# Foundations of Modern Macroeconomics Third Edition

## Chapter 12: Exogenous economic growth – Solow-Swan

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# Outline

# Aims of this chapter

- Stylized facts of economic growth
- How well does the Solow-Swan model explain these stylized facts?
- Adding human capital to the Solow-Swan model
- Growth models based on dynamically optimizing consumers
- Fiscal policy and Ricardian equivalence in various traditional growth models?

# Kaldor's stylized facts of economic growth

(1) (*) Output per worker shows continuing growth "with no tendency for a falling rate of growth of productivity"

(2) Capital per worker shows continuing growth

(3) The rate of return on capital is steady

(4) (*) The capital-output ratio is steady

(5) (*) Labour and capital receive constant shares of total income

(6) (*) There are wide differences in the rate of productivity growth across countries

- **Note**: Not all these stylized facts are independent:
  - (SF1) and (SF4) imply (SF2)
  - (SF4) and (SF5) imply (SF3)
- Hence, the starred facts are fundamental

# Romer's additional stylized facts of economic growth

(7) In cross section, the mean growth rate shows no variation with the level of per capita income

(8) The rate of growth of factor inputs is not large enough to explain the rate of growth of output; that is, growth accounting always finds a residual

(9) Growth in the volume of trade is positively correlated with growth in output

(10) Population growth rates are negatively correlated with the level of income

(11) Both skilled and unskilled workers tend to migrate toward high-income countries

# The neoclassical growth model: Solow-Swan (1)

- *Key notion*: capital and labour are substitutable
- Technology (neoclassical part of the model):

$$Y(t) = F\left(K(t), L(t), t\right)$$

  - $t$ is time-dependent shift in technology
  - CRTS:

$$F\left(\lambda K(t), \lambda L(t), t\right) = \lambda F\left(K(t), L(t), t\right), \quad \text{for } \lambda > 0 \quad \text{(P1)}$$

- Saving ("Keynesian" part of the model):

$$S(t) = Y(t) - C(t) = sY(t), \quad 0 < s < 1$$

where $s$ is the constant propensity to save (exogenous)

# The neoclassical growth model: Solow-Swan (2)

- Goods market (closed economy, no government consumption):

$$Y(t) = C(t) + I(t)$$

- Gross investment:

$$I(t) = \delta K(t) + \dot{K}(t)$$

where $\delta K(t)$ is replacement investment ($\delta$ is the constant depreciation rate), and $\dot{K}(t)$ is net addition to the capital stock

- Labour supply is exogenous but the population grows as a whole at a constant exponential rate $n_L$:

$$\frac{\dot{L}(t)}{L(t)} = n_L \quad \Leftrightarrow \quad L(t) = L(0)e^{n_L t}$$

where we can normalize $L(0) = 1$

# Case 1: No technical progress (1)

- Time drops out of technology:

$$Y(t) = F(K(t), L(t))$$

  - Positive and diminishing marginal products:

$$F_K, F_L > 0, \quad F_{KK}, F_{LL} < 0, \quad F_{KL} > 0 \qquad \text{(P2)}$$

  - Inada conditions:

$$\lim_{K \to 0} F_K = \lim_{L \to 0} F_L = +\infty, \quad \lim_{K \to \infty} F_K = \lim_{L \to \infty} F_L = 0 \quad \text{(P3)}$$

# Case 1: No technical progress (2)

- Solve model by writing in per capita form, i.e. $y(t) \equiv Y(t)/L(t)$, $k(t) \equiv K(t)/L(t)$, etcetera. Here are some steps:
  - *Step 1*: Savings equal investment:

$$\begin{aligned} S(t) &= I(t) \\ sY(t) &= \delta K(t) + \dot{K}(t) \\ sF\left(K(t), L(t)\right) &= \delta K(t) + \dot{K}(t) \qquad \Rightarrow \\ s\frac{F\left(K(t), L(t)\right)}{L(t)} &= \delta \frac{K(t)}{L(t)} + \frac{\dot{K}(t)}{L(t)} \end{aligned} \tag{S1}$$

  - *Step 2*: Since $k(t) \equiv K(t)/L(t)$ it follows that:

$$\begin{aligned} \dot{k}(t) &\equiv \frac{\dot{K}(t)}{L(t)} - \frac{K(t)}{L(t)} \frac{\dot{L}(t)}{L(t)} \qquad \Rightarrow \\ \frac{\dot{K}(t)}{L(t)} &= \dot{k}(t) + k(t) n_L \end{aligned} \tag{S2}$$

