Foundations of Modern Macroeconomics Third Edition

Chapter 8: Search in the labour market

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Outline

Aims of this lecture

- How can we explain unemployment *duration*?
- What policies can be used to reduce equilibrium unemployment?
- Can the search model explain the persistence in the unemployment rate?

Searching and matching (1)

- Matching function:

$$MN = G(\underset{+}{UN}, \underset{+}{VN})$$

  - M is the matching rate
  - N is the number of workers
  - U is the unemployment rate
  - V is the vacancy rate
  - $G(\cdot,\cdot)$ features CRTS (i.e. $G(UN, VN) = N \cdot G(U, V)$ $= NV \cdot G(U/V, 1)$)
  - Example: Cobb-Douglas matching function: $MN = (UN)^{\alpha}(VN)^{1-\alpha}$
  - Further properties: $G_U, G_V > 0$; $G_{UU}, G_{VV} < 0$; $G_{UU}G_{VV} - G_{UV}^2 > 0$

Searching and matching (2)

- Instantaneous probability of a vacancy being filled:

$$q \equiv \frac{\text{number of matches}}{\text{number of vacancies}} = \frac{G(UN, VN)}{VN}$$
$$= \frac{VN \cdot G(UN/VN, 1)}{VN} = G(U/V, 1) \equiv q(\underset{-}{\theta})$$

where θ is the indicator for labour market pressure:

$$\theta \equiv \frac{V}{U}$$

  - If θ is high then there are relatively many vacancies so firms with a vacancy find it hard to get a match with an unemployed job seeker (q is low)
  - If θ is low then there are relatively few vacancies so firms with a vacancy find it easy to get a match with an unemployed job seeker (q is high)

Searching and matching (3)

- Continued
  - For later use: the elasticity of the $q(\theta)$ function:

$$\eta(\theta) \equiv -\frac{\theta}{q}\frac{dq}{d\theta} = \frac{G_U}{\theta q} \Rightarrow \quad 0 < \eta(\theta) < 1$$

- Inst. prob. of an unemployed job seeker finding a job:

$$f \equiv \frac{\text{number of matches}}{\text{number of unemployed}} = \frac{G(UN, VN)}{UN}$$
$$= \frac{VN \cdot G(UN/VN, 1)}{UN} = \theta q(\theta) \equiv f(\underset{+}{\theta})$$

  - If θ is high then there are relatively few unemployed workers so unemployed job seekers find it easy to locate a firm with a vacancy (f is high)
  - If θ is low then there are relatively many unemployed workers so unemployed job seekers find it hard to locate a firm with a vacancy (f is low)

Searching and matching (4)

- Continued
  - For later use: the elasticity of the $f(\theta)$ function:

$$\frac{\theta}{f}\frac{df}{d\theta} = \left[q(\theta) + \theta\frac{dq}{d\theta}\right]\frac{\theta}{\theta q(\theta)} = 1 + \frac{\theta}{q}\frac{dq}{d\theta} = 1 - \eta(\theta) > 0$$

- Note the intimate link between the probabilities facing the two searching parties, i.e. firms with a vacancy and unemployed job seekers [Two sides of the same coin]
- We now already have some duration definitions:
  - Expected duration of a job vacancy:

$$\frac{1}{q(\theta)}$$

  - Expected duration of unemployment spell:

$$\frac{1}{f(\theta)}$$

Searching and matching (5)

- Inflow/outflow equilibrium

$$\underbrace{\delta_m(1-U)Ndt}_{(a)} = \underbrace{\theta q(\theta)UNdt}_{(b)} \tag{S1}$$

where δ_m is the (exogenous) job destruction rate (due to idiosyncratic match-productivity shocks

(a) (expected) flow into unemployment
(b) (expected) flow out of unemployment

- Note: Large numbers, so frequencies and probabilities coincide
- Equation (S1) implies equilibrium unemployment rate:

$$U = \frac{\delta_m}{\delta_m + \theta q(\theta)} = \frac{\delta_m}{\delta_m + f(\theta)}$$

