantaneous mortality rate of an individual of age s. This rate is strictly increasing and convex in age, $\mu'\left(s\right) > 0$ and $\mu''\left(s\right) > 0$, and features $\lim_{s\rightarrow\bar{D}} \mu\left(s\right) = +\infty$.

The agent's budget identity is given by:

$$\dot{A}(v,\tau) = r^A\left(\tau-v\right) A(v,\tau) + w(v,\tau)L\left(v,\tau\right) - C(v,\tau) + TR\left(v,\tau\right), \tag{9}$$

where $A\left(v,\tau\right)$ is the stock of financial assets, $r^A\left(\tau-v\right)$ is the age-dependent rate of interest on annuities, $w\left(v,\tau\right) \equiv w\left(\tau\right) E\left(\tau-v\right)$ is the age-dependent wage rate, $E\left(\tau-v\right)$ is the exogenous labour productivity profile, and $TR\left(v,\tau\right)$ are lump-sum transfers from the government (see below).

Earlier studies argue that productivity is positive and hump-shaped over the life-cycle – see, e.g., Hansen (1993) and Rios-Rull (1996). In terms of $E\left(\tau-v\right)$ this implies that $E\left(\tau-v\right) > 0$, $E'\left(\tau-v\right) > 0$ for $\tau-v < \bar{u}$ and $E'\left(\tau-v\right) < 0$ for $\tau-v > \bar{u}$, where $\bar{u}$ is the age at which labour productivity is at its peak.

Following Yaari (1965), we postulate the existence of annuity markets, but unlike Yaari we allow the annuities to be less than actuarially fair. Since the agent is subject to lifetime uncertainty and has no bequest motive, he/she will fully annuitize so that $A\left(v,\tau\right)$ also represents the demand for annuities and the annuity rate of interest facing the agent is given by:

$$r^A\left(\tau-v\right) \equiv r + \theta\mu\left(\tau-v\right), \tag{10}$$

where r is the time-invariant real interest rate (see, equation (7)) and θ is a parameter ($0 < \theta \leq 1$). In addition, we assume that there is no market for life-insured loans, i.e. the demand for annuities must be non-negative at all times:

$$A(v, \tau) \geq 0. \tag{11}$$

Our specification for the annuity rate in (10) can be rationalized in three ways. First, $1-\theta$ may be interpreted as a load factor needed to cover the administrative costs of organizing the annuity firm – see Horneff *et al.* (2008, p. 3595). Second, as Hansen and İmrohoroğlu (2008, p. 569) suggest, θ may represent the fraction of assets that are annuitized. Provided θ is strictly less that unity, there will be unintended bequests under this interpretation. Third, annuity firms may possess some market power, allowing them to make a profit by offering a less than actuarially fair annuity rate. In this paper we adopt the market-power interpretation. We shall refer to $1 - \theta$ as the degree of imperfection in the annuity market.

Our specification is very general and incorporates two important cases:

- *Perfect annuities (PA):* The case of perfect (actuarially fair) annuities is obtained by setting $\theta = 1$. Annuity companies break even, and $TR(v, \tau) = 0$.
- *Imperfect annuities (IA):* The case of imperfect (less than actuarially fair) annuities is obtained by assuming $0 < \theta < 1$. Annuity firms make excess profits which are taxed away by the government and distributed in a lump-sum fashion to surviving agents.[1]

The agent chooses time profiles for $C(v, \tau)$, $A(v, \tau)$, and $L(v, \tau)$ (for $\tau \geq t$) in order to maximize (8), subject to (i) the budget identity (9), (ii) the initial asset position in the planning period, $A(v, \tau)$ and (iii) the borrowing constraint (11).

We restrict attention to the optimal individual life-cycle decisions in the context of an economy moving along the steady-state balanced growth path. Along this path, labour productivity grows at a constant exponential rate, γ (see below), and individual agents face the

[1]Naturally, if $\theta = 0$ there no longer exists an annuities market. In that case it is not the profits of the annuity firms that are being redistributed but accidental bequests. Leung (1994) shows that for the no annuities case agents hit the asset constraint (11) towards the end of their life. Simulations, however, show that this point is extremely close to the maximum attainable age, $\bar{D}$. So close, indeed, that the probability of actually hitting it is negligible. For this reason we ignore the case with $\theta = 0$ in this paper.

following profile for real wages over their lifetimes:

$$w(v,\tau) = w(v)\,E(\tau - v)\,e^{\gamma(\tau-v)}. \tag{12}$$

Using (12) we can define the *scaled wage rate* facing individuals aged $u = \tau - v$:

$$\frac{w(v,v+u)}{w(v)} = E(u)\,e^{\gamma u}. \tag{13}$$

Whereas the unscaled wage rate, $w(v,\tau)$, is both time- and age-dependent, the scaled wage rate, $w(v,\tau)/w(v)$ only depends on the individual's age, u. Effectively, the wage rate at birth, $w(v)$, acts as a scale factor that pins down the individual's initial condition.

With imperfect annuities ($0 < \theta < 1$) we must confront the issue of redistribution of excess profits and recognize the fact that $TR(v,\tau)$ will be positive in general. To keep things simple we assume that the transfers are set according to $TR(v,\tau) = z \cdot w(\tau) \cdot e^{\phi(\tau-v)}$ or:

$$\frac{TR(v,v+u)}{w(v)} = z \cdot e^{(\phi+\gamma)u}, \tag{14}$$

where z is a positive indexing parameter which is taken as given by individual agents but determined endogenously in general equilibrium via the balanced budget requirement of the redistribution scheme (see, equation (T1.5)). The parameter ϕ determines the *skew* of the transfers, where $\phi = 0$ implies a neutral regime, whereas $\phi > 0$ implies a skew toward the elderly and $\phi < 0$ implies a skew toward the young. In the remainder of this section we focus on the neutral case with $\phi = 0$.

Agents pass through three regimes over their life-cycle, depending on whether or not the asset constraint is binding and whether or not the agent is retired. We demark the three regimes by calling the age at which the asset constraint no longer holds F_b and calling the age at which agents retire R. Using these dates in combination with the age at birth (0) and the maximum attainable age ($\bar{D}$) we can describe the regimes as follows:

- *Regime 1:* For $0 \leq u < F_b$ the asset constraint is binding and labour supply is positive.
- *Regime 2:* For $F_b \leq u < R$ the asset constraint is not binding and labour supply is positive.
- *Regime 3:* For $R \leq u \leq \bar{D}$ the asset constraint is not binding and labour supply is zero.

The life-cycle consumption-leisure choice is illustrated in Figure 1, where $C(v,v+u)/w(v)$ and $L(u)$ stand for, respectively, scaled consumption and labour supply of an agent at age u. The left-hand panel shows the choices made by asset constrained agents whereas the right-hand panel shows the choices of the agents that are not asset constrained.