# Case 1: No technical progress (3)

- Continued
  - *Step 3*: Since $F\left(K(t), L(t)\right)$ features CRTS we have:

$$Y(t) = F\left(K(t), L(t)\right) = L(t)F\left(\frac{K(t)}{L(t)}, 1\right) \qquad \Rightarrow$$

$$y(t) = f(k(t)) \tag{S3}$$

(e.g. Cobb-Douglas $Y = K^{\alpha}L^{1-\alpha}$ implies $y = k^{\alpha}$)

- By substituting (S2) and (S3) into (S1) we obtain the *fundamental differential equation* (FDE) for $k(t)$:

$$\dot{k}(t) = \underbrace{sf(k(t))}_{(a)} - \underbrace{(\delta + n_L)k(t)}_{(b)}$$

(a) Per capita saving
(b) To maintain constant $k(t)$ one must replace and expand the *level* of the capital stock

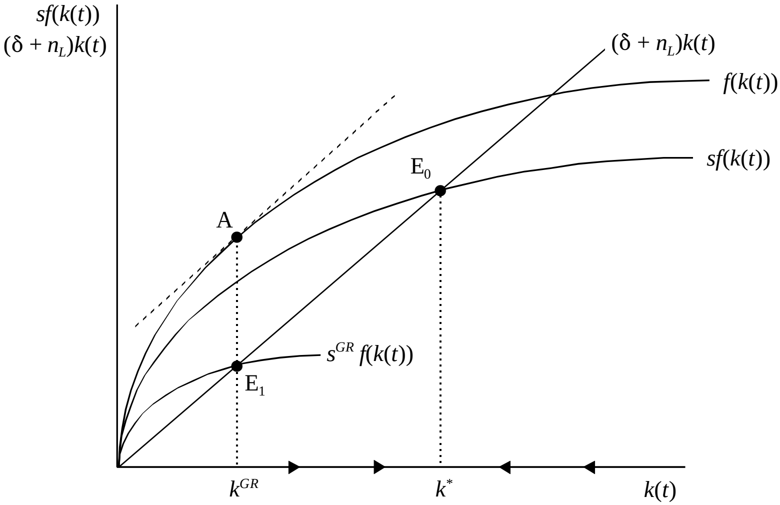
# Case 1: No technical progress (4)

- In **Figure 12.1** we illustrate the FDE. The Inada conditions imply:
  - $f(k(t))$ vertical for $k(t) \to 0$
  - $f(k(t))$ horizontal for $k(t) \to \infty$
  - Unique steady state at $\mathrm{E}_0$
  - Stable equilibrium
- In the balanced growth path (BGP):

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{I}(t)}{I(t)} = \frac{\dot{S}(t)}{S(t)} = \frac{\dot{L}(t)}{L(t)} = n_L$$

Hence the name *exogenous growth*

# Figure 12.1: The Solow-Swan model



# Case 2: With technical progress (1)

- Focus on *disembodied* technological progress

$$Y(t) = F(\underbrace{Z_K(t)K(t)}_{(a)}, \underbrace{Z_L(t)L(t)}_{(b)})$$

(a) "Effective" capital input
(b) "Effective" labour input

- Three types of progress:
  - Harrod neutral: $Z_K(t) \equiv 1$
  - Hicks neutral: $Z_K(t) \equiv Z_L(t)$
  - Solow neutral: $Z_L(t) \equiv 1$
- Cases are indistinguishable for Cobb-Douglas

# Case 2: With technical progress (2)

- For non-CD case progress must be Harrod neutral to have a steady state with constant growth rate (otherwise one of the shares goes to zero, contra (SF5))
- Define $N(t) \equiv Z(t)L(t)$ and assume that technical progress occurs at a constant exponential rate:

$$\frac{\dot{Z}(t)}{Z(t)} = n_Z, \quad Z(t) = Z(0)e^{n_Z t}$$

so that the *effective* labour force grows at a constant exponential rate $n_L + n_Z$

# Case 2: With technical progress (3)

- Measuring output and capital per unit of effective labour, i.e. $y(t) \equiv Y(t)/N(t)$ and $k(t) \equiv K(t)/N(t)$, the FDE for $k(t)$ is obtained:

$$\dot{k}(t) = sf(k(t)) - (\delta + n_L + n_Z)k(t)$$

- In the BGP we have:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{I}(t)}{I(t)} = \frac{\dot{S}(t)}{S(t)} = \frac{\dot{N}(t)}{N(t)} = \frac{\dot{L}(t)}{L(t)} + \frac{\dot{Z}(t)}{Z(t)} = n_L + n_Z$$