Remainder of the model solved as follows

(A) Firm behaviour
- Firm with a vacancy
- Firm without a vacancy

(B) Worker behaviour
- Employed worker
- Unemployed worker

(C) Wage setting
- What happens when a match occurs?
- Wage as the instrument to share the rents

(D) Market equilibrium

(A) Firm behaviour (1)

- Analyze single-job firms (risk-neutral owner)
- Focus on intuitive "derivation"
- Firms with a vacancy have the following arbitrage equation:

$$\underbrace{rJ_V}_{(a)} = \underbrace{-c + q(\theta)\left[J_O - J_V\right]}_{(b)}$$

  - J_V is the value of a (firm with a) vacancy; r is the interest rate
  - c is the search cost of the firm with a vacancy
  - J_O is the value of (a firm with) an occupied job

(a) capital cost of the asset

(b) return on the asset: "dividend" [search costs] plus expected capital gain [finding a worker, upgrading from vacancy to a filled job]

(A) Firm behaviour (2)

- Assumption: free entry of firms with a vacancy:

$$J_V = 0 \;\Rightarrow\; 0 = -c + q(\theta) J_O \;\Rightarrow$$
$$J_O = \frac{c}{q(\theta)}$$

  Hence, the value of a filled job equals the expected cost of creating it [i.e. the cost of filling a vacancy]
- Firms with an occupied job have the following arbitrage equation:

$$\underbrace{r J_O}_{(a)} = \underbrace{[F(K,1) - (r+\delta_k)K - w] - \delta_m J_O}_{(b)} \qquad \text{(S2)}$$

  - $F(K,1)$ is the output of the single-job firm
  - Firm rents capital at rental rate $r + \delta_k$
  - Firm hires labour at wage rate w [to be determined below]

(A) Firm behaviour (3)

- Continued

$$\underbrace{rJ_O}_{(a)} = \underbrace{[F(K,1) - (r+\delta_k)K - w] - \delta_m J_O}_{(b)} \qquad \text{(S2)}$$

(a) Capital cost of the asset
(b) Return on the asset, consisting of the "dividend" [profit, i.e. output left over after capital and labour have been paid] plus the expected capital gain [experiencing a shock by which the match is destroyed: downgrading from filled job to vacancy]

- The firm hires capital such that J_O is maximized:

$$\max_{\{K\}}(r+\delta_m)J_O \equiv F(K,1) - (r+\delta_k)K - w \quad \Rightarrow$$

$$F_K(K,1) = r + \delta_k \qquad \text{(S3)}$$

(A) Firm behaviour (4)

- Since $J_O = c/q(\theta)$ and $F(K,1) = F_K K + F_L$ we can combine (S2) and (S3):

$$\frac{(r+\delta_m)c}{q(\theta)} = F(K,1) - F_K(K,1)K - w \quad \Rightarrow$$

$$\underbrace{\frac{F_L(K,1) - w}{r+\delta_m}}_{(a)} = \underbrace{\frac{c}{q(\theta)}}_{(b)} \qquad \text{(ZP condition)}$$

(a) The value of an occupied job, equalling the present value of rents (accruing to the firm during the job's existence) using the risk-of-job-destruction-adjusted discount rate, $r+\delta_m$, to discount future rents

(b) Expected search costs

- Since firm search costs are positive ($c > 0$) it follows that $w < F_L$ (workers do not get their marginal product!)

(B) Worker behaviour (1)

- Risk-neutral / infinitely-lived worker
- Cares only for the present value of present and future income stream
- Receives wage w when employed and "unemployment benefit" b when unemployed
- Unemployed worker's arbitrage equation is:

$$\underbrace{rY_U}_{(a)} = \underbrace{b + \theta q(\theta)\left[Y_E - Y_U\right]}_{(b)} \tag{S4}$$

  - Y_U is the human wealth of the unemployed worker (who is looking for a job)
  - Y_E is the human wealth of the employed worker

(a) Capital cost of the asset

(b) Return on the asset: "dividend" [unemployment benefits] plus expected capital gain [finding a job and upgrading from unemployment to being employed]

(B) Worker behaviour (2)