During **Regime 1** the agent equates the marginal rate of substitution between leisure and consumption to the scaled wage rate,

$$\frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v+u)/w(v)}{1-L(u)}=\frac{w(v,v+u)}{w(v)}, \tag{15}$$

but faces a binding asset constraint which implies a flow constraint on scaled full consumption, $\frac{X(v,v+u)}{w(v)}$:

$$\left[\frac{X(v,v+u)}{w(v)}\equiv\right]\quad\frac{C(v,v+u)}{w(v)}+\frac{w(v,v+u)}{w(v)}\left[1-L(u)\right]=\frac{w(v,v+u)}{w(v)}+\frac{TR(v,v+u)}{w(v)}. \tag{16}$$

Solving (15)–(16) gives the consumption and labour supply profiles in the asset constrained regime:

$$\frac{C(v,v+u)}{w(v)}=\varepsilon_C\left[\frac{w(v,v+u)}{w(v)}+\frac{TR(v,v+u)}{w(v)}\right], \tag{17a}$$

$$L(u)=\varepsilon_C-(1-\varepsilon_C)\frac{TR(v,v+u)}{w(v,v+u)}. \tag{17b}$$

Figure 1(a) can be used the illustrate the mechanisms at work. MRSC(0) and BC(0) depict, respectively, equations (15) and (16) evaluated at birth ($u=0$). Ideally, the agent would like to operate along the budget curve BC_0', postpone labour market entry (i.e. set $L(0)=0$) and consume at point A. But full consumption at that point is $X(v,v)=w(v,v)/(1-\varepsilon_C)$ which exceeds the available income $w(v,v)+TR(v,v)$, i.e. point A is unattainable due to the borrowing constraint. The best choice available to the agent is at point E_0, described by the expressions in (17).

Over time, $X(v,v+u)/w(v)$ increases because the agent's productivity, and therefore, $w(v,v+u)$ increases in the initial part of the life-cycle (see, equation (13)). The increase in the wage rate causes relative transfers $TR(v,v+u)/w(v,v+u)=z/E(u)$ to fall and, hence, labour supply to increase. This process is illustrated by the shifts of the BC and MRSC curves for increasing ages u_1 and u_2. Of course, in the absence of transfers (i.e. if annuities are perfect), the optimal choices lie along the dotted line through E_0 as income and substitution effects of scaled wage changes on labour supply exactly cancel out. In contrast, with positive transfers (i.e. imperfect annuities) the substitution effect dominates the income effect so that both consumption and labour supply increase with age in Regime 1 – see points E_1 and E_2.

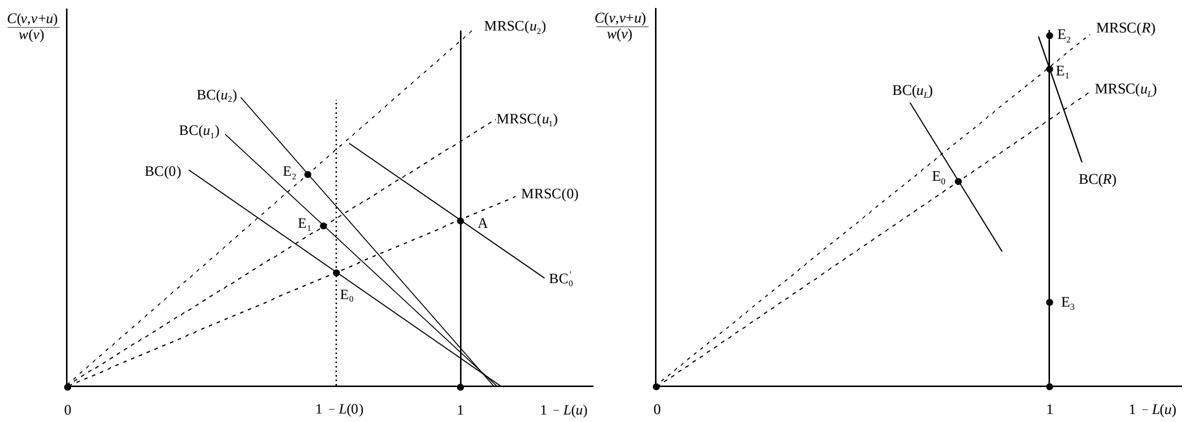
Figure 1: Consumption and leisure choices over the life cycle

(a) Regime 1

$0 \leq u \leq F_b$

(b) Regimes 2-3

$F_b \leq u \leq \bar{D}$



The agent remains in the asset-constrained regime as long as consumption growth satisfies the following inequality:

$$\frac{\dot{C}\left(v, v+u\right)}{C\left(v, v+u\right)} > r - \rho - \left(1-\theta\right)\mu\left(\tau - v\right), \tag{18}$$

that is, as long as consumption grows faster than it would in the non-constrained regime (see equation (22) below). Using (17a) and noting (13) and (14), we find that actual consumption growth in this regime equals:

$$\frac{\dot{C}\left(v, v+u\right)}{C\left(v, v+u\right)} = \gamma + \frac{\dot{E}\left(u\right) + \phi \cdot z \cdot e^{\phi u}}{E\left(u\right) + z \cdot e^{\phi u}} \tag{19}$$

Combining (18) and (19) we find that Regime 1 exists provided:

$$\gamma + \frac{\dot{E}\left(0\right) + \phi \cdot z}{E\left(0\right) + z} > r - \rho - \left(1-\theta\right)\mu\left(0\right) \tag{20}$$

and that Regime 2 is entered for the lowest $u = F_b$ such that:

$$\gamma + \frac{\dot{E}\left(F_b\right) + \phi \cdot z}{E\left(F_b\right) + z} = r - \rho - \left(1-\theta\right)\mu\left(F_b\right). \tag{21}$$

The relationships in (20) and (21) allow us to establish that, ceteris paribus, imperfect annuities increase (i) the likelihood of being asset constrained and (ii) the length of the asset constrained period. In general equilibrium, however, this relationship is moderated by the impact of annuity market imperfections on the macroeconomic variables z and γ and by the redistribution parameter ϕ.

In **Regime 2** the agent still sets labour supply according to (15) but is no longer asset constrained and chooses optimal (full) consumption growth according to the well-known Euler equation:

$$\frac{\dot{C}(v,v+u)}{C(v,v+u)} = \frac{\dot{X}(v,v+u)}{X(v,v+u)} = r - \rho - (1-\theta)\,\mu(\tau - v)\,. \tag{22}$$

Observe that with imperfect annuities, (full) consumption growth is affected by the mortality rate, a result first demonstrated for the case with $\theta = 0$ by Yaari (1965, p. 143). Optimal choices for scaled consumption and labour supply are given by:

$$\frac{C(v,v+u)}{w(v)} = \frac{\tilde{C}(v,v)}{w(v)} e^{(r-\rho)u-(1-\theta)M(u)}, \tag{23a}$$

$$\frac{\tilde{C}(v,v)}{w(v)} \equiv \frac{C(v,v+F_b)}{w(v)} e^{-(r-\rho)F_b+(1-\theta)M(F_b)}, \tag{23b}$$

$$L(u) = 1 - \frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v+u)}{w(v,v+u)}, \tag{23c}$$

where $\tilde{C}(v,v)/w(v)$ is the *hypothetical* level of consumption at birth that would have prevailed if the agent would not have been credit constrained, i.e. if (22) had been relevant right from the agent's birth date.

