Human capital formation and macroeconomic performance in an ageing small open economy: Further Results

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Abstract

This paper contains some supplementary material on B. J. Heijdra and W. E. Romp (2008). First, we show the details on how we estimate the Gompertz-Makeham process from a Lee-Carter model for the Netherlands. Second, we show the macroeconomic effects of fiscal and demographic shocks for the endogenous growth version of the model. Though unrealistic, this knife-edge case has been used a lot throughout the literature. Third, we demonstrate how the optimal schooling decision is affected by mortality in the presence of imperfect annuity markets.

JEL codes: E10, D91, O40, F41, J11.

Keywords: demography, education, human capital, economic growth, fertility rate, ageing, overlapping generations, small open economy.

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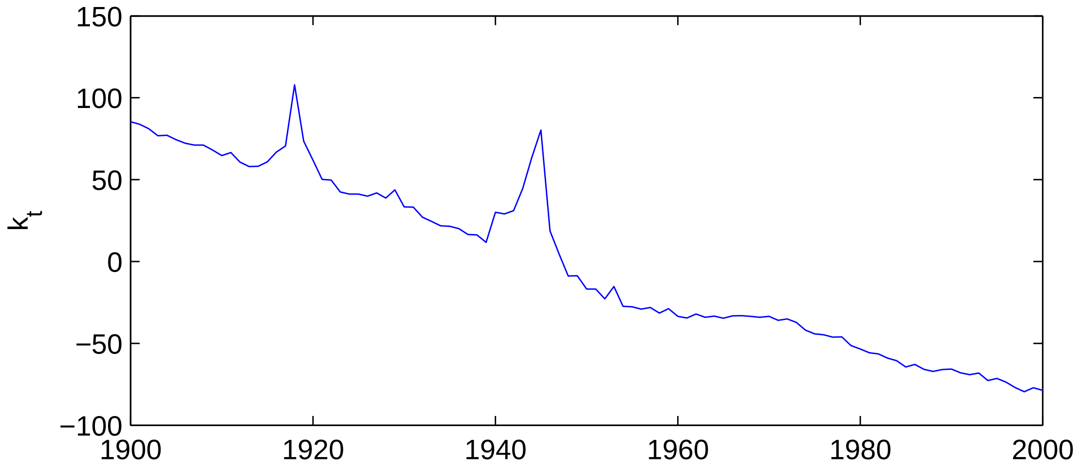
Figure F.1: Lee-Carter mortality index for the Netherlands



F.1 Estimating the Gompertz-Makeham process

Our simulations require cohort life tables, which are only available (more or less complete) for pre-1918 cohorts. Life tables for more recent cohorts are incomplete for obvious reasons. Using published period life tables to estimate cohort mortality profiles leads to an overestimation of mortality because of the well documented downward trend in mortality. To solve this we use a two-stage procedure in this paper. In the first step we estimate the Lee-Carter model for the period 1920–2000. Following Lee and Carter (1992), we estimate the logarithm of the period mortality rates using the singular value decomposition:

$$\ln m^{p}_{u,t} = a_u + k_t b_u + \varepsilon_{u,t}, \tag{F.1}$$

where $m^{p}_{u,t}$ is the period mortality rate, i.e. the probability that an individual of age u will die this year, given that he has survived until time t, a_u and b_u are age-varying vectors and k_t is a time-varying vector. The relative size of the first eigenvalue in the singular value decomposition confirms the Lee-Carter (1992) conclusion that a one-factor model is sufficient and a plot of the Lee-Carter mortality index[1] (see Figure F.1) also confirms that for the Netherlands an AR(1) process with trend, $k_t = k_{t-1} + \gamma_0 + \eta_t$ with γ_0 the trend and η_t an error term, properly describes the behaviour of the Lee-Carter mortality index k_t (more formal test statistics are available upon request).

From the period life table estimates and the trend of the mortality index we calculate the cohort life tables as follows:

$$\ln m^{c}_{u,v} = a_u + (k_v + \gamma_0 u) b_u, \tag{F.2}$$

[1] As Lee and Carter (1992) note, the parameters of their model are not uniquely identified, we use their proposed restrictions, so k_t add up to zero.

where v is the year of birth and γ_0 is the trend in k_t.[2]

In principle, we could use a continuous time version of these discrete time 'expected' cohort mortality rates directly in the paper to calculate the mortality process and the mortality-rate-adjusted function $\Delta(\cdot)$. The main problem with this approach is that the Lee-Carter model does not restrict the age mortality profiles a_u and b_u and we would have one parameter for each age-class. For computational reasons we need a more parsimonious specification of the mortality process. Moreover, there is no explicit function for the corresponding $\Delta(\cdot)$-function. To solve this we approximate the Lee-Carter cohort mortality rates for the *relevant* ages (16–80 years, we exclude youth and very old age mortality) with a Gompertz-Makeham mortality process. As is well known in the literature (see e.g. Lee & Carter, 1992), the GM mortality process is not capable of capturing mortality at low ages and very high ages. For low ages observed mortality is decreasing and GM gives an underestimate, whilst for very high ages mortality seems to increase at a lower than exponential rate and GM gives an overestimation. Since the GM model is not intended to capture child mortality and mortality at very high age, estimating the GM parameters without 'cleaning' the data first will lead to errors. Fortunately, we are only interested mortality rates in the range of 16–80, the range relevant for most of our economic decisions.