- Employed worker's arbitrage equation is:

$$\underbrace{rY_E}_{(a)} = \underbrace{w - \delta_m \left[Y_E - Y_U \right]}_{(b)} \qquad \text{(S5)}$$

  - Capital cost of the asset
  - Return on the asset, consisting of the "dividend" [the wage] plus the expected capital gain [losing one's job due to a shock and downgrading from being employed to being unemployed]

- Combining (S4) and (S5) yields:

$$rY_U = \frac{(r+\delta_m)b + \theta q(\theta) w}{r + \delta_m + \theta q(\theta)}$$

$$rY_E = \frac{\delta_m b + \left[r + \theta q(\theta) \right] w}{r + \delta_m + \theta q(\theta)} = \frac{r(w-b)}{r + \delta_m + \theta q(\theta)} + rY_U$$

(C) Wage setting (1)

- Generalized wage bargaining over the wage between the firm and the worker
- Expected gain from striking a deal
  - To the firm:

$$rJ_O^i = F(K_i, 1) - (r + \delta_k)K_i - w_i - \delta_m J_O^i \quad \Rightarrow$$

$$J_O^i = \frac{F_L(K_i, 1) - w_i}{r + \delta_m}$$

  - To the worker:

$$r\left(Y_E^i - Y_U\right) = w_i - \delta_m \left[Y_E^i - Y_U\right] - rY_U$$

(C) Wage setting (2)

- Bargaining is over a wage, w_i, which maximizes Ω:

$$\max_{\{w_i\}} \Omega \equiv \beta \ln \left[Y_E^i - Y_U \right] + (1-\beta) \ln \left[J_O^i - \underbrace{J_V}_{=0} \right]$$

where $0 < \beta < 1$ represents the (relative) bargaining power of the worker and Y_U and $J_V = 0$ are the threat points of, respectively the worker and the firm

- Maximization yields the *rent sharing rule*:

$$Y_E^i - Y_U = \frac{\beta}{1-\beta} \left[J_O^i - J_V \right] \tag{S6}$$

(C) Wage setting (3)

- There are two ways to turn the rent sharing rule into a wage equation [details in the book]
  1) After some substitutions we get:

$$w_i = (1-\beta)rY_U + \beta F_L(K_i, 1)$$

  - Worker gets a weighted average of the reservation wage (rY_U) and the marginal product of labour (F_L)

(C) Wage setting (4)

- Continued

2) In symmetric situation we have $K_i = K$ and $w_i = w$ for all firm/worker pairs:

$$w = (1-\beta)b + \beta\left[F_L(K,1) + \theta c\right] \qquad \text{(WS curve)}$$

- Worker gets a weighted average of the unemployment benefit (b) and the match surplus ($F_L + c\theta$)
- The match surplus consists of the marginal product of labour plus the expected search costs that are saved if the deal is struck [$\theta \equiv V/U$ so that $c\theta \equiv cV/U$ represents the average hiring costs per unemployed worker]

(D) Market equilibrium

- Summary of the model

$$F_K(K,1) = r + \delta_k \quad \text{(T1)}$$

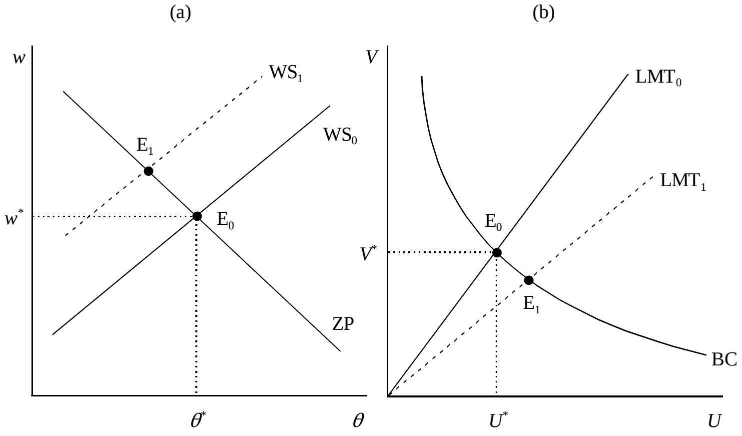
$$\frac{c}{q(\theta)} = \frac{F_L\left[K(r+\delta_k),1\right] - w}{r+\delta_m} \quad \text{(T2)}$$

