Foundations of Modern Macroeconomics
Third Edition

Chapter 16: Overlapping generations in discrete time (sections 16.3 – 16.4)

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Outline

1. Human capital accumulation
   - Azariadis-Drazen model
   - Eckstein-Zilcha model

2. Public investment

3. Endogenous fertility

Extensions to the basic Diamond-Samuelson model

- Human capital accumulation
  - Automatic knowledge transfer and endogenous growth
  - A family externality and the benefit of a mandatory education system
- Public investment
  - Macroeconomic effects
  - Some modified golden rules
- Endogenous fertility
  - What determines the population growth rate?
  - Is the Ricardian Equivalence Theorem still valid?

Human capital accumulation

- Human capital creation may be an important engine of growth in the economy
- We study an OLG version of the Lucas-Uzawa model (also studied in Chapter 14) proposed by Azariadis-Drazen

Households: utility

- Work full time during second period of life
- Divide time between training and working during first period
- Lifetime utility:

$$\Lambda_t^{Y,i} \equiv \Lambda^Y(C_t^{Y,i}, C_{t+1}^{O,i})$$

  - No direct utility attached to leisure and to training (knowledge not value *per se*)

Households: constraints

- Budget identities:

$$C_t^{Y,i} + S_t^i = w_t H_t^i N_t^i$$
$$C_{t+1}^{O,i} = (1 + r_{t+1}) S_t^i + w_{t+1} H_{t+1}^i$$

  - w_t is the wage rate per efficiency unit of labour
  - H_t^i is the level of human capital of worker i at time t
  - N_t^i is the amount of time spent working (rather than training) during youth
  - $C_t^{Y,i}$, $C_{t+1}^{O,i}$, and S_t^i have their usual meaning
- Time constraint during youth:

$$E_t^i = 1 - N_t^i \geq 0$$

  - Time endowment is unity
  - E_t^i is time spent on training during youth

Households: constraints

- Training technology:

$$H_{t+1}^i = G(E_t^i) \cdot H_t^i$$

  - Positive but non-increasing returns to training ($G' > 0 \geq G''$)
  - No knowledge depreciation ($G(0) = 1$)
- Household optimization in two steps:
  - Choose training level to maximize lifetime income
  - Choose consumption and savings (subject to lifetime income)

Step 1: Training decision (1)

- Household chooses E_t^i such that lifetime income is maximized:

$$I_t^i(E_t^i) \equiv H_t^i \cdot \left[w_t(1 - E_t^i) + \frac{w_{t+1} G(E_t^i)}{1 + r_{t+1}} \right]$$

- First-order (Kuhn-Tucker) condition:

$$\frac{dI_t^i}{dE_t^i} = H_t^i \cdot \left[-w_t + \frac{w_{t+1} G'(E_t^i)}{1 + r_{t+1}} \right] \leq 0$$

$$E_t^i \geq 0, \qquad E_t^i \cdot \frac{dI_t^i}{dE_t^i} = 0$$

Step 1: Training decision (2)

- Two possible solutions
  - No-training solution:

$$G'(0) < \frac{w_t(1+r_{t+1})}{w_{t+1}} \qquad \Rightarrow \qquad E_t^i = 0$$

    Corner solution because training technology not productive enough!
  - Training solution:

$$E_t^i > 0 \qquad \Rightarrow \qquad 1 + r_{t+1} = \frac{w_{t+1}}{w_t} \cdot G'(E_t^i)$$

    Invest in human capital until its yield equals the yield on financial assets

Step 2: Consumption-saving decision

- Household chooses $C_t^{Y,i}$, $C_{t+1}^{O,i}$ and S_t^i in order to maximize lifetime utility subject to the lifetime budget constraint:

$$C_t^{Y,i} + \frac{C_{t+1}^{O,i}}{1 + r_{t+1}} = I_t^i$$

where I_t^i is now maximized lifetime income (see Step 1)

- Key expression is the savings function:

$$S_t^i = S\Big(r_{t+1}, (1 - E_t^i) w_t H_t^i, w_{t+1} H_{t+1}^i\Big)$$

Further elements of the model

- Initial condition for household i:

$$H_t^i = H_t$$

  Household “inherits” average level of human capital in the economy (*osmotic* human capital transfer across generations)
- Model is symmetric so index i can be dropped
- Constant population ($L_t = L_{t-1} = 1$)
- Total labour supply in efficiency units is $N_t = (1 - E_t)H_t + H_t$