Specifically, we calculate the non-linear-least-squares estimates of the cohort specific GM parameters. The model we estimate for each year is:[3]

$$m^c_{u,v} = 1 - e^{-m(u,v)} + \epsilon_{u,v}, \quad \text{with} \quad m(u,v) = \mu_0(v) + \mu_1(v)e^{\mu_2(v)u} \tag{F.3}$$

Figure F.5 shows the estimates for μ_0, μ_1 and μ_2 for the cohort 1920 to 2000, excluding the war years.[4]

The first panel of Figure F.2 clearly shows the well documented downward trend in μ_0. This shows that for all ages the constant probability of dying has nearly vanished over time for the 16-80 years old domain. The second and third panels show the path of μ_1 (the multiplicative term) and μ_2 (the exponential term). μ_1 is clearly downward sloping, whereas μ_2 is constant at first and upward sloping from the 1950s. This signals that for ages where the multiplicative term matters most mortality decreases, but later in life the exponential term dominates and mortality increases. For the observed pattern of the GM parameters our assumption of $m(u, v+s) < m_(u,v)$ obviously do not hold for all ages between all cohorts. However, for any v and s combination within our domain, the *growth* of $m_{u,v+s}$ only

[2]This procedure assumes that an individual born in year v assumes that the observed k_v is the actual or proper mortality index. This is a rather strong assumption for clear outliers (e.g. the war years). Fortunately, this is not an issue for the cohorts of interest.

[3]Note that we approximate the discrete time hazard rate $m_u = \frac{\Phi(u+1)-\Phi(u)}{1-\Phi(u)}$ by $1 - e^{-m(u)}$ instead of $m(u)$. The latter amplifies approximation errors, whereas the first gives a very close approximation.

[4]The observed Lee-Carter mortality index k_t during these years is clearly an outlier and blur the picture, see Figure F.1.

dominates $m_{u,v}$ after ages well beyond the age of 100 and the *level* only after 110. These age domains are not relevant from a macroeconomic viewpoint.

As we point out in footnote 24, the μ_2 effect may dominate for very high ages. (Of course, the G-M process is not suited to describe such mega-old people.) Figures F.3 and F.4 show remaining lifetimes. Figure F.3 shows the average of males and females as given by Statistics Netherlands (CBS), Figure F.4 shows the expected remaining lifetime calculated using the procedure used in the paper. We only show ages 50, 60, 70, 80 and 90 to avoid that younger ages suppress the axes too much (the profiles for younger ages are comparable to the profile of age 50, but steps of 10 years higher). There is one striking difference between the two graphs: our estimates have (more or less) the same gradient, but are structurally higher. This difference is caused by the fact that Statistics Netherlands uses period life tables for their estimates and we use estimated cohort life tables. Period life tables do not take lower mortality over time into account, hence expected remaining lifetimes based on them are lower. For our purposes we prefer cohort life tables.

Figure F.3 clearly shows that the gradient of expected remaining lifetimes diminishes as one gets older. Our estimates also have this feature due to the higher values of μ_2. The μ_2 parameter affects mortality rates exponentially and only kicks in at high age. Before that G-M mortality is mostly determined by the constant term μ_0 and the linear term μ_1. The upward trend of μ_2 will only dominate the downward trends of μ_0 and μ_1 after the age of 110.

Figure F.5 shows the long term forecasts of the Gompertz-Makeham parameters. It shows the same estimates as Figure F.2, but this time with extrapolated data until 2050. We admit that this extrapolation is highly questionable (it involves estimation of the Lee-Carter Mortality index well beyond 2050), but it does give a nice picture. One striking feature is that μ_2 will stabilize and even decline after 2015. According to our estimates, μ_2 will not continue to increase until it also dominates the downward trends of μ_0 and μ_1.

F.2 Endogenous growth

In the main body of the paper we have restricted attention to the case for which the intergenerational knowledge externality is relatively weak (i.e. $0 \leq \phi < 1$) and the system reaches a steady state in terms of per capita levels. In this paper we study the knife-edge case for which the intergenerational knowledge transfer is very strong and subject to constant returns ($\phi = 1$). It has been studied extensively in the literature, despite a lack of empirical support for this knife-edge case. See, among others, Azariadis and Drazen (1990), de la Croix and Licandro (1999), Boucekkine *et al.* (2002), Echevarría (2004), and Echevarría and Iza (2006).

Figure F.2: Gompertz-Makeham parameters based on Lee-Carter cohort table estimates (excluding war-year outliers)