$$w = (1-\beta)b + \beta\left[F_L\left(K(r+\delta_k),1\right) + \theta c\right] \quad \text{(T3)}$$

$$U = \frac{\delta_m}{\delta_m + \theta q(\theta)} \quad \text{(T4)}$$

- Endogenous: K, w, θ, and U. Exogenous: r, b, c, δ_m, and δ_k
- Model is recursive and can thus be solved sequentially:
  - (T1) yields K^* as a function of $r+\delta_k$ $[K^* = F_K^{-1}(r+\delta_k)]$
  - (T2)-(T3) with $K = K^*$ inserted only depend on (and determine) w^* and θ^*
  - Once θ^* is known equation (T4) determines U^*

Figure 8.1: Search equilibrium in the labour market



Graphical analysis (1)

- The model can be represented graphically in **Figure 8.1**
- ZP curve: [equation (T2)] supply of vacancies under free entry/exit of firms
  - Slopes downwards in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{ZP} = \frac{(r+\delta_m)c}{q(\theta)^2}q'(\theta) < 0$$

    Intuition: $w\downarrow$ increases the value of an occupied job [raises the right-hand side of (T2)]. To restore the zero-profit equilibrium the expected search cost for firms (the left-hand side of (T2) must also increase, i.e. $q(\theta)\downarrow$ and $\theta\uparrow$
  - Shifts up as $c\downarrow$ or as $\delta_m\downarrow$

Graphical analysis (2)

- WS curve: [equation (T3)] wage setting curve
  - Upward sloping in (w, θ) space:

$$\left(\frac{dw}{d\theta}\right)_{WS} = \beta c > 0$$

    Intuition: the worker receives part of the search costs that are foregone when he strikes a deal with a firm with a vacancy
  - Shifts up as $b \uparrow$ or $c \uparrow$
- In panel (a) the intersection of ZP and WS yields the equilibrium (w^*, θ^*) combination. This is the ray from the origin in panel (b)

Graphical analysis (3)

- The Beveridge curve (BC) is given by equation (T4). It can be linearized in (V, U) space as follows:

$$\tilde{V} = \frac{1}{1-\eta}\tilde{\delta}_m - \frac{\delta_m + f\eta}{f(1-\eta)}\tilde{U}$$

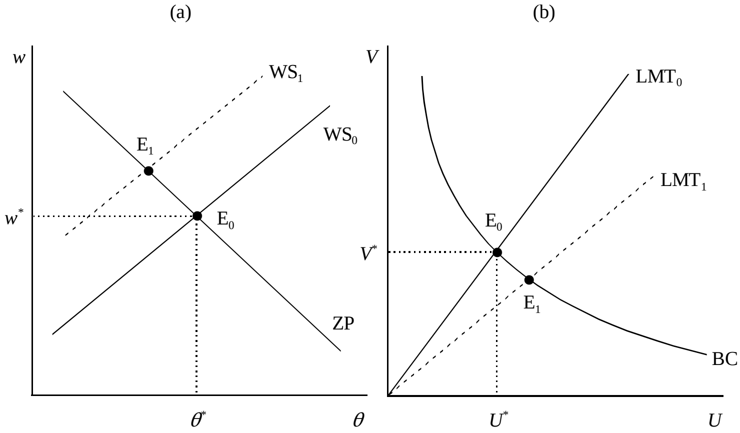
where $\tilde{U} \equiv dU/U$, $\tilde{V} \equiv dV/V$, and $\tilde{\delta}_m \equiv d\delta_m/\delta_m$

  - BC slopes down: for a given unemployment rate, $V \downarrow$ leads to a fall in the instantaneous probability of finding a job ($f \downarrow$), i.e. for points below the BC curve the unemployment rate is less than the rate required for flow equilibrium in the labour market ($U < \delta_m/(\delta_m + f)$). To restore flow equilibrium the $U \uparrow$
  - Shifts to the right as $\delta_m \uparrow$