Table 16.2: Growth, human capital, and overlapping generations

$$N_{t+1}k_{t+1} = S(r_{t+1}, (1-E_t)w_t H_t, w_{t+1}H_{t+1}) \tag{T2.1}$$

$$r_{t+1} + \delta = f'(k_{t+1}) \tag{T2.2}$$

$$w_t = f(k_t) - k_t f'(k_t) \tag{T2.3}$$

$$N_t = (2 - E_t)H_t \tag{T2.4}$$

$$1 + r_{t+1} = \frac{w_{t+1}}{w_t} G'(E_t) \tag{T2.5}$$

$$H_{t+1} = G(E_t)H_t \tag{T2.6}$$

Summary of the Azariadis-Drazen model (1)

- Model displays endogenous growth in the steady state. This is illustrated in **Figure 16.7** for the unit-elastic case with technology and log-linear utility:

$$y_t = Z_0 k_t^\alpha$$

$$\Lambda_t^Y = \ln C_t^Y + \frac{1}{1+\rho} \ln C_{t+1}^O$$

  - PB is portfolio balance line: (k, E) combinations for which yields on physical and human capital equalize
  - SI is the savings-investment line: (k, E) combinations for which savings equals investment
  - equilibrium at E_0
  - Growth rate is $\gamma \equiv G(E) - 1$ is depicted in bottom panel

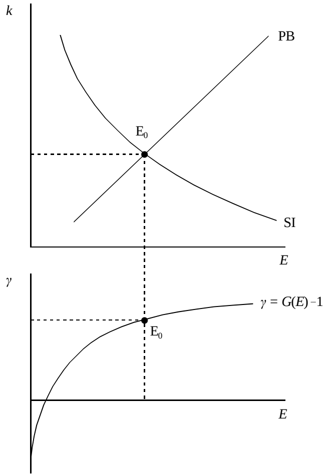
Summary of the Azariadis-Drazen model (2)

- Engine of growth in the model is the training technology:

$$H^i_{t+1} = G(E^i_t)H^i_t$$

  - *Level* of training explains *growth rate* in human capital
  - Knowledge/technical skills are disembodied (live on after agent has died)
  - Edogenous growth vanishes if knowledge/technical skills are embodied

Figure 16.9: Endogenous growth due to human capital formation



Choosing your offspring's education (1)

- Eckstein-Zilcha: why do we have compulsory education systems?
- *Key idea*: Parents may under-invest in the human capital of their children (*intra*-family external effect)
- We discuss a simple version of the EZ model to demonstrate underinvestment result
- Utility function:

$$\Lambda_t^Y \equiv \Lambda^Y(C_t^Y, C_{t+1}^O, M_t, O_{t+1})$$

  - C_t^Y is consumption when young
  - C_{t+1}^O is consumption when old
  - M_t is leisure during youth
  - $O_{t+1} \equiv (1+n)H_{t+1}$ is total human capital of the agent's offspring (H_{t+1} is human capital per child, $1+n$ is the number of children)

Choosing your offspring's education (2)

- In-house training technology (no schools)

$$H_{t+1} = G(E_t) \cdot H_t^{\beta}$$

  - E_t is educational effort per child
  - $G(\cdot)$ is the training curve (satisfying $0 < G(0) \leq 1$, $G(1) > 1$, $G' > 0 \geq G''$)
  - Positive but diminishing marginal product of human capital: $0 < \beta \leq 1$
  - Note difference with A-D model: now parent chooses human capital of children (costs and benefits accrue to different agents)

Choosing your offspring's education (3)

- Time endowment: households has two units of time available:
  - One unit is supplied inelastically to the labour market
  - One is divided over leisure and training: $M_t + (1+n)E_t = 1$
- Household's lifetime budget constraint:

$$C_t^Y + \frac{C_{t+1}^O}{1+r_{t+1}} = w_t \cdot H_t$$

  - $w_t = F_N(K_t, N_t)$ where $N_t \equiv L_t H_t$ (efficiency units of labour)
  - $r_t + \delta = F_K(K_t, N_t)$

Choosing your offspring's education (4)