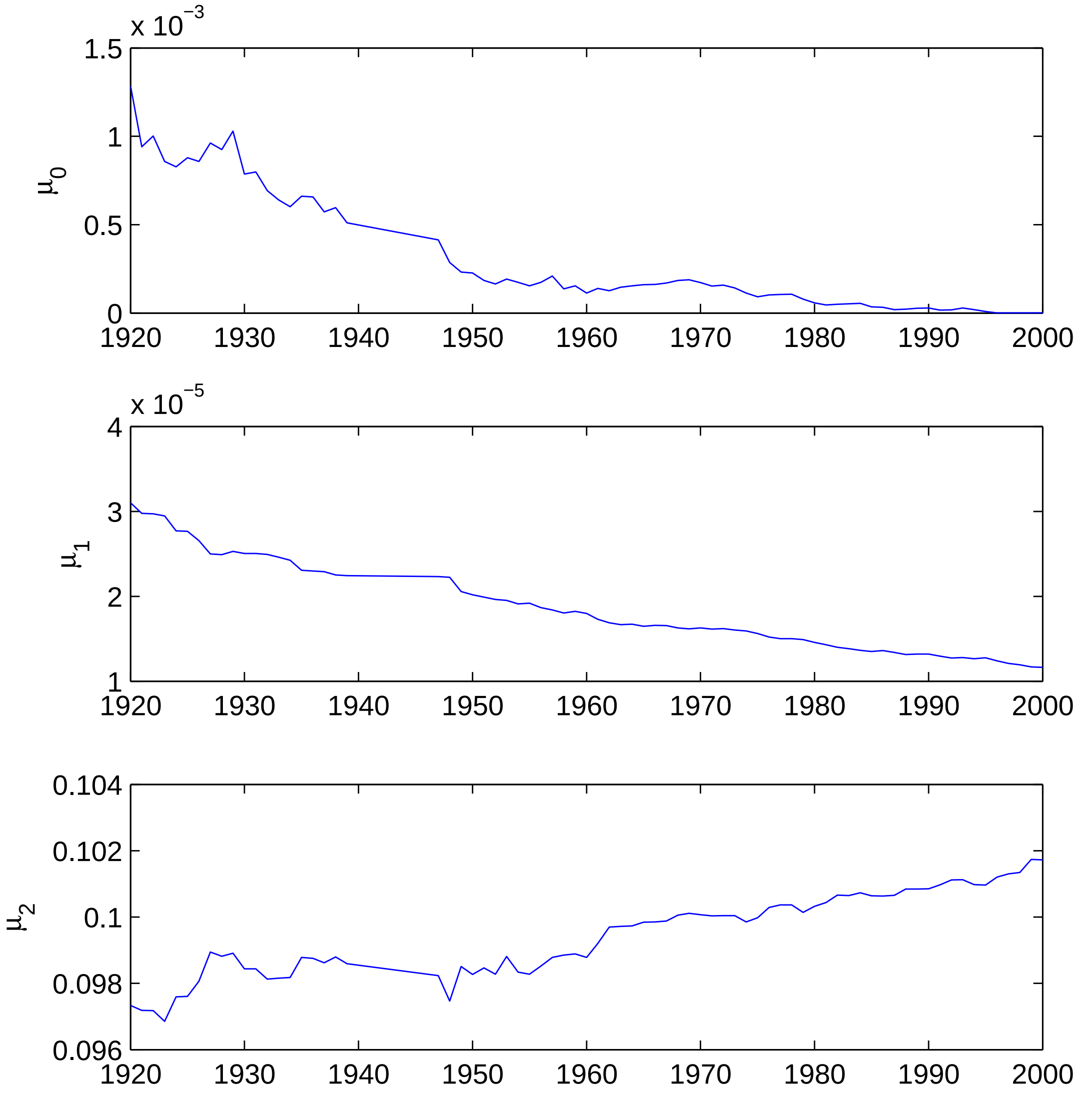


Figure F.3: Expected remaining lifetimes according to Statistics Netherlands

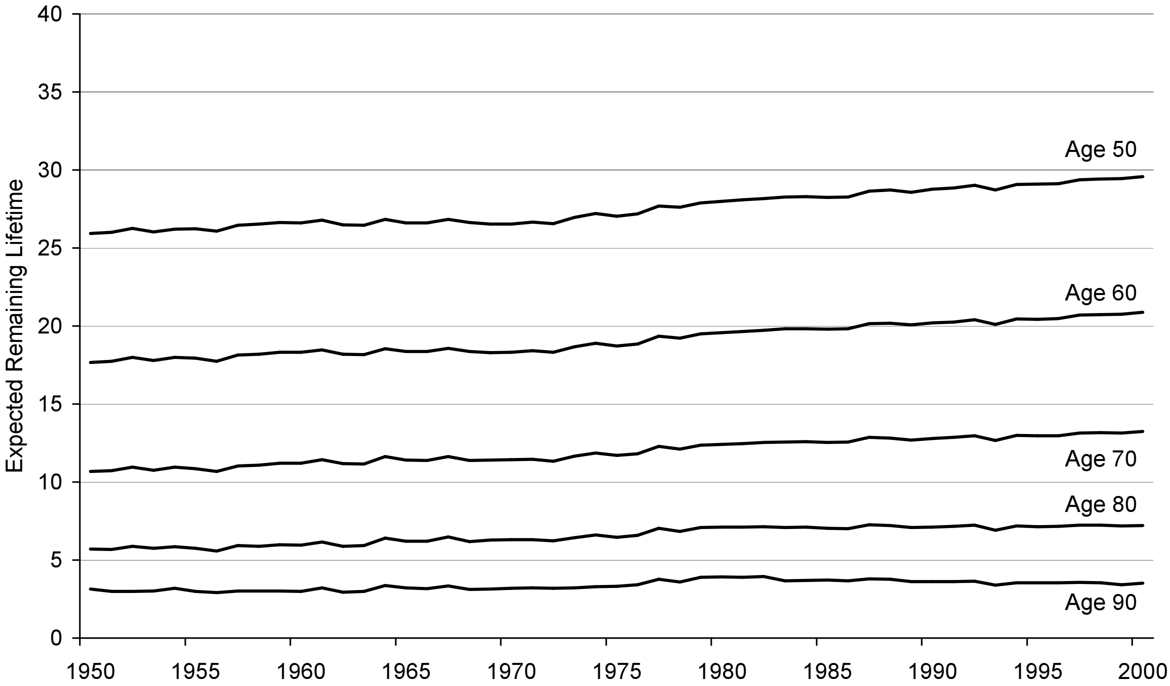