- Exogenous growth rate now equals $n_L + n_Z$

# Further properties of the Solow-Swan model

(A) The golden rule of capital accumulation: dynamic inefficiency possible

(B) Transitional dynamics: conditional growth convergence seems to hold

(C) Speed of adjustment: too fast. Model can be rescued

(D) Rescuing the Solow-Swan model

# (A) The golden rule (1)

- Golden rule: maximum steady-state consumption per capita
- For each savings rate there is a unique *steady-state* capital-labour ratio (assume $n_Z = 0$ for simplicity):

$$k^* = k^*(s)$$

  with $dk^*/ds = y^*/[\delta + n - sf'(k^*)] > 0$. The higher is $s$, the larger is $k^*$
- Since $C(t) \equiv (1-s)Y(t)$ we have for per capita consumption:

$$\begin{aligned} c^*(s) &= (1-s)f\left(k^*(s)\right) \\ &= f\left(k^*(s)\right) - (\delta + n_L)k^*(s) \end{aligned}$$

# (A) The golden rule (2)

- The golden-rule savings rate is such that $c(s)$ is maximized:

$$\frac{dc^*(s)}{ds} = \left[ f'\left(k^*(s)\right) - (\delta + n_L) \right] \frac{dk^*(s)}{ds} = 0$$

- Since $\frac{dk^*(s)}{ds} > 0$ we get that:

$$f'\left(k^*(s^{GR})\right) = \delta + n_L \tag{S4}$$

One interpretation: the produced asset (the physical capital stock) yields an own-rate of return equal to $f' - \delta$, whereas the non-produced primary good (labour) can be interpreted as yielding an own-rate of return $n_L$. Intuitively, the efficient outcome occurs if the rate of return on the two assets are equalized

- Recall that in the steady state::

$$s^{GR} f\left(k^*(s^{GR})\right) = (\delta + n_L) k^*(s^{GR}) \tag{S5}$$

# (A) The golden rule (3)

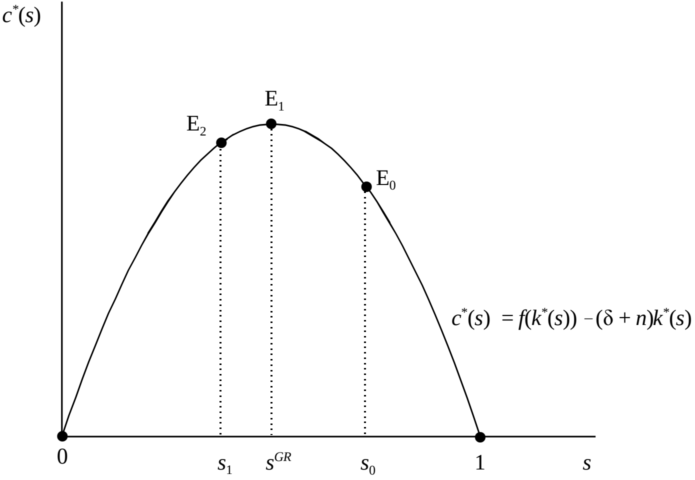
- By using (S4) we can rewrite (S5) in terms of a national income share:

$$s^{GR} = \frac{(\delta + n_L)k^*(s^{GR})}{f\left(k^*(s^{GR})\right)} = \frac{k^*(s^{GR})f'(k^*(s^{GR}))}{f\left(k^*(s^{GR})\right)}$$

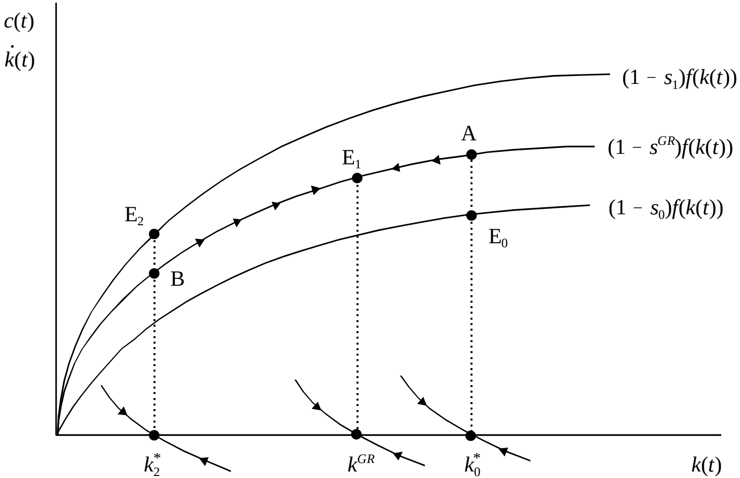
(e.g. for Cobb-Douglas $f(.) = k(t)^\alpha$, $\alpha$ represents the capital income share so that the golden rule savings rate equals $s^{GR} = \alpha$)