- Household chooses C_t^Y, C_{t+1}^O, M_t, E_t, and H_{t+1} to maximize lifetime utility subject to (a) the training technology, (b) the time constraint, and (c) the consolidated budget constraint. Key first-order conditions:

$$\frac{\partial \Lambda^Y / \partial C_t^Y}{\partial \Lambda^Y / \partial C_{t+1}^O} = 1 + r_{t+1}$$

$$\frac{\partial \Lambda^Y}{\partial O_t} G'(E_t) H_t^{\beta} - \frac{\partial \Lambda^Y}{\partial M_t} < 0 \quad \Longrightarrow \quad E_t = 0$$

$$\frac{\partial \Lambda^Y}{\partial O_t} G'(E_t) H_t^{\beta} - \frac{\partial \Lambda^Y}{\partial M_t} = 0 \quad \Longleftarrow \quad E_t > 0 \qquad \text{(S1)}$$

  - Corner solution if the net marginal benefit of training is negative
  - For interior solution training provided until net marginal benefit of training is zero (all gains exhausted)

Choosing your offspring's education (5)

- Assume that interior solution (S1) obtains. Note that (S1) only contains costs and benefits of the parent! First hint at underinvestment problem. Not all benefits are taken into account
- Formal analysis of underinvestment problem: Social Welfare Function approach
- The social welfare function is:

$$SW_0 \equiv \sum_{t=0}^{\infty} \lambda_t \Lambda_t^Y = \sum_{t=0}^{\infty} \lambda_t \Lambda^Y (C_t^Y, C_{t+1}^O, M_t, O_{t+1})$$

  - SW_0 is social welfare in the planning period $(t = 0)$
  - $\{\lambda_t\}_{t=0}^{\infty}$ is a positive monotonically decreasing sequence of weights attached to the different generations (which satisfies $\sum_{t=0}^{\infty} \lambda_t < \infty$)

Choosing your offspring's education (6)

- Resource constraint:

$$C_t^Y + \frac{C_t^O}{1+n} + (1+n)k_{t+1} = F(k_t, H_t) + (1-\delta)k_t$$

  where $k_t \equiv K_t/L_t$

- Social planner chooses sequences for consumption ($\{C_t^Y\}_{t=0}^{\infty}$ and $\{C_{t+1}^O\}_{t=0}^{\infty}$), the stocks of human and physical capital ($\{K_{t+1}\}_{t=0}^{\infty}$ and $\{H_{t+1}\}_{t=0}^{\infty}$), and the educational effort ($\{E_t\}_{t=0}^{\infty}$) in order to maximize SW_0 subject to (a) the training technology, (b) the time constraint, and (c) the resource constraint

Choosing your offspring's education (7)

- Most interesting (for our purposes) first-order condition:

$$\frac{\partial \Lambda^Y(\hat{x}_t)}{\partial M_t} = G'(\hat{E}_t)\hat{H}_t^{\beta} \cdot \left[\frac{\partial \Lambda^Y(\hat{x}_t)}{\partial O_t} \right.$$
$$+ \frac{\partial \Lambda^Y(\hat{x}_t)}{\partial C_{t+1}^O} F_N(\hat{k}_{t+1}, \hat{H}_{t+1})$$
$$\left. + \frac{\beta(1+n)\hat{H}_{t+2}}{G'(\hat{E}_{t+1})\hat{H}_{t+1}^{1+\beta}} \frac{\partial \Lambda^Y(\hat{x}_t)}{\partial C_{t+1}^O} \cdot \frac{\partial \Lambda^Y(\hat{x}_{t+1})/\partial Z_{t+1}}{\partial \Lambda^Y(\hat{x}_{t+1})/\partial C_{t+1}^Y} \right]$$

  - Marginal social costs (LHS) must be equated to marginal social benefits (RHS)
  - Marginal social benefits consist of three terms:
    - "Own" term, affecting decision maker directly (line 1)
    - "Induced" term, affecting earning power of children (line 2)
    - "Induced" term, affecting incentive of children to educate *their* children provide (line 3)

Choosing your offspring's education (8)

- Second and third effects are ignored by parents which leads to under-investment in human capital. Policy options:
  - Complex set of incentives (taxes/subsidies) to correct the parent's behaviour
  - Compulsory education

Public investment

- Empirical work by Aschauer prompts a number of questions:
  - What are the macroeconomic effects of public investment?
  - How much public capital should a country possess?
- Study these questions with a modified D-S model
  - Exogenous labour supply / lump-sum taxes
  - Public capital is a stock variable
  - Public capital affects factor productivity (e.g. bridges, roads, airports, etc.)