Figure F.4: Expected remaining lifetimes according to own estimates

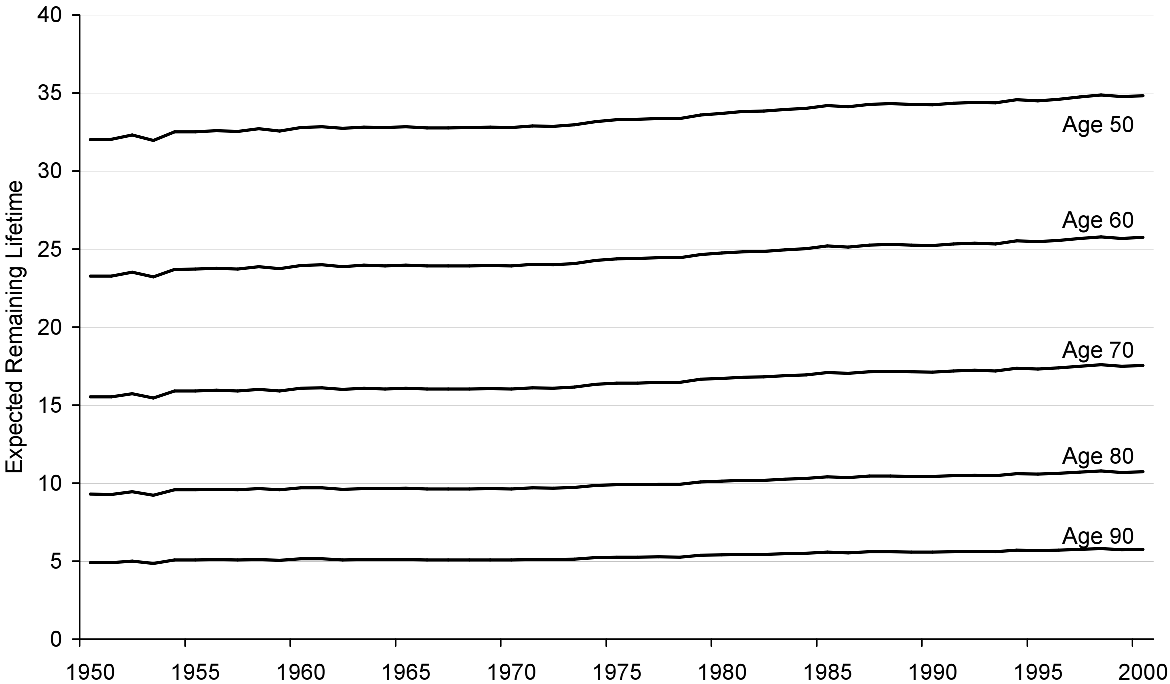
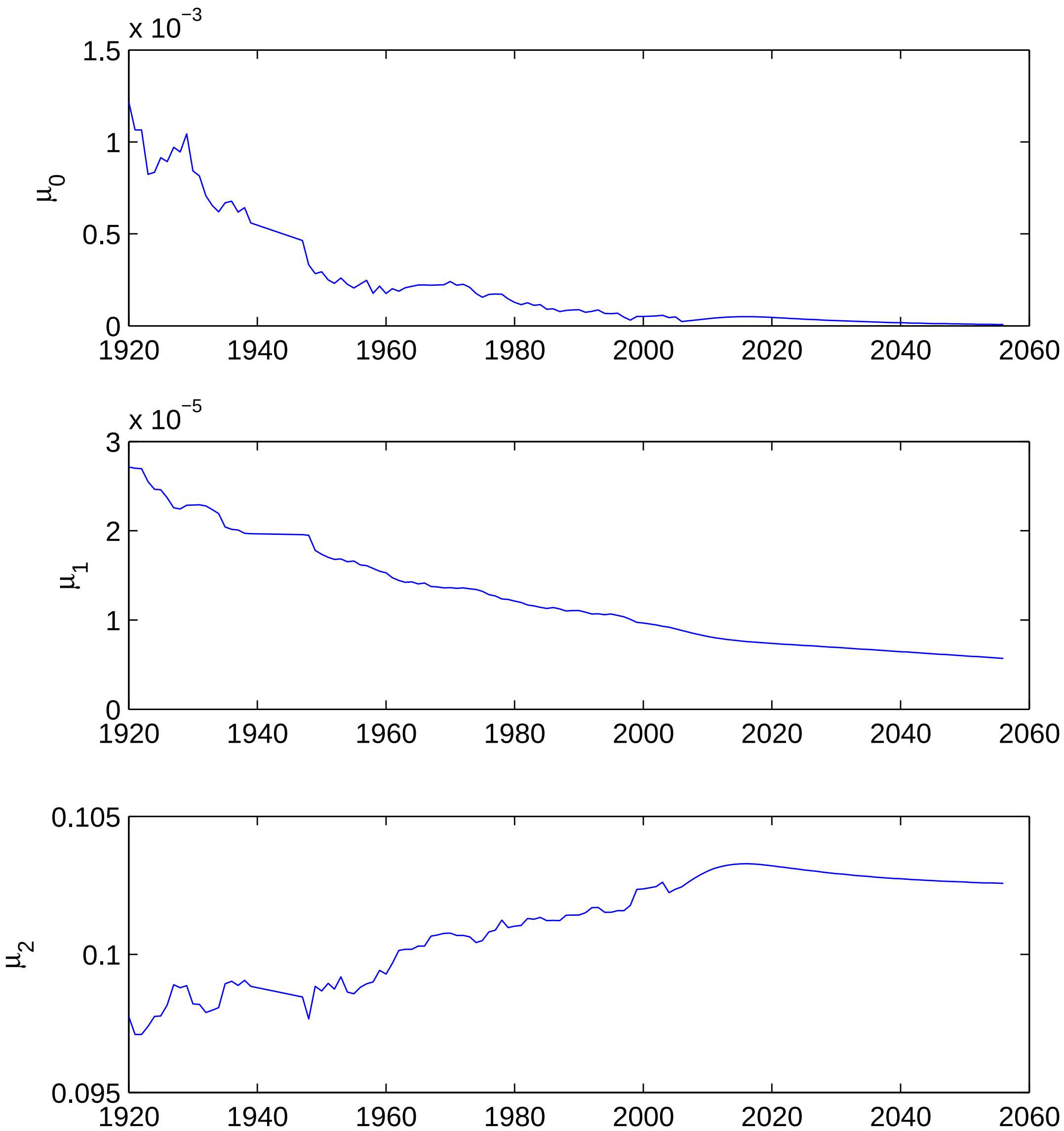