Taking the time derivative of (23c) we obtain the labour supply dynamics:

$$\dot{L}(u) = \frac{1-\varepsilon_C}{\varepsilon_C}\frac{C(v,v+u)}{w(v,v+u)}\left[\gamma + \frac{\dot{E}(u)}{E(u)} + \rho + (1-\theta)\,\mu(u) - r\right] \tag{24}$$

where the term in brackets represents the difference between wage and consumption growth. During the initial phase of Regime 2, wage growth outstrips consumption growth and labour supply increases with age. Labour supply peaks at age u_L such that $\gamma + \frac{\dot{E}(u_L)}{E(u_L)} + \rho + (1-\theta)\,\mu(u_L) = r$ and falls thereafter. The maximum at u_L is unique because mortality rate does not rise very fast during Regime 2 (see Figure 2(a)) and productivity is single peaked (see Figure 2(b)). Eventually, labour supply becomes zero and the agent retires.

Figure 1(b) can be used to illustrated the later phase of the agent's employment spell. Labour supply is at its maximum at point E_0 where MRSC(u_L) and BC(u_L) intersect. As the agent gets older, the budget curve gradually shifts in a north-easterly direction and becomes steeper as scaled wages continue to rise. Retirement takes place at point E_1, where MRSC(R) and BC(R) intersect.

In **Regime 3** the agent no longer participates in the labour market so (15) is not relevant anymore. Scaled consumption, $C(v, v+u)/w(v)$, still grows according to the Euler equation in (22). In this regime the agent's consumption is financed out of his financial assets complemented with government transfers (if annuities are imperfect). In terms of Figure 1(b), after retirement consumption gradually moves from E_1 to E_2.

If annuities are imperfect, the sharp increase in mortality towards the end of life gradually reduces the growth rate of consumption and, eventually, leads to a *decrease* in consumption. Indeed, as is clear from (22) consumption reaches its maximum at age u_C which is defined implicitly in $\mu(u_C) = (r-\rho)/(1-\theta)$. Since $\mu'(u) > 0$ we find that $du_C/d\theta > 0$ and $du_C/d(r-\rho) > 0$. Hence, the smaller is θ or $r-\rho$, the lower the age at which consumption peaks. In terms Figure 1(b), u_C is at point E_2. Beyond that point consumption rapidly decreases and moves toward E_3. Financial assets are slowly decumulated and – provided the agent lives long enough – run out at the maximum attainable age $\bar{D}$.

The fact that both consumption and labour supply are smooth over the life-cycle allows us to determine F_b, R and $\tilde{C}(v,v)/w(v)$ as an implicit system of equations that only takes the parameters and macroeconomic variables (γ and z) as input. For convenience we summarize all equations necessary to solve the model in Table 1.

For F_b, observe that consumption at the end of the constrained regime must coincide with consumption at the beginning of the unconstrained regime. That is, (17a) must equal (23a) at F_b, which, using (13), gives equation (T1.2).[2] To obtain (T1.3) we note that $L(R) = 0$ and use (23a)–(23c) which gives an expression relating R to F_b and $\tilde{C}(v,v)/w(v)$. Finally, in (T1.1) we notice that $\tilde{C}(v,v)/w(v)$ itself is determined by integrating (9) from F_b to $\bar{D}$ and substituting (15) and (22).

The system of equations (T1.1)–(T1.3) allows us to observe that F_b, R, and $\tilde{C}(v,v)/w(v)$ are independent of v. We summarize this important result in the following proposition.

[2] Alternatively we could have (21) which is the time-derivative of (T1.3).

Proposition 1 Consider lump-sum redistribution of excess profits of life-insurance companies, of the form $TR(v,t) = z \cdot w(t) \cdot e^{\phi(t-v)}$ and productivity, $E(t-v)$, and mortality profiles, $M(t-v)$, that are independent of v. In that case: (i) F_b is independent of v; (ii) R is independent of v; (iii) $\tilde{C}(v,v)/w(v)$ is independent of v.

Proof. Observe that only age s appears in (T1.1)–(T1.3) but not v. ■

2.2.2 Aggregate household behaviour

In this subsection we derive expressions for per-capita average consumption, saving, and labour supply. As is shown in Heijdra and Romp (2008, p. 94) the demographic steady-state equilibrium has the following features:

$$\frac{1}{\beta} = \int_0^{\bar{D}} e^{-\pi s - M(s)} ds \tag{25a}$$

$$p(v,t) \equiv \frac{P(v,t)}{P(t)} \equiv \beta e^{-\pi(t-v)-M(t-v)} \tag{25b}$$

where β is the crude birth rate of the population, $p(v,t)$ and $P(v,t)$ are, respectively, the relative and absolute size of cohort v at time $t \geq v$, and $P(t)$ is the population size at time t. For a given birth rate, equation (25a) determines the unique population growth rate consistent with the demographic steady-state or vice verse. The average population-wide mortality rate, $\bar{\mu}$, follows residually from the fact that $\pi \equiv \beta - \bar{\mu}$.

Due to the existence of asset-constrained agents we must define two different types of per capita aggregate values. On the one hand, there are per capita averages relating to the *full* population which we define generically as:

$$b(t) \equiv \int_{t-\bar{D}}^{t} p(v,t) B(v,t) dv, \tag{26}$$

where $B(v,t)$ denotes the variable in question at the individual level, and $b(t)$ is the per capita average value of that same variable. On the other hand, there are per capita averages relating only to the *non-asset constrained* population which we define as:

$$\hat{b}(t) \equiv \int_{t-\bar{D}}^{t-F_b} p(v,t) B(v,t) dv. \tag{27}$$

Using (23a) in combination with (25b) and (27) we can write per capita average consumption of the non-asset constrained population as (T1.10). Efficiency units of labour of vintage

$t-v$ are defined as $N(t-v) \equiv E(t-v) L(v,t)$. Using this expression as well as (17) and (23) allows us to write per capita average supply of labour as in (T1.8), where $\hat{n}$ is defined in (T1.9) and is the per capita average supply of labour by non-asset constrained population. The maximum labour potential in the economy is given by $\bar{n} \equiv \beta \int_0^{\bar{D}} E(s) e^{-\pi s - M(s)} ds$. The various elements in (T1.8) and (T1.9) provide the rational for why actual labour supply, n, falls short of $\bar{n}$. First, as indicated by the first composite term in (T1.9), agents only work for a part of their lives. After age R they consume their full time-endowment in the form of leisure. Second, as indicated by the second composite term in (T1.9), during their productive career, workers never supply their full time-endowment. Finally, as indicated by second and third element in (T1.8), during the asset constrained period workers only supply as much labour as is strictly necessary to cover current consumption.