Public investment

- Accumulation identity:

$$G_{t+1} - G_t = I_t^G - \delta_g G_t$$

  - G_{t+1} is public capital stock
  - I_t^G is public investment
  - δ_g is depreciation rate on public capital

- Technology:

$$Y_t = F(K_t, L_t, g_t)$$

  where $g_t \equiv G_t / L_t$
  - CRTS in (K_t, L_t)
  - Positive but diminishing marginal product of public capital, $F_g > 0$, $F_{gg} < 0$

Public investment

- Competitive production yields rental expressions:

$$r_t = r(k_t, g_t) \equiv f_k(k_t, g_t) - \delta_k,$$
$$w_t = w(k_t, g_t) \equiv f(k_t, g_t) - k_t f_k(k_t, g_t),$$

  where $f(k_t, g_t) \equiv F(K_t/L_t, 1, g_t)$ is the intensive-form production function. By assumption, g_t affects r_t and w_t positively. (Example: $Y_t = Z_0 K_t^{\alpha} L_t^{1-\alpha} g_t^{\eta}$ with $0 < \eta < 1 - \alpha$.)
- Household utility:

$$\Lambda_t^Y = \ln C_t^Y + \frac{1}{1+\rho} \ln C_{t+1}^O$$

- Household lifetime budget constraint:

$$\hat{w}_t \equiv w_t - T_t^Y - \frac{T_{t+1}^O}{1+r_{t+1}} = C_t^Y + \frac{C_{t+1}^O}{1+r_{t+1}}$$

Public investment

- Savings function:

$$S_t = (1-c)\left(w_t - T_t^Y\right) + c\frac{T_{t+1}^O}{1+r_{t+1}}$$

where $c \equiv \frac{1+\rho}{2+\rho}$

Model summary (1)

- Model is:

$$(1+n)g_{t+1} = i_t^G + (1-\delta_g)g_t \tag{S2}$$

$$i_t^G = T_t^Y + \frac{T_t^O}{1+n} \tag{S3}$$

$$(1+n)k_{t+1} = (1-c)\left[W(k_t, g_t) - T_t^Y\right] + \frac{cT_{t+1}^O}{1+r(k_{t+1}, g_{t+1})} \tag{S4}$$

  - Eq. (S2): Accumulation identity for public capital per worker ($i_t^G \equiv I_t^G / L_t$)
  - Eq. (S3): Government budget constraint
  - Eq. (S4): Link between savings and private capital formation
- Immediately obvious that financing method critically affects the model: who pays for i_t^G affects (S4) and thus the private capital stock and the rest of the economy

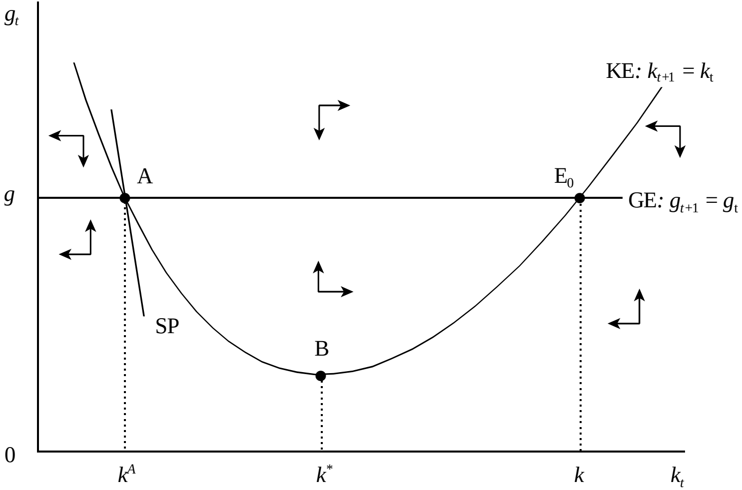
Model summary (2)

- In Figure 16.8 we consider the case in which the old are untaxed ($T_t^O = 0$ for all t)
  - GE line: (g, k) combinations for which $g_{t+1} = g_t$
    - Horizontal
    - g_t rises (falls) for points above (below) the GE line–see the vertical arrows
  - KE line: (g, k) combinations for which $k_{t+1} = k_t$
    - Slope is negative (positive) for low (high) private capital stock (Intuition: slope determined by $1 + n - (1 - c) W_k$; W_k high (low) if k is low (high))
    - k_t increases (decreases) for points above (below) the KE line–see the horizontal arrows
  - Two steady-state equilibria:
    - E_0: stable node (stable monotonic or cyclical adjustment)
    - A: saddle point (unstable because both g_t and k_t are predetermined variables)