Figure F.5: Gompertz-Makeham parameters based on Lee-Carter cohort table estimates (excluding war-year outliers)



F.2.1 Long-run effects

The steady-state growth path for per capita human capital can be written as follows:

$$\begin{aligned}\hat{h}(t) &= \int_{-\infty}^{t-e^*} l(t-v)\hat{\bar{h}}(t-v)dv \\ &= bA_H e^* \int_{-\infty}^{t-e^*} \mathrm{e}^{-[\hat{n}(t-v)+M(t-v,\psi)]}\,\hat{h}(v)dv, \end{aligned} \tag{F.4}$$

where we have used (6) and (18) to arrive at the second expression. As Romp (2007, pp. 91-94) shows, the steady-state growth path features the following key properties: (i) there is a unique steady-state growth rate of per capita variables, and (ii) all per capita variables feature uniform convergence to their respective steady-state growth path.

Denoting the steady-state growth rate by $\hat{\gamma}$, it follows that along the balanced growth path we have $\hat{h}\,(v) = \hat{h}\,(t)\,\mathrm{e}^{-\hat{\gamma}(t-v)}$. By using this result in (F.4) and simplifying we obtain the implicit definition for $\hat{\gamma}$:

$$1 = bA_H e^* \int_{e^*}^{\infty} \mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du. \tag{F.5}$$

Clearly, the model implies a *scale effect* in the growth process, i.e. a productivity improvement in the human capital production function gives rise to an increase in the steady-state growth rate ($\partial\hat{\gamma}/\partial A_H > 0$). Equation (F.5) can also be used to compute the effect on the asymptotic growth rate of the fiscal and demographic shocks.

Pure schooling shock Just as in Subsection 5.1 in the paper, the interpretation of our results is facilitated by first considering a pure schooling shock. By differentiating (F.5) with respect to $\hat{\gamma}$ and e^*, and gathering terms we find:

$$\begin{aligned}\frac{\partial\hat{\gamma}}{\partial e^*} &= \frac{\mathrm{e}^{-[(\hat{\gamma}+\hat{n})e^*+M(e^*,\psi)]}\cdot\left[\Delta\left(e^*,\hat{\gamma}+\hat{n}\right)-e^*\right]}{e^*\int_{e^*}^{\infty} u\,\mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du} \\ &= \frac{\mathrm{e}^{-[(\hat{\gamma}+\hat{n})e^*+M(e^*,\psi)]}}{e^*\int_{e^*}^{\infty} u\,\mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du}\cdot\left[\Delta\left(e^*,\hat{\gamma}+\hat{n}\right)-\Delta\left(e^*,r\right)-\frac{s_E}{1-t_L}\right] > 0, \end{aligned} \tag{F.6}$$

where we have used equation (13) to arrive at the final expression. The sign of $\partial\hat{\gamma}/\partial e^*$ is determined by the term in square brackets on the right-hand side of (F.6). By appealing to the endogenous-growth counterpart of Assumption 2 (with $\hat{n}$ replaced by $\hat{n}+\hat{\gamma}$) we find that the steady-state growth rate increases as a result of the pure schooling shock.

Fiscal shock An increase in the educational subsidy or the labour income tax affects the steady-state growth rate via its positive effect on the schooling period. Indeed, we deduce from (38)-(39) and (F.6) that:

$$\frac{\partial\hat{\gamma}}{\partial\left[s_E/\left(1-t_L\right)\right]} = \frac{\partial\hat{\gamma}}{\partial e^*}\cdot\frac{\partial e^*}{\partial\left[s_E/\left(1-t_L\right)\right]} > 0. \tag{F.7}$$

Birth rate shock The growth effects of a birth rate change are computed most readily by restating the shock in terms of the steady-state population growth rate, $\hat{n}$, and noting the monotonic relationship between $\hat{n}$ and b stated in (34) in the paper. Indeed, by substituting the steady-state version of (19) into (F.5) we find an alternative implicit expression for $\hat{\gamma}$:

$$\int_0^\infty \mathrm{e}^{-[\hat{n}u+M(u,\psi)]}\,du = A_H e^* \int_{e^*}^\infty \mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du. \tag{F.8}$$

Since the birth rate shock leaves the schooling period unchanged, it follows from (F.8) that:

$$\frac{\partial\hat{\gamma}}{\partial b} = \frac{\partial\hat{\gamma}}{\partial\hat{n}}\cdot\frac{\partial\hat{n}}{\partial b} = \frac{\partial\hat{n}}{\partial b}\cdot\left[\frac{\int_0^\infty u\,\mathrm{e}^{-[\hat{n}u+M(u,\psi)]}\,du}{A_H e^*\int_{e^*}^\infty u\,\mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du} - 1\right] \gtreqless 0. \tag{F.9}$$