- In **Figures 12.2-12.3** we illustrate the possibility of *dynamic inefficiency* (oversaving: If $s_0 > s^{GR}$ then a Pareto-improving transition from $E_0$ to $E_1$ is possible)

# Figure 12.2: Per capita consumption and the savings rate



# Figure 12.3: Per capita consumption during transition to its golden rule level



# (B) Transitional dynamics towards the steady state (1)

- Defining the growth rate of $k(t)$ as $\gamma_k(t) \equiv \dot{k}(t)/k(t)$, we derive from the FDE:

$$\gamma_k(t) \equiv s\frac{f(k(t))}{k(t)} - (\delta + n)$$

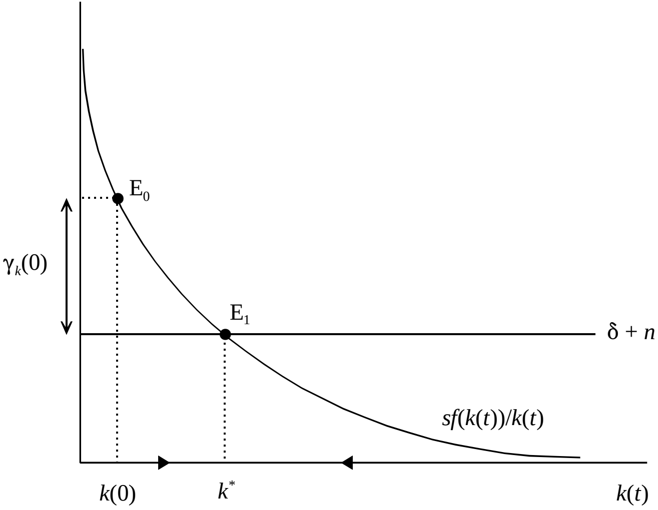
where $n \equiv n_L + n_Z$

- In **Figure 12.4** this growth rate is represented by the vertical difference between the two lines. (The Inada conditions ensure that $\lim_{k\to 0} sf(k)/k = \infty$ and $\lim_{k\to\infty} sf(k)/k = 0$)
- Countries with little capital (in efficiency units) grow faster than countries with a lot of capital. In other words, poor and rich countries should converge! (Link between $\gamma_k(t)$ and $\gamma_y(t)$ is easily established, especially for the CD case)

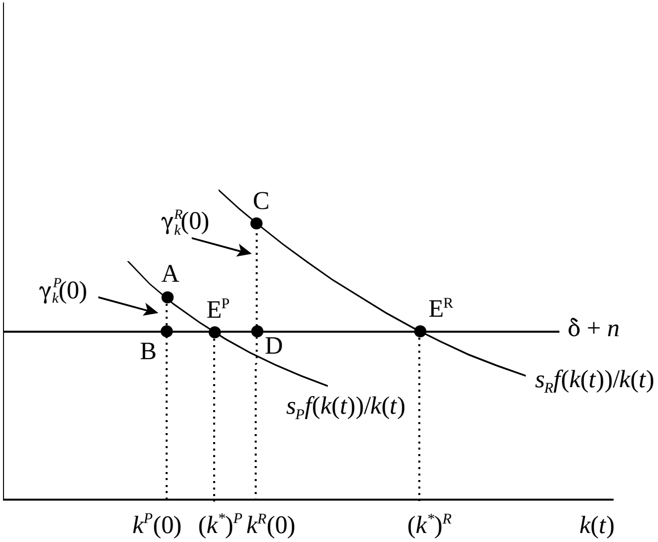
# (B) Transitional dynamics towards the steady state (2)

- This suggests that there is a simple empirical test of the Solow-Swan model which is based on the convergence property of output in a cross section of many different countries
  - *Absolute convergence hypothesis* (ACH): poor countries should grow faster than rich countries. Barro and Sala-i-Martin regress $\gamma_y(t)$ on $\ln y(t)$ for a sample of 118 countries. The results are dismal: instead of finding a negative effect as predicted by the ACH, they find a slight positive effect. Absolute convergence does not seem to hold and (Romer's) stylized fact (SF7) is verified by the data
  - More refined test: *Conditional convergence hypothesis* (CCH): *similar* countries should converge. Confirmed by the data. In **Figure 12.5** we show case where poor country is closer to its steady state than the rich country is to its own steady state. Hence, rich country grows at a faster rate.