Regarding assets, we observe that $\hat{a}(t) = a(t)$ because – by definition – asset-constrained agents have no assets. Using (26) we can define per capita average assets as $a(t) \equiv \int_{t-\bar{D}}^{t} p(v,t)$ $A(v,t)\, dv$, so that their rate of change is given by:

$$\dot{a}(t) = \int_{t-\bar{D}}^{t-F_b} p(v,t) \dot{A}(v,t)\, dv - \int_{t-\bar{D}}^{t-F_b} [\pi + \mu(t-v)] A(v,t)\, dv, \tag{28}$$

where we have used the fact that up until F_b agents have zero assets and that they deplete their assets at $\bar{D}$. Furthermore, the relative cohort size evolves according to $\dot{p}(v,t) = -[\pi + \mu(t-v)] p(v,t)$. Using (9) and (11) we find that:

$$\begin{aligned} \dot{a}(t) &= (r-\pi) a(t) + w(t) \hat{n} - \hat{c}(t) \\ &\quad + \int_{t-\bar{D}}^{t-F_b} p(v,t)\, TR(v,t)\, dv - (1-\theta) \int_{t-\bar{D}}^{t-F_b} p(v,t) \mu(t-v) A(v,t)\, dv. \end{aligned} \tag{29}$$

The balanced-budget requirement for the redistribution scheme is given by:

$$\int_{t-\bar{D}}^{t} p(v,t)\, TR(v,t)\, dv = (1-\theta) \int_{t-\bar{D}}^{t-F_b} p(v,t) \mu(t-v) A(v,t)\, dv. \tag{30}$$

Hence, we can rewrite (29) as:

$$\dot{a}(t) = (r-\pi) a(t) + w(t) \hat{n} - \hat{c}(t) - \int_{t-F_b}^{t} p(v,t)\, TR(v,t)\, dv. \tag{31}$$

Like in the standard case with perfect annuities, the aggregate per capita annuity receipts, $\theta \int_{t-\bar{D}}^{t-F_b} p(v,t) \mu(t-v) A(v,t)\, dv$, do not feature directly in (31) because they constitute pure transfers from the dead to the living. In each period, life insurance companies receive

$\int_{t-\bar{D}}^{t-F_b} p(v,t)\,\mu(t-v)\,A(v,t)\,dv$ from the estates of the deceased and pay $\theta\int_{t-\bar{D}}^{t-F_b} p(v,t)\,\mu(t-v)$ $A(v,t)\,dv$ to their surviving customers. The resulting profit, $(1-\theta)\int_{t-\bar{D}}^{t-F_b} p(v,t)\,\mu(t-v)$ $A(v,t)\,dv$, is taxed away by the government and redistributed to the surviving agents. A lion's share of the transfers constitute a transfer between agents that are not asset constrained and debudget from the per capita average asset accumulation equation. Some of the transfers, $\int_{t-F_b}^{t} p(v,t)\,TR(v,t)\,dv$, flow to the asset-constrained agents and, therefore, retard aggregate capital accumulation. Finally, using (14) in (30) we can write the value of the redistribution rate, z, implicitly as in (T1.5).

2.3 Balanced growth path

In the absence of government bonds, the capital market equilibrium condition is given by $A(t)=K(t)$. In per capita average terms we thus find:

$$a(t)=k(t) \tag{32}$$

where $k(t)\equiv K(t)/P(t)$ is the per capita stock of capital. From (5)–(6) we easily find:

$$y(t)=\Omega_0 k(t), \tag{33a}$$

$$w(t)\,n(t)=(1-\varepsilon_K)\,y(t), \tag{33b}$$

where $y(t)\equiv Y(t)/P(t)$ is per capita output. The expression in (33b) highlights the necessity of considering both n and $\hat{n}$ as aggregates. While n is the amount of labour that is relevant for the production process, $\hat{n}$ is the amount of labour that is relevant for the capital accumulation process (see, equation (31)).

The macroeconomic growth model has been written in compact form in Table 1. For the microeconomic part, equations (T1.1)–(T1.3) have been discussed above. The scaled asset path, (T1.4), has been derived by solving (9) and features two branches depending on the agent's life-cycle phase. Equation (T1.6) is (31) with (32) imposed and (14) and (25b) substituted in, whilst (T1.7) is (33b) with (33a) substituted in. Finally, also equations (T1.8)–(T1.10) have been discussed above.

The model features a two-way interaction between the microeconomic decisions and the macroeconomic outcomes. Equations (T1.1)–(T1.4) determine $\tilde{C}(v,v)/w(v)$, F_b, R and the life-cycle path for assets as function of the macroeconomic variables. Equations (T1.5)–

(T1.10) determine z, γ, $w(t)/k(t)$, n, $\hat{n}$ and $\hat{c}(t)/w(t)$ as a function of the microeconomic variables.

3 Parameterization

The key virtue of our model is that it allows us to pinpoint the various places where life-cycle elements affect individual choices and aggregate outcomes. The model contains two features that independently, but also jointly, give rise to life-cycle effects. First, the mortality process is age-dependent. That is, the instantaneous and cumulative hazard rates ($\mu(u)$ and $M(u)$) are both age-dependent. Second, labour productivity ($E(u)$) depends on the worker's age. In the remainder of this section we add empirical content to the model by estimating the main features of the two life-cycle processes and embedding the model in a realistic macroeconomic environment.

To capture the salient features of the demographic process we use the demographic model suggested by Boucekkine *et al.* (2002). In this model, the surviving fraction up to age u (from the perspective of birth) is given by:

$$1 - \Phi(u) \equiv \frac{\eta_0 - e^{\eta_1 u}}{\eta_0 - 1}, \tag{34}$$

with $\eta_0 > 1$ and $\eta_1 > 0$. For this demographic process, $\bar{D} = (1/\eta_1)\ln\eta_0$ is the maximum attainable age, whilst the instantaneous mortality rate at age u is given by:

$$\mu(u) \equiv \frac{\Phi'(u)}{1 - \Phi(u)} = \frac{\eta_1 e^{\eta_1 u}}{\eta_0 - e^{\eta_1 u}}. \tag{35}$$

Thus, the mortality rate is increasing in age and becomes infinite at $u = \bar{D}$.

Following Heijdra and Romp (2008), we use mortality data from age 18 onward for the Dutch cohort that was born in 1960 and estimate the parameters of the mortality function (34) by means of non-linear least squares. We find the following estimates (with t-statistics in brackets): $\hat{\eta}_0 = 122.643$ (11.14), $\hat{\eta}_1 = 0.0680$ (48.51). The standard error of the regression is $\hat{\sigma} = 0.02241$, and the implied estimate for $\bar{D}$ is 70.75 in economic years (i.e., the maximum age in biological years is 88.75). Figure 2(a) depicts the actual and fitted survival rates with, respectively, solid and dashed lines. Up to age 69, the model fits the data rather well. For higher ages the fit deteriorates as the model fails to capture the fact that some people are expected to live to very ripe old ages in reality.

Table 1: Balanced growth and retirement with age-dependent productivity and mortality

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(a) Microeconomic relationships:[†]

$$\frac{\tilde{C}(v,v)}{w(v)} \cdot \Delta = \int_{F_b}^{R} E(s)\, e^{-(r-\gamma)s-\theta M(s)} ds + z \cdot \int_{F_b}^{\bar{D}} e^{-(r-\gamma-\phi)s-\theta M(s)} ds \qquad \text{(T1.1)}$$

$$\text{with } \Delta \equiv \left[\int_{F_b}^{\bar{D}} e^{-\rho s-M(s)} ds + \frac{1-\varepsilon_C}{\varepsilon_C}\int_{F_b}^{R} e^{-\rho s-M(s)} ds\right]$$

$$\frac{\tilde{C}(v,v)}{w(v)} = \varepsilon_C \left[E(F_b) + z \cdot e^{\phi F_b}\right] e^{-(r-\rho-\gamma)F_b+(1-\theta)M(F_b)} \qquad \text{(T1.2)}$$

$$\frac{\tilde{C}(v,v)}{w(v)} = \frac{\varepsilon_C}{1-\varepsilon_C} E(R)\, e^{-(r-\rho-\gamma)R+(1-\theta)M(R)} \qquad \text{(T1.3)}$$

$$e^{-ru-\theta M(u)} \frac{A(v,v+u)}{w(v)} = \left[\int_{F_b}^{u} E(s)\, e^{-(r-\gamma)s-\theta M(s)} ds \right.$$

$$\left. -\frac{1}{\varepsilon_C}\frac{\tilde{C}(v,v)}{w(v)} \int_{F_b}^{u} e^{-\rho s-M(s)} ds + z \cdot \int_{F_b}^{u} e^{-(r-\gamma-\phi)s-\theta M(s)} ds\right] \qquad \text{(T1.4a)}$$

$$= \left[\frac{\tilde{C}(v,v)}{w(v)} \int_{u}^{\bar{D}} e^{-\rho s-M(s)} ds - z \cdot \int_{u}^{\bar{D}} e^{-(r-\gamma-\phi)s-\theta M(s)} ds\right] \qquad \text{(T1.4b)}$$