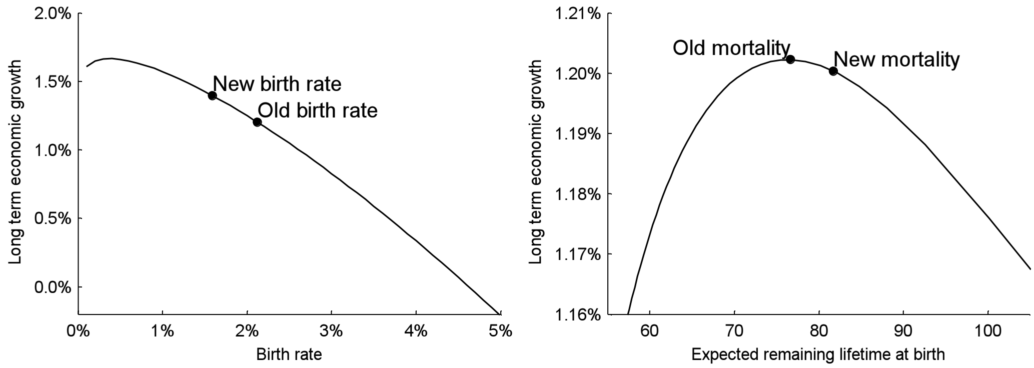
Despite the fact that $\partial\hat{n}/\partial b > 0$, the growth effect of a birth rate change is ambiguously because the term in square brackets on the right-hand side of (F.9) cannot be signed a priori. Indeed, using the calibrated version of the model, we find that the relationship between $\hat{\gamma}$ and b is hump-shaped. As is illustrated in Figure F.2.1(a), the growth rate rises with the birth rate for low birth rates, but is decreasing for higher birth rates. For the calibrated model, the maximum growth rate of 1.67% per annum is attained at a (very low) birth rate of about 0.4% per annum.

Mortality shock Just as in the exogenous growth model, increased longevity constitutes by far the most complicated shock studied here. Indeed, as can be seen from equation (F.5) above, a mortality shock affects three distinct items featuring in the implicit expression for the steady-state growth rate, $\hat{\gamma}$, namely (a) the optimal schooling period, e^*, (b) the steady-state growth rate of the population, $\hat{n}$, and (c) the cumulative mortality factor, $M(u,\psi)$. By differentiating (F.5) with respect to $\hat{\gamma}$ and ψ (and recognising the dependence of e^* and $\hat{n}$ on ψ) we find after some steps:

$$\frac{\partial\hat{\gamma}}{\partial\psi} = \frac{\partial\hat{\gamma}}{\partial e^*}\cdot\frac{\partial e^*}{\partial\psi} - \frac{\partial\hat{n}}{\partial\psi} + \frac{\int_{e^*}^\infty -\frac{\partial M(u,\psi)}{\partial\psi}\,\mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du}{\int_{e^*}^\infty u\,\mathrm{e}^{-[(\hat{\gamma}+\hat{n})u+M(u,\psi)]}\,du} \gtreqless 0. \tag{F.10}$$

The overall growth effect of increased longevity is ambiguous. The first composite term on the right-hand side of (F.10) represents the schooling effect, which is positive (see (37) and (F.6)). The third term on the right-hand side represents the cumulative mortality effect and is also positive (given Proposition 2(i)). The ambiguity thus arises because the second term on the right-hand side exerts a negative influence on growth, i.e. increased longevity boosts the steady-state population growth rate (see equation (35)) which in turn slows down growth.

In Figure F.2.1(b) we use the calibrated version of the model to plot the relationship between the steady-state growth rate and a measure of longevity, namely life expectancy at birth, $R(0,\psi) \equiv \Delta(0,0,\psi)$. Interestingly, there is a inverse U-shaped relationship between long-term growth and longevity. For the calibrated model, a maximum growth rate of 1.20% per annum is attained for a life expectancy of $R(0,\psi) = 76$ years.



(a) Growth and the birth rate (b) Growth and life expectancy at birth

Figure F.6: Growth and demography

F.2.2 Transitional dynamics

In this subsection we visualise the transitional effects of fiscal and demographic shocks in the endogenous growth model. Unlike Boucekkine *et al.* (2002), we are able to study shocks which change the optimal schooling period. For reasons of space we ignore the adjustment paths for the remaining macroeconomic variables and restrict attention to the growth rate of per capita human wealth, $\gamma(t) \equiv \dot{h}(t)/h(t)$. Except for ϕ and A_H, we use the same calibration values as before (see Subsection 5.2). Because the model contains a scale effect, we set $A_H = 0.13$ and obtain a realistic steady-state growth rate, $\hat{\gamma}_0 = 1.2\%$. The discussion here can be quite brief because, following a shock, the transition proceeds along the same phases as in the exogenous growth model.

Fiscal shock Figure F.2.2(a) illustrates the path for $\gamma(t)$ following a 20% increase in the educational subsidy. For $0 \leq t < e_1^* - e_0^*$ there are no new labour market entrants and the growth rate collapses. Then, for $e_1^* - e_0^* \leq t < e_1^*$ pre-shock students enter the labour market and the growth rate jumps above its initial steady-state level. Finally, for $t \geq e_1^*$ the growth rate converges in a non-monotonic fashion to its long-run value, i.e. $\lim_{t\to\infty} \gamma(t) = \hat{\gamma}_1 = 1.24\%$, where $\hat{\gamma}_1$ exceeds the initial steady-state growth rate $\hat{\gamma}_0$ (see equation (F.7) above).

Birth rate shock In Figure F.2.2(b) the transitional effects of a baby bust are illustrated. There is no effect on the optimal schooling period but the population growth rate falls from $\hat{n}_0$ to $\hat{n}_1$—see Figure F.2.2(a). Growth jumps sharply due to the fast reduction in $n(t)$ that occurs at impact and immediately hereafter. Intuitively, pre-shock students enter the labour market but their human capital is spread out over fewer people than before the shock so that growth in per capita terms increases sharply. About twenty-two years after the shock, $n(t) \approx \hat{n}_1$ and there is a sharp decline in growth. This is because the post-shock students start to enter the labour market. Despite the fact that they have higher human capital than existing workers, as a group they are not large enough to maintain the previous growth in per capita human capital. Thereafter, the growth rate converges in a non-monotonic fashion to its long-run level $\hat{\gamma}_1 = 1.4\%$, which is higher than the initial steady-state growth rate, i.e. $\hat{\gamma}_1 > \hat{\gamma}_0$. Given our calibration, the economy lies to the right of the peak in the curve for $\hat{\gamma}$ in Figure F.2.1(a) so that a baby bust increases long-run growth.