# Figure 12.4: Growth convergence



# Figure 12.5: Conditional growth convergence



# (C) Speed of adjustment (1)

- How *fast* is the convergence in a Solow-Swan economy?
- Focus on the Cobb-Douglas case for which $f(\cdot) = k(t)^{\alpha}$, and the FDE is:

$$\dot{k}(t) = sk(t)^{\alpha} - (\delta + n)k(t) \qquad \text{(S6)}$$

- First-order Taylor approximation around $k^*$:

$$\begin{aligned} sk(t)^{\alpha} &\approx s \cdot (k^*)^{\alpha} + s\alpha \cdot (k^*)^{\alpha-1} \cdot [k(t) - k^*] \\ &= (\delta + n) \cdot k^* + \alpha(\delta + n) \cdot [k(t) - k^*] \end{aligned} \qquad \text{(S7)}$$

- Using (S7) in (S6) we obtain the *linearized* differential equation for $k(t)$:

$$\dot{k}(t) = -\beta \cdot [k(t) - k^*], \qquad \beta \equiv (1-\alpha)(\delta + n) > 0 \qquad \text{(S8)}$$

# (C) Speed of adjustment (2)

- Solving (S8) with initial condition $k(0)$, we find:

$$k(t) = k^* + [k(0) - k^*] \cdot e^{-\beta t} \qquad \text{(S9)}$$

where $\beta$ measures the speed of convergence / adjustment

- Speed of adjustment in the growth rate of output for the Cobb-Douglas case. Divide both sides of (S8) by $k(t)$, note that $\dot{k}(t)/k(t) = d\ln k(t)/dt$, $d\ln y(t)/dt = \alpha d\ln k(t)/dt$, and use the approximation $\ln(k(t)/k^*) = 1 - k^*/k(t)$:

$$\frac{d\ln y(t)}{dt} = -\beta \cdot [\ln y(t) - \ln y^*] \qquad \text{(S10)}$$

# (C) Speed of adjustment (3)

- Solving (S10) with initial condition $y(0)$, we find:

$$\ln y(t) = \ln y^* + [\ln y(0) - \ln y^*] \cdot e^{-\beta t} \qquad \text{(S11)}$$

  - $\beta \equiv (1-\alpha)(\delta+n)$ is the *common* (approximate) adjustment speed for $k(t)$, $\ln k(t)$, $y(t)$, and $\ln y(t)$ toward their respective steady-states
  - Interpretation of $\beta$: $\zeta \times 100\%$ of the difference between, say, $y(t)$ and $y^*$ is eliminated after a time interval of $t_\zeta$:

$$t_\zeta \equiv -\frac{1}{\beta} \cdot \ln(1-\zeta)$$

For example, the **half-life** of the convergence ($\zeta = \frac{1}{2}$) equals $t_{1/2} = \ln 2/\beta = 0.693/\beta$

# (C) Speed of adjustment (4)

- Back-of-the-envelope computations: $n_L = 0.01$ (per annum), $n_Z = 0.02$, $\delta = 0.05$, and $\alpha = 1/3$ yield the value of $\beta = 0.0533$ (5.33 percent per annum) and an estimated half-life of $t_{1/2} = 13$ years. Fast transition
- Estimate is way too high to accord with empirical evidence: actual $\beta$ is in the range of 2 percent per annum (instead of 5.33 percent)
- Problem with the Solow-Swan model. Solutions:
  - Assume high capital share (for $\alpha = \frac{3}{4}$ we get $\beta = 0.02$)!
  - Assume a broad measure of capital to include human as well as physical capital (Mankiw, Romer, and Weil (1992))