(b) Macroeconomic relationships:

$$z \cdot \int_{0}^{\bar{D}} e^{-(\pi-\phi)s-M(s)} ds = (1-\theta)\int_{F_b}^{\bar{D}} e^{-(\pi+\gamma)s-M(s)} \mu(s)\, \frac{A(v,v+s)}{w(v)} ds \qquad \text{(T1.5)}$$

$$\gamma = \frac{\dot{k}(t)}{k(t)} = r - \pi + \left[\hat{n} - \frac{\hat{c}(t)}{w(t)} - \beta z \int_{0}^{F_b} e^{(\phi-\pi)s-M(s)} ds\right] \cdot \frac{w(t)}{k(t)} \qquad \text{(T1.6)}$$

$$\frac{w(t)\, n}{k(t)} = (1-\varepsilon_K)\, \Omega_0 \qquad \text{(T1.7)}$$

$$n = \hat{n} + \beta\varepsilon_C \int_{0}^{F_b} e^{-\pi s-M(s)} E(s)\, ds - \beta(1-\varepsilon_C)\, z \int_{0}^{F_b} e^{(\phi-\pi)s-M(s)} ds \qquad \text{(T1.8)}$$

$$\hat{n} = \beta \int_{F_b}^{R} e^{-\pi s-M(s)} E(s)\, ds$$

$$-\frac{1-\varepsilon_C}{\varepsilon_C}\frac{\tilde{C}(v,v)}{w(v)} \beta \int_{F_b}^{R} e^{(r-\pi-\rho-\gamma)s-(2-\theta)M(s)} ds \qquad \text{(T1.9)}$$

$$\frac{\hat{c}(t)}{w(t)} \equiv \frac{\tilde{C}(v,v)}{w(v)} \beta \int_{F_b}^{\bar{D}} e^{(r-\pi-\rho-\gamma)s-(2-\theta)M(s)} ds \qquad \text{(T1.10)}$$

[†]The expressions (T1.4a)-(T1.4b) are valid for, respectively, $F_b \le u < R$ and $R \le u \le \bar{D}$.

(a) mortality process $1 - \Phi(u)$ (b) productivity profile $E(u)$

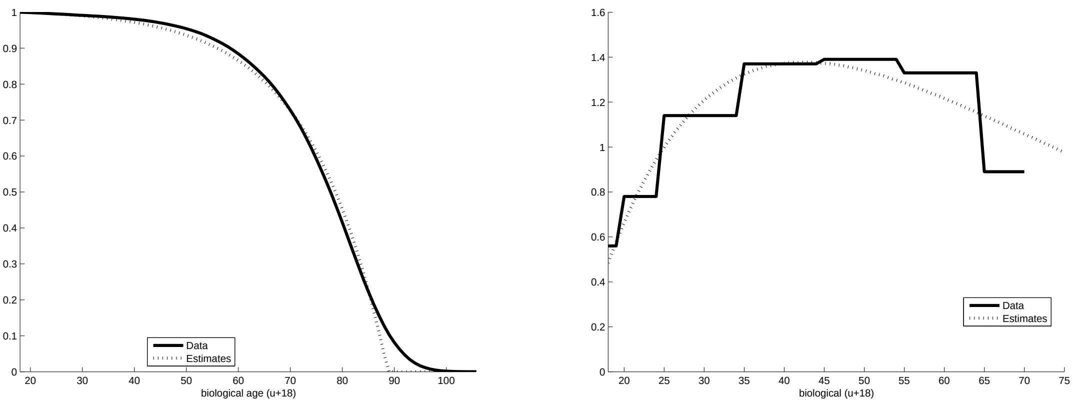

Figure 2: Life-cycle features

An analytically useful age profile for productivity involves exponential terms:

$$E(u) = \alpha_0 e^{-\zeta_0 u} - \alpha_1 e^{-\zeta_1 u}, \tag{36}$$

where $E(u)$ is labour productivity of a u-year old worker. Assuming that $\alpha_0 > \alpha_1 > 0$, $\zeta_1 > \zeta_0 > 0$, and $\alpha_1 \zeta_1 > \alpha_0 \zeta_0$, we easily find that the labour productivity profile adheres to the properties outlined above. That is, labour productivity is non-negative throughout life, starts out positive, is rising during the first life phase, and declines thereafter.

Using cross-section efficiency data for male workers aged between 18 and 70 from Hansen (1993, p. 74) we find the solid pattern in Figure 2(b). We interpolate these data by fitting equation (36) using non-linear least squares. We find the following estimates (t-statistics in brackets): $\alpha_0 = 4.494$ (fixed), $\hat{\alpha}_1 = 4.010$ (71.04), $\hat{\zeta}_0 = 0.0231$ (24.20), and $\hat{\zeta}_1 = 0.050$ (17.81). The fitted productivity profile is illustrated with dashed lines in Figure 2(b).

The remainder of the model is parameterized to replicate the macroeconomic features of the Netherlands. In this respect we take the population growth rate to be 0.5%. For the process described in (35) the demographic steady-state (25a) yields a birth rate of $\beta = 0.0204$. Since $\bar{\mu} \equiv \beta - \pi$, this implies that the average mortality rate is $\bar{\mu} = 0.0154$. Capital receives roughly one third of aggregate output, therefore, $\varepsilon_K = \frac{1}{3}$, the long-run real interest rate is 4% per year ($r = 0.04$) and capital depreciates at a rate of 6% per year ($\delta = 0.06$). Using (7) the combination of ε_K, r and δ imply an aggregate productivity index equal to $\Omega_0 = 0.3$. The

Table 2: Parameter values in the core model

Description	Parameter	Value
Crude birth rate	β	0.0204
Aggregate mortality rate	$\bar{\mu}$	0.0154
Population growth rate	$\pi \equiv \beta - \bar{\mu}$	0.0050
Rate of interest	r	0.0400
Pure rate of time preference	ρ	0.0231
Capital depreciation rate	δ	0.0600
Capital share parameter	ε_K	0.3333
Consumption taste parameter	ε_C	0.0733
Production function constant	Ω_0	0.3000

steady-state growth rate of the economy is 2% per year ($\gamma = 0.02$). We use the time-preference rate ρ and the consumption taste, ε_C, as calibration parameters to give an implied economic retirement age, R, of 47, which implies a biological retirement age of 65, the eligibility age for social security in the Netherlands. All the parameter values are summarized in Table 2 for convenience.