Mortality shock In Figure F.2.2(c) the effect on the growth rate of increased longevity of generations born after time $t = 0$ is illustrated. Just as for the exogenous growth model, nothing happens to growth for the period $0 \leq t < e_0^*$ because only pre-shock agents enter the labour market and the same type of agents die off. For $e_0^* \leq t < e_1^*$ there are no new labour market entrants and the growth rate collapses. At time $t = e_1^*$ the oldest of the post-shock cohorts enter the labour market and as a result growth is boosted again. For $t > e_1^*$, the growth rate converges non-monotonically towards the new steady-state growth rate $\hat{\gamma}_1 = 1.2\% \approx \hat{\gamma}_0$. In terms of Figure F.2.1(b), the calibration places the economy on the flat segment of the $\hat{\gamma}$ curve (near the top). This explains why there is a negligible effect on the long-run growth rate.

F.2.3 Discussion

The main findings of this section are as follows. For the calibrated model, the long-run growth rate in per capita human capital increases as a result of a positive fiscal impulse or a fall in the birth rate. Increased longevity, however, reduces this long-run growth rate. The transition path in the growth rate is cyclical and rather complex for all shocks considered, and the new equilibrium is reached only very slowly.

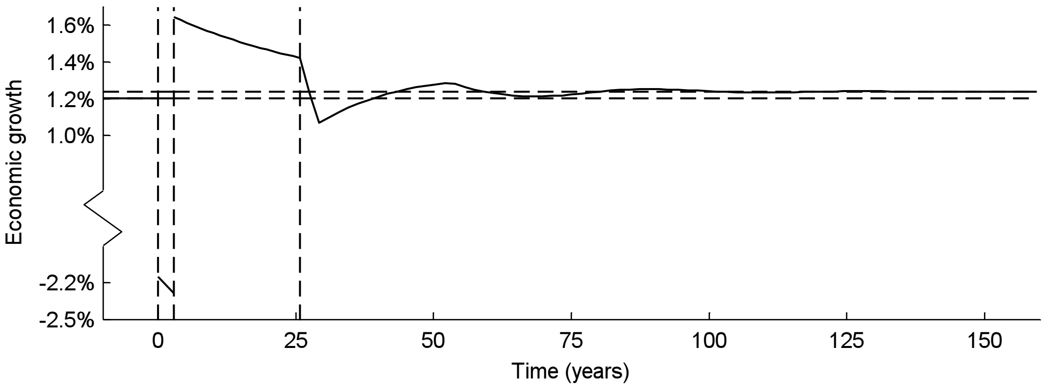
F.3 Schooling decision with imperfect annuity markets

In this section we study the individual schooling decision when annuities are imperfect. Following the paper, we assume that time is continuous and that people do not know how long they will live. To prevent borrowing constraints (which would render the analysis intractable) we assume that life insurance is available, but allow this type of insurance to be less than actuarially fair (Yaari's case C). People maximize lifetime utility by choosing their time spent at school e and a path for consumption $c(u)$.

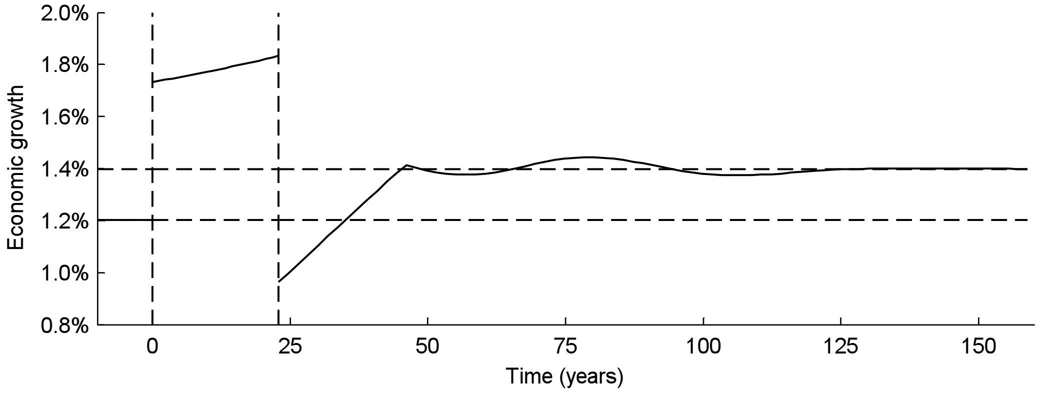
$$\max_{e,c(u)} \Lambda = \int_0^\infty \mathcal{U}(c(u)) \mathrm{e}^{-\theta u - M(u)} du, \tag{F.11}$$

subject to budget identity and a transversality condition

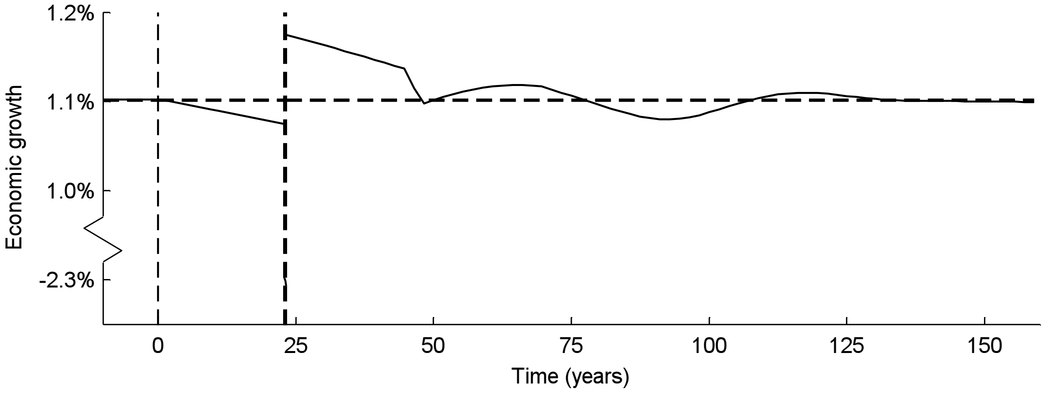
$$\begin{aligned} \dot{a}(u) &= [r + q(u)]a(u) + w(u,e) + z - c(u), && \text{(F.12)} \\ 0 &= \lim_{u\to\infty} a(u)\mathrm{e}^{-ru-Q(u)} = 0, && \text{(F.13)} \end{aligned}$$