Shock 1: Increase in the unemployment benefit

- Suppose that $b \uparrow$
- In Figure 8.1 this shock is illustrated
  - WS curve to the left
  - Equilibrium from E_0 to E_1
  - $w^* \uparrow$ and $\theta^* \downarrow$
  - In panel (b) the LMT ratio rotates clockwise
  - $V \downarrow$ and $U \uparrow$

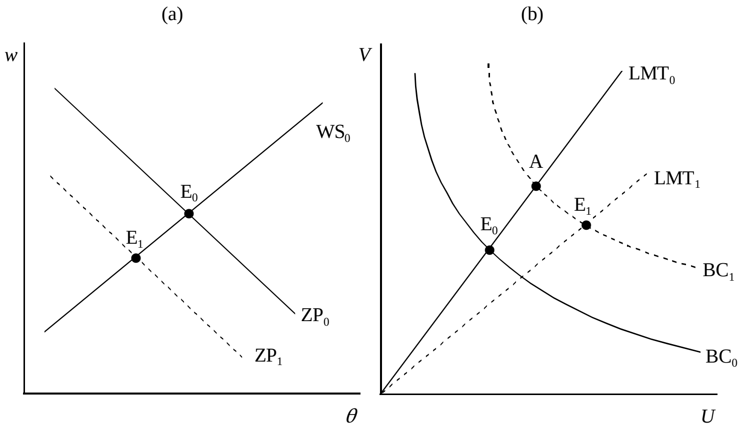
Figure 8.1: Search equilibrium in the labour market



Shock 2: Increase in the job destruction rate

- Suppose that $\delta_m \uparrow$
- ZP curve down in panel (a) of **Figure 8.2**
- Equilibrium from E_0 to E_1
- $w^* \downarrow$ and $\theta^* \downarrow$
- In panel (b) the LMT ratio rotates clockwise **and** BC shifts outwards [dominant effect]
- $V \uparrow$ and $U \uparrow$

Figure 8.2: The effects of a higher job destruction rate



Labour taxes

- The effects of labour taxes; t_E levied on firms t_L levied on households
- The model becomes:

$$\frac{c}{q(\theta)} = \frac{F_L\left(K(r+\delta_k),1\right) - w(1+t_E)}{r+\delta_m}$$

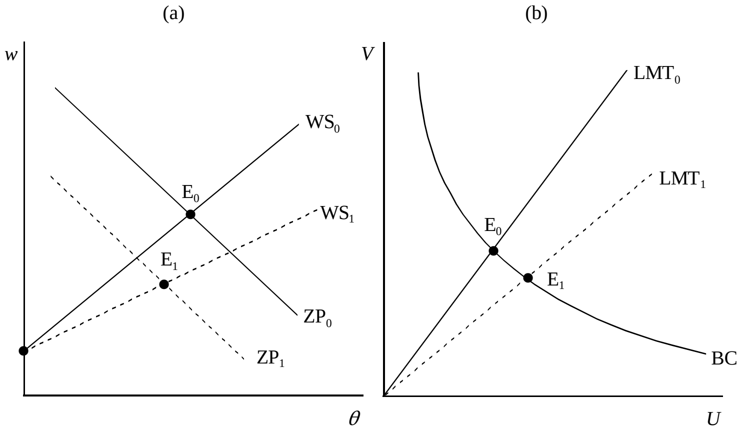
$$w = (1-\beta)\frac{b}{1-t_L} + \beta\frac{F_L\left(K(r+\delta_k),1\right) + \theta c}{1+t_E}$$

$$U = \frac{\delta_m}{\delta_m + \theta q(\theta)}$$

Labour taxes

- In **Figure 8.3** the effects of the payroll tax increase are analyzed ($t_E \uparrow$)
  - WS curve to the right
  - ZP curve to the left
  - equilibrium from E_0 to E_1 and $w^* \downarrow$ and $\theta^* \downarrow$
  - In panel (b) the LMT ratio rotates clockwise
  - $V \downarrow$ and $U \uparrow$