For the *core* model we assume that annuities are perfect (PA, with $\theta = 1$) and illustrate the optimal life-cycle choices in Figure 3. Figure 3(a) shows the life-cycle pattern of consumption. The dashed line, labeled KCF can be used to illustrate the implications of the asset-constraint over the life-cycle. KCF is the "Keynesian consumption function" faced by the agent at the start of life resulting from the binding borrowing constraint in combination with active labour market participation. Mathematically, KCF is given by (17a) above. Up until age $F_b = 18.5$ the optimal consumption path coincides with KCF but thereafter the paths diverge because the agent becomes a net saver. A disturbing feature of the consumption profile is that it predicts that consumption will increase indefinitely over the life-cycle whereas in reality consumption is hump shaped (Alessie and de Ree, 2009; Gourinchas and Parker, 2002; Fernández-Villaverde and Krueger, 2007).. However, the discussion surrounding (22) above and the numerical simulation below reveal that the introduction of imperfect annuities can easily remedy this caveat.

Figure 3(b) shows the age pattern of labour supply. Labour market entry is immediate and due to the absence of transfers labour supply remains constant until the agent reaches age F_b (see the discussion surrounding (17b) above). Beyond F_b labour supply follows a hump-shaped path, reaching its maximum at age $u_L = 27.9$ and retirement setting in at age $R = 47$ as calibrated. The labour supply profile is broadly consistent with the United States evidence presented by McGrattan and Rogerson (2004).

Figure 3(c) depicts the scaled asset profile. Naturally, until age F_b assets are zero because the agents are asset constrained. In fact, agents would like to borrow at this stage but are barred from doing so by the asset constraint. After F_b assets follow a typical life-cycle pattern (see, Huggett, 1996), reaching their peak at $u_A = 44.7$ (just ahead of retirement) and dropping thereafter so that they run out exactly at the maximum attainable life-time, $\bar{D}$.

Finally, Figure 3(d) shows the combined consumption and labour supply choices made over the life-cycle with arrows giving the direction through time. This figure provides the calibrated equivalent to the discussion surrounding Figure 1 above. Segment A through B gives the choice during the asset constrained period. After that, labour supply increases rapidly until point C from whereon it declines. Retirement occurs at D where the agent consumes the full time endowment in the form of leisure.

Table 3(a) summarizes the main features of the core-model. With perfect annuities, there are no transfers because annuity firms are breaking even ($z = 0$). The macroeconomic growth rate is two percent as calibrated.

4 The individual life-cycle, growth and annuities

In this section we consider the general equilibrium impact of imperfect annuity markets. Instead of setting $\theta = 1$, we simulate the model with a value of $\theta = 0.7$. This degree of imperfection of the annuity market follows from the empirical analysis of Friedman and Warshawsky (1988, p. 59) who estimate a load factor of 48 cents per dollar of expected present value. They suggest that 15 cents of this amount may be due to adverse selection and the remaining 33 cents due to costs, taxes, and profit. We assume that the profits made by the annuity firms are redistributed equally to all agents (denoted by **TA**). In terms of (14) this means that $\phi = 0$. In the sensitivity analysis in subsection 4.1 we consider different degrees of

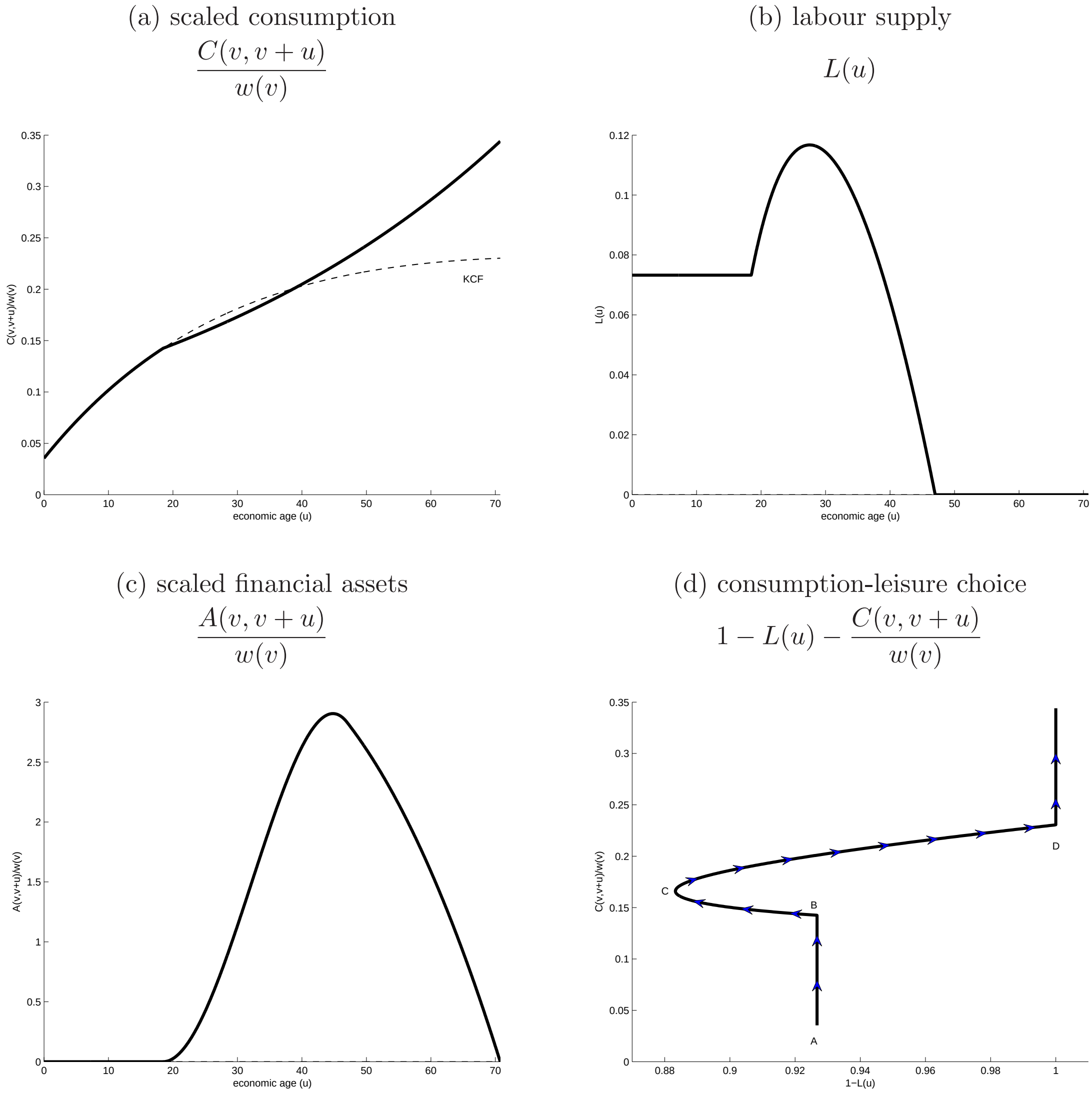

Figure 3: Optimal life-cycle plans with perfect annuities

Table 3: Growth and retirement: quantitative effects

	(a) PA	(b) TA	(c) TY	(d) TO	(e) WE	(f) TA ($\theta = \frac{1}{2}$)
$\frac{\tilde{C}(v,v)}{w(v)}$	0.1044	0.1009	0.1010	0.1006	0.0856	0.0985
$F_b + 18$ (years)	36.48	35.87	35.91	35.81	31.01	35.40
$R + 18$ (years)	65	66.25	66.44	66.02	57.13	68.06
z (g)	0	0.0019	0.0026	0.0012	(0.0025)	0.0029
γ ($\times 100\%$)	2.00	1.81	1.82	1.80	1.22	1.68
$\frac{w(t)}{k(t)}$	2.4449	2.4956	2.4955	2.4956	2.5518	2.5268
n	0.0818	0.0801	0.0801	0.0801	0.0784	0.0792
$\hat{n}$	0.0523	0.0524	0.0525	0.0523	0.0591	0.0526
$\frac{\hat{c}(t)}{k(t)}$	0.0564	0.0565	0.0564	0.0566	0.0686	0.0568

annuity market imperfection and different assumptions regarding the redistribution of profits.