(a) Fiscal impulse



(b) Baby bust



(c) Reduced adult mortality

Figure F.7: Per capita human capital growth where $q(u) \geq 0$ denotes the net annuity payment and $Q(u)$ its primitive. (The annuity rate of interest in $r + q(u)$.) The net annuity payment may or may not depend on mortality (see below). With perfect annuities (the case discussed in the paper), we set $q(u) = m(u)$. In reality, annuities are typically less-than-actuarially fair, i.e. $q(u) < m(u)$, and the insurance companies make excess profits. In a general equilibrium setting we assume that such excess profits are distributed in a lump-sum fashion to surviving households, i.e. z denotes such transfers.

The wage rate $w(u,e)$ is defined as:

$$w(u,e) = \begin{cases} 0 & \text{for } u < e \\ w(e) & \text{for } u \geq e \end{cases} \tag{F.14}$$

Integration of the budget identity (F.12) subject to the initial condition $a(0) = 0$ and using the transversality condition (F.13) gives the lifetime budget constraint:

$$\int_0^\infty c(u)\mathrm{e}^{-ru-Q(u)}du = z\int_0^\infty \mathrm{e}^{-ru-Q(u)}du + w(e)\int_e^\infty \mathrm{e}^{-ru-Q(u)}du \equiv \overline{li}(e). \tag{F.15}$$

If instantaneous utility is given by a power utility function (as in the paper), the Euler equation can be written as

$$\frac{\dot{c}(u)}{c(u)} = \sigma[r - \theta - m(u) + q(u)]. \tag{F.16}$$

By solving the Euler equation and substituting the resulting expression into the lifetime budget constraint (F.15) we find:

$$c(u) = \overline{li}(e)\Phi(u), \tag{F.17}$$

$$\Phi(u) = \frac{\mathrm{e}^{\sigma[(r-\theta)u-M(u)+Q(u)]}}{\int_0^\infty \mathrm{e}^{-ru-Q(u)+\sigma[(r-\theta)u-M(u)+Q(u)]}du}. \tag{F.18}$$

Substituting (F.17) into the utility function (F.11) we find the *concentrated* utility function to be maximized by choice of e:

$$\max_e \Lambda = \int_0^\infty \mathcal{U}(\overline{li}(e)\Phi(u))\mathrm{e}^{-\theta u-M(u)}du \tag{F.19}$$

Differentiation with respect to time spent at school gives the first-order condition:

$$\frac{d\Lambda}{de} = \frac{d\overline{li}}{de}\cdot\int_0^\infty \Phi(u)\mathcal{U}'(c(u))\mathrm{e}^{-\theta u-M(u)}du = 0. \tag{F.20}$$

The integral is always positive so this first order condition reduces to $d\overline{li}/de = 0$ which implies that the optimal schooling choice maximizes lifetime income.

Recall (from (F.15)) that lifetime income is equal to:

$$\overline{li}(e) \equiv z\int_0^\infty \mathrm{e}^{-ru-Q(u)}du + w(e)\int_e^\infty \mathrm{e}^{-ru-Q(u)}du. \tag{F.21}$$

If mortality is to exert an effect on the schooling decision, it must do so via the $Q(u)$ term. Several points are worth noting.

- With perfect annuities, $q(u) = m(u)$, $Q(u) = M(u)$, and the schooling choice is affected by mortality. This is the case discussed in the paper (and in the literature).

- With imperfect annuities there are two sub-cases.

  – The net annuity payment depends on mortality, i.e. $q(u) = f(m(u))$. If mortality changes, insurance companies change the annuity payment, $Q(u)$ is affected, as is the schooling choice.

  – The net annuity payment does not depend on mortality at all. In this rather unrealistic case, changes in mortality do not affect $q(u)$ or $Q(u)$ and the schooling choice does not depend on mortality. The excess profits of insurance companies do not influence the schooling decision because they are distributed in a lump-sum fashion to surviving agents (see above). Mortality changes do not have an effect via this general equilibrium channel either.

We conclude that mortality affects the schooling decision also with imperfect annuities, unless the net annuity payment bears no relationship whatsoever with the mortality rate. In the paper follow the literature and choose to work with the simplest case of perfect annuities. Of course, in reality annuity markets are not perfect. But they are not absent either–see for example Cannon and Tonks (2006) for a survey on the UK experience. To summarize, we defend our perfect annuity assumption on two grounds:

- Much (if not all) of the literature adopts this assumption. Assuming imperfect annuities would make our paper difficult to relate to existing work.

- As Cannon and Tonks (2006, p. 5) argue, annuities are less than actuarially fair but not outrageously so. They cite "money's worth" ratios in the range of 0.85-1.05. Of course there is such a thing called the "annuity puzzle." Voluntary annuity demand is quite low despite the fact that annuities are theoretically very attractive. But this puzzle cannot be explained with the standard expected utility approach upon which our model (and much of the literature we contribute to) is based.

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