# (D) Rescuing the Solow-Swan model (1)

- *Key idea*: add human capital to the model
- Technology:

$$Y(t) = K(t)^{\alpha_K} H(t)^{\alpha_H} \left[Z(t)L(t)\right]^{1-\alpha_K-\alpha_H}, \quad 0 < \alpha_K + \alpha_H < 1$$

  where $H(t)$ is the stock of human capital and $\alpha_K$ and $\alpha_H$ are the efficiency parameters of the two types of capital ($0 < \alpha_K, \alpha_H < 1$)
- In close accordance with the Solow-Swan model, productivity and population growth are both exponential ($\dot{Z}(t)/Z(t) = n_Z$ and $\dot{L}(t)/L(t) = n_L$)

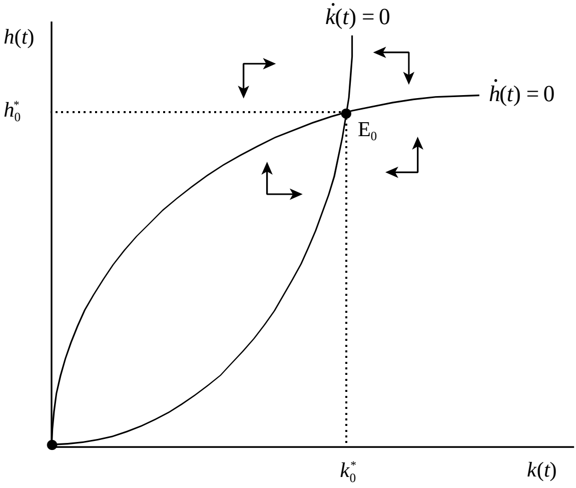
# (D) Rescuing the Solow-Swan model (2)

- The accumulation equations for the two types of capital can be written in effective labour units as:

$$\dot{k}(t) = s_K y(t) - (\delta_K + n)k(t)$$
$$\dot{h}(t) = s_H y(t) - (\delta_H + n)h(t)$$

  where $h(t) \equiv H(t)/[Z(t)L(t)]$, $n \equiv n_Z + n_L$, and $s_K$ and $s_H$ represent the propensities to accumulate physical and human capital, respectively. The depreciation rates are $\delta_K$ and $\delta_H$
- Phase diagram in **Figure 12.6**

# Figure 12.6: Augmented Solow-Swan model



# (D) Rescuing the Solow-Swan model (3)

- Since there are decreasing returns to the two types of capital in combination ($\alpha_K + \alpha_H < 1$) the model possesses a steady state for which $\dot{k}(t) = \dot{h}(t) = 0$, $k(t) = k^*$, and $h(t) = h^*$:

$$k^* = \left( \left( \frac{s_K}{\delta_K + n} \right)^{1-\alpha_H} \left( \frac{s_H}{\delta_H + n} \right)^{\alpha_H} \right)^{1/(1-\alpha_K-\alpha_H)}$$

$$h^* = \left( \left( \frac{s_K}{\delta_K + n} \right)^{\alpha_K} \left( \frac{s_H}{\delta_H + n} \right)^{1-\alpha_K} \right)^{1/(1-\alpha_K-\alpha_H)}$$

- By substituting $k^*$ and $h^*$ into the (logarithm of the) production function we obtain an estimable expression for per capita output along the balanced growth path:

$$\ln \left( \frac{Y(t)}{L(t)} \right)^* = \ln Z(0) + n_Z t - \frac{\alpha_K \ln(\delta_K + n_Z + n_L) + \alpha_H \ln(\delta_H + n_Z + n_L)}{1 - \alpha_K - \alpha_H} + \frac{\alpha_K}{1 - \alpha_K - \alpha_H} \ln s_K + \frac{\alpha_H}{1 - \alpha_K - \alpha_H} \ln s_H$$

# (D) Rescuing the Solow-Swan model (4)

- Mankiw et al. (1992, p. 417) suggest approximate guesses for $\alpha_K = \frac{1}{3}$ and $\alpha_H$ between $\frac{1}{3}$ and $\frac{4}{9}$
- The extended Solow-Swan model is much better equipped to explain large cross-country income differences for relatively small differences between savings rates ($s_K$ and $s_H$) and population growth rates ($n$) (Multiplier factor is $\frac{1}{1-\alpha_K-\alpha_H}$ instead of $\frac{1}{1-\alpha_K}$)
- The inclusion of a human capital variable works pretty well empirically; the estimated coefficient for $\alpha_H$ is highly significant and lies between 0.28 and 0.37

# (D) Rescuing the Solow-Swan model (5)

- The convergence property of the augmented Solow-Swan model is also much better. For the case with $\delta_K = \delta_H = \delta$, the convergence speed is defined as $\beta \equiv (1 - \alpha_K - \alpha_H)(n + \delta)$ which can be made in accordance with the observed empirical estimate of $\hat{\beta} = 0.02$ without too much trouble
- Hence, by this very simple and intuitively plausible adjustment (adding human capital) the Solow-Swan model can be salvaged from the dustbin of history. The speed of convergence it implies can be made to fit the real world

# Macroeconomic applications of the Solow-Swan

(A) Fiscal policy: long-run crowding out of private by public consumption?