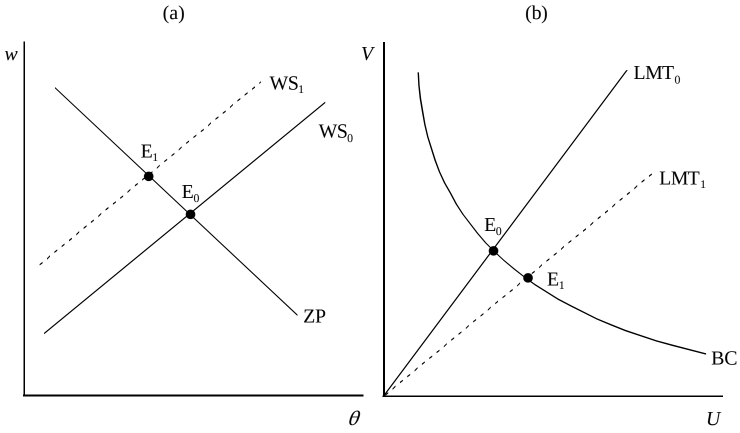
Figure 8.3: The effects of a payroll tax



Labour taxes

- In **Figure 8.4** the effects of the labour income tax increase are analyzed ($t_L \uparrow$)
  - WS curve to the left [z untaxed!]
  - Equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \downarrow$
  - In panel (b) the LMT ratio rotates clockwise
  - $V \downarrow$ and $U \uparrow$

Figure 8.4: The effects of a labour income tax



Deposits on labour

- Workers as empty pop bottles
- Deposit scheme: firm pays a deposit s_H to the government when it fires a worker, to be refunded s_H when it (re-) hires that (or another) worker
- Model becomes:

$$\frac{F_L(K,1) - w + rs_H}{r + \delta_m} = \frac{c}{q(\theta)}$$

$$w = (1-\beta)b + \beta\left[F_L(K,1) + rs_H + \theta c\right]$$

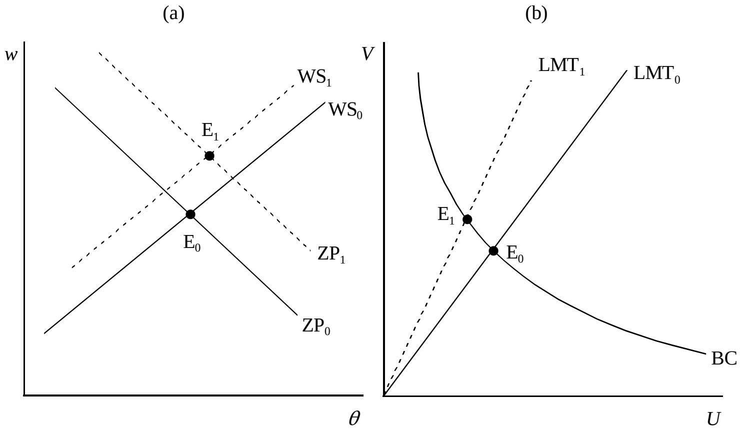
$$U = \frac{\delta_m}{\delta_m + \theta q(\theta)}$$

Hence, the capital value of the deposit (rs_H) acts as a subsidy on the use of labour!

Deposits on labour

- In **Figure 8.5** we show the effects of $s_H \uparrow$
  - ZP curve to the right
  - WS curve up
  - Equilibrium from E_0 to E_1 and $w^* \uparrow$ and $\theta^* \uparrow$
  - In panel (b) the LMT ratio rotates counterclockwise
  - $V \uparrow$ and $U \downarrow$
- The system works to combat unemployment!

Figure 8.5: The effects of a deposit on labour



Encore: Unemployment persistence in the search model

- One of the stylized facts of the labour market: high persistence in the unemployment rate
- Pissarides argues that loss of skills during unemployment can explain this phenomenon
  - Unemployed lose human capital [“skills”]
  - Are thus less attractive to firms, vacancy supply falls
  - More long-term unemployment

Punchlines

- Central elements of the search model:
  - Search frictions
  - Matching function
  - Wage negotiations
  - Beveridge curve
- Attractive model which abandons notion of the aggregate labour market
- Holds up well empirically