Model summary (3)

- Focus on equilibrium E_0: what happens if i_t^G is increased?
  - Both GE and KE shift up
  - Effect on g unambiguously positive
  - Effect on k depends on relative scarcity of public capital ($w \uparrow$ but $T_t^Y \uparrow$ so net effect ambiguous): k rises (falls) if $i^G/y < \eta(1-\alpha)$ ($> \eta(1-\alpha)$), i.e. if public capital is initially relatively scarce (abundant)

Figure 16.10: Public and private capital



How much public capital should a country have? (1)

- SWF approach in an OLG setting
- Social welfare function:

$$SW_0 \equiv \left(\frac{1+n}{1+\rho_G}\right)^{-1} \Lambda^Y(C_{-1}^Y, C_0^O) + \sum_{t=0}^{\infty} \left(\frac{1+n}{1+\rho_g}\right)^t \Lambda^Y(C_t^Y, C_{t+1}^O)$$

  - Benthamite format ("...greatest happiness of the greatest number...")
  - ρ_g is the planner's discount rate ($\rho_g > n$). May or may not equal ρ
  - Special treatment of current old generation to avoid dynamic inconsistency of the social optimum (see Intermezzo)
- Resource constraint:

$$C_t^Y + \frac{C_t^O}{1+n} + (1+n)\left[k_{t+1} + g_{t+1}\right] = f(k_t, g_t) + (1-\delta_k)k_t + (1-\delta_g)g_t$$

How much public capital should a country have? (2)

- Social planner chooses $\{C_t^Y\}_{t=0}^{\infty}$, $\{C_t^O\}_{t=0}^{\infty})$, $\{g_{t+1}\}_{t=0}^{\infty}$, and $\{k_{t+1}\}_{t=0}^{\infty})$, in order to maximize SW_0 subject to the resource constraint, taking k_0 and g_0 as given
- Key first-order conditions for the social optimum:

$$\frac{\partial \Lambda^Y(\hat{x}_t)/\partial C_t^Y}{\partial \Lambda^Y(\hat{x}_t)/\partial C_{t+1}^O} = 1 + \hat{r}_{t+1} \tag{S5}$$

$$\hat{r}_{t+1} = f_k(\hat{k}_{t+1}, \hat{g}_{t+1}) - \delta_k = f_g(\hat{k}_{t+1}, \hat{g}_{t+1}) - \delta_g \tag{S6}$$

$$\frac{\partial \Lambda^Y(\hat{x}_t)/\partial C_t^Y}{\partial \Lambda^Y(\hat{x}_{t-1})/\partial C_t^O} = 1 + \rho_{sp} \tag{S7}$$

  - Eq. (S5): Socially optimal Euler equation; MRS between present and future consumption equated to gross interest factor
  - Eq. (S6): Yields on private and public capital should be equalized (efficient investment)

How much public capital should a country have? (3)

- Continued
  - Eq. (S7): ρ_{sp} determines optimal *intra*temporal division of consumption. With additively separable preferences we get:

$$\frac{U'(\hat{C}_t^Y)}{U'(\hat{C}_t^O)} = \frac{1+\rho_{sp}}{1+\rho}$$

    - If $\rho_{sp} > \rho$ then planner ensures that $U'(\hat{C}_t^Y) > U'(\hat{C}_t^O)$ i.e. that $\hat{C}_t^Y < \hat{C}_t^O$ (favour the old)
    - If $\rho_{sp} = \rho$ then planner ensures that $U'(\hat{C}_t^Y) = U'(\hat{C}_t^O)$ i.e. that $\hat{C}_t^Y = \hat{C}_t^O$ (egalitarian solution)
    - If $\rho_{sp} < \rho$ then planner ensures that $U'(\hat{C}_t^Y) < U'(\hat{C}_t^O)$ i.e. that $\hat{C}_t^Y > \hat{C}_t^O$ (favour the young)

Some final remarks on public capital

- In the steady state, $\hat{r}_t = \rho_{sp}$ so (b) simplifies to:

$$[\hat{r} \equiv] \quad f_k(k,g) - \delta_k = \rho_{sp} = f_g(k,g) - \delta_g$$

  Hence, modified golden rules for private and public capital accumulation feature the social planner's discount rate
- First-best social optimum can be decentralized if and only if the right policy instruments are available:
  - i^G (and thus g) is set correctly
  - *Age-specific* lump-sum taxes are available (even stronger requirement than in the representative-agent model)
- If one or more of the policy variables is not available, the problem becomes a second-best optimization problem (Ramsey taxation, modified Samuelson rule)