- Balanced-budget: without government debt
- Deficit financing: with government debt

(B) Ricardian non-equivalence: government debt is not neutral

# (A) Fiscal policy in the Solow-Swan model (1)

- The government consumes $G(t)$ units of output so that aggregate demand in the goods market is:

$$Y(t) = C(t) + I(t) + G(t)$$

- Aggregate saving is proportional to after-tax income:

$$S(t) = s\left[Y(t) - T(t)\right]$$

where $T(t)$ is the lump-sum tax

- Since $S(t) \equiv Y(t) - C(t) - T(t)$ any primary government deficit must be compensated for by an excess of private saving over investment, i.e.

$$G(t) - T(t) = S(t) - I(t)$$

# (A) Fiscal policy in the Solow-Swan model (2)

- The government budget identity is given by:

$$\dot{B}(t) = r(t)B(t) + G(t) - T(t)$$

where $B(t)$ is public debt and $r(t)$ is the real interest rate

- Under the competitive conditions the interest rate equals the net marginal productivity of capital (see also below):

$$r(t) = f'(k(t)) - \delta$$

- By writing all variables in terms of effective labour units, the model can be condensed to the following two equations:

$$\begin{aligned} \dot{k}(t) &= f(k(t)) - (\delta + n)k(t) - c(t) - g(t) \\ &= sf(k(t)) - (\delta + n)k(t) + (1-s)\tau(t) - g(t), \end{aligned} \tag{S12}$$

$$\dot{b}(t) = \Big[f'(k(t)) - \delta - n\Big]b(t) + g(t) - \tau(t), \tag{S13}$$

with: $\tau(t) \equiv T(t)/N(t)$, $g(t) \equiv G(t)/N(t)$, $b(t) \equiv B(t)/N(t)$

# (A) Fiscal policy in the Solow-Swan model (3)

- Under pure tax financing we have $\dot{b}(t) = b(t) = 0$ so that government budget identity (b) reduces to $g(t) \equiv \tau(t)$. The FDE becomes:

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) - sg(t)$$

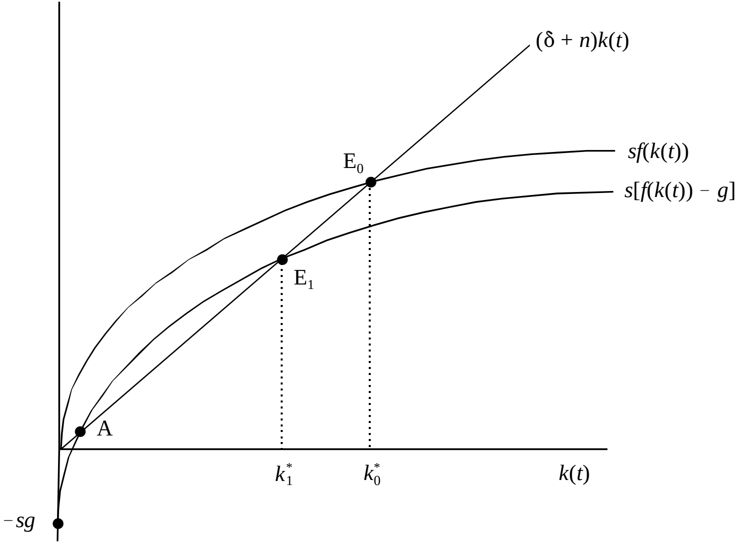
- In **Figure 12.7** we show stability and illustrate the effects of an increase in government consumption
- Qualitative effects:

$$\frac{dy(\infty)}{dg} = \frac{f'(k_0^*)dk(\infty)}{dg} = \frac{sf'(k_0^*)}{sf'(k_0^*) - (\delta + n)} < 0$$

$$\frac{dc(\infty)}{dg} = (1 - s)\left[\frac{dy(\infty)}{dg} - 1\right] = \frac{(1 - s)(\delta + n)}{sf'(k_0^*) - (\delta + n)} < 0$$