The quantitative impact of the annuity market imperfection is visualized in Table 3(b) and Figure 4. Figure 4(a) shows the consequences of the annuity market imperfection for individual life-cycle consumption. The solid line gives the perfect annuity market equilibrium as a benchmark and the dashed line shows the imperfect annuity market equilibrium. In line with the discussion surrounding (22) we find that consumption now becomes hump-shaped over the life-cycle. This implies that by introducing an annuity market imperfection the model is able to yield a more realistic consumption profile.

Figure 4(b) exhibits the consequences of imperfect annuities for labour supply. As before, the solid line is the perfect annuities equilibrium and the dashed line is the imperfect annuities equilibrium. As can be seen, labour supply is now slightly increasing during the asset constrained regime because relative transfers are decreasing. Over the remainder of the life-cycle, labour supply is somewhat less than before but retirement is now marginally later. The asset constraint, on the other hand, is binding for a shorter length of time. Below, in Table 4, we study the effects of the imperfection on retirement and the asset constraint in more detail.

Figure 4(c) maps out the asset path. Asset accumulation starts slightly earlier and over the life-cycle agents accumulate less capital. The decrease in capital accumulation can be traced to two factors. First, less assets need to be accumulated for consumption in old age because agents now have a hump-shaped consumption profile. In addition, the individual agent's income declines due to the decrease in the return on savings and the decline in the growth rate of output and wages (see below).

Finally, 4(d) traces the consumption-leisure choice over the life-cycle. For the sake of clarity we only show the equilibrium path from the imperfect annuity equilibrium. Along the trajectory between A and B labour supply increases slightly. From point B to point D (i.e. from F_b to R) the agent behaves roughly as in the perfect annuity equilibrium. However, after point D consumption now increases for a little while longer and reaches its maximum at E after which consumption drops.

In Table 3(b) we also report the macroeconomic consequences of the annuity market imperfection. Naturally, due to the profits made by annuity firm, the redistribution parameter now becomes positive. As a consequence of the decrease in capital accumulation over the life-cycle we find that the growth rate of output drops by 19 basis points. Aggregate labour supply drops slightly because, although agents retire later, they supply less labour during their active life-cycle. Also we can see that the main impact on labour supply comes from changes in the labour supply of the asset-constrained agents. That is, while $\hat{n}$ is nearly constant n changes substantially. Finally, both the consumption-capital and wage-capital ratios remain roughly constant.

In order to gain a better understanding of the different forces acting upon the individual decisions we present a decomposition analysis in Table 4. In that table, columns (a) and (b) present the general equilibrium outcomes with, respectively, perfect and imperfect annuities. These outcomes coincide with columns (a) and (b) in Table 3. In Table 4(b) we consider the pure partial equilibrium effect of the annuity market imperfection by keeping both z and γ fixed (at their perfect annuities level) but setting $\theta = 0.7$ and solving (T1.1)–(T1.3). In Table 4(c) we add the impact of the general equilibrium change in γ, whilst in Table 4(d) we add the impact of the general equilibrium change in z.

In Table 4(b) we see that the pure partial equilibrium effect of an imperfect annuity market

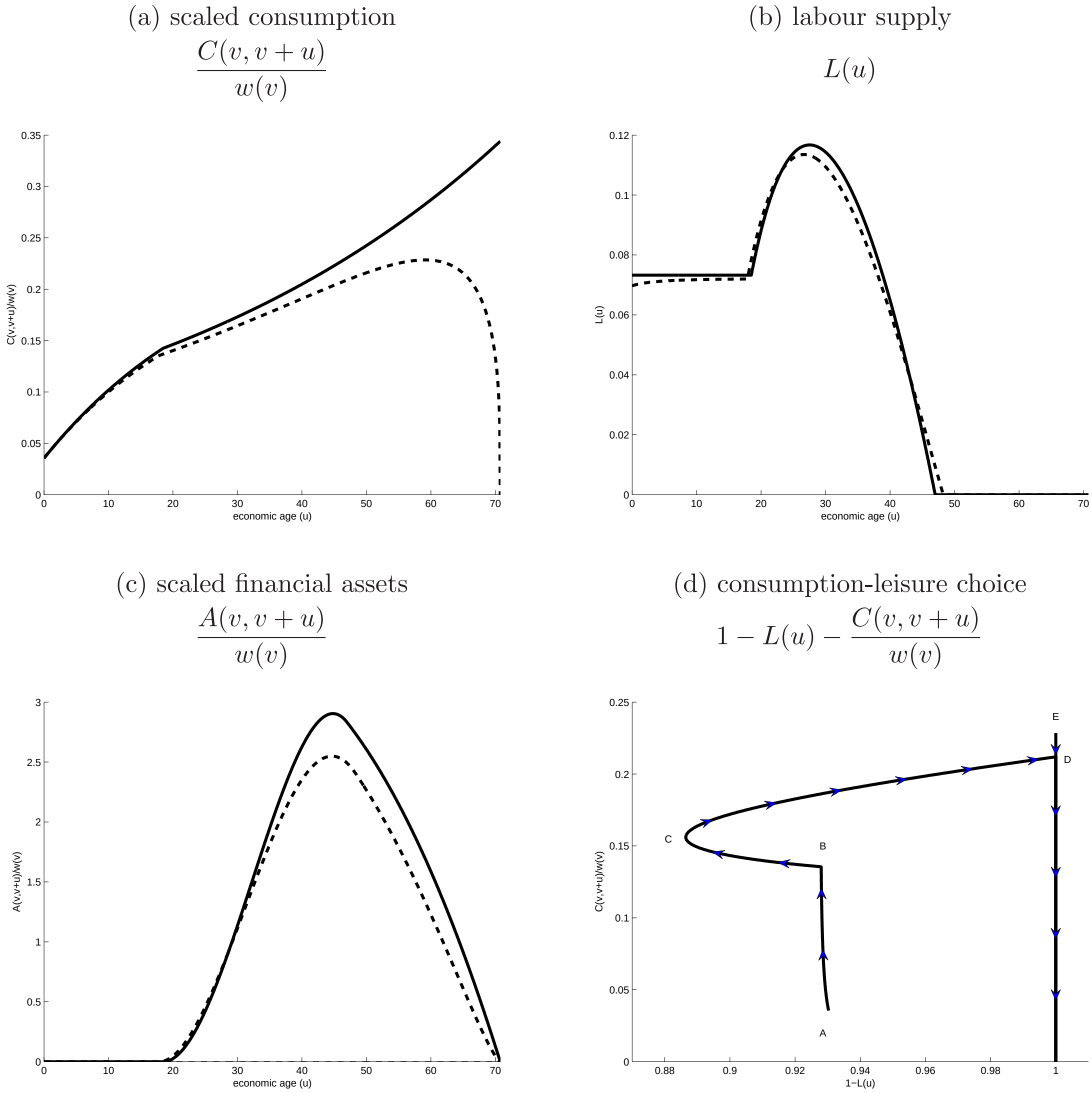

Figure 4: Optimal life-cycle plans with imperfect annuities

Table 4: Decomposing the general equilibrium effects

	(a) *PA (GE)*	(b) *IA (PE)*	(c) *IA (PE+γ)*	(d) *IA (PE+z)*	(e) *IA (GE)*
$\frac{\tilde{C}(v,v)}{w(v)}$	0.1044	0.1070	0.1007	0.1072	0.1009
$F_b + 18$	36.48	38.17	35.83	38.20	35.87
$R + 18$	65	74.14	66.49	73.22	66.25

is very substantial. Initial consumption is much higher than before and the asset constraint binds for a longer period of time. Most dramatically, however, retirement now occurs much later. Going to 4(c) reveals that the strong effects of the annuity market imperfection are actually largely counteracted by the change in the economy-wide growth rate. That is, of the 9 year increase in retirement due the annuity market imperfection only 1.5 are left if proper account is taken of the change in the output growth rate. The entries in 4(d) show that not taking into account the redistribution of annuity profits in itself already leads to an overestimation of the impact on retirement of 10%. Finally, from 4(e) we learn that the general equilibrium impact of annuity market imperfections on individual choices are much less pronounced than a partial equilibrium impact would lead us to believe. This exercise allows us to draw the conclusion that as much as a macroeconomics needs to be microfounded, microeconomics needs to be macrofounded.

4.1 Sensitivity analysis

In this section we briefly reflect on the robustness of our analysis to alternative assumptions concerning the size of the annuity market imperfection and the way that the profits of the annuity firms are redistributed. As far as redistribution is concerned, we study what happens if the profits are distributed with a skew toward either the young or the old. Alternatively, we consider what happens if the profits are wasted by the government. Regarding the magnitude of the imperfection, we consider what happens if the annuity market suffers from very heavy imperfections by setting $\theta = \frac{1}{2}$.

In Table 3(c) and 3(d) we consider the impact of redistributing the profits of the annuity firms to young agents (denoted by **TY**) and to old agents (**TO**), respectively.[3] Although the effects are mild, a clear pattern arises. If the profits are distributed with a skew to the young we observe a positive growth impact while when the profits are distributed to the old we observe a negative growth effect.

To understand these effects it is necessary to observe two facts. First, the middle-aged and elderly contribute substantially more to the profits of the annuity firms because both their asset holdings $(A(v, v+u)/w(v))$ and their instantaneous probability of death $(\mu(u))$ are much higher. Second, apart from those that are asset constrained, young agents are accumulators of capital whereas old agents are decumulators of capital. In concert, these two facts assure that the TY scenario channels funds from the dissaving old to the saving young. Hereby, the capital accumulation rate increases and, thus, output growth. In contrast, the TO scenario channels funds in the opposite direction and, thereby, decreases the output growth rate.

In line with the observations around (21) we find that the age until which the asset constraint binds is increased slightly under the TY scenario whereas it is decreased somewhat under the TO scenario. Effectively, by giving the transfers to the young agents they are receiving additional funds which decreases the desire to step out of the borrowing constrained regime. As before, giving the transfers to the old has the opposite effect.

In keeping with the analysis of the redistribution of profits we next analyze what happens if the profits are used for wasteful expenditures (denoted by **WE**). In that case the aggregate capital accumulation equation (T1.6) becomes:

$$\frac{\dot{k}(t)}{k(t)} = \gamma = r - \pi + \left[\hat{n} - \frac{\hat{c}(t)}{w(t)}\right]\frac{w(t)}{k(t)} - \frac{g(t)}{k(t)}, \tag{37}$$

where $g(t)$ stands for government expenditures and is determined by the balanced-budget requirement as:

$$g(t) = (1-\theta)\int_{F_b}^{\bar{D}} e^{-(\pi+\gamma)s - M(s)}\mu(s)\frac{A(v, v+s)}{w(v)}ds. \tag{38}$$

From the individual perspective, the WE scenario implies that the redistribution parameter z is zero even if the annuity market is imperfect.

[3]We construct the two redistribution scenarios by setting ϕ in (14) equal to $-1/\bar{D}$ for the TY scenario and equal to $1/\bar{D}$ for the TO scenario.

We study the effects of the WE scenario in Table 3(e). It is immediately clear both the macroeconomic and microeconomic implications of annuity market imperfections are much more pronounced. For instance, in comparison to the TA scenario the growth rate of output now drops by 78 basis points instead of 19. Naturally, this is a direct consequence of the fact that productive assets are now being drained from the economy. The growth effect is now so large that agents will actually retire 7 years earlier instead of a year later. This larger effect is best understood by focusing on Table 4. There we see that, in isolation, the retirement age is negatively affected by θ and positively affected by γ. Faced with imperfect annuity markets both θ and γ decrease so that the total impact on the retirement age is determined by the balance of these two effects. If the profits of the annuity firms are redistributed, the impact of θ on retirement is always larger than that of γ (see Table 3(b)-(d)). If, however, the proceeds are wasted the impact of γ is larger than that of θ (see Table 3(e)). Similar reasoning can be used to reconcile the impact on F_b.

As a final exercise, in Table 3(f) we study the impact of having an extremely imperfect annuity market by setting $\theta = \frac{1}{2}$. By and large this exercise, but also unreported simulations for other values of θ, shows that the impact of the annuity market imperfections is monotonic. That is, it is only the magnitude of the effects that changes, not the sign.

5 Conclusion

We study the impact of imperfect annuity markets on the individual life-cycle and macroeconomic outcomes. On the individual level we find that imperfect annuities lead to a hump-shaped consumption profile which brings the predictions of the model closer to the empirically observed consumption profiles (Alessie and de Ree, 2009; Gourinchas and Parker, 2002; Fernández-Villaverde and Krueger, 2007). Regarding labour supply we find that agents retire slightly later but supply less hours over their active career. The hump-shaped consumption profile reduces the need for retirement savings and, thereby, decreases capital accumulation. From an aggregate perspective the decrease in capital accumulation leads to a reduction in economic growth. In addition, we find that the mode of redistributing the profits has important implications at both the individual and the aggregate level. Finally, in terms of both sign and magnitude, we find that a partial equilibrium analysis grossly overestimates the impact of annuity market imperfections.

The focus of the current paper has been on the development of a model that allows us to study the implications of annuity market imperfections on both the individual life-cycle and the macroeconomic environment. The model is, however, much more versatile than the current paper would lead one to conclude. Indeed, in Heijdra and Mierau (2010, 2011) we have used alternative versions of this model to study important public finance issues such as the different impact of consumption and labour taxes and the moderating effect of a public pension system in the wake of a mortality shock. In future work we will introduce an active human-capital accumulation phase into the model and study the transitional dynamics of various policy measures.

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