Endogenizing the birth rate

- Simplified version of the Lapan-Enders model
- Large number of dynastic families
- Youth:
    - fully dependent on parent
    - no economic decisions
- Adulthood:
    - inherits wealth from parent
    - supplies one unit of labour
    - decides on consumption
    - decides on number of kids (born at beginning of the period)
    - decides on bequest to each child
- There are L_t adults

Choices of adult i

- Lifetime utility of adult i:

$$\Lambda_t^i \equiv U(c_t^i, n_t^i) + \xi \Lambda_{t+1}^i, \qquad 0 < \xi < 1$$

  - ξ is the altruism parameter
  - c_t^i is consumption
  - n_t^i is the number of children
  - Λ_{t+1}^i is the *maximized* lifetime utility per child

- Budget constraint of adult i:

$$(1 + r_t) \cdot a_t^i + w_t = c_t^i + tax_t^i + n_t^i \cdot \left[\bar{c} + a_{t+1}^i\right]$$

  - r_t is the real interest rate
  - a_t^i is the bequest received at the beginning of adulthood
  - w_t is the wage rate
  - tax_t^i is the lump-sum tax
  - $\bar{c}$ is the cost of raising a child
  - a_{t+1}^i is the bequest granted to each child at the end of life

Dynastic choices

- Provided bequests remain operative ($a^i_{t+\tau} > 0$ for $\tau = 1, 2, \cdots$) we have an "as if" infinitely-lived agent with lifetime utility function:

$$\Lambda^i_t \equiv \sum_{\tau=0}^{\infty} \xi^\tau U(c^i_{t+\tau}, n^i_{t+\tau})$$

- Choice variables: $c^i_{t+\tau}$, $n^i_{t+\tau}$, and $a^i_{t+\tau}$
- First-order condition for consumption:

$$\frac{U_c(c^i_{t+\tau+1}, n^i_{t+\tau+1})}{U_c(c^i_{t+\tau}, n^i_{t+\tau})} = \frac{n^i_{t+\tau}}{\xi\left[1 + r_{t+\tau+1}\right]} \qquad \text{(S8)}$$

  - Eq. (S8): Euler equation depends on the capital interest rate, the biological interest rate, and the altruism parameter ("impatience")

Dynastic choices

- First-order conditions for kids:

$$\frac{U_n(c^i_{t+\tau}, n^i_{t+\tau})}{U_c(c^i_{t+\tau}, n^i_{t+\tau})} = \bar{c} + a^i_{t+\tau+1} \tag{S9}$$

  - Eq. (S9): marginal benefit of a kid equals marginal cost

- Two financial assets; capital and government bonds (perfect substitutes):

$$a^i_{t+\tau} = k^i_{t+\tau} + b^i_{t+\tau}$$

Aggregate outcomes

- future population:

$$L_{t+1} \equiv \sum_{i=1}^{L_t} n_t^i = \bar{n}_t L_t$$

- debt per adult:

$$\bar{n}_t b_{t+1} = (1 + r_t)\, b_t + g_t - tax_t$$

- features of production:

$$\begin{aligned} y_t &= f(k_t) \\ w_t &= f(k_t) - k_t f'(k_t) \\ r_{t+1} + \delta &= f'(k_{t+1}) \end{aligned}$$

Ricardian Equivalence Theorem

- Ricardian Equivalence Theorem valid iff all conditions are satisfied:
  (a) The chain of bequests is unbroken, i.e. $a^i_{t+\tau} > 0$ for all τ and i. This ensures that each dynasty is effectively infinitely lived;
  (b) Fertility is not a choice variable but is exogenously given, i.e. $n^i_{t+\tau} = n_0$, where n_0 is exogenous (and assumed to be constant for notational convenience);
  (c) The government does not engage in redistribution between dynasties, i.e. $tax^i_{t+\tau} = tax_{t+\tau}$ for all i and τ, so that the government solvency condition implies (at the individual and per capita level) that:

$$b^i_t = b_t = \sum_{\tau=0}^{\infty} n_0^{\tau} R_{t-1,\tau} \left[tax_{t+\tau} - g_{t+\tau} \right], \quad R_{t-1,\tau} \equiv \prod_{s=0}^{\tau} \frac{1}{1+r_{t+s}}$$

  With a symmetric fiscal treatment of dynasties, per capita and individual debt coincide

Ricardian non-equivalence: debt matters

- With endogenous fertility:

$$\frac{U_c(c^i_{t+1}, n^i_{t+1})}{U_c(c^i_t, n^i_t)} = \frac{n^i_t}{\xi\left[1 + r_{t+1}\right]} \tag{S10}$$

$$\frac{U_n(c^i_t, n^i_t)}{U_c(c^i_t, n^i_t)} = \bar{c} + k^i_{t+1} + b_{t+1} \tag{S11}$$

$$(1 + r_{t+\tau})\, k^i_t + w_t = c^i_t + g_t + \left(n^i_t - \bar{n}_t\right) b_{t+1} + n^i_t \left[\bar{c} + k^i_{t+1}\right] \tag{S12}$$

- Debt non-neutral because:
  - Eq. (S10): it affects the relative price of children
  - Eq. (S12): "fiscal externality" economy-wide average fertility, $\bar{n}_t$, reduces the tax burden of individual agents (who treat $\bar{n}_t$ parametrically). Free riding on child production by others thus explains that children will be underproduced (and fertility will be too low) in the presence of public debt

- Assume that $U(c_t, n_t) = \varepsilon \ln c_t + (1-\varepsilon) \ln n_t$
- Symmetric steady-state equilibrium (unit-elastic):

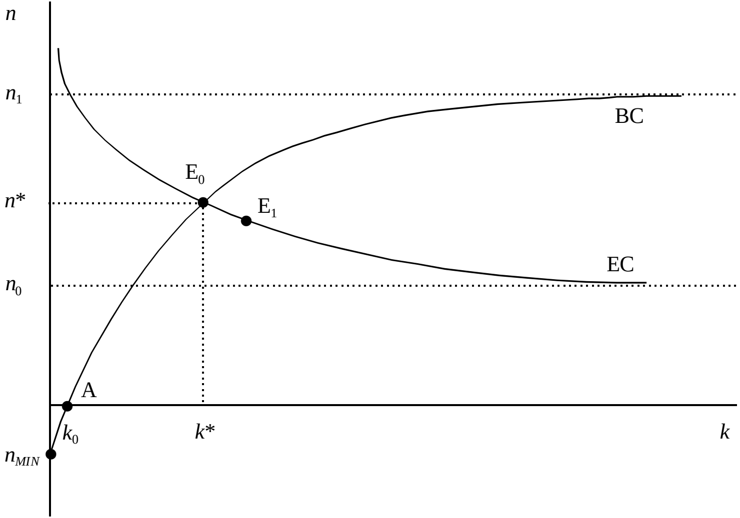
$$n = \xi \cdot \left[1 - \delta + \alpha Z_0 k^{\alpha-1}\right] \qquad \text{(S13)}$$

$$c = \frac{\varepsilon}{1-\varepsilon} n \cdot [\bar{c} + k + b] \qquad \text{(S14)}$$

$$c = k^{\alpha} + (1-\delta)\, k - n \cdot [\bar{c} + k] - g \qquad \text{(S15)}$$

  - See **Figure 16.9** for equilibrium
  - EC: efficiency condition Eq. (S13)
  - BC: budget constraint Eqs. (S14)–(S15)
  - increase in $\bar{c}$ or b rotates BC clockwise around point A: $dn/d\bar{c} < 0$, $dn/db < 0$, $dk/d\bar{c} > 0$, and $dk/db > 0$

Figure 16.11: Steady-state fertility rate and capital intensity



Punchlines

- Studied workhorse model of macroeconomics and public finance
  - Life-cycle saving
  - Dynamic inefficiency quite possible
  - Wide set of applications
- Pensions
  - Fully funded: neutral (saving by the government)
  - PAYG: not neutral (welfare and crowding-out effects)
  - Transitional problems
  - Population ageing may lead to extra saving under PAYG system

Punchlines

- Human capital
  - Osmotic transfer and growth
  - Mandatory education
- Public capital
  - Macroeconomic effects
  - Some more golden rules
- Endogenous fertility and economic incentives
  - Dynastic model with operative bequests
  - Ricardian Equivalence unlikely
  - Economic effects on fertility rate