- Capital and consumption are both crowded out in the long run! Model is Classical in the long run (despite its Keynesian consumption function)

# Figure 12.7: Fiscal policy in the Solow-Swan model



# (B) Ricardian non-equivalence in the S-S model (1)

- If the economy is dynamically efficient we have:

$$r(t) \equiv f'(k(t)) - \delta > n$$

- This means that debt process in (S13) is inherently unstable (explosive); an economically uninteresting phenomenon
- *Buiter rule* ensures that debt is stabilized:

$$\tau(t) = \tau_0 + \xi b(t), \quad \xi > r - n$$

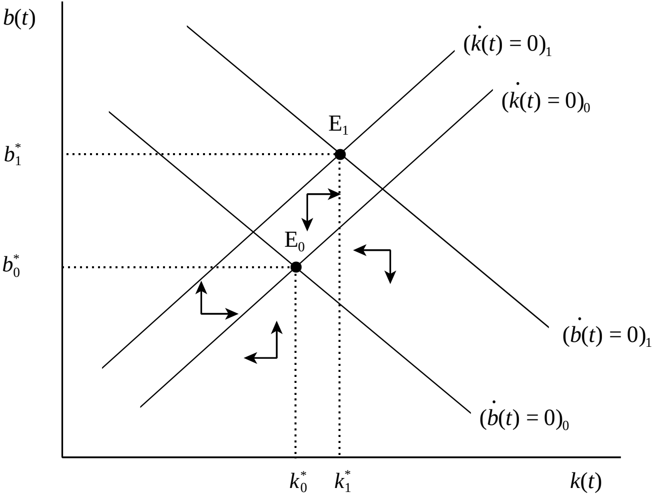
- The system of fundamental differential equations becomes:

$$\dot{b}(t) = \Big[ f'(k(t)) - \delta - n - \xi \Big] b(t) + g(t) - \tau_0$$
$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) + (1 - s)\left[\tau_0 + \xi b(t)\right] - g(t)$$

# (B) Ricardian non-equivalence in the S-S model (2)

- The model can be analyzed graphically with the aid of **Figure 12.8**
- The $\dot{k} = 0$ line:
  - Upward sloping in $(k, b)$ space
  - Points above (below) the line are associated with positive (negative) net investment, i.e. $\dot{k} > 0$ $(< 0)$
- The $\dot{b} = 0$ line:
  - Downward sloping in $(k, b)$ space
  - For points above (below) the $\dot{b} = 0$ line there is a government surplus (deficit) so that debt falls (rises)
- Equilibrium at $E_0$ is inherently stable

# Figure 12.8: Ricardian non-equivalence in the S-S model



# (B) Ricardian non-equivalence in the S-S model (3)

- *Ricardian experiment:* postponement of taxation
  - In the model this amounts to a reduction in $\tau_0$. This creates a primary deficit at impact ($g(t) > \tau_0$) so that government debt starts to rise
  - In terms of **Figure 12.8**, both the $\dot{k} = 0$ line and the $\dot{b} = 0$ line shift up, the latter by more than the former
  - In the long run, government debt, the capital stock, and output (all measured in efficiency units of labour) rise as a result of the tax cut

$$\frac{dy(\infty)}{d\tau_0} = \frac{f'(k_0^*)dk(\infty)}{d\tau_0} = -\frac{(1-s)(r_0^* - n)f'(k_0^*)}{|\Delta|} < 0$$

$$\frac{db(\infty)}{d\tau_0} = \frac{sf'(k_0^*) - (\delta + n) + (1-s)b_0^* f''(k_0^*)}{|\Delta|} < 0$$

  - Ricardian equivalence does not hold in the Solow-Swan model. A temporary tax cut boosts consumption, depresses investment and thus has real effects

# Punchlines

- We have looked at some stylized facts on economic growth
- Solow-Swan model features (a) substitutability between capital and labour and (b) an exogenous savings rate
- The Solow-Swan model can account for all stylized facts (but long-run growth is exogenously determined)
- The Solow-Swan model (a) allows for oversaving to occur, (b) does not feature Ricardian equivalence, and (c) predicts that fiscal policy crowds out the private capital stock
- By adding human capital accumulation to the Solow-Swan model its empirical performance is greatly enhanced
- In the long run the Solow-Swan model